

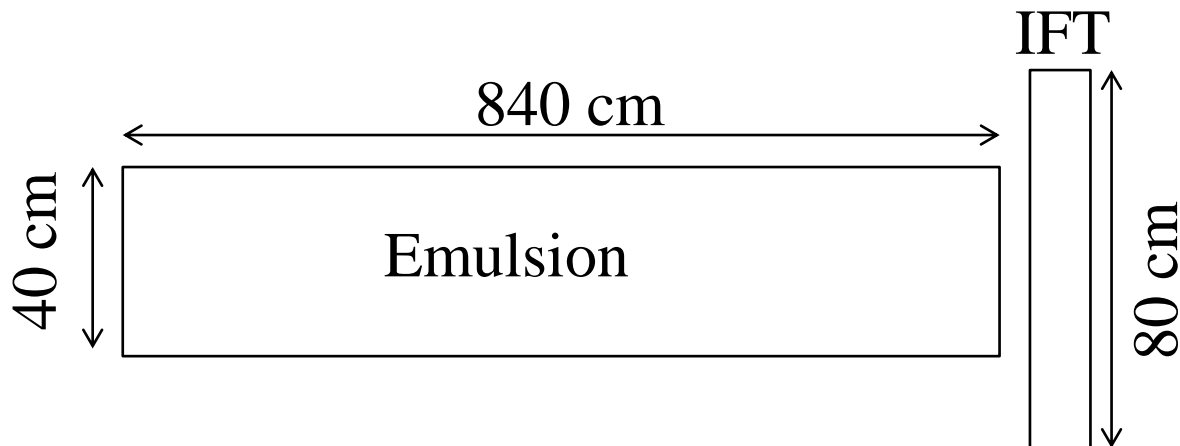
Tracking performance studies for FASER_{v2}/FASER2

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Y. Takubo (KEK)

Track matching with Emulsion/IFT

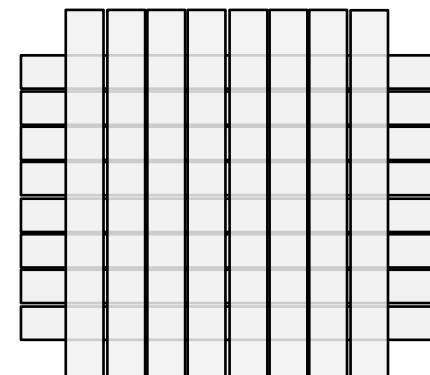
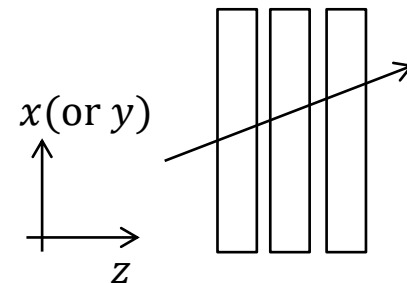
IFT for FASERv2

- IFT will be used as an interface tracker between emulsion detector and FASER2 tracker.
 - Detector candidates : silicon strip detector (SCT), scintillating fiber
- The studies on tracking performance and capability of track matching between emulsion and tracker were just started.
 - Assuming one IFT with 3 layers just behind the emulsion detector.
 - Emulsion: $40 \times 40 \times 840 \text{ cm}^3$, IFT: $80 \times 80 \text{ cm}^2$



Tracking performance in IFT (1)

- Performance of the tracking with IFT only was evaluated with a linear fitting of hit positions in IFT: $x(\text{or } y) = \alpha z + \beta$
- A charged particle is assumed to come from $(x, y, z) = (0, 0, 0)$ with $\vec{p} = (0, 0, p_z)$.
- Fitting is done 1000 times for each p_z , fluctuating the hit position with the position resolution.
- Position resolution: 16 μm (SCT), 50 and 100 μm (scinti. fiber)
 - Two strip sensors in a module / scintillating fiber layers are considered to be placed with 90 degrees of the stereo-angle.



Tracking performance in IFT (2)

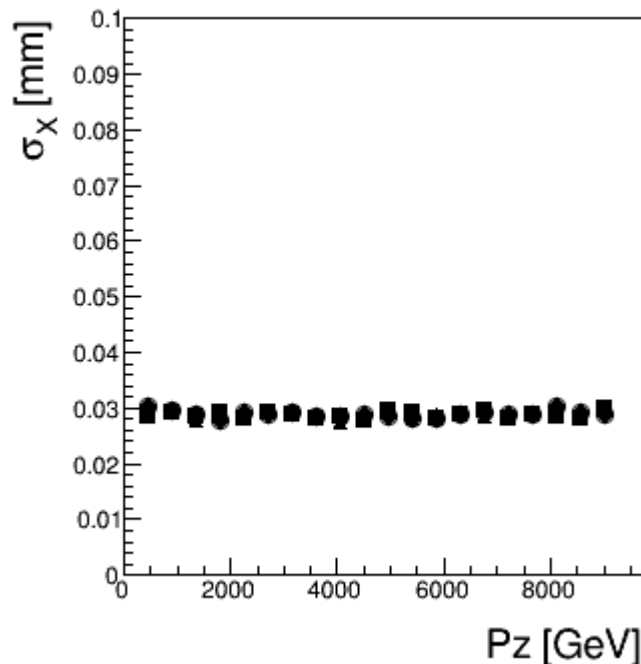
$$x(\text{or } y) = \alpha z + \beta$$

- σ_α : angular resolution
- σ_β : offset resolution

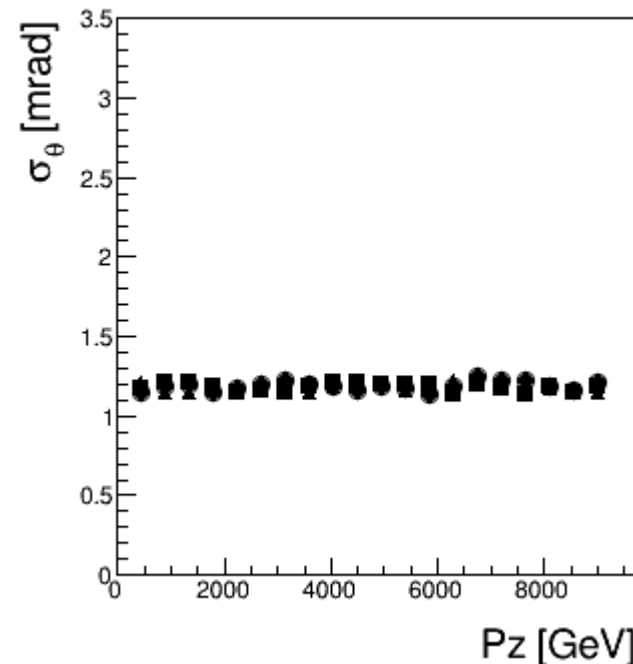
σ_{xy}	Offset reso.	Angular reso.
16 μm	$\sim 10 \mu\text{m}$	$\sim 0.4 \text{ mrad}$
50 μm	$\sim 30 \mu\text{m}$	$\sim 1.2 \text{ mrad}$
100 μm	$\sim 58 \mu\text{m}$	$\sim 2.4 \text{ mrad}$

Offset/angular reso. with 50 μm of IFT position reso.

X offset resolution



X angle resolution



Track matching capability (Introduction)

- Matching probability of signal-signal and signal-background was studied with Emulsion and IFT ($R_{em1/2} = 12.5/20$ cm).
- $\left[4k \times \frac{R_{em2}^2 L_{em2}}{R_{em1}^2 L_{em1}} \text{ signal} \right]$ v.s. $\left[40k \times \frac{R_{em2}^2}{R_{em1}^2} \text{ background} \right]$ (@30fb⁻¹)
- The signal tracks were generated with distributions in Backup which was used for the studies on the current FASER.
 - Angular distribution: $\sigma_\theta \sim 35$ (ν_μ), 27 ($\bar{\nu}_\mu$) mrad.
- Backgrounds were injected with $\sigma_\theta = 2$ mrad. and uniformly in radius.
- The position and angular resolution of the emulsion are assumed to be 1 μ m and 0.5 mrad., respectively.
- 3.5 cm was used for the gap between the last layer of emulsion and the first layer of IFT.

Track matching capability (Method)

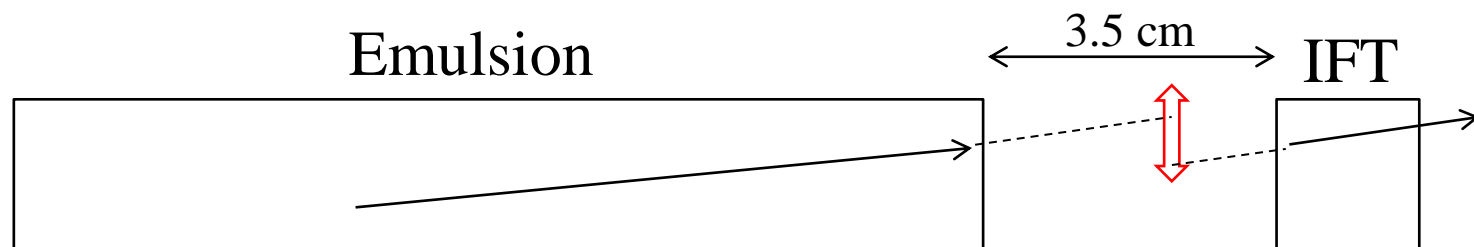
- $(x, y, \theta_x, \theta_y)$ was fluctuated by the resolutions of Emulsion and IFT, separately.
- If $(\Delta x, \Delta y, \Delta\theta_x, \Delta\theta_y)$ is within $(\sigma_x, \sigma_y, \sigma_{\theta_x}, \sigma_{\theta_y})$, the track is assumed as matching.

➤ $\Delta x = x(\text{Emulsion}) - x(\text{IFT})$

➤ $\sigma_{x/y} = \sqrt{(\alpha \cdot \sigma)_{Emulsion}^2 + (\beta \cdot \sigma)_{IFT}^2 + \sigma_{x/y(gap)}^{min,2}}$

➤ $\sigma_{\theta_x/\theta_y} = \sqrt{(\alpha \cdot \sigma)_{Emulsion}^2 + (\beta \cdot \sigma)_{IFT}^2}$

- Matching is checked by changing α/β (=1, 2, 3).

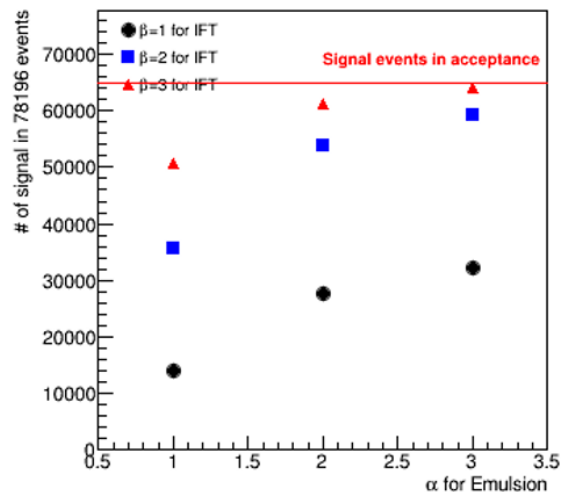


Track matching capability (ν_μ)

Signal-signal
matching

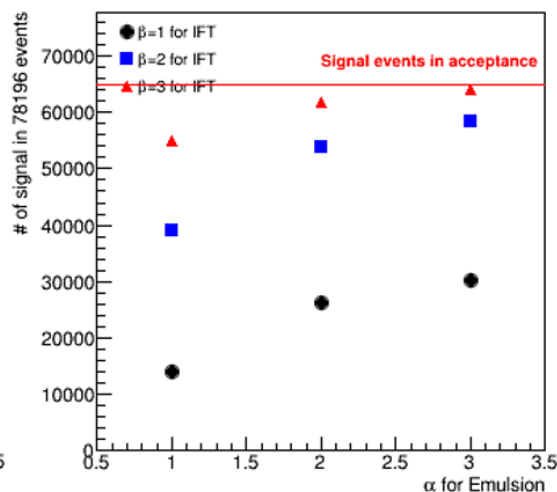
$\sigma_{xy}(\text{IFT}) = 16 \text{ um}$

Correct matching



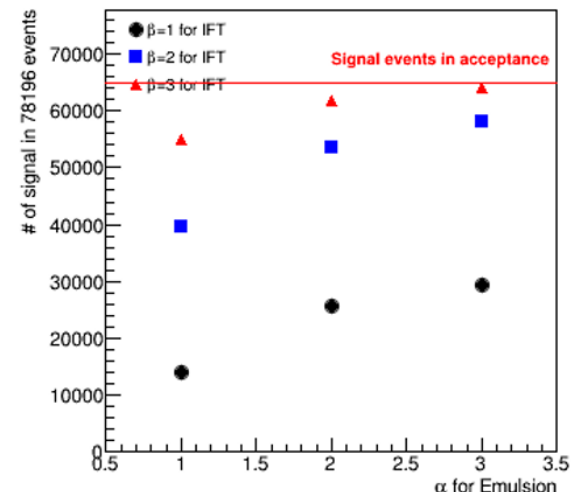
$\sigma_{xy}(\text{IFT}) = 50 \text{ um}$

