



Pietro's supergravity

A tale of geometry

Torino, 7/10/22

Antoine Van Proeyen

KU Leuven

Stony Brook 1979



Tom
Cockroft
 Peter West Peter Lindenstrom
 Nichols Nielsen Zachar
 Luciano Grandella Jerry
 Lichneri
 Frensdorff
 Cox
 Jacob Perk Stanley Derer
 Ergin Keegan
 Monica Mae
 Edzell
 Anagone John Dixon
 Amos
 Bernard de Wit
 Jan Willem Van Holten
 Jurek
 Jan Willem Van Holten
 Sunit Mukhi
 Alf Lindstrom Mark Gusarov
 Mike Huff
 Julius Weiss
 Peter Van Haevenhansen
 Dan Friedman
 Peter Jacob
 Bernard de Wit
 Peter Freund
 Schwarz
 Steven Saper Kellog Skille Paul Townsend
 Rolf Ashok Das
 Sergio Ferraro T. V. P.
 Mark Fuchler
 Neil Schwab
 Martin Roick
 Annoult
 Paul Hase
 Bron Nash

Stony Brook – Torino connection



Independently:

I stayed in Stony Brook for several months in 1979,
and started to work on matter couplings in $N=1$
supergravity

First encounters

- D=11 search for auxiliary fields;
'the hidden supergroup' (Riccardo D'Auria and Pietro Frè)
- 5 sept. - 8 nov. 1985, Torino, conference in villa Gualino,
Start of my stay for 2 months.
Start of friendship with the whole group in Torino.
I often came, and certainly with much pleasure
- Pietro often came to Leuven.
- EC collaborations

One of our contact persons in European networks

- Gauge theories, applied supersymmetry and quantum gravity, 1/9/1992- 31/8/1996
- Quantum aspects of Gauge Theories, Supersymmetry and Unification, 1/9/1996 - 31/8/2000
- The quantum structure of spacetime and the geometric nature of fundamental interactions, 1/10/2000 - 30/9/2004
- Constituents, Fundamental Forces and Symmetries of the Universe, MRTN-CT-2004-005104, EC Marie Curie, 1/11/2004 - 31/10/2008,

Structure of the network



9 main nodes

10 subcontractors

Slide
mid-term
meeting 1998,
Corfu

Contact person meetings were occasions to meet



pictures , sept 2003,
Kobenhavn: network
contact person meeting

Contact person meetings

Barcelona, 2004



Present continuations of network activities

- networks were the start of European string workshops;
2023: nr. 25: Oviedo – Gijon
(there was already a SCIENCE contact meeting in Torino in 1994)
- Schools
now yearly CERN winter school
- Common Postdoc applications.
First time 1998-99. Still continuing.
- Many scientific collaborations

1999 TMR School	Leuven 18-23 jan
2000 TMR School	Torino 2000 26 Jan-2 Feb
2001 RTN School	PARIS 2001 1-7 Feb
2002 RTN School	Utrecht 2002 17-22 Jan
2003 RTN School	Torino 2003 7-11 Jan
2004 RTN School	Barcelona 2004 12-16 Jan
2005 MRTN School	TRIESTE 2005 31 Jan-4 Feb
2006 MRTN School	CERN
... "CERN winter school"	

Dubna

we all hope that these friendly encounters with the Russians will come back

Herwig Schopper, CERN Director-General from 1981 to 1988.

“what I consider most important is to prepare for the situation after the war. Somehow and sometime there will be a solution to the Russian invasion. On that “day after”, it will be necessary to talk to each other again and build a new world out of the ruins. This was facilitated after World War II because, despite the Nazi reign of terror, some far-sighted scientists maintained human relations as well as scientific ones

A vision for the day after requires courage and more Science for Peace than ever before. ”



Different methods for supergravity

- Noether method (Stony Brook)
- Superspace
- Group manifold, R. D'Auria, P. Frè, T. Regge
(or rheonomic approach)
- superconformal tensor calculus

