How Geometric Approach to Supergravity Was Born in Torino

Riccardo D'Auria

Dipartimento DISAT
Politecnico di Torino

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Talk for the celebration of the 70-th anniversary birthday of Pietro Fre'.

It is a pleasure to have the possibility to give a talk in honor of the 70-th birthday of Pietro.
The pleasure is in my case double since Pietro is not only a high class physicist, but also an ancient collaborator of mine , a collaboration which lasted many years and was the source of a profound friendship between us generated by empathy, easy character and full generosity. I can honestly say that I am much indebted to him since our collaboration had much influenced my scientific career.

Pietro's career is really impressive with many very important achievements:
Let me quote just a sample of them besides many others

- Geometric Supergravity in D=11 (Introduction in Physics of Free Differential Algebras and their iterative construction with Chevalley-Eilenberg Lie Algebra Cohomology; Reduction of D=11 FDA to an ordinary Lie Algebra)(with RD'A).
- N=2 Supergravity and N=2 superYang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map.
  (with L.Andrianopoli, A.Ceresole S. Ferrara, T. Magri, RD'A).
- N=8 Gaugings revisited: An Exhaustive classification (with F.Cordaro, L. Gualtieri, P. Termonia, M. Trigiante); This paper contains the first introduction of the Embedding tensor in supersymmetry.

- Stable de Sitter vacua from N=2 supergravity, (with M. Trigiante, A.Van Proeyen)(relevant for inflationary cosmology);
- G / H M-branes and AdS(p+2) geometries, (with L.Castellani, A. Ceresole S. Ferrara(CERN), R.D'A.)
- Consistent Supergravity in D=6 without Action Invariance, (with T.Regge,RD'A);
- Solvable Lie Algebras in Type IIA, IIB, and M-Theory (with L.Andrianopoli,S.Ferrara, M. Trigiante, Minasian.RDA);
- Special and quaternionic isometries: General couplings in N=2 supergravity and the scalar potential, (with S.Ferrara,RDA);
- SU(3) x SU(2) x U(1) From D = 11 Supergravity,(with L. Castellani ,RDA.)

and many others.....

Books:

- Supergravity and Superstrings: A Geometric perspective, World Scientific (with L. Castellani,RDA);
- The N=2 Wonderland: From Calabi- Yau manifolds to Topological Field Theories. World Scientific, (with P.Soriani);
- Gravity a Geometrical Course,Springer.

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- Coordinator of italian knots of the quadriennal European research contracts:
- Twice coordinator of the PRIN projects;
- Member and four years President of the SIGRAV;
- Addetto Scientifico at the Italian Embassy in Moscow;
- Collaborator of Joint Institute of Nuclear Research in Dubna;
- Sabbatical Course of Supersymmetry and Supergravity at Kiev Academy of Sciences.

Prizes:
Together with A. Sorin he was honored in 2008, with the First Prize of Joint Institute of Nuclear Research di Dubna for their researches and results of the cosmic billiards and integrable Systems.

- Given these premises it is quite obvious to conclude that Pietro has been, and I am sure he will continue to be, one of the most brilliant and of high profile Italian physicists of its generation with a deep influence in the international research of supersymmetric theories.
In general he has given important contributions to the physicist community not only in terms of scientific papers, but also in terms of organizing meetings and collaborations, establishing scientific contacts and giving precious suggestions also to groups not directly involved in supersymmetric theories.

Our collaboration was almost exclusively based on the Geometrical Approach to Supergravity theories.
Therefore my talk will be concerned on the birth of this approach which has its origin in the seminal paper of Ne'eman and Regge.

The Ne'eman-Regge proposal.

- A geometric understanding of Supergravity requires the use of differential forms and group theoretical principles. (Cartan)
- One starts from the super- Poincare'group, the Dual Lie algebra of Maurer-Cartan forms and their associated supercurvatures (1- and 2-forms superfields,respectively, in terms of which a 4-form Lagrangian is constructed.
- The Action is obtained by integration of the 4-form Lagrangian on a four dimensional hypersurface immersed in superspace;
- Note: The minimal action principle does not require the variation of the hypersurface embedding functions, since any variation of the hypersurface can be identified with a diffeomorphism under which the lagrangian is invariant.

