A tale of homogeneous spaces

Alberto Zaffaroni

University of Milano-Bicocca

Some personal recollections

Gravity, Geometry and Symmetry: a celebration for Pietro Fre's 70's

When I was a student in SISSA

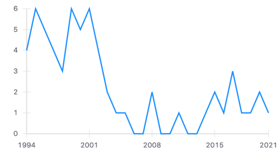
ALE manifolds and conformal field theories

Damiano Anselmi (SISSA, Trieste and INFN, Trieste), Marco Billo (SISSA, Trieste and INFN, Trieste), Pietro Fre (SISSA, Trieste and INFN, Trieste), Luciano Girardello (Milan U. and INFN, Milan), Alberto Zaffaroni (SISSA, Trieste and INFN, Trieste)

Apr 30, 1993

52 pages
Published in: *Int.J.Mod.Phys.A* 9 (1994) 3007-3058
e-Print: hep-th/9304135 [hep-th]
DOI: 10.1142/S0217751X94001199
Report number: IFUM-443-FT, SISSA-44-92-EPA
View in: AMS MathSciNet, ADS Abstract Service

Citations per year



Abstract: (arXiv)
We address the problem of constructing the family of (4,4) theories associated with the sigma-model on a parametrized family $\mathcal{M}_\zeta$ of Asymptotically Locally Euclidean (ALE) manifolds. We rely on the ADE classification of these manifolds and on their construction as HyperK\"ahler quotients, due to Kronheimer. So doing we are able to define the family of (4,4) theories corresponding to a $\mathcal{M}_\zeta$ family of ALE manifolds as the deformation of a solvable orbifold $\mathbf{C}^2/\Gamma$ conformal field-theory, Γ being a Kleinian group. We discuss the relation among the algebraic structure underlying the topological and metric properties of self-dual 4-manifolds and the algebraic properties of non-rational (4,4)-theories admitting an infinite spectrum of primary fields. In particular, we identify the Hirzebruch signature τ with the dimension of the local polynomial ring {\cal R}=\o {{\bf C}[x,y,z]}{\partial W} associated with the ADE singularity, with the number of non-trivial conjugacy classes in the corresponding Kleinian group and with the number of short representations of the (4,4)-theory minus four.

field theory: conformal | dimension: 2 | sigma model: nonlinear | field theory: orbifold | coset space | operator product expansion | supersymmetry | bosonization | group theory: representation

When I took my first examination for a "concorso" in Italy:

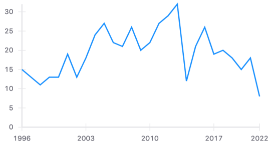
N=2 supergravity and N=2 superYang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map

L. Andrianopoli (Genoa U.), M. Bertolini (Turin Polytechnic), Anna Ceresole (Turin Polytechnic), R. D'Auria (Turin Polytechnic), S. Ferrara (CERN and UCLA) Show All(7) Pietro Fre'

May, 1996

81 pages
Published in: *J.Geom.Phys.* 23 (1997) 111-189
e-Print: hep-th/9605032 [hep-th]
DOI: 10.1016/S0393-0440(97)00002-8
Report number: POLFIS-TH-03-96, UCLA-96-TEP-9
View in: AMS MathSciNet, CERN Document Server, ADS Abstract Service

Citations per year



Abstract:
The general form of N=2 supergravity coupled to an arbitrary number of vector multiplets and hypermultiplets, with a generic gauging of the scalar manifold isometries is given. This extends the results already available in the literature in that we use a coordinate independent and manifestly symplectic covariant formalism which allows to cover theories difficult to formulate within superspace or tensor calculus approach. We provide the complete lagrangian and supersymmetry variations with all fermionic terms, and the form of the scalar potential for arbitrary quaternionic manifolds and special geometry, not necessarily in special coordinates. Lagrangians for rigid theories are also written in this general setting and the connection with local theories elucidated. The derivation of these results using geometrical techniques is briefly summarized.