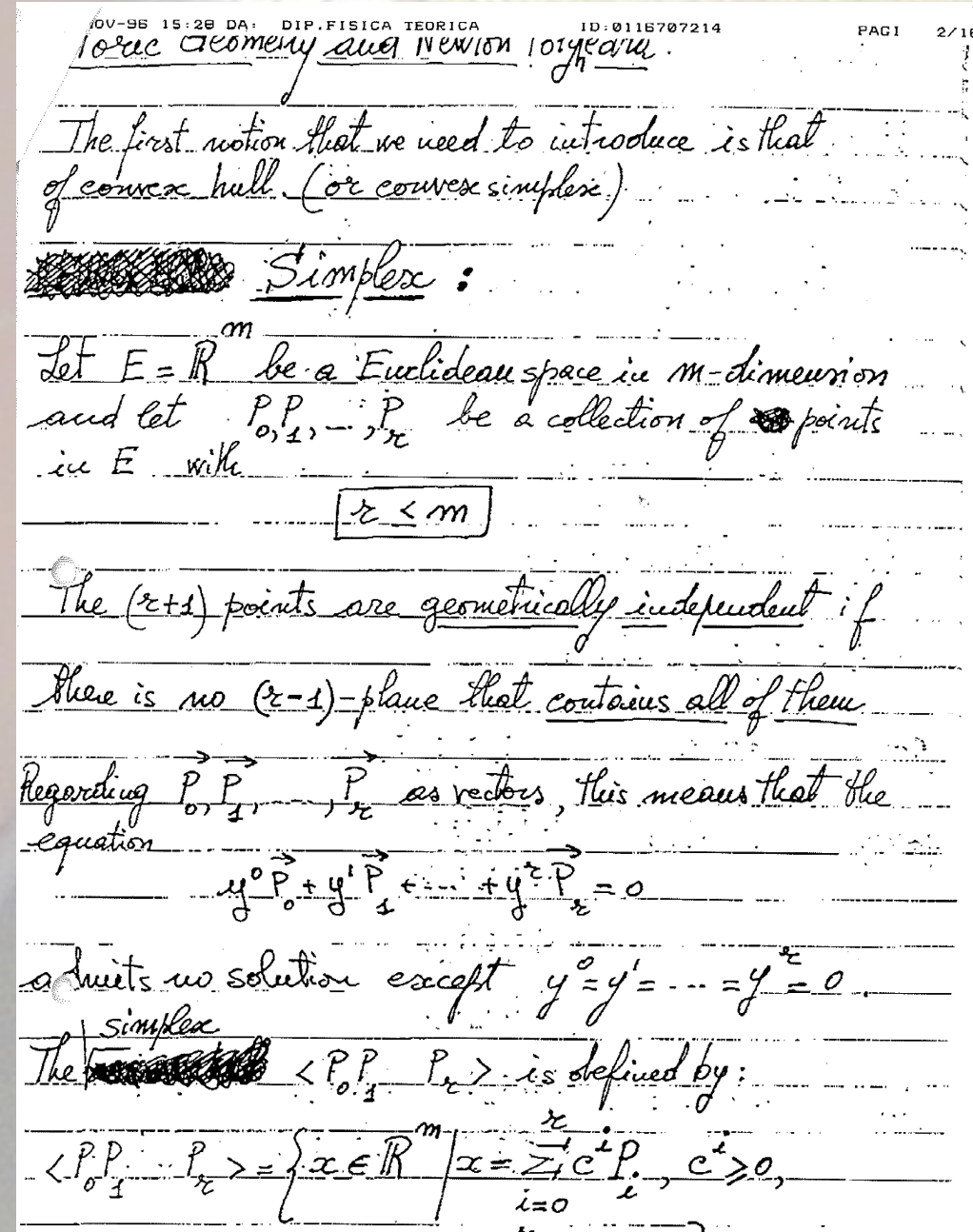
Structures are related and can easily be compared.
Collaborations were easy.