Correct matching



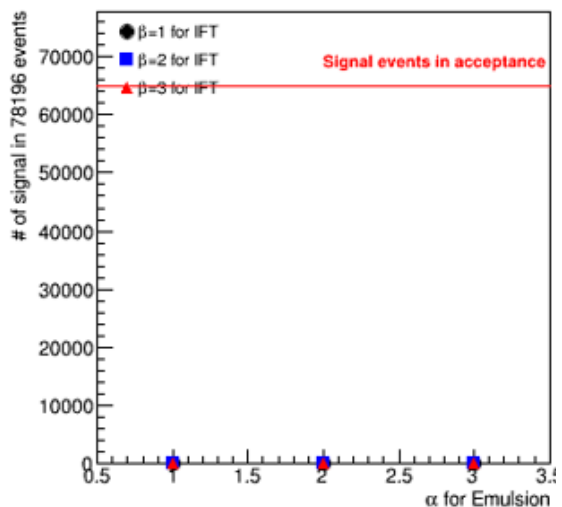
$\sigma_{xy}(\text{IFT}) = 100 \text{ um}$

Correct matching

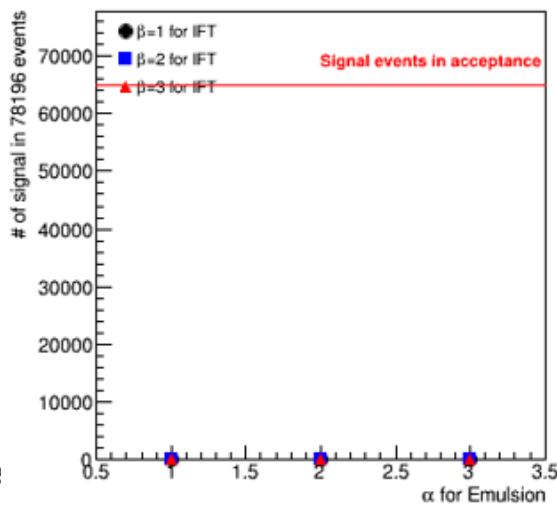


Signal-BG
matching

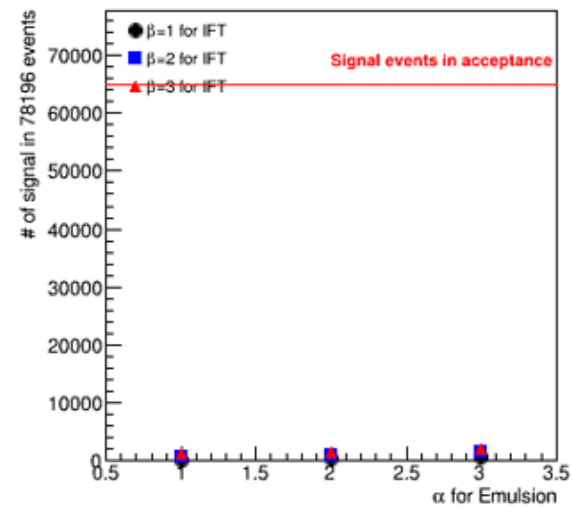
All matching



All matching

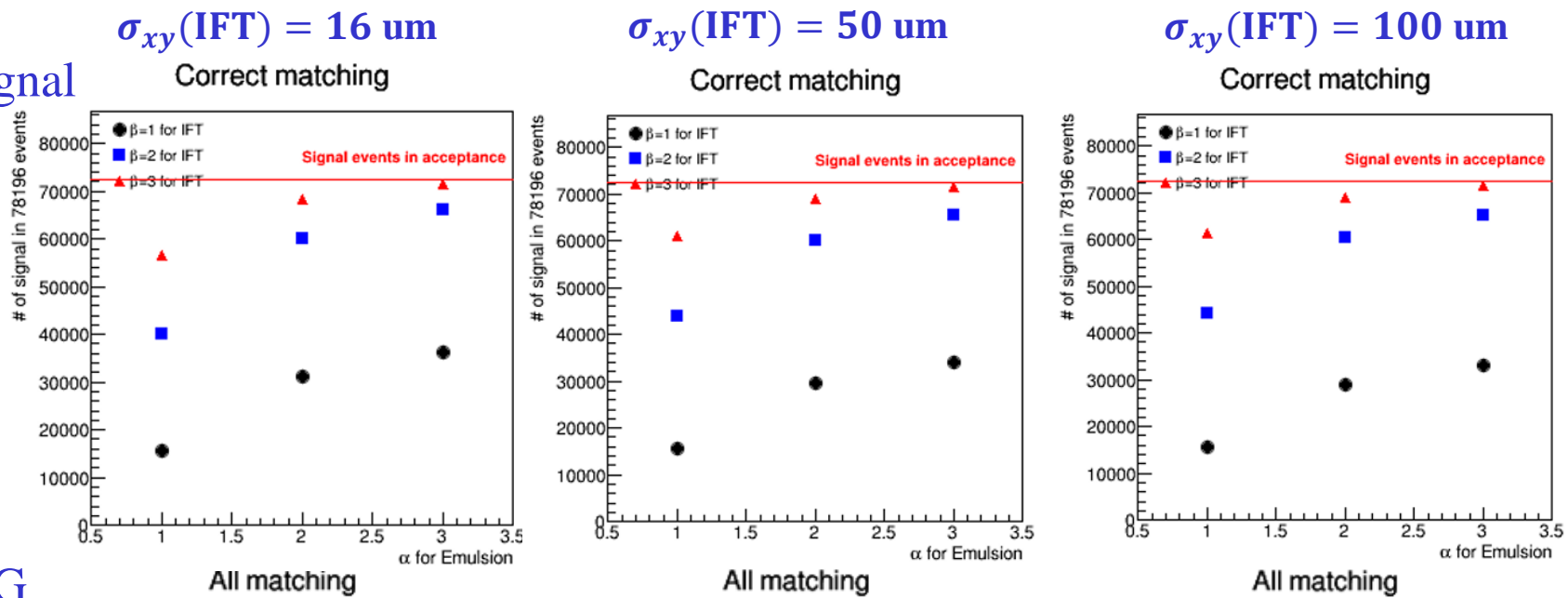


All matching

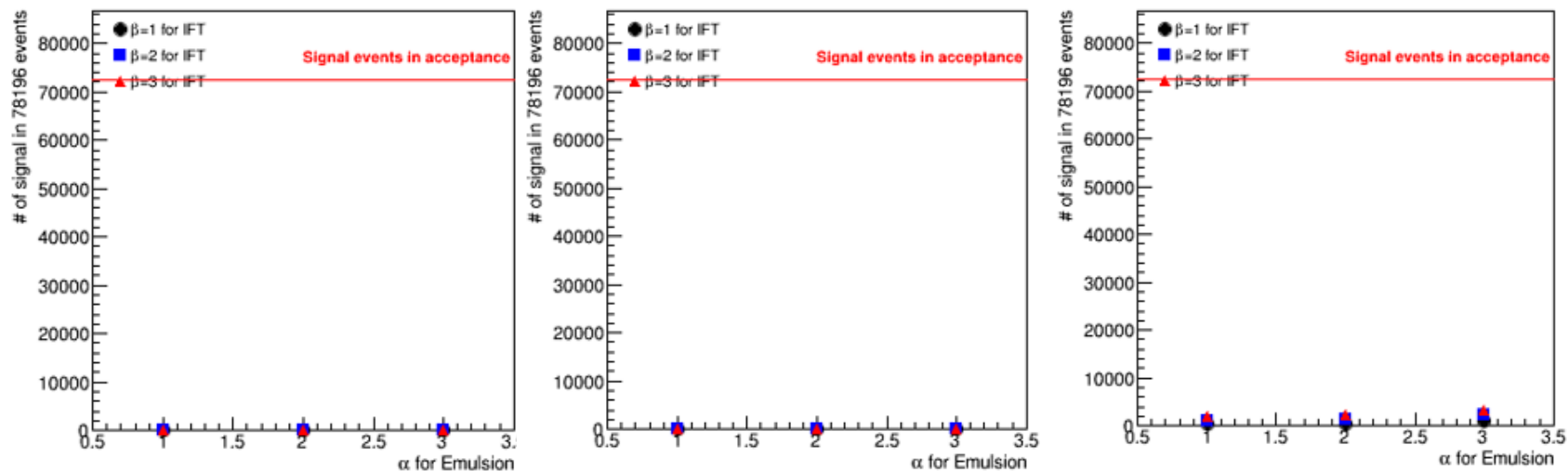


Track matching capability (\bar{v}_μ)

Signal-signal
matching



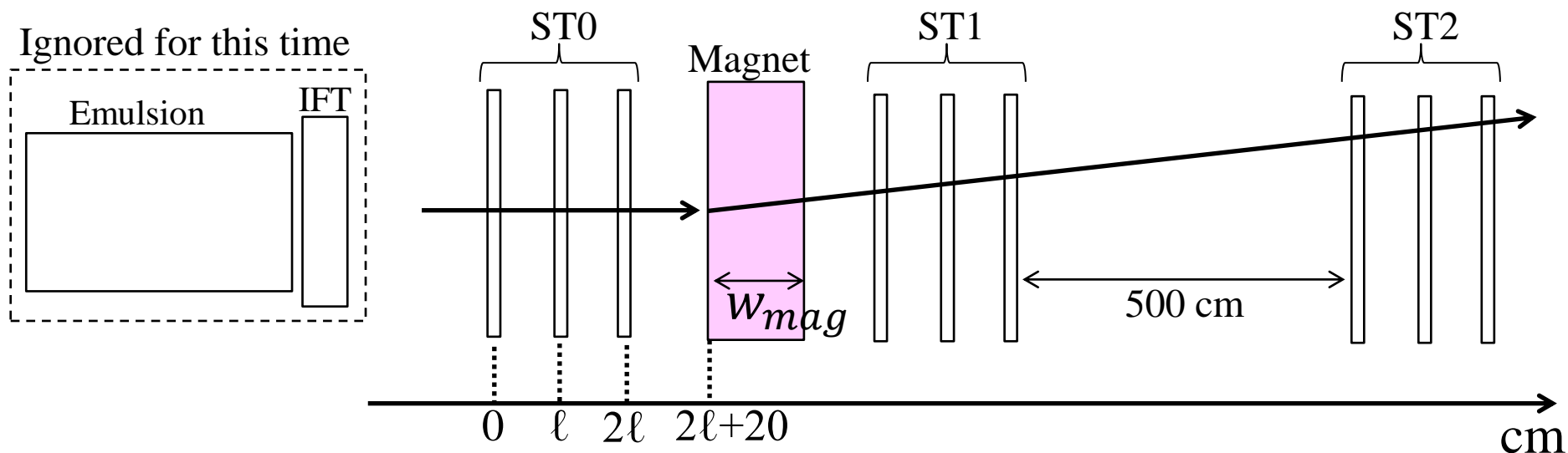
Signal-BG
matching



Tracking performance with FASER2 tracker

Charge ID capability and momentum resolution

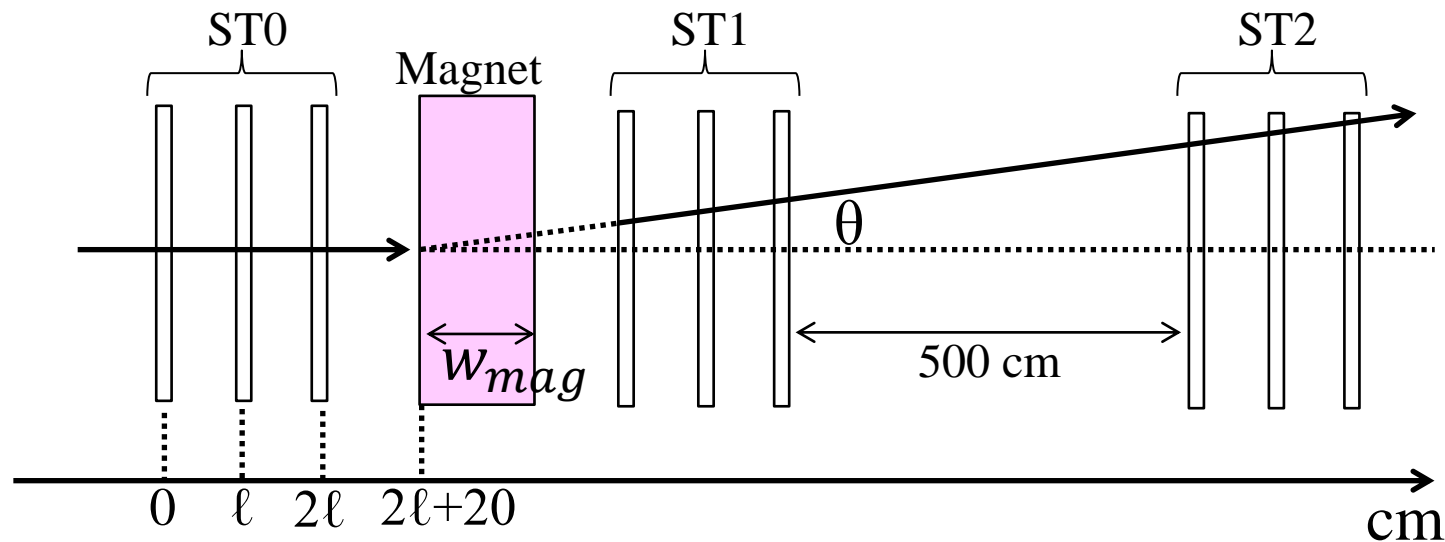
- 3 tracker stations with 3 single-layers (ST0-2) are placed before and after the magnet.
 - 80 μm position resolution
 - 50 or 100 cm gap between each layer (ℓ)
 - Distance between ST1 and ST2 is set to 500 cm.
- Magnetic field is changed to 1/2/4 Tm with 50 cm length (w_{mag}).