- The equations of motion are 3-forms obtained by variation of the superfields $\Phi(x, \theta)$ and can be naturally extended to the full superspace.
- They can be analyzed along the different basis elements of the of the supervielbeins 3-forms, namely VVV; $VV\psi$; $V\psi\psi$; $\psi\psi\psi$.
- The projection of the equations of motion along VVV gives the (supercovariant) equations of motion on space-time.
- All the other projections along a basis having at least a fermionic vielbein ψ give instead linear relations between the supercovariant space-time field strengths and the supercurvatures with at least one leg in the ψ direction are given in terms of the supercovariant space time field-strengths. This property was named by us *rheonomy*.

Rheonomy is a crucial property of the new action principle: Indeed it gives an on-shell parametrization of the supercurvatures whose component along any of the 3-form basis is written in terms of the space-time field-strengths only. For example in N=2 Supergravity, the on shell curvature of the gravitino turns out to be:

$$\mathcal{D}\psi^A = (\mathcal{D}\psi^A)_{ab} V^a V^b + (\gamma^a F_{ab} + i\gamma^5\gamma^a * F_{ab})\epsilon_{AB}\psi^B V^b. \quad (1)$$

The Susy transformation of the superfields is a Lie derivative in superspace , which can be expressed as

$$\delta\Phi^A \equiv L_\epsilon \Phi^A = \nabla\epsilon^A + \iota(\epsilon)R^A \quad (2)$$

which in our example ($\Phi = \psi^A$) gives

$$\delta\psi_A = \mathcal{D}\epsilon_A + (\gamma^a F_{ab} + i\gamma^5\gamma^a * F_{ab})\epsilon_{AB}\epsilon^B V^b \quad (3)$$

and this gives the required Susy gravitino transformation.

This approach is now accepted as one of the several ways to adress supergravity theory, together with:

- The Noether approach: (construction of the first supergravities ($\rightarrow$ the seminal papers on $N = 8$ supergravity of De Wit and Nicolai and D=11 dimensional Supergravity of Cremmer and Scherk) ;
- The Superconformal Approach mainly developed by Antoine Van Proeyen and summarized in the Supergravity book of Antoine with D. Freedman; (is group-theoretically minded and allows a general method of construction of supergravity theories($\rightarrow$ first formulation of matter coupled N=2 Supergravity and the concept of Special Kaehler Geometry.
- The superspace approach based on the Berezin integral (and expansion of the superfields in Grassmann coordinates.)

Birth of The geometric Approach to Supergravity .

- 1978-1979 Two simultaneous events: Tullio Regge coming back from IAS of Princeton, and simultaneously I met for the first time Pietro also coming back from a post-doc position in Caltech.
- Tullio asked me if I would join him to the development of a project of constructing a Geometric approach to supegravity, as proposed in a recent paper of him together with Y. Ne'eman.I proposed to Pietro to join to the same project and we both began to collaborate with Tullio.
- Tullio presence was not only scientifically important, but sometimes amusing because of his making fun character: as an example I remember the Cukoo-Clock...

- 1979-1982: During the first years of our collaboration together with Regge, our attention was mainly directed to establish the fundamental rules for the construction of pure supergravities.
- We formulated a list of few building priciples of a geometric supergravity theory for any $N \leq 8$ and any $D \leq 11$
- We put in evidence the important and alternative role of the Bianchi Identities of the Supercurvatures for the construction of the theory and the role of Cohomology of Lie Algebras in the geometric approach.
- We constructed in the geometric approach higher dimensional supergravities: D=5,6 and finally D=11 where the new important concept of Free Differential Algebra was discovered. It was a period of intense work, but our friendship made it sometimes amusing.

A nice iterative event: Pietro versus Attila, Flagellum Dei (Scourge of God).
During the first years of our collaboration Pietro and I were often working at my home; in the morning we were alone, except for the presence of my cat, whose name was Attila....