Note: LaTeX, 80 pages, extended version of hep-th/9603004 Report-no: POLFIS-TH.03/96, UCLA/96/TEP/9

gauge field theory: Yang-Mills | supersymmetry: transformation | supergravity | differential geometry: symplectic | space-time: Kaehler | field theory: action | bibliography

After my post-doc years I met Pietro walking on a road at Vietri sul Mare

3-D superconformal theories from Sasakian seven manifolds: New nontrivial evidences for AdS(4) / CFT(3)

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre' (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Cesare Reina (SISSA, Trieste), Alessandro Tomasiello (SISSA, Trieste) Show All(7) Alberto Zaffaroni (CERN) Alessandro Zampa (SISSA)

Jul, 1999

62 pages
Published in: *Nucl.Phys.B* 577 (2000) 547-608
e-Print: hep-th/9907219 [hep-th]
DOI: 10.1016/S0550-3213(00)00098-5
Report number: CERN-TH-99-231
View in: AMS MathSciNet, ADS Abstract Service, CERN Document Server

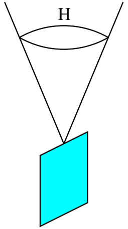
Citations per year

Abstract:
In this paper we discuss candidate superconformal N=2 gauge theories that realize the AdS/CFT correspondence with M--theory compactified on the homogeneous Sasakian 7-manifolds M^7 that were classified long ago. In particular we focus on the two cases M^7=Q^{1,1,1} and M^7=M^{1,1,1}, for the latter the Kaluza Klein spectrum being completely known. We show how the toric description of M^7 suggests the gauge group and the supersingleton fields. The conformal dimensions of the latter can be independently calculated by comparison with the mass of baryonic operators that correspond to 5-branes wrapped on supersymmetric 5-cycles and are charged with respect to the Betti multiplets. The entire Kaluza Klein spectrum of short multiplets agrees with these dimensions. Furthermore, the metric cone over the Sasakian manifold is a conifold algebraically embedded in some C^p. The ring of chiral primary fields is defined as the coordinate ring of C^p modded by the ideal generated by the embedding equations; this ideal has a nice characterization by means of representation theory. The entire Kaluza Klein spectrum is explained in terms of these vanishing relations. We give the superfield interpretation of all short multiplets and we point out the existence of many long multiplets with rational protected dimensions, whose presence and pattern seem to be universal in all compactifications.

11.30.Pb 04.65 Supergravity AdS/CFT M-theory gauge field theory field theory: conformal field theory: anti-de Sitter supersymmetry dimension: 3 Show all (17)

Klebanov-Witten paper

D3 branes probing a conical Calabi-Yau with base a Sasaki-Einstein H_5:



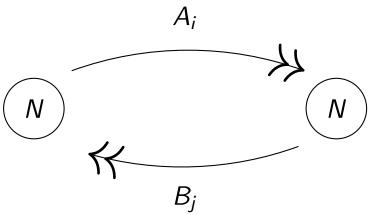
- Near horizon geometry $\mathrm{AdS}_5 \times H_5$
- Superconformal field theory on the world-volume

Symmetries come from bulk massless gauge fields:

- isometries of H_5 – mesonic symmetries
- reduction of $A_{(4)}$ on non-trivial three cycles in H_5 – baryonic symmetries – Betti multiplets in supergravity

Klebanov-Witten paper

Everyone favorite is the conifold $C(T^{1,1})$: $T^{1,1} = SU(2) \times SU(2)/U(1)$



Symmetries:

mesonic: $U(1)_R \times SU(2) \times SU(2)$

baryonic: $+1$ for A_i and -1 for B_i

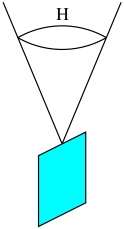
$$W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right)$$

Baryons = D3-branes wrapped on three cycles S [Gubser-Klebanov 98]

$$\det A\,, \det B \qquad \Longrightarrow \qquad \Delta = \frac{\pi \mathrm{Vol}(S)}{2\mathrm{Vol}(H_5)} = \frac{3N}{4}$$

M2 branes in M theory

M2 branes probing a conical Calabi-Yau with base a Sasaki-Einstein H_7:



- Near horizon geometry $\mathrm{AdS}_4 \times H_7$
- Superconformal field theory on the world-volume

So we decided to investigate the case of homogeneous Sasaki-Einstein H_7. A perfect combination of gravity, geometry and symmetry.