Momentum reconstruction

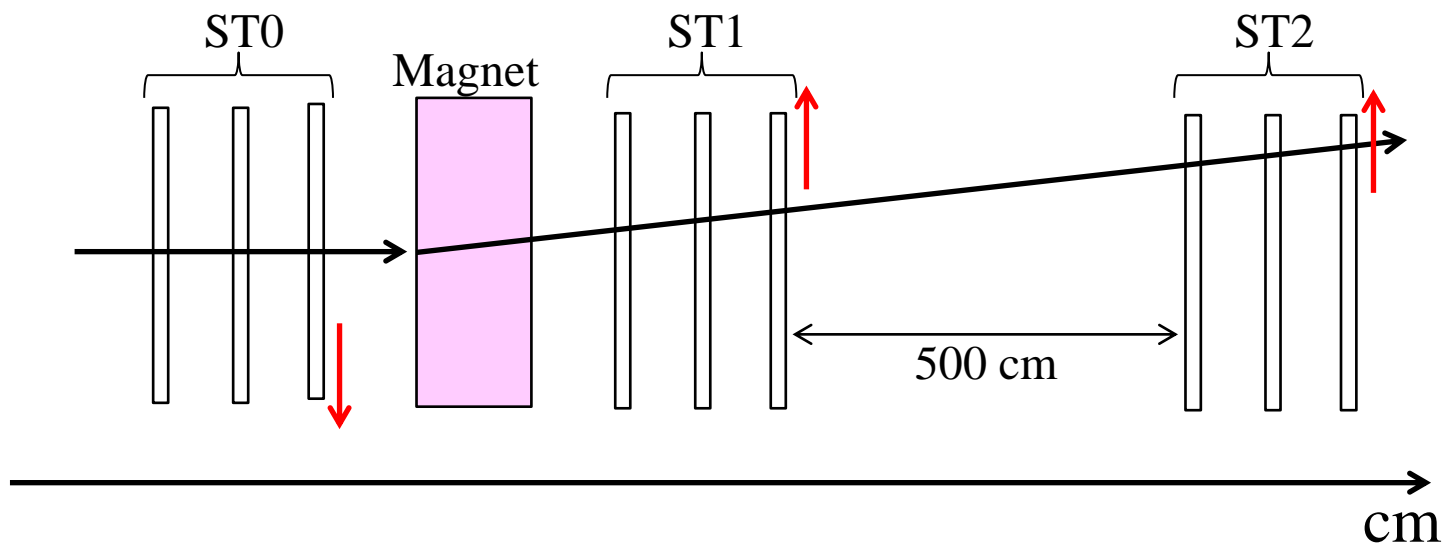
- The linear fitting is done with ST0 and ST1-2 separately.
- θ is calculated with a residual of slope of the linear function.
- The charge is identified with a sign of θ (+ or -).

$$r = \frac{w(\theta^2 + 1)}{2\theta} \Rightarrow p[\text{MeV}] = 3 \cdot r[\text{cm}] \cdot B[\text{T}]$$



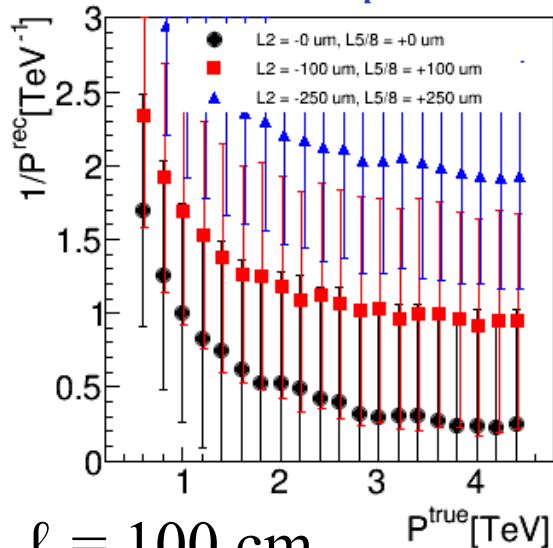
Alignment shift

- The alignment shift (100 μm and 250 μm) was applied to the 3rd layers in each station.
 - “-” (down) for ST0 and “+” (up) for ST1/2

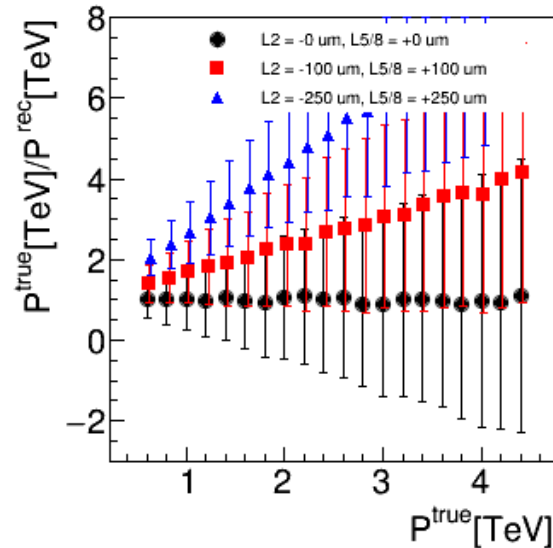


Performance (1Tm, single layer)

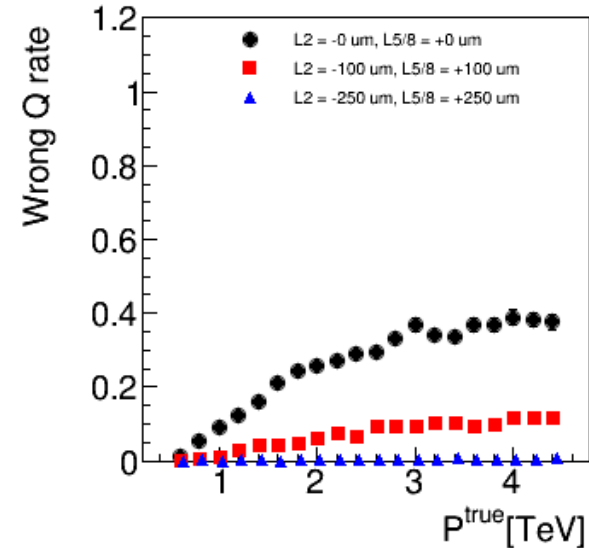
$\ell = 50$ cm $1/p$ v.s. momentum



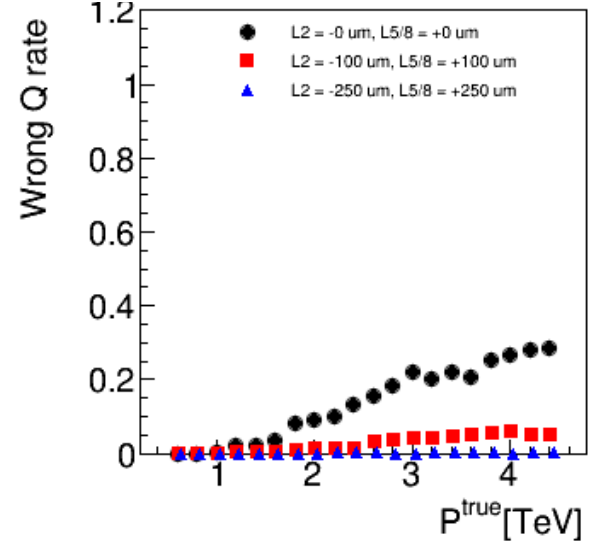
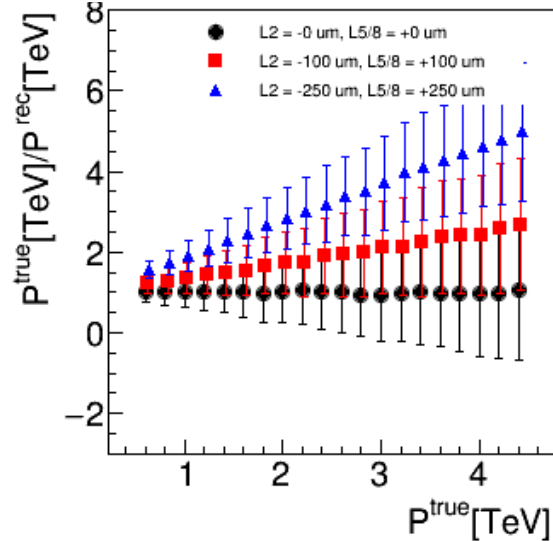
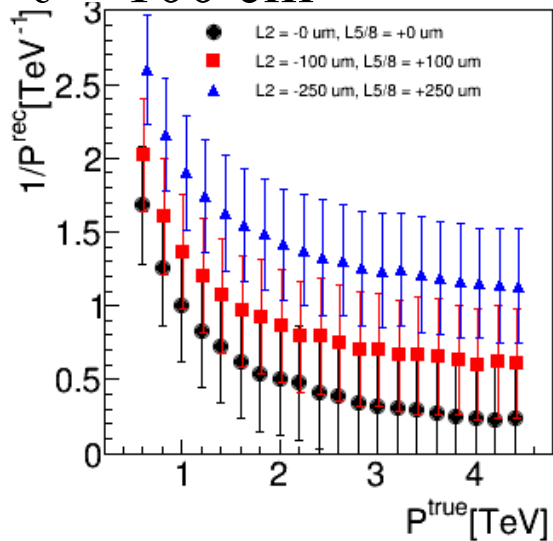
Momentum resolution



Wrong charge rate

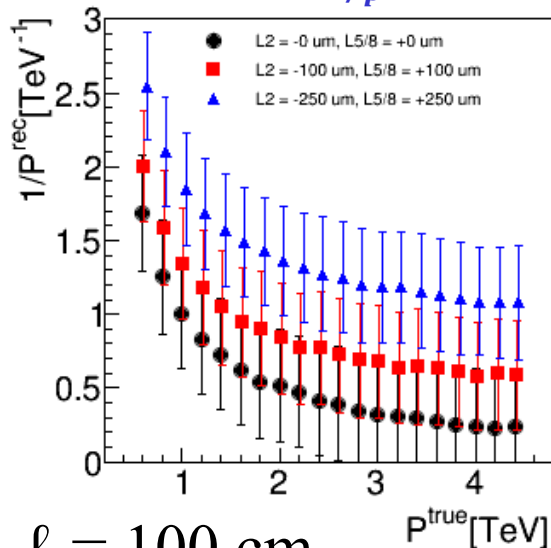


$\ell = 100$ cm

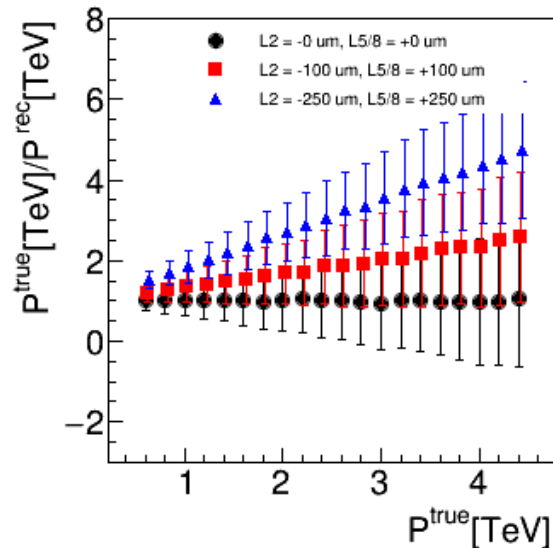


Performance (2Tm, single layer)