Collaborations

- Mostly in context of $N=2$ supergravity.
- Because there the geometry plays a big role.
- Pietro is expert in applications of geometry and Lie groups in physics

Papers with Pietro and me

- **1995:** M. Billò, R. D'Auria, S. Ferrara, **P. Frè**, P. Soriani, AVP,
R symmetry and the topological twist of N=2 effective supergravities of heterotic strings
- **1996:** M. Billò, A. Ceresole, R. D'Auria, S. Ferrara, **P. Frè**, AVP,
A Search for nonperturbative dualities of local N=2 Yang-Mills theories from Calabi-Yau threefolds
- **1998:** M. Billò, F. Denef, **P. Frè**, I. Pesando, W. Troost, AVP, D. Zanon
 - A detailed case study of the rigid limit in Special Kähler geometry using K3
 - Special geometry of Calabi-Yau compactifications near a rigid limit
 - The rigid limit in Special Kähler geometry; From K3-fibrations to Special Riemann surfaces: a detailed case study



Papers with Pietro and me

- **1999:** M. Billò, S. Cacciatori, F. Denef, **P. Frè**, AVP, D. Zanon
The 0-brane action in a general $D = 4$ supergravity background
- **2003:** **P. Frè**, M. Trigiante, AVP
 - Stable de Sitter vacua from $N=2$ supergravity
 - $N=2$ supergravity models with stable de Sitter vacua
- **2006:** **P. Frè**, F. Gargiulo, J. Rosseel, K. Rulik, M. Trigiante, AVP
Tits-Satake projections of homogeneous special geometries

N=2 supergravity-matter

- supergravity multiplet: spin $(2, \frac{3}{2}, \frac{3}{2}, 1)$
- n gauge multiplets : $n * (1, \frac{1}{2}, \frac{1}{2}, 0, 0)$
- m hypermultiplets $m * (\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0)$

Important features:

- R-symmetry: $SU(2) \times U(1)$
- $(n+1)$ vectors , which will imply duality transformations in $Sp(2(n+1))$
- **n complex** scalars, transforming under the $U(1)$: Kähler manifold, but due to the symplectic symmetry: extra structure: **Special Kähler manifold**
- **m quaternion** scalars: transforming under the $SU(2)$:
Quaternionic-Kähler manifold

Geometry became clear after several steps:

Potentials and symmetries of general gauged $N = 2$ supergravity-Yang-Mills models,

B. de Wit, AVP, April 1984

Vector multiplets coupled to $N = 2$ supergravity :

super-Higgs effect, flat potentials and geometric structure

E. Cremmer, C. Kounnas, AVP, J.P. Derendinger, S. Ferrara, B. de Wit, L. Girardello, Sept 1984

Lagrangians of $N = 2$ supergravity-matter systems,

B. de Wit, P. Lauwers, AVP, Dec. 1984

Special Kähler Geometry: An intrinsic formulation from $N=2$ Space-time Supersymmetry,

L.Castellani, R. D'Auria and S. Ferrara, Febr. 1990

Duality Transformations in Supersymmetric Yang-Mills Theories coupled to Supergravity,

A. Ceresole, R. D'Auria, S. Ferrara, AVP, 9502072

$N=2$ Supergravity and $N=2$ Super Yang-Mills Theory on General Scalar Manifolds:

Symplectic Covariance, Gaugings and the Momentum Map

L.Andrianopoli, M.Bertolini, A. Ceresole,

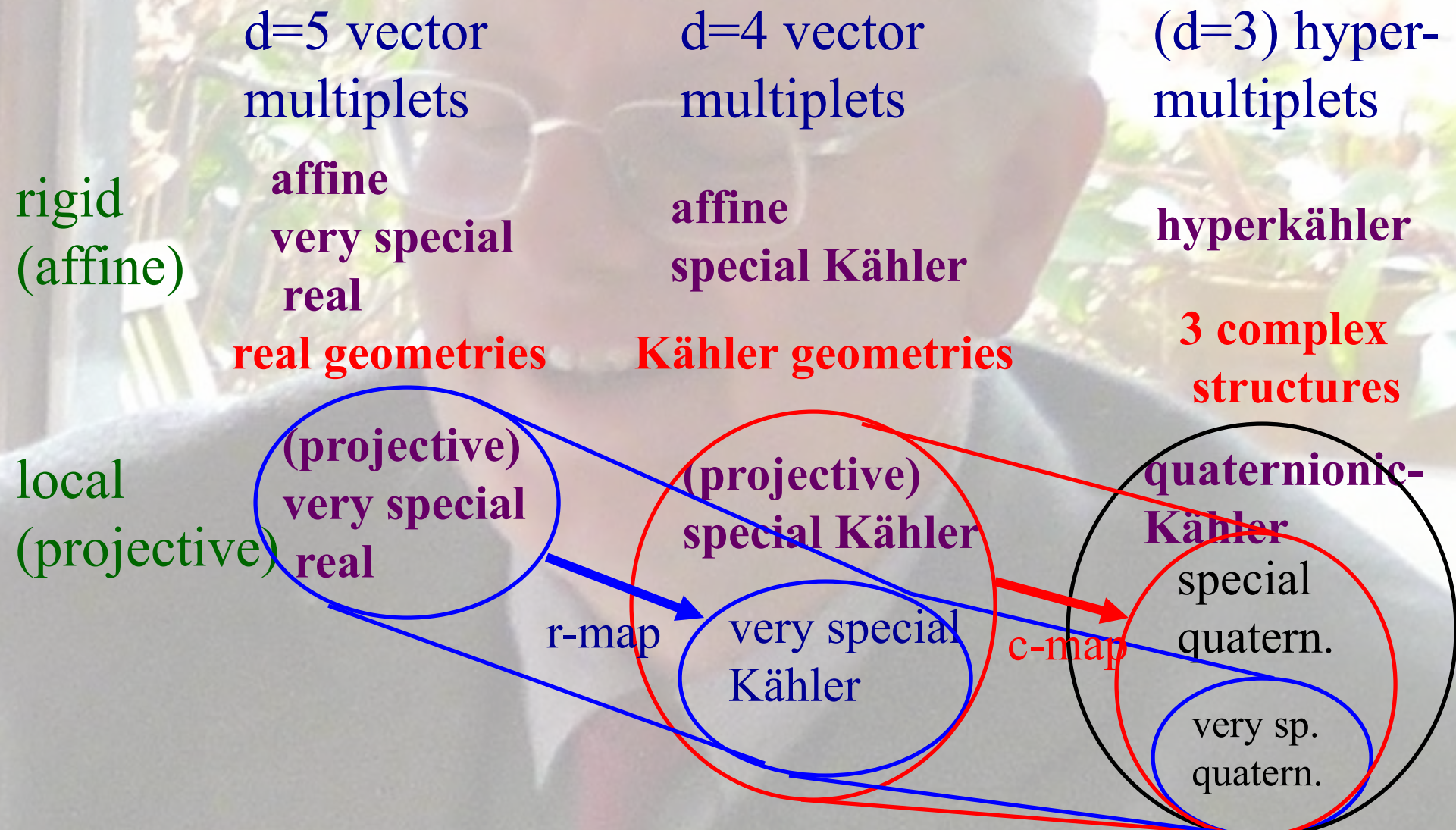
R. D'Auria, S.Ferrara, **P.Frè**, T. Magri, 9605032

In string theory from moduli of Calabi-Yau manifolds

- Special Kähler scalars are the complex structure moduli of Calabi-Yau manifolds
- complex 3-dim manifolds with (p, q) forms with Hodge diamond

$$\begin{array}{cccccc} & & & h^{00} = 1 & & \\ & & & 0 & & 0 \\ & & 0 & h^{11} = m & & 0 \\ h^{30} = 1 & & h^{21} = n & & h^{12} = n & h^{03} = 1 \\ & & 0 & h^{22} = m & & 0 \\ & & 0 & & & 0 \\ & & & h^{33} = 1 & & \end{array}$$

The family of special geometries from reductions of multiplets in $d=5,4,3$



Homogeneous, symmetric ?

- In general, these special manifolds are not homogeneous spaces.
- But there are homogeneous manifolds called $L(q, P)$ or $L(4n, P, \dot{P})$. These are in 1-to-1 correspondence with representations of P irreps of Euclidean real Clifford algebras in dimension $q+1$.