Homogeneous Sasaki-Einstein H_7:

$$Q^{1,1,1} = \frac{SU(2) \times SU(2) \times SU(2)}{U(1) \times U(1)}$$

$$M^{1,1,1} = \frac{SU(3) \times SU(2) \times U(1)}{SU(2) \times U(1) \times U(1)}$$

$$N^{0,1,0} = \frac{SU(3)}{U(1)}$$

$$V^{5,2} = \frac{SO(5)}{SO(3)}$$

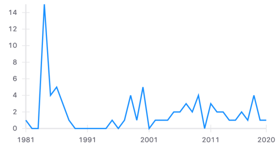
And of course Pietro wrote:

SU(3) x SU(2) x U(1) FROM D = 11 SUPERGRAVITY

L. Castellani (Turin U. and INFN, Turin), R. D'Auria (Turin U. and INFN, Turin), P. Fre (Turin U. and INFN, Turin)
1983

43 pages
Published in: *Nucl.Phys.B* 239 (1984) 610-652
Published: 1984
DOI: 10.1016/0550-3213(84)90265-7
Report number: IFTT 427
View in: AMS MathSciNet

Citations per year



Abstract: (Elsevier)
In this paper we show that D = 11 supergravity admits an infinite discrete class of solutions having the phenomenological group SU(3) ⊗ SU(2) ⊗ U(1) as a symmetry of the internal space M 7 . These solutions lead, in dimensional reduction, to SU(3) ⊗ SU(2) ⊗ U(1) gauge fields.

SUPERGRAVITY: ELEVEN-DIMENSIONAL | FIELD THEORY: compactification | SYMMETRY: SU(3) X SU(2) X U(1)

And started working on it again:

Osp(N|4) supermultiplets as conformal superfields on partial AdS(4) and the generic form of N=2, D = 3 gauge theories #4

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Piet Termonia (Turin U. and INFN, Turin) (May, 1999)

Published in: *Class.Quant.Grav.* 17 (2000) 55-92 • e-Print: hep-th/9905134 [hep-th]

pdf DOI cite 44 citations

M theory on AdS(4) x M111: The Complete Osp(2|4) x SU(3) x SU(2) spectrum from harmonic analysis** #5

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Piet Termonia (Turin U. and INFN, Turin) (Mar, 1999)

Published in: *Nucl.Phys.B* 560 (1999) 617-682 • e-Print: hep-th/9903036 [hep-th]

pdf DOI cite 57 citations

I was still in CERN, Pietro and Leonardo in Torino, Reina and Tomasiello in SISSA and we were meeting half-way in Milano, invading poor Luciano Girardello's office

3-D superconformal theories from Sasakian seven manifolds: New nontrivial evidences for AdS(4) / CFT(3)

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre' (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Cesare Reina (SISSA, Trieste), Alessandro Tomasiello (SISSA, Trieste) Show All(7) Alberto Zaffaroni (CERN) Alessandro Zampa (SISSA)

Jul, 1999

62 pages
Published in: *Nucl.Phys.B* 577 (2000) 547-608
e-Print: hep-th/9907219 [hep-th]
DOI: 10.1016/S0550-3213(00)00098-5
Report number: CERN-TH-99-231
View in: AMS MathSciNet, ADS Abstract Service, CERN Document Server

pdf cite claim 114 citations

Citations per year

Abstract:
In this paper we discuss candidate superconformal N=2 gauge theories that realize the AdS/CFT correspondence with M--theory compactified on the homogeneous Sasakian 7-manifolds M^7 that were classified long ago. In particular we focus on the two cases M^7=Q^{1,1,1} and M^7=M^{1,1,1}, for the latter the Kaluza Klein spectrum being completely known. We show how the toric description of M^7 suggests the gauge group and the supersingleton fields. The conformal dimensions of the latter can be independently calculated by comparison with the mass of baryonic operators that correspond to 5-branes wrapped on supersymmetric 5-cycles and are charged with respect to the Betti multiplets. The entire Kaluza Klein spectrum of short multiplets agrees with these dimensions. Furthermore, the metric cone over the Sasakian manifold is a conifold algebraically embedded in some C^p. The ring of chiral primary fields is defined as the coordinate ring of C^p modded by the ideal generated by the embedding equations; this ideal has a nice characterization by means of representation theory. The entire Kaluza Klein spectrum is explained in terms of these vanishing relations. We give the superfield interpretation of all short multiplets and we point out the existence of many long multiplets with rational protected dimensions, whose presence and pattern seem to be universal in all compactifications.