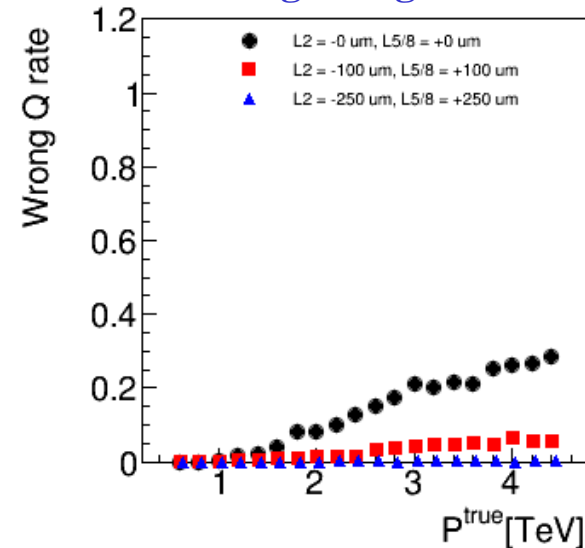
$\ell = 50$ cm $1/p$ v.s. momentum



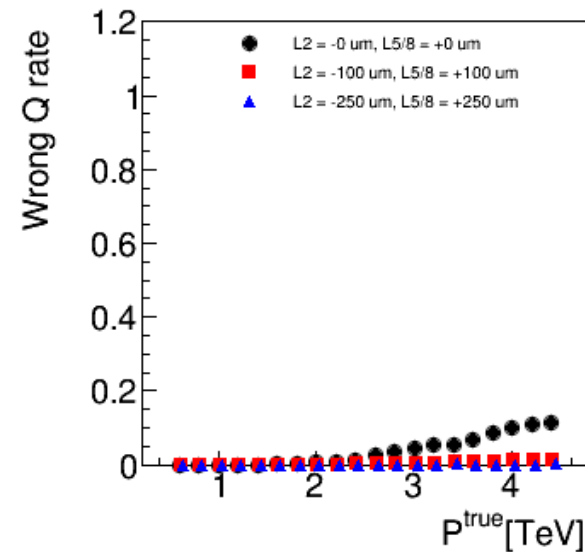
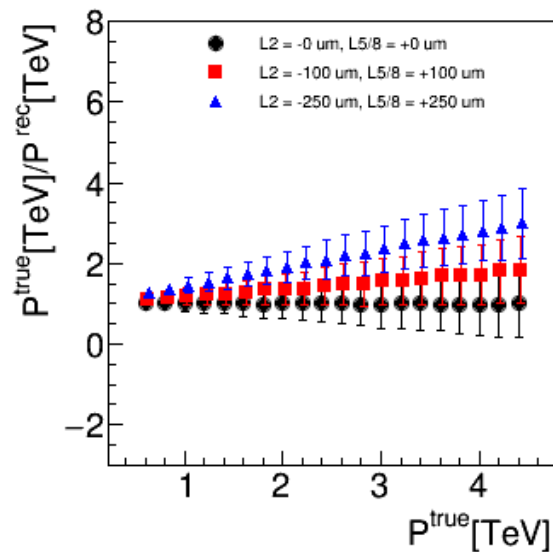
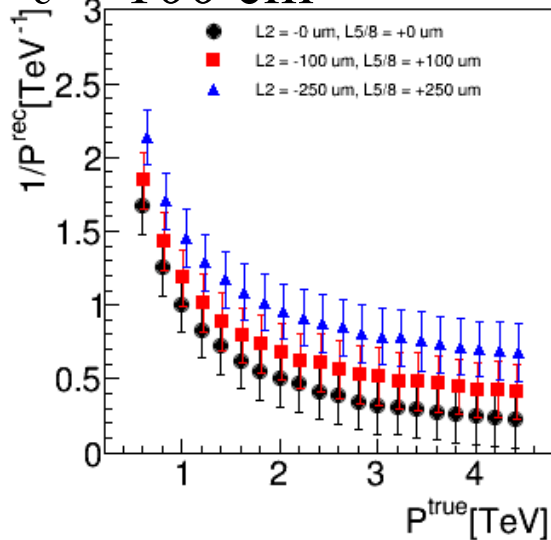
Momentum resolution



Wrong charge rate

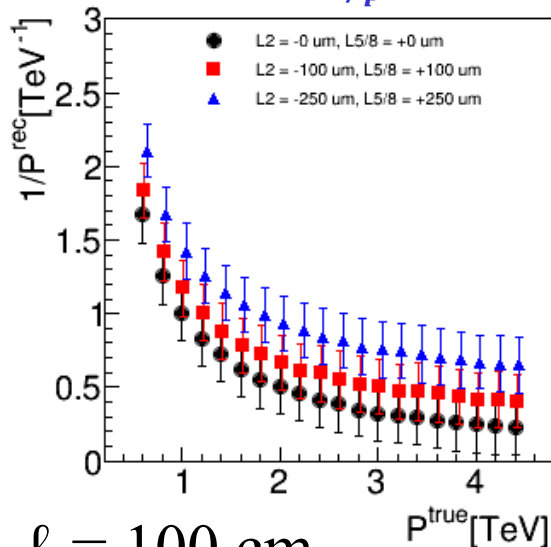


$\ell = 100$ cm

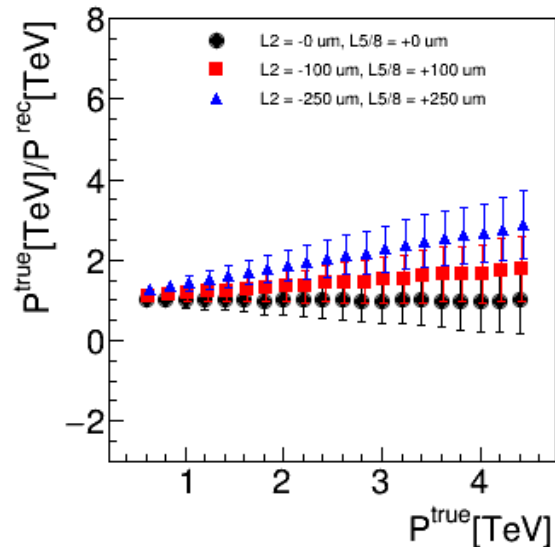


Performance (4Tm, single layer)

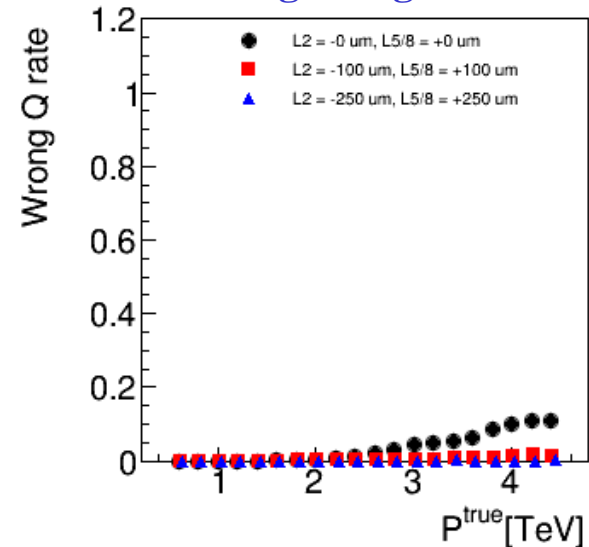
$\ell = 50$ cm $1/p$ v.s. momentum



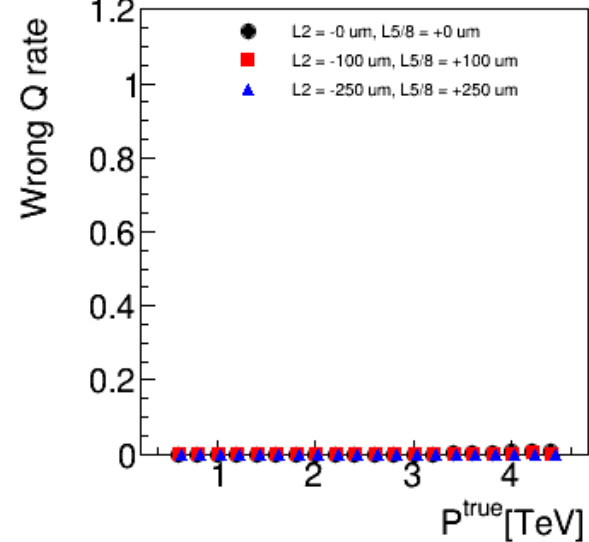
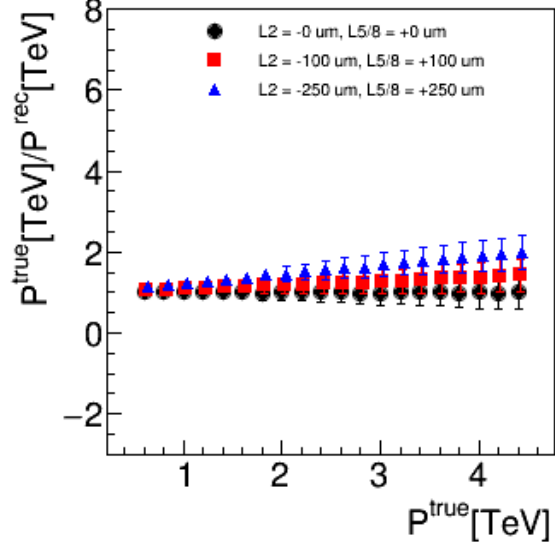
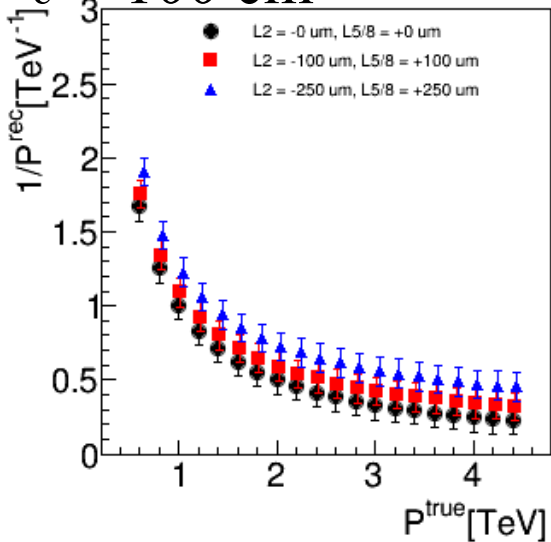
Momentum resolution



Wrong charge rate

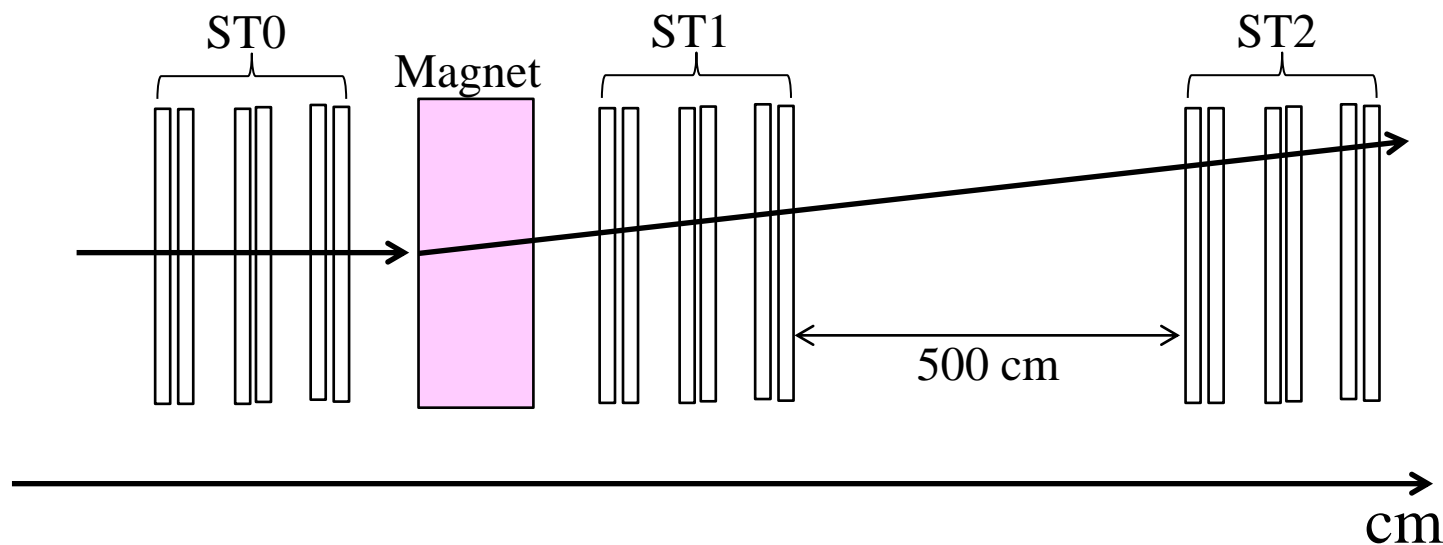


$\ell = 100$ cm



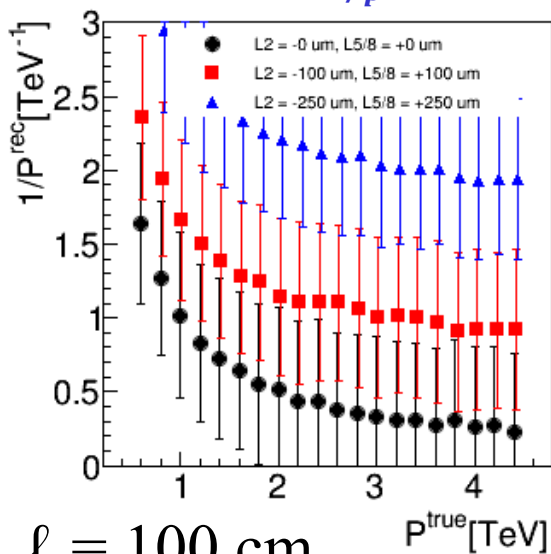
Performance with double layer

- The performance was also check with 3 tracker stations with 3 **double-layers** (ST0-2).
 - 3 cm gap between top and bottom layers in a double-layer.