Special geometry, cubic polynomials and homogeneous quaternionic spaces,
B. de Wit, AVP, 9112027

□ For special values of q, P : symmetric

	real	Kähler	quaternionic
$L(-1, 0)$	$SO(1, 1)$	$\left[\frac{SU(1, 1)}{U(1)} \right]^2$	$\frac{SO(3, 4)}{(SU(2))^3}$
$L(0, P)$	$SO(1, 1) \otimes \frac{SO(P+1, 1)}{SO(P+1)}$	$\frac{SU(1, 1)}{U(1)} \otimes \frac{SO(P+2, 2)}{SO(P+2) \otimes SO(2)}$	$\frac{SO(P+4, 4)}{SO(P+4) \otimes SO(4)}$
$L(1, 1)$	$\frac{Sl(3, R)}{SO(3)}$	$\frac{Sp(6)}{U(3)}$	$\frac{F_4}{USp(6) \otimes SU(2)}$
$L(2, 1)$	$\frac{Sl(3, C)}{SU(3)}$	$\frac{SU(3, 3)}{SU(3) \otimes SU(3) \otimes U(1)}$	$\frac{E_6}{SU(6) \otimes SU(2)}$
$L(4, 1)$	$\frac{SU^*(6)}{Sp(3)}$	$\frac{SO^*(12)}{SU(6) \otimes U(1)}$	$\frac{E_7}{SO(12) \otimes SU(2)}$
$L(8, 1)$	$\frac{E_6}{F_4}$	$\frac{E_7}{E_6 \otimes U(1)}$	$\frac{E_8}{E_7 \otimes SU(2)}$

making contact with

Exceptional supergravity theories
and the magic square,
M. Günaydin, G. Sierra, P.
Townsend, 1983

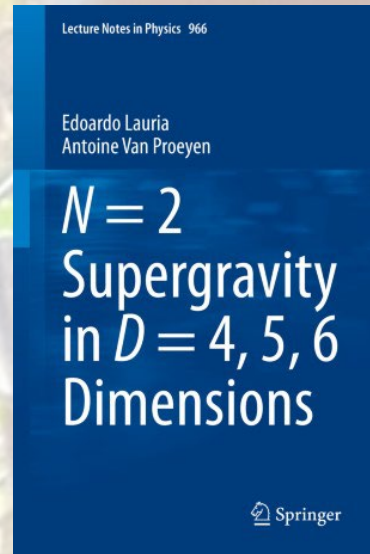
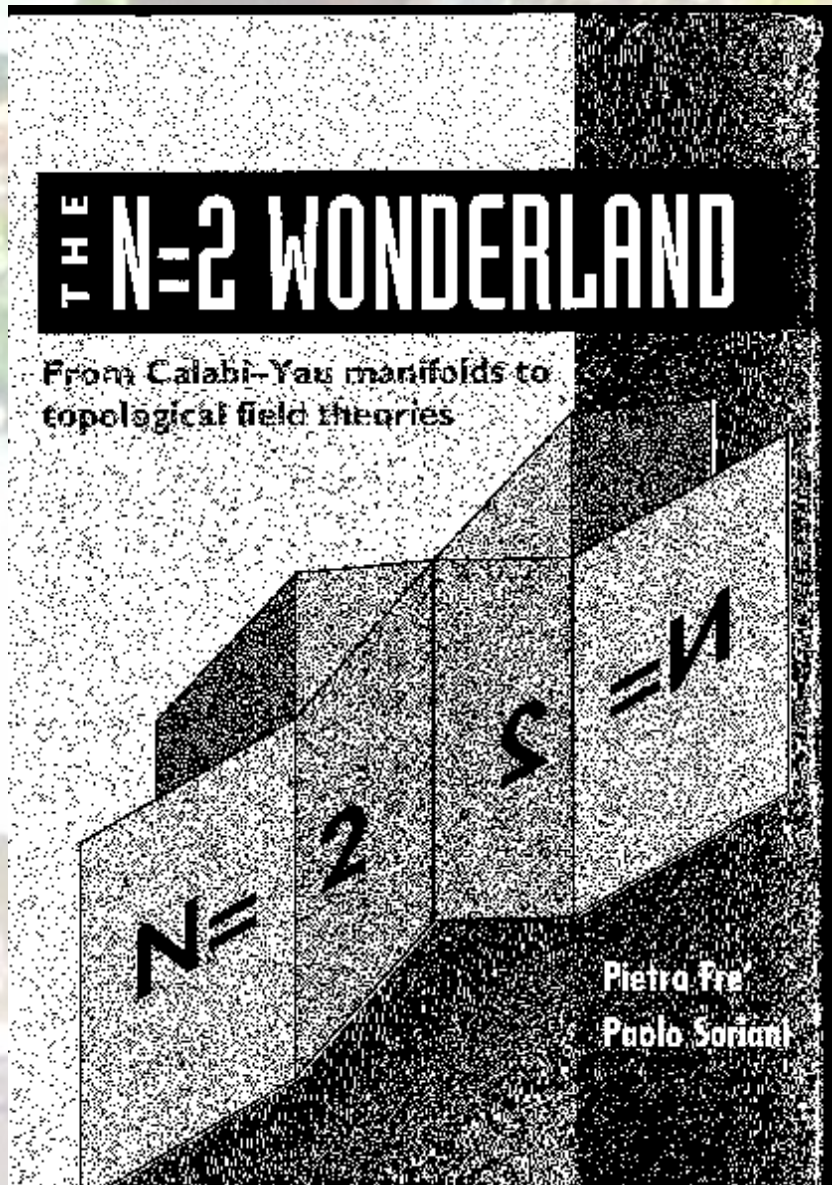
Further studied in our last paper with Pietro

