String-Inspired Dynamics with Sharp Transitions

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Mostly Based on:

- P. Fré, AS and A. S. Sorin, ``Integrable Scalar Cosmologies I. Foundations and links with String Theory,'' Nucl. Phys. B877 (2013) 1028 [arXiv:1307.1910 [hep-th]].
- P. Pelliconi and AS, `Integrable Models and Supersymmetry Breaking,'' Nucl. Phys. B965 (2021) 115363 [arXiv:2102.06184 [hep-th]].

(Two recent reviews with J. Mourad : 2107.04064, 1711.11494)

GRAVITY, GEOMETRY AND SYMMETRY
A CELEBRATION FOR PIETRO FRE' 70'S
OCTOBER 7, 2022 (ONLINE)

My Long-Lasting Link with Pietro

- A long-lasting friendship that goes well beyond our profession

- We met at Caltech, when I was a student and Pietro looked (more or less) as above …
- I have felt consistently close to Pietro, over the years
- Our interests overlapped, finally, in 2012-13, and we had a fruitful collaboration

Vacuum Energy in Field Theory

- BOSE (FERMI) OSCILLATOR:

$$H = \frac{p^2}{2m} + \frac{1}{2}\, m\, \omega^2\, q^2 \longrightarrow \mathcal{E}_0 = (-)\frac{\hbar\,\omega}{2}$$

- QUANTUM FIELD THEORY:

$$\frac{\mathcal{E}_0}{V} = \sum_i (-1)^{F_i} \int \frac{d^d\mathbf{k}}{(2\pi)^d}\, \frac{\hbar\, c}{2}\, \sqrt{\mathbf{k}^2 + m_i^2}$$

- THE COSMOLOGICAL CONSTANT ISSUE:

$$\frac{\mathcal{E}_0}{V} \sim \frac{\mathcal{E}_{Pl}}{V_{Pl}} \sum_i (-1)^{F_i} + \ldots$$

(Zeldovich, 1968)

- (Exact, GLOBAL) SUPERSYMMETRY removes problem

(Zumino, 1975)

The (SUSY) 10D-11D Zoo

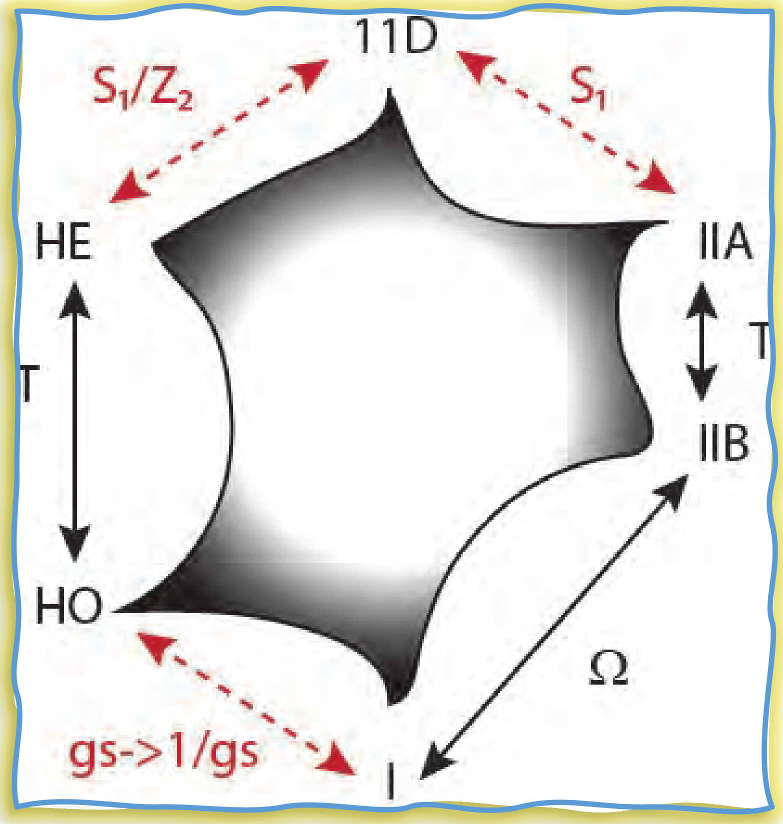
- Highest point of (SUSY) String Theory

BUT:

- Exhibits dramatically our limitarions
- perturbative → Solid arrows
- [10&11D supergravity → Dashed arrows]