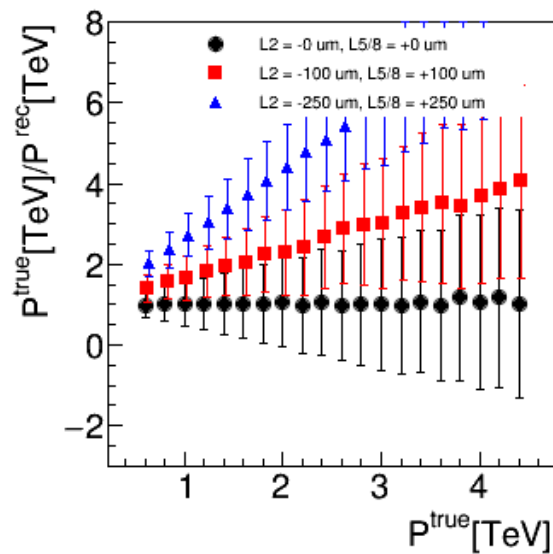


Performance (1Tm, double layer)

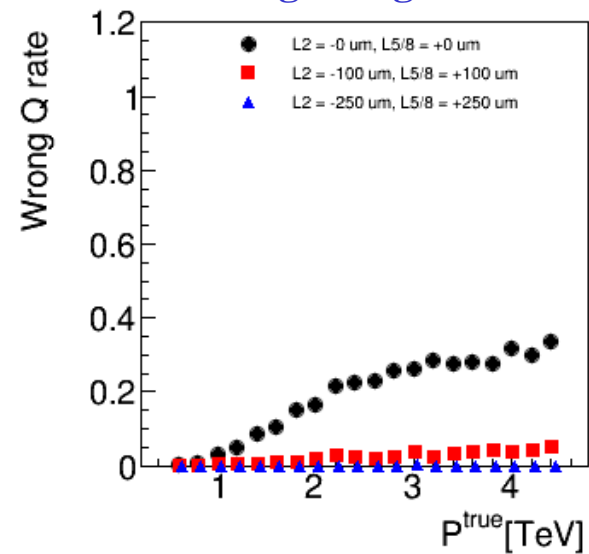
$\ell = 50 \text{ cm}$ $1/p$ v.s. momentum



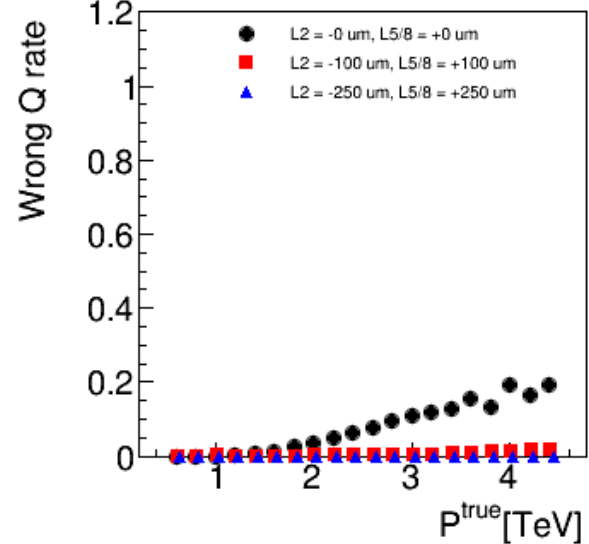
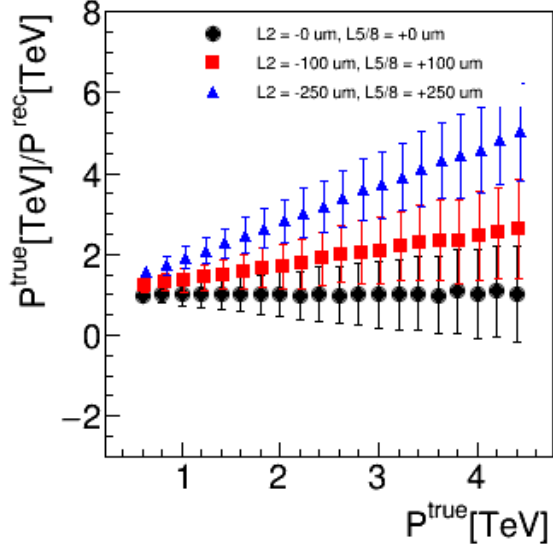
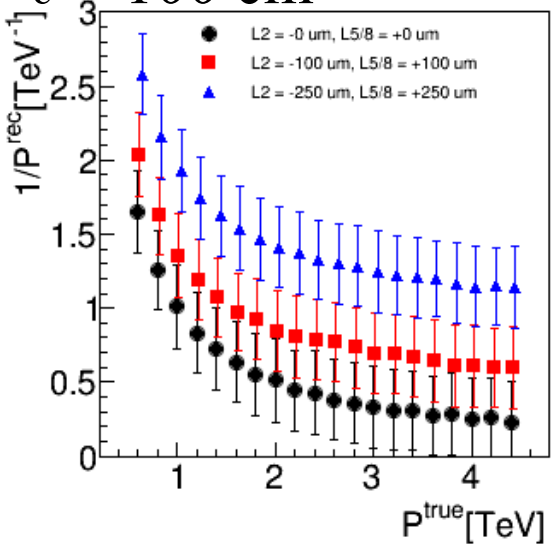
Momentum resolution



Wrong charge rate

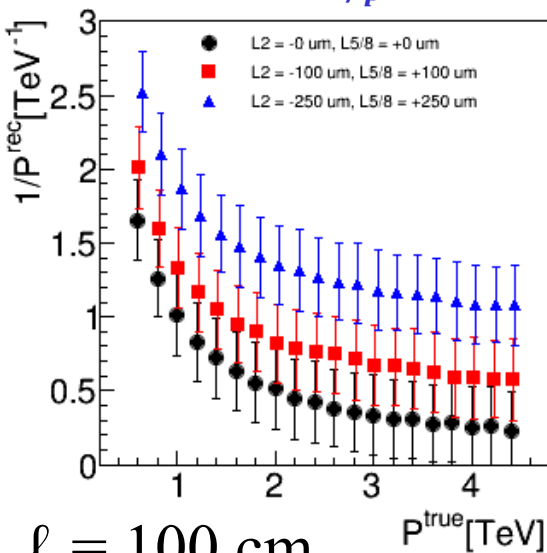


$\ell = 100 \text{ cm}$

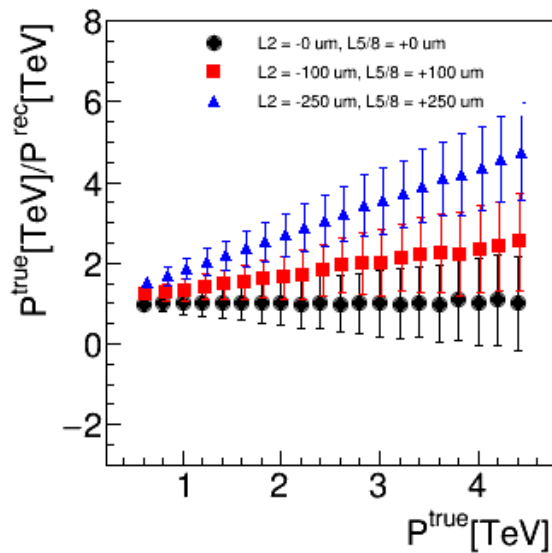


Performance (2Tm, double layer)

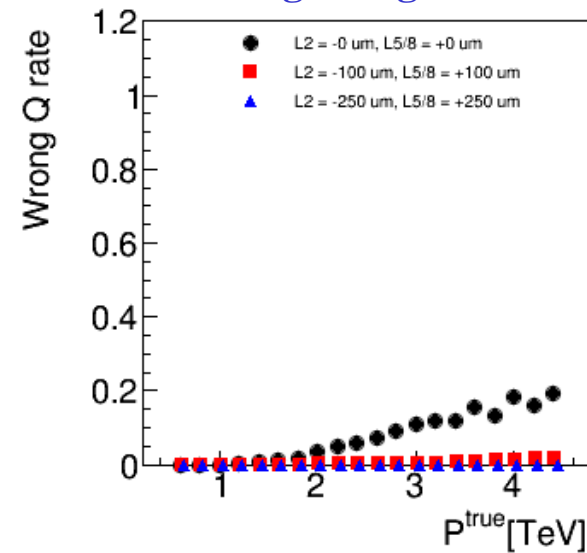
$\ell = 50$ cm $1/p$ v.s. momentum



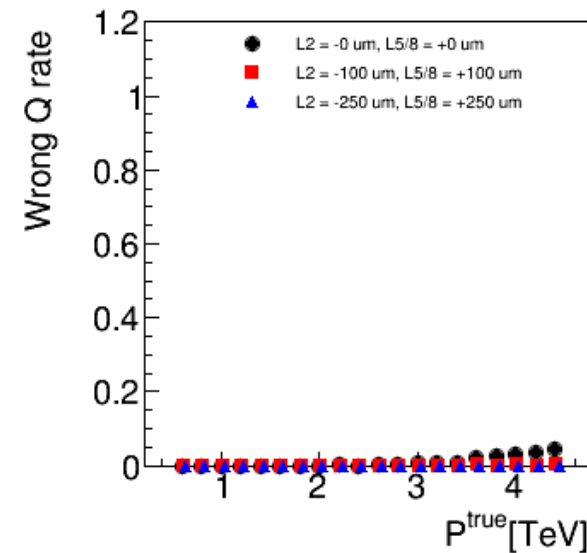
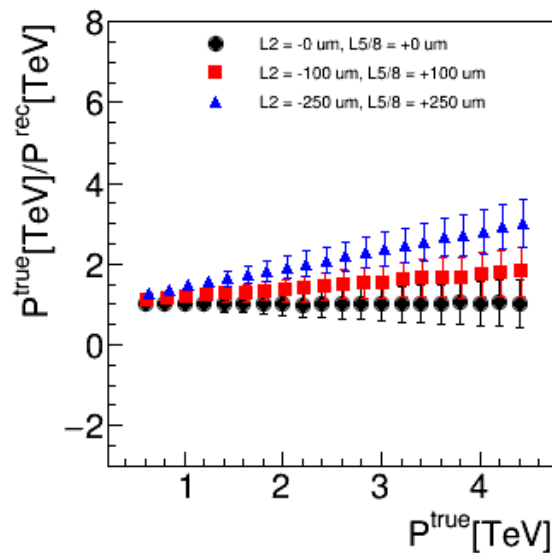
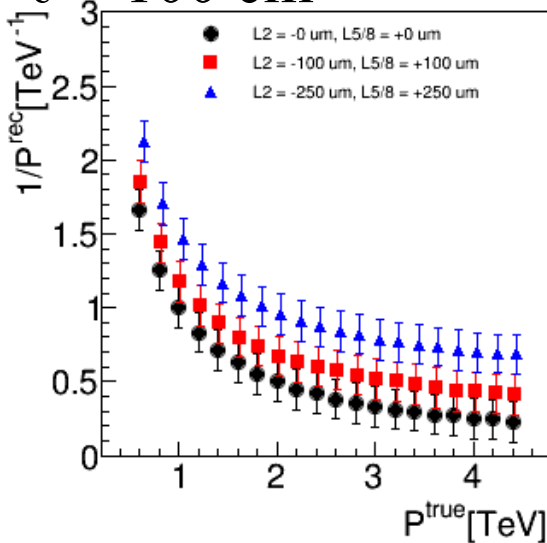
Momentum resolution



Wrong charge rate

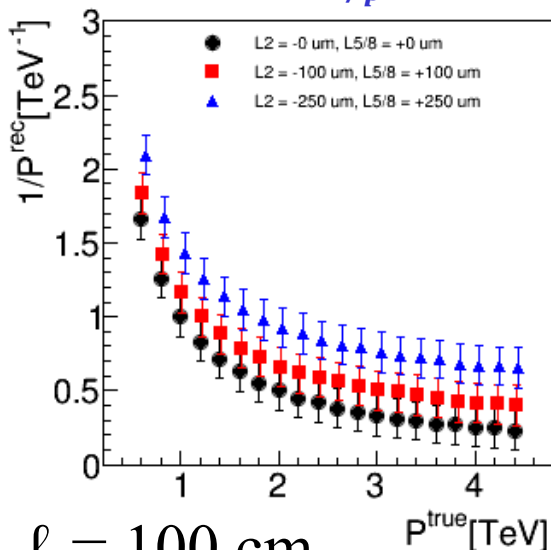


$\ell = 100$ cm

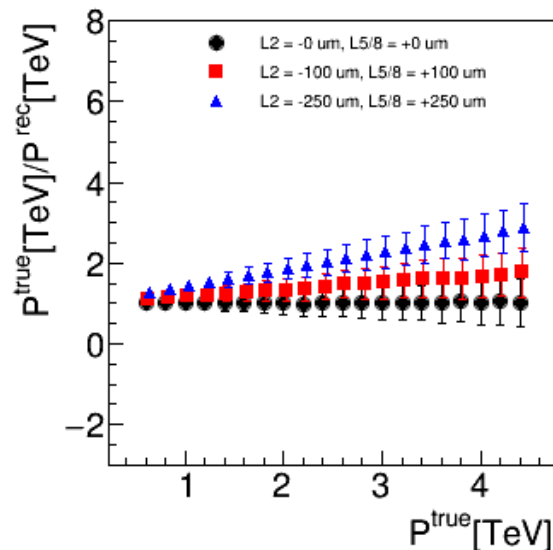


Performance (4Tm, double layer)