Tits-Satake projections of homogeneous special geometries

P. Frè, F. Gargiulo, J. Rosseel, K. Rulik, M. Trigiante, AVP, 0606173

- Tits-Satake projection of simple algebras (defining the real forms).
Extended to all solvable Lie algebras occurring in the homogeneous special geometries.
These then act as isometries of these homogeneous spaces.
- makes use of Paint groups, first looked at in the context of *billiard cosmologies* (*Paint group = compact group, commuting with the generating solvable algebra*)
in string theory interpretation : permutation of coloured branes

Structure of $N=2$ contains much more



Springer book, or
arxiv 2004.11433

Recently

- new possibilities for supersymmetry breaking, using new types of Fayet-Iliopoulos terms.
- new conformal multiplets, allowing higher-derivative terms and off-shell formulations for supersymmetric localization.

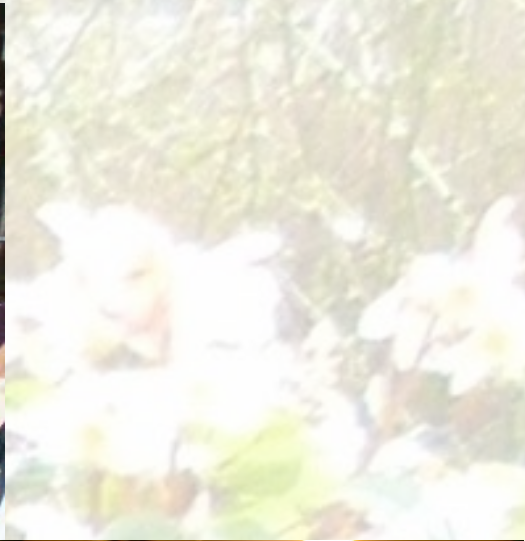
Many conferences

- 2005: The legacy of supergravity, in honour of Sergio Ferrara, Villa Mondragone



2016: GGI, workshop "Supergravity in Action"







A close-up photograph of an elderly man with white hair and glasses, smiling warmly. He is wearing a dark suit jacket, a white shirt, and a red tie. The background is slightly out of focus, showing a window with a view of greenery and a potted plant with white flowers.

And now ...

- participate to what you like, and refrain from other obligations.
- Pietro, enjoy the memories and the future (your $N=2$ life) !