11.30.Pb | 04.65 | Supergravity | AdS/CFT | M-theory | gauge field theory | field theory: conformal | field theory: anti-de Sitter | supersymmetry | dimension: 3 | Show all (17)

We computed the dimension of baryons

Baryons are M5-branes wrapped over five-cycles and

$$b_5(Q^{1,1,1}) = 2\,, \qquad b_5(M^{1,1,1}) = 1$$

We identified supersymmetric cycles:

$Q^{1,1,1}$

$$\Delta = \frac{2\pi \mathrm{Vol}(S_5)}{2\mathrm{Vol}(Q^{1,1,1})} = \frac{N}{3}$$

$\Longrightarrow$ fields of dimension $1/3$

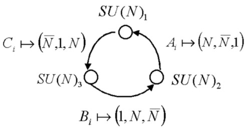
$M^{1,1,1}$

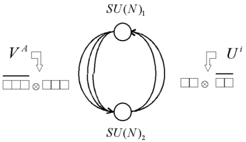
$$\Delta = \frac{2\pi \mathrm{Vol}(S_5)}{2\mathrm{Vol}(M^{1,1,1})} = \frac{N}{3}, \frac{4N}{9}$$

$\Longrightarrow$ fields of dimension $1/3, 4/9$

We were perhaps too naive

$Q^{1,1,1}$ $M^{1,1,1}$





- Nicely matching the KK spectrum: $\mathrm{Tr}(ABC)^k$, $\mathrm{Tr}(U^3V^2)^k$
- Nicely compatible with the dimension of baryonic operators
- Failing to reproduce the non-abelian moduli space $\mathrm{Sym}(H_7)^N$

"Since we do not have a superpotential here, we have to suppose that both the elimination of the unwanted colored massless states as well as the disappearing of the non-symmetric chiral operators emerges as a non-perturbative IR effect. "

The $\mathcal{N}_3 = 3 \to \mathcal{N}_3 = 4$ enhancement of Super Chern-Simons theories in $D = 3$, Calabi HyperKähler metrics and M2-branes on the $\mathcal{C}(\mathrm{N}^{0,1,0})$ conifold #1

P. Fré (Turin U. and INFN, Turin and Piemonte Orientale U., Alessandria), A. Giambrone (Turin U.), P.A. Grassi (Turin U. and INFN, Turin and Piemonte Orientale U., Alessandria), P. Vaško (Turin U. and INFN, Turin and Piemonte Orientale U., Alessandria) (Jun 27, 2019)

Published in: *J.Geom.Phys.* 160 (2021) 103962 • e-Print: 1906.11672 [hep-th]

pdf DOI cite 1 citation

Shadow multiplets and superHiggs mechanism #1

Davide Fabbri (Leuven U.), Pietro Fre (Turin U. and INFN, Turin) (Jan, 2001)

Published in: *Fortsch.Phys.* 49 (2001) 475-483 • Contribution to: Workshop on the Quantum Structure of Spacetime and the Geometric Nature of Fundamental Interactions (1st Workshop of RTN Network and 34th International Symposium Ahrenshoop on the Theory of Elementary Particle) • e-Print: hep-th/0101050 [hep-th]

pdf links DOI cite 2 citations

Rings of short N=3 superfields in three-dimensions and M theory on AdS(4) x N**(0,1,0) #2

M. Billo (Turin U. and INFN, Turin), D. Fabbri (Turin U. and INFN, Turin), P. Fre (Turin U. and INFN, Turin), P. Merlatti (Turin U. and INFN, Turin), A. Zaffaroni (INFN, Milan) (May, 2000)

Published in: *Class.Quant.Grav.* 18 (2001) 1269-1290 • e-Print: hep-th/0005219 [hep-th]

pdf DOI cite 48 citations

Shadow multiplets in AdS(4) / CFT(3) and the superHiggs mechanism: Hints of new shadow supergravities #3

M. Billo (Turin U. and INFN, Turin), D. Fabbri (Turin U. and INFN, Turin), P. Fre (Turin U. and INFN, Turin), P. Merlatti (Turin U. and INFN, Turin), A. Zaffaroni (INFN, Milan) (May, 2000)

Published in: *Nucl.Phys.B* 591 (2000) 139-194 • e-Print: hep-th/0005220 [hep-th]

pdf DOI cite 30 citations

Osp(N|4) supermultiplets as conformal superfields on partial AdS(4) and the generic form of N=2, D = 3 gauge theories #4

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Piet Termonia (Turin U. and INFN, Turin) (May, 1999)