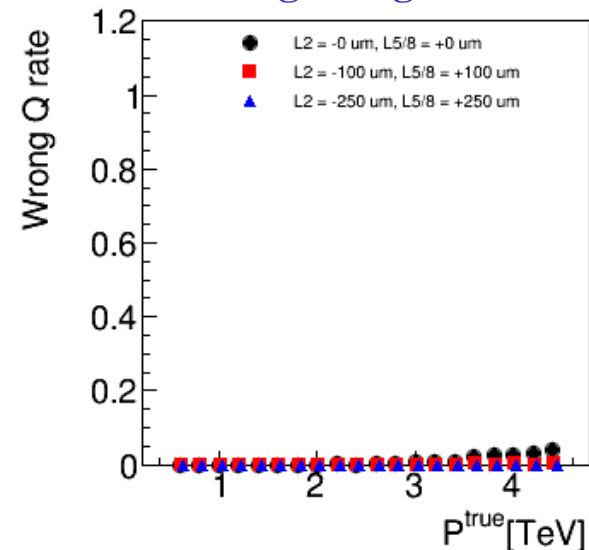
$\ell = 50$ cm $1/p$ v.s. momentum



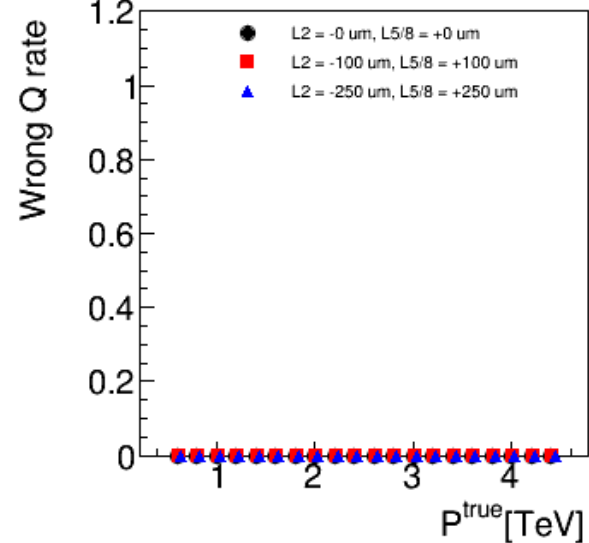
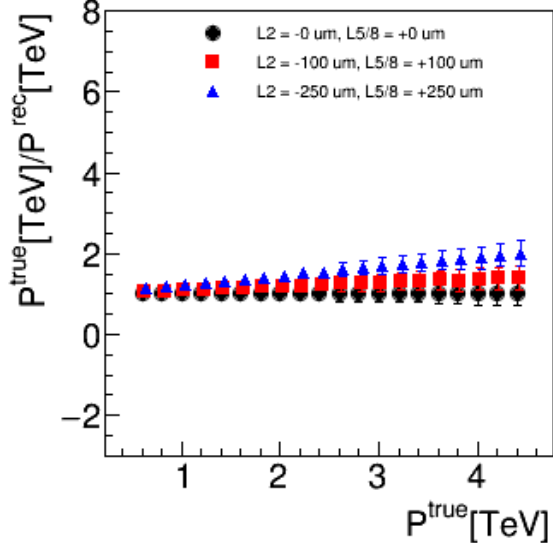
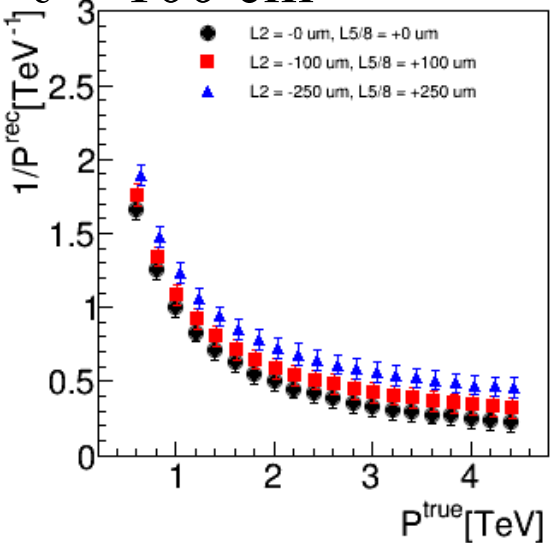
Momentum resolution



Wrong charge rate



$\ell = 100$ cm



Summary of momentum resolution

$\frac{\sigma_{1/p_T}}{1/p_T^{true}}$ is summarized here:

$\ell = 50 \text{ cm}$

	Single layer			Double layer		
	1 T·m	2 T·m	4 T·m	1 T·m	2 T·m	4 T·m
1 TeV	0.74	0.37	0.19	0.56	0.28	0.14
2 TeV	1.53	0.76	0.38	1.10	0.55	0.27
3 TeV	2.30	1.15	0.58	1.63	0.82	0.41

$\ell = 100 \text{ cm}$

	Single layer			Double layer		
	1 T·m	2 T·m	4 T·m	1 T·m	2 T·m	4 T·m
1 TeV	0.38	0.19	0.09	0.28	0.14	0.07
2 TeV	0.78	0.39	0.19	0.56	0.28	0.14
3 TeV	1.17	0.59	0.29	0.83	0.41	0.21

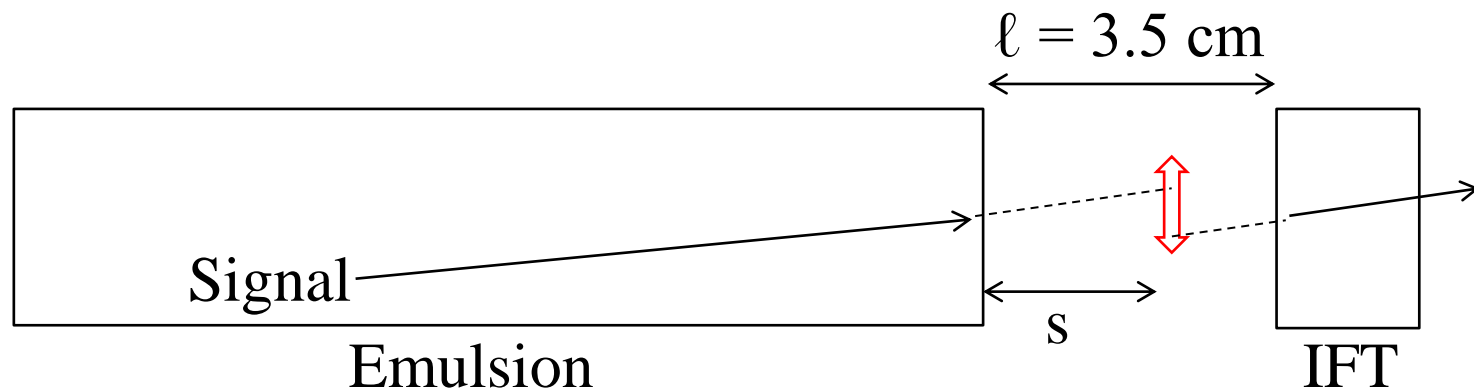
Backup

Position resolution caused by gap

- Define ℓ as gap between the last layer of emulsion and the first layer of IFT ($\ell = 3.5$ or 4.5 cm was used for this study).
- Fluctuation on x/y caused by extrapolating tracks with angular resolution of emulsion/IFT:

$$\sigma_{x/y(gap)} = \sqrt{\{\sigma_{\theta(emulsion)} \cdot s\}^2 + \{\sigma_{\theta(IFT)}(\ell - s)\}^2}$$

$$\rightarrow \sigma_{x/y(gap)}^{min} = \frac{\sigma_{\theta(emulsion)} \cdot \sigma_{\theta(IFT)} \cdot \ell}{\sqrt{\sigma_{\theta(emulsion)}^2 + \sigma_{\theta(IFT)}^2}}$$



Checking items

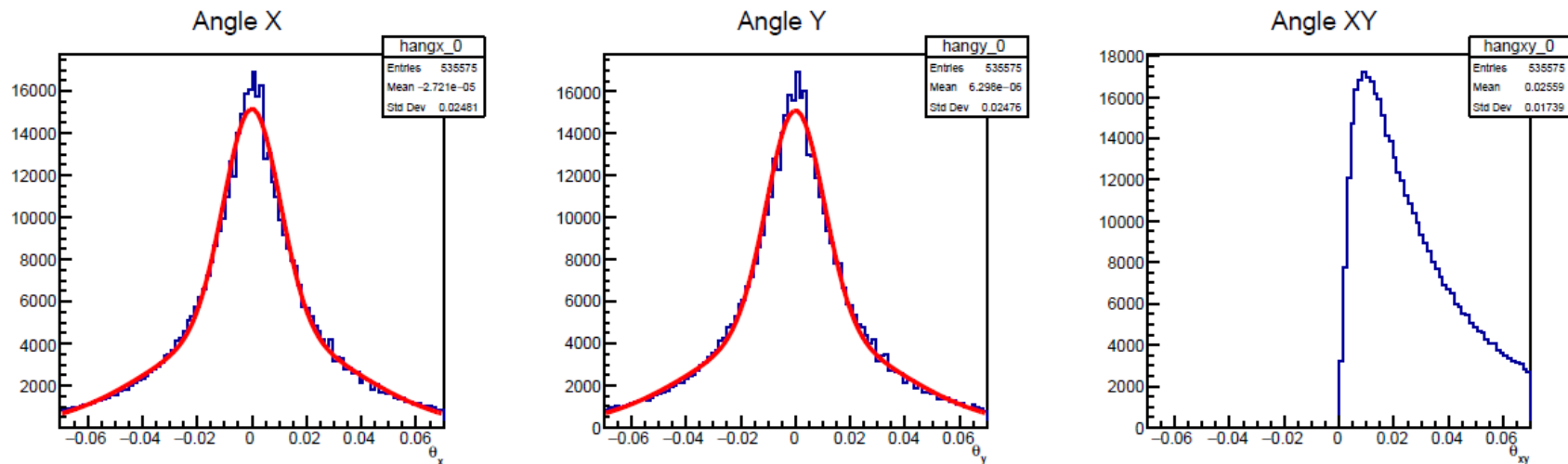
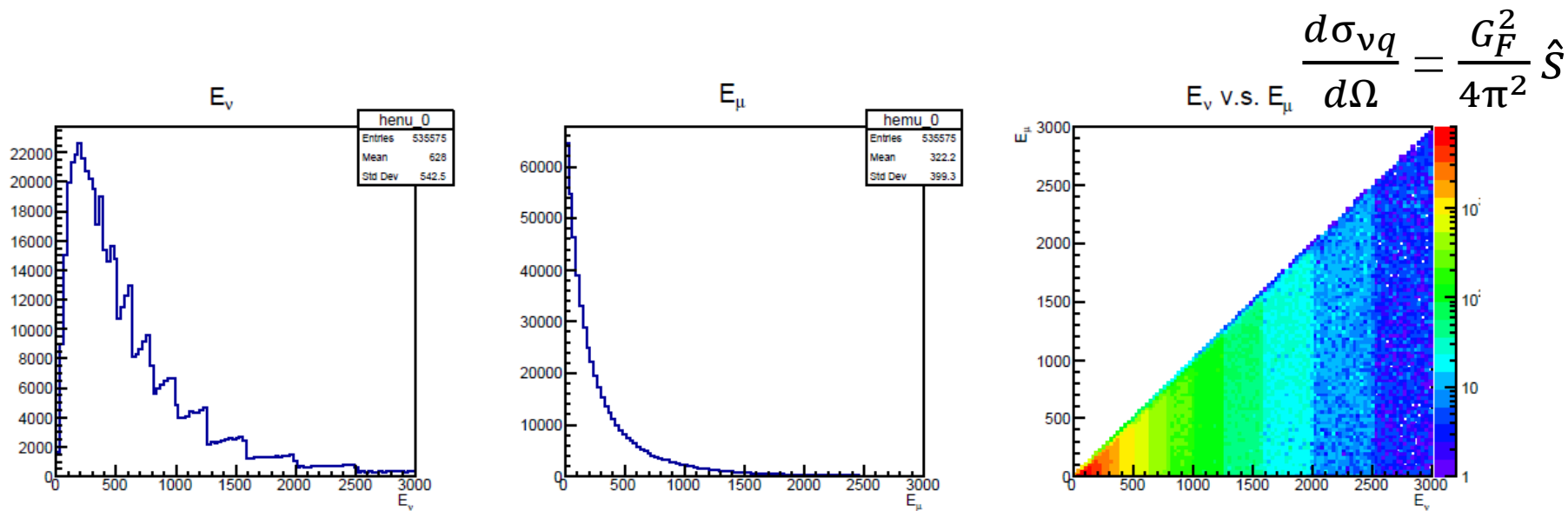
Signal-signal matching

- # of signal tracks correctly matching (“Correct matching”) and matching to another signal track (“Wrong matching”)