- Starting from 1981 other collaborators joined; Pietro obtains a NATO contract for scientific exchange between Stony Brook University and Torino University and collaboration with P. Van Nieuwenhuizen begins. During the 1983-1985 period we were several times in Stony Brook and several times Peter visited Torino Department of Physics.
- In this period there was an intense activity in the supergravity people in the field of Kaluza Klein compactification, mainly from D=11 down to D=4.
- We mainly focused on the generalization of the Kaluza-Klein reduction of the Freund-Rubin solutions of the D=11 supergravity, generally made on a T^7 torus, to more general coset manifolds G/H,and in deriving through harmonic analysis the corresponding spectrum of states in D=4. Our collaboration with Peter was intense and productive sometimes amusing. For example:

Peter and the good Flamish roasted Leg of Lamb.
During one of our visits in Stony Brook we lived in Peter home as guests (paying a part of the rent...) We cooked the lunch and....

- We had of course often more collaborators to the project. In 1983 we had our first student: Anna Ceresole made his Laurea thesis collaborating to the paper of Pietro and H. Nicolai : Multiplet Structure and Spectra of N=2 Supersymmetric Compactifications.
- In 1984 L. Castellani takes a position in the Torino Physics Department and joins our group.
- A. D'Adda, E. Maina and other people also concurred to the initial development of the project.

During the next years (1985-1990) the activity of Pietro was manly devoted both to the supergravity and related topics and also to superstrings. At same time he wrote this famous(?) book (or mattone?) on the Geometric approach, together with Castellani and myself, edited in 1990.

There were changes in our positions: I was called to Padua university for a full professorship (1986-1989) and in 1990 I reentered in Torino at the Politecnico.
In the same year also Pietro was called for a full professorship at SISSA in Trieste.
Notwithstanding this our collaboration continued, albeit with new collaborators and young people.
Among them*Anna Ceresole*, coming back from the States, won a research position in Politecnico, and *M. Trigiante* who was in fact one of the Pietro students at Sissa; Some years later a student of mine, *L. Andrianopoli* reinforced our group in Politecnico and in 2003 , after two post-doc positions, also M. Trigiante increased the theoretical group of Politecnico.

- Meantime the Geometric Approach actually showed its full power during the last decade of the century. As everybody knows, in the nineties there was an enormous increase of our understanding in String Theory and Supergravity. The treatment of new concepts, like duality symmetry, Calabi-Yau compactifications, conformal field theories gauging of supergravity symmetries and so on, needed a clear geometrical interpretation:
- Indeed global and local supersymmetric theories exhibit deep geometrical structures of the non-linear interactions of matter multiplets. Almost all of these couplings can be rephrased in a geometrical language as coupling of some non-linear sigma-model to gravity and gauge fields.
- The Geometric Approach appeared particularly indicated and advisable for the interpretation of the new discovered properties, distinguishing itself in many respects from the other approaches.

- As a matter of fact it can explain in a very clear and natural way the close interrelations between geometry, topology and supersymmetry which is in fact one of the most striking results of supersymmetric quantum field theories.
- As an example, the origin of the Special Kaehler Geometry and its relation with the Picard-Fuchs equations associated to Calabi Yau compactifications emerges quite naturally in this approach, and the same happens for the quaternionic isometries associated to hypermultiplets.
  These results were the basic premises for the full general construction of the N=2 matter coupled supegravity, (quoted 500+ in the Archive).

- Most of the results in this context were obtained by Pietro in collaboration with many italian and mostly European people, many of them present today.I can quote: L. Andrianopoli, D. Anselmi, M.Billò, L. Castellani, A. Ceresole , S. Ferrara, Girardello, F. Gliozzi, P.A. Grassi, T.Regge, P. Soriani, M. Trigiante, K. Pilch, P.K.Townsend P. Van Nieuwenhuizen, A. Van Proeyen, A. Zaffaroni, A. Zanon, RDA, etc.

Let me conclude this short excursion in the past: I hope that my recollection of the beginning and of the growing importance of the Geometrical approach initially proposed by T. Regge and in the sequel developed in all his power by Pietro and collaborators can aid to give the right perspective to the actual research in this field.

Personally I think that the role played by Pietro in the development of the approach through several years of work, first with with my collaboration, and later also with other eminent physicists, most of them present today, has been quite essential for the success of the Geometric approach and for its becoming a well established procedure to analyze the very intricate group theoretical properties of supergravity theories.
The extra bonus I got from this scientific adventure was a everlasting friendship with him.
Therefore: Thank you Pietro and many wishes for your scientific future.
Thank you!