Published in: *Class.Quant.Grav.* 17 (2000) 55-92 • e-Print: hep-th/9905134 [hep-th]

pdf DOI cite 44 citations

M theory on AdS(4) x M**111: The Complete Osp(2|4) x SU(3) x SU(2) spectrum from harmonic analysis #5

Davide Fabbri (Turin U. and INFN, Turin), Pietro Fre (Turin U. and INFN, Turin), Leonardo Gualtieri (Turin U. and INFN, Turin), Piet Termonia (Turin U. and INFN, Turin) (Mar, 1999)

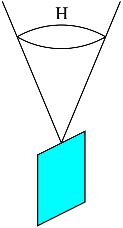
Published in: *Nucl.Phys.B* 560 (1999) 617-682 • e-Print: hep-th/9903036 [hep-th]

pdf DOI cite 57 citations

SINCE THEN

Superconformal field theories zoo

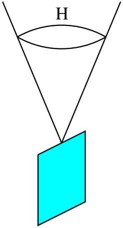
D3 branes probing a conical Calabi-Yau with base a Sasaki-Einstein H_5:



- Near horizon geometry $\mathrm{AdS}_5 \times H_5$
- Superconformal field theory on the world-volume
- Complete correspondence between CY and CFT in the toric case
  [Franco-Hanany-Kennaway-Vegh-Wecht; Feng-He-Kennaway-Vafa, 2005]

Superconformal field Theories Zoo in 3d

M2 branes probing a conical Calabi-Yau with base a Sasaki-Einstein H_7:

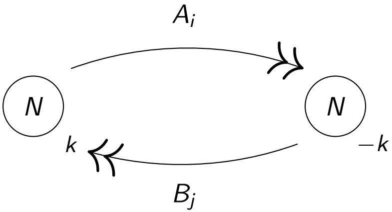


- Near horizon geometry $\mathrm{AdS}_4 \times H_7$
- Superconformal field theory on the world-volume
- Correspondence between CY and CFT still missing even in the toric case

Most examples are obtained by reducing dimensionally D3-branes and adding Chern-Simons couplings and/or flavoring with fundamentals.

Superconformal Field Theories Zoo in 3d

Everyone favorite is the ABJM theory: $C(H_7) = \mathbb{C}^4/\mathbb{Z}_k$



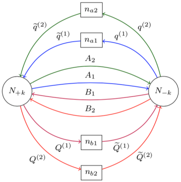
Symmetries:

mesonic: $U(1)_R \times SU(2) \times SU(2) \times U(1)$

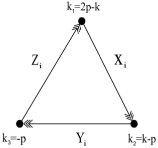
$$W = \text{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right)$$

Homogeneous Sasaki-Einstein

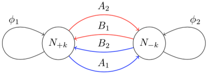
$Q^{1,1,1}$ [Benini,Closset,Cremonesi;Jafferis '10]



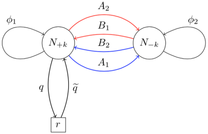
$M^{1,1,1}$ [Martelli,Sparks, 09]



$V^{5,2}$ [Martelli,Sparks, 09]



$N^{0,1,0}$ [Gaiotto,Jafferis 09]



All good?

Basic holographic dictionary

$$F_{S^3} = N^{3/2}\sqrt{\frac{2\pi^6}{27\mathrm{Vol_S}(H_7)}} \qquad \Delta_a = \frac{2\pi}{3}\frac{\mathrm{Vol_S}(S_a)}{\mathrm{Vol_S}(H_7)}$$

where F_{S^3} is the free energy on S^3.

- The quantum field theory large N limit F_{S^3} has been successfully compared with the gravitational result: $N^{3/2}$ big success!
- The QFT method miserably fails for "chiral" quivers like $M^{1,1,1}$

[Martelli-Sparks;Herzog-Jafferis-Klebanov-Pufu-Safdi;Amariti-Klare-Siani, ...]

With refinement

If R_0 is a $U(1)$ R-symmetry

$$R = R_0 + \sum_a \Delta_a J_a$$

also is, for all $U(1)$ global symmetries J_a.

- The exact R-charges Δ_a of a three-dimensional $\mathcal{N} = 2$ CFT can be obtained by extremizing the free energy on S^3, $F_{S^3}(\Delta_a)$, that depends on a trial R-charge [Jafferis '10]
- The large N limit $F_{S^3}(\Delta_a)$ has been computed only for a subclass of theories and curiously depends only on mesonic symmetries

[Martelli-Sparks;Herzog-Jafferis-Klebanov-Pufu-Safdi;Amariti-Klare-Siani, ...]