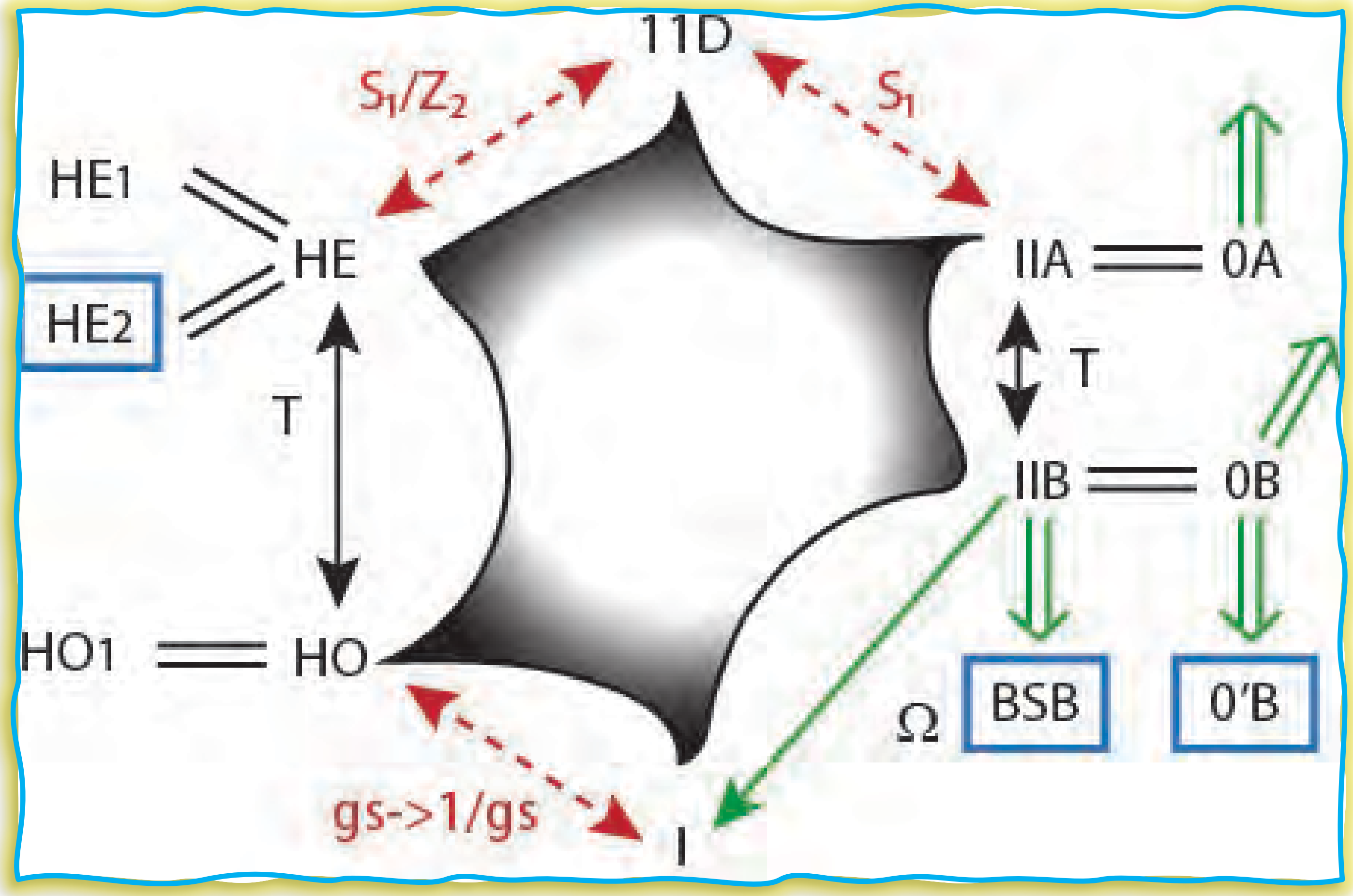
(Witten, 1995)

- SUSY: stabilizes these 10D Minkowski vacua



BROKEN SUSY ?

The 10D-11D Zoo



- 3 D=10 **non-SUSY non-tachyonic** strings
- SO(16)xSO(16) *(Dixon, Harvey, 1987) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)*
- 0'B U(32) *(AS, 1995)*
- [BSB Usp(32)] *(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)*

- **String consistency rules OK**
- **BUT: vacuum modified** (Tadpole potential)

- **QUESTIONS:**
  - Compactifications? Stability?
  - NON-PERTURBATIVE LINKS?