Signal-BG matching

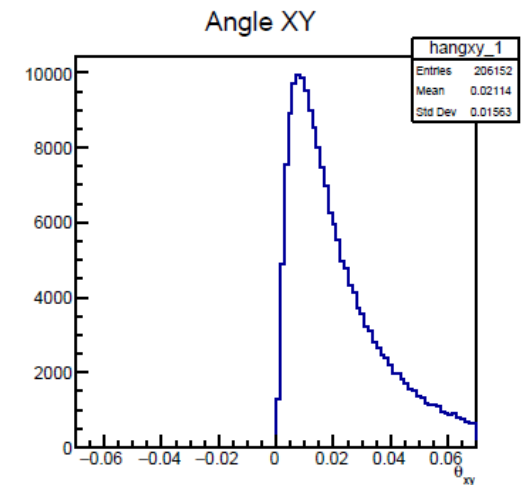
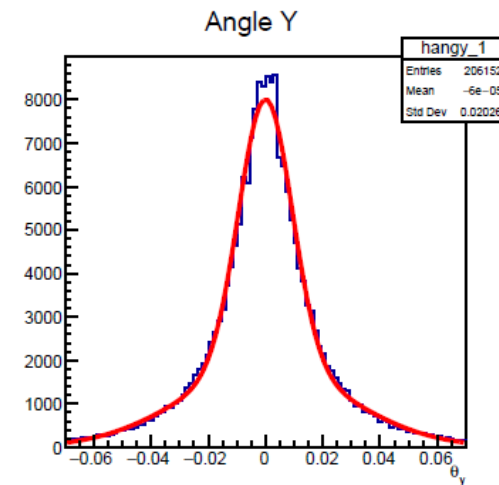
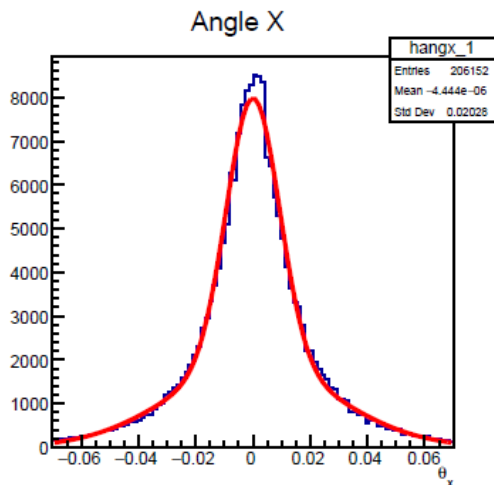
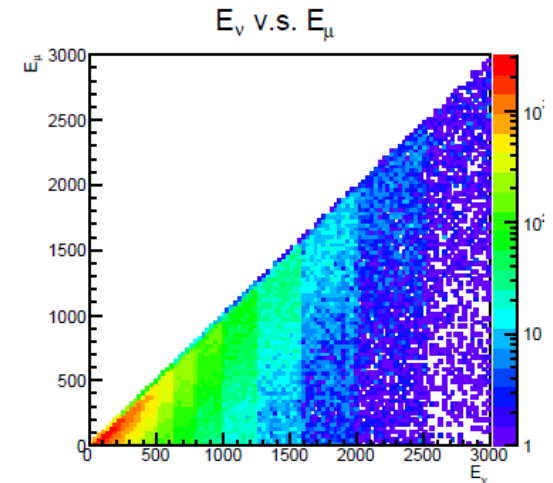
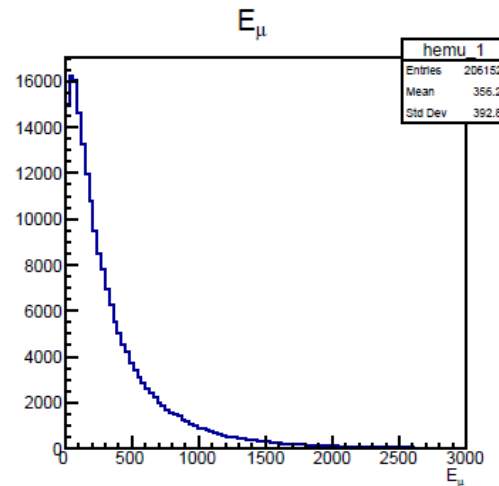
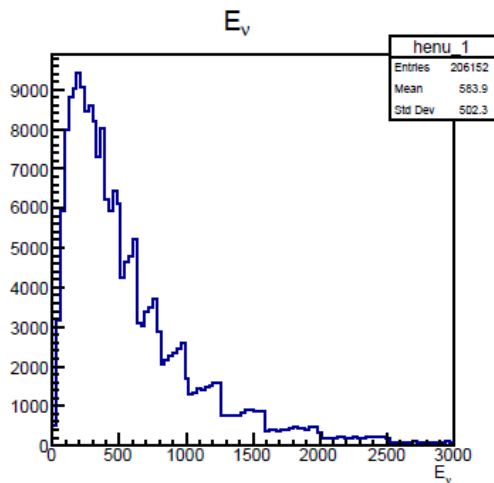
- # of signal tracks matching to a single BG track (“Single matching”), matching to more than one BG tracks (“Multiple matching”) and either of them (“All matching”).

Muon distribution from neutrino



Muon distribution from anti-neutrino

$$\frac{d\sigma_{\nu q}}{d\Omega} = \frac{G_F^2}{16\pi^2} (1 + \cos\theta)^2 \hat{s}$$



Calculation method


$$dx = r - \sqrt{r^2 - w_{mag}^2}$$

$$(dx - r)^2 = r^2 - w_{mag}^2$$

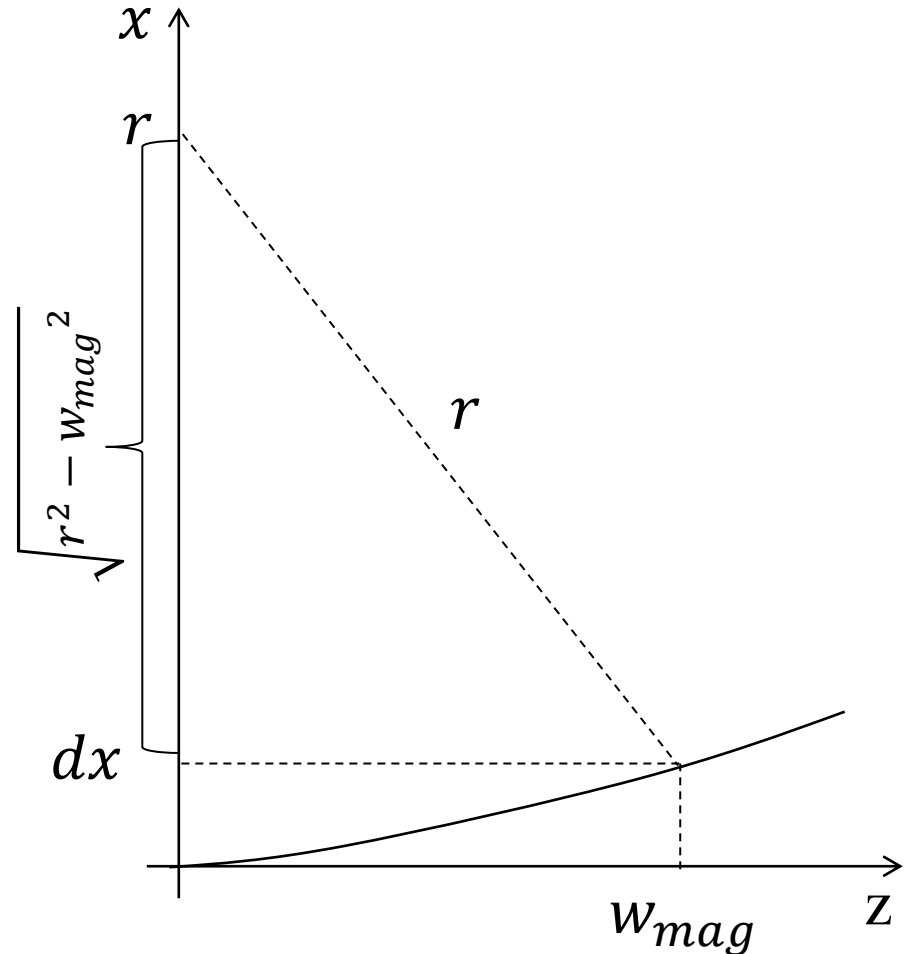
$$dx^2 - 2dx \cdot r = -w_{mag}^2$$

$$2dx \cdot r = dx^2 + w_{mag}^2$$

$$r = \frac{dx^2 + w_{mag}^2}{2dx}$$

$$\theta = \frac{dx}{w_{mag}}$$


$$r = \frac{w_{mag}(\theta^2 + 1)}{2\theta}$$



Confirmation by hand calculation (1)

Let's try rough estimation of momentum resolution for 1 TeV/c.

The largest uncertainty on θ comes from that of linear fitting of track1 ($x = \alpha z + \beta$).

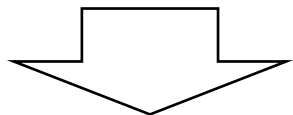
- $\sigma_{\alpha}^2 = \sigma^2 \frac{n}{\Delta}$

- $\sigma_{\beta}^2 = \sigma^2 \frac{\sum z_i}{\Delta}$

- $\Delta = n \sum z_i^2 - (\sum z_i)^2$

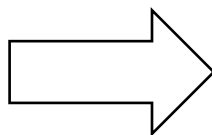
- $\sigma = 100/\sqrt{3}$ [um] (i.e., position resolution of each station)

- z_i : center position of each station

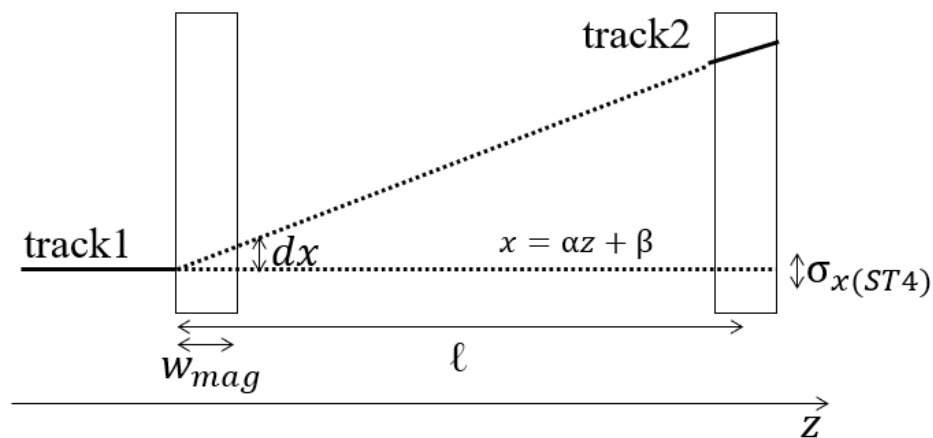


- $\sigma_{\alpha} = 0.00041$

- $\sigma_{\beta} = 5.3 \times 10^{-5}$ [m]

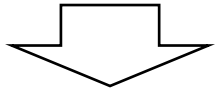


$$\sigma_{x(ST4)} = 8.2 \text{ mm}$$



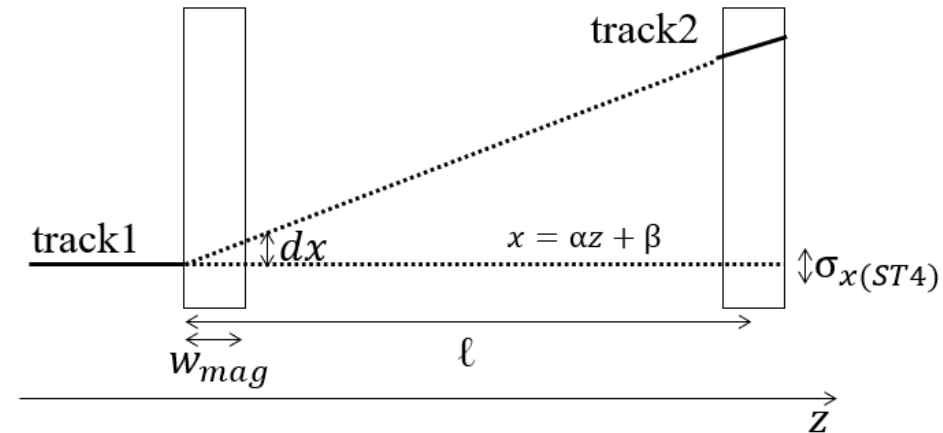
Confirmation by hand calculation (2)

$$\sigma_{x(ST4)} = 8.2 \text{ mm}$$

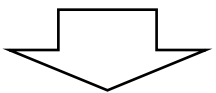


$$\sigma_{\theta} = \frac{\sigma_{x(ST4)}}{\ell} = 4.2 \times 10^{-3}$$

The resolution of track2 is ignored since it is much smaller than $\sigma_{x(ST4)}$.



$$r = \frac{w_{mag}(\theta^2 + 1)}{2\theta}$$

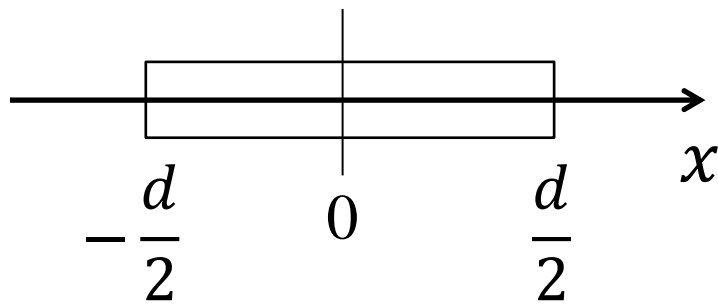


$$\sigma_r = \frac{w_{mag}}{2} \left(\frac{1}{\theta^2} - 1 \right) \sigma_{\theta} \sim 1500 \text{ [m]}$$

$$\frac{\sigma_{1/p}}{1/p} = \frac{\sigma_{1/r}}{1/r} = \frac{\sigma_r}{r} = 1.7 \quad (r = 833 \text{ m at 1 TeV})$$