And we can put black holes in AdS4

Compactify the M2-brane theory on a Riemann surface $\Sigma_{\mathfrak{g}}$ by turning turn on magnetic fluxes for all symmetries:

$$\int_{\Sigma_{\mathfrak{g}}} F_a = \mathfrak{n}_a \qquad \sum_{a=1}^{d} \mathfrak{n}_a = 2 - 2\mathfrak{g}.$$

The dual is four-dimensional magnetically charged black holes in $\text{AdS}_4 \times H_7$ whose horizon is a fibration $\text{AdS}_2 \times \Sigma_{\mathfrak{g}} \times H_7$.

- Explicit solutions for $Q^{1,1,1}$,$M^{1,1,1}$ with baryonic charges [Halmagyi,Petrini,Zaffaroni 13]
- General construction for computing the entropy from toric data [Couzens,Gauntlett,Martelli,Sparks, 18]

$\mathcal{I}$-extremization

The entropy of such black holes can be found by extremizing

$$\boxed{\mathcal{I}(\Delta_I, \mathfrak{n}_I) = \log Z_{\Sigma_{\mathfrak{g}} \times S^1}(\Delta_a, \mathfrak{n}_a)}$$

where the topologically twisted index of the M2 theory [Benini-Hristov-AZ 15]

$$Z_{\Sigma_{\mathfrak{g}} \times S^1}(\Delta_I, \mathfrak{n}_I) = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{iJ_I \Delta_I} e^{-\beta H} = \mathrm{Tr}_{\mathcal{H}}(-1)^{R(\Delta_I)}$$

computes the equivariant Witten index of the IR quantum mechanics.

There is is a simple formula valid at large N [Hossein-AZ 16]

$$\boxed{\mathcal{I}(\Delta_I, \mathfrak{n}_I) = -\frac{1}{2} \sum_I \mathfrak{n}_I \frac{\partial F_{S^3}(\Delta_I)}{\partial \Delta_I}}$$

puzzling again, all baryonic symmetries are invisible at large N.

Holographic test

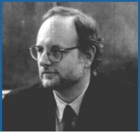
Partial answer. Large N computations exists only for non-chiral theories and are blind to baryonic symmetries but when they exist they match the BH entropy:

$$S(\mathfrak{n}_a) \equiv -\frac{1}{2}\sum_{a=1}^{d} \mathfrak{n}_a \frac{\partial F_{S^3}(\Delta_a)}{\partial \Delta_a}\Big|_{\text{extremized on } \Delta_a}$$

- full equivalence for all theories without baryonic symmetries: S^7, $V^{5,2}$
- partial equivalence (turning on only mesonic $\mathfrak{n}_a$) for vector-like theories with baryonic symmetries: $Q^{1,1,1}$, $N^{0,1,0}$
- no check available for chiral theories: $M^{1,1,1}$

[Hosseini, A.Z; Gauntlett, Martelli, Sparks; Kim, Kim 19]

Conclusions



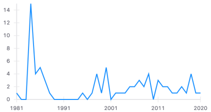
Gravity, Geometry and Symmetry: a celebration for Pietro Fre's 70's

SU(3) x SU(2) x U(1) FROM D = 11 SUPERGRAVITY

L. Castellani (Turin U. and INFN, Turin), R. D'Auria (Turin U. and INFN, Turin), P. Fre (Turin U. and INFN, Turin)
1983

43 pages
Published in: *Nucl.Phys.B* 239 (1984) 610-652
Published: 1984
DOI: 10.1016/0550-3213(84)90265-7
Report number: IFTT 427
View in: AMS MathSciNet

Citations per year



Abstract: (Elsevier)
In this paper we show that D = 11 supergravity admits an infinite discrete class of solutions having the phenomenological group SU(3) ⊗ SU(2) ⊗ U(1) as a symmetry of the internal space M 7 . These solutions lead, in dimensional reduction, to SU(3) ⊗ SU(2) ⊗ U(1) gauge fields.

SUPERGRAVITY: ELEVEN-DIMENSIONAL | FIELD THEORY: compactification | SYMMETRY: SU(3) X SU(2) X U(1)