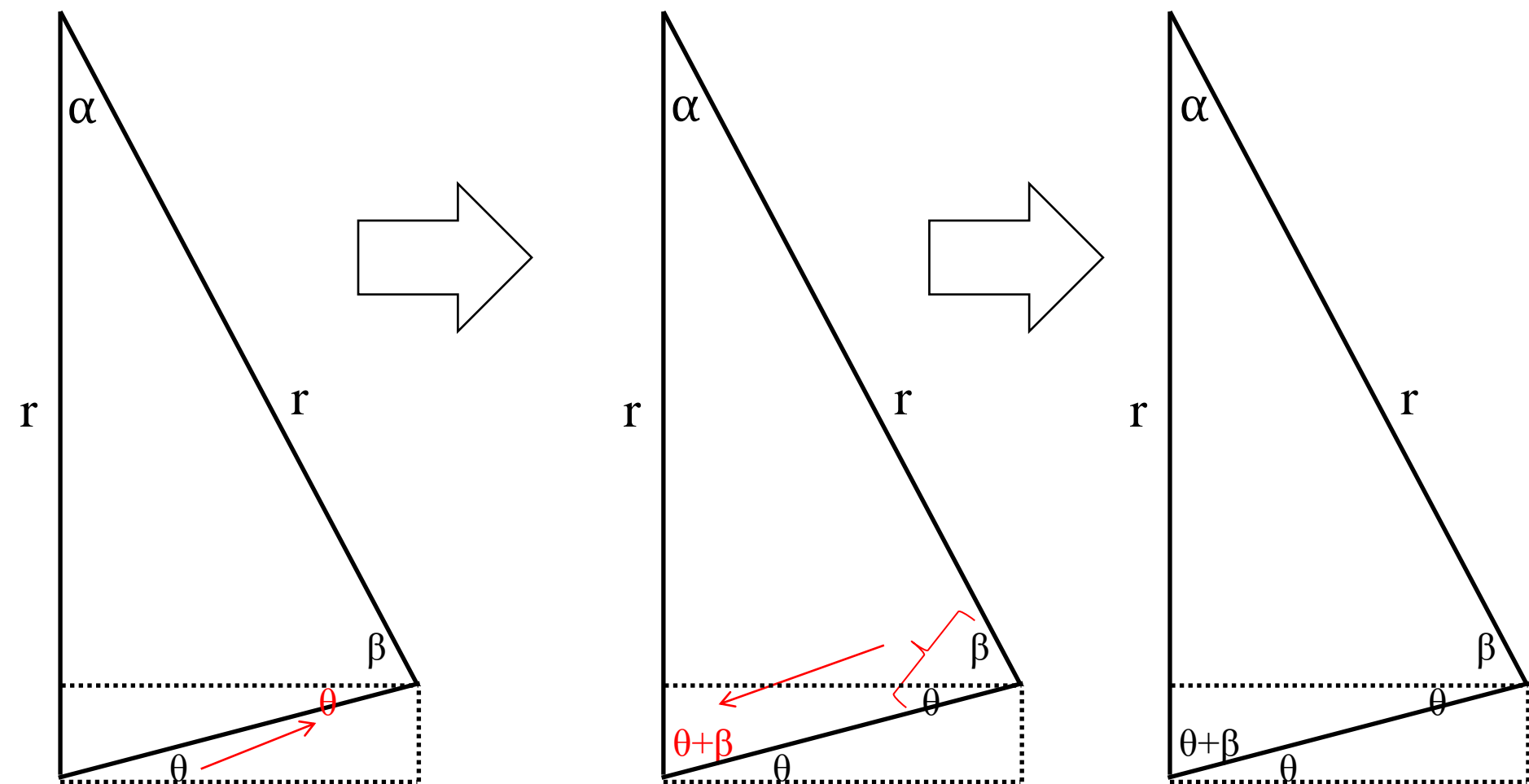
Hit position resolution

If a particle penetrates a strip with the width d uniformly, the probability to pass position x is $1/d$.

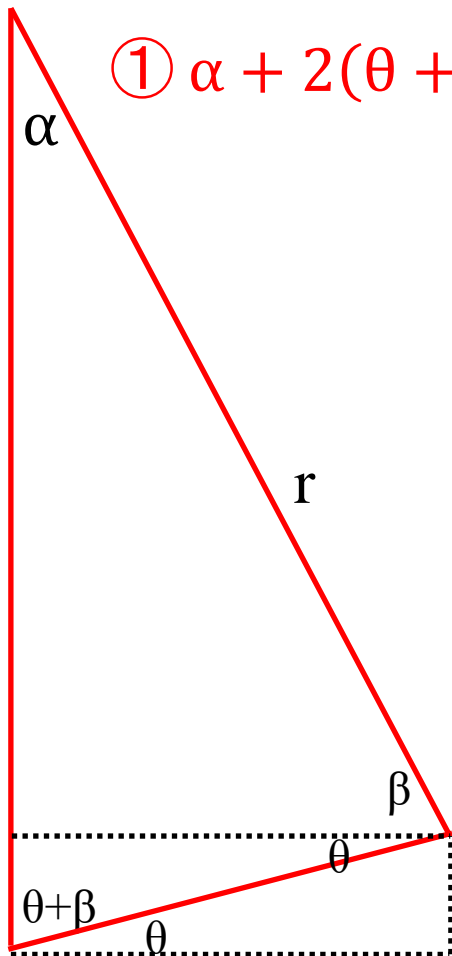


$$\begin{aligned}\sigma_x^2 &= \int_{-d/2}^{+d/2} x^2 p(x) dx \\ &= \int_{-d/2}^{+d/2} \frac{x^2}{d} dx \\ &= \frac{d^2}{12} \\ \sigma_x &= \frac{d}{\sqrt{12}}\end{aligned}$$

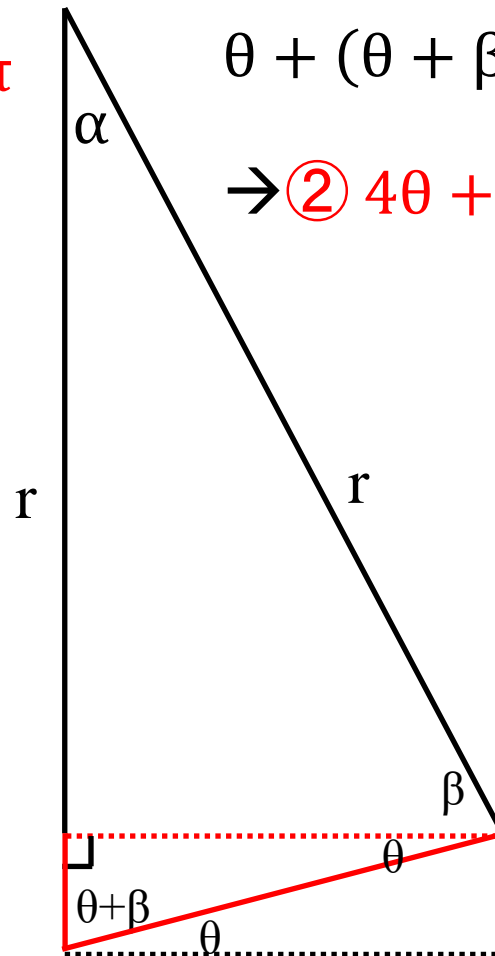
Comparison with AdvSND (1)



Comparison with AdvSND (2)



$$\textcircled{1} \alpha + 2(\theta + \beta) = \pi$$



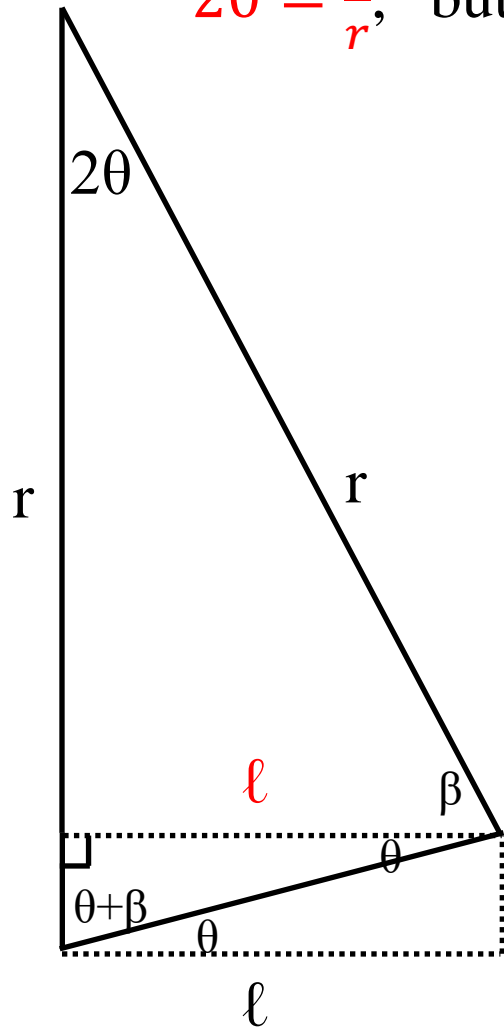
$$\theta + (\theta + \beta) + \frac{\pi}{2} = \pi$$

$$\rightarrow \textcircled{2} 4\theta + 2\beta = \pi$$

$$\text{With } \textcircled{1} + \textcircled{2}, 2\alpha = \theta$$

Comparison with AdvSND (3)

$2\theta = \frac{\ell}{r}$, but, written as $\theta = \frac{\ell}{r}$ in Sec. 3.3.2 in [[FPF White Paper](#)]



Comparison with AdvSND (4)

Anyway, let's compare with my calculation.

Condition of my calculation:

$$\left\{ \begin{array}{l} \bullet \theta = \frac{dx}{\ell} \\ \bullet dx = r - \sqrt{r^2 - \ell^2} \end{array} \right.$$

$$dx = r - \sqrt{r^2 - \ell^2}$$

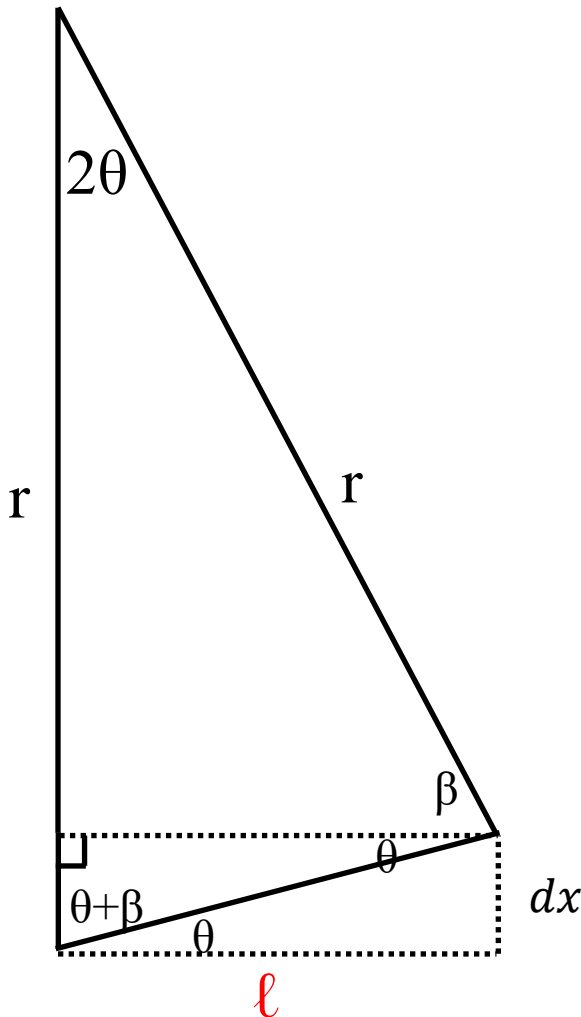
$$\rightarrow \textcircled{1} dx \sim r - r \left(1 - \frac{\ell^2}{2r^2} \right) = \frac{\ell^2}{2r}$$

$$\theta = \frac{dx}{\ell}$$

$$\rightarrow \textcircled{2} dx = \theta \ell$$

$$\text{With } \textcircled{1} \text{ and } \textcircled{2}, \theta \ell = \frac{\ell^2}{2r} \rightarrow 2\theta = \frac{\ell}{r}$$

The result is the same as the method of AdvSND (except for factor of 2).



Comparison with AdvSND (5)

$$p[\text{MeV}] = 3 \cdot r[\text{cm}] \cdot B[\text{T}]$$

(“3” is missing in calculation of Sec. 3.3.2 in [[FPF White Paper](#)])

$$\rightarrow r = \frac{p}{3B}$$

$$\Rightarrow 2\theta = \frac{\ell}{r} = \frac{3B\ell}{p}$$

$$\Rightarrow \theta = \frac{3B\ell}{2p} \quad (\theta = \frac{B\ell}{p} \text{ in Sec. 3.3.2 in [[FPF White Paper](#)])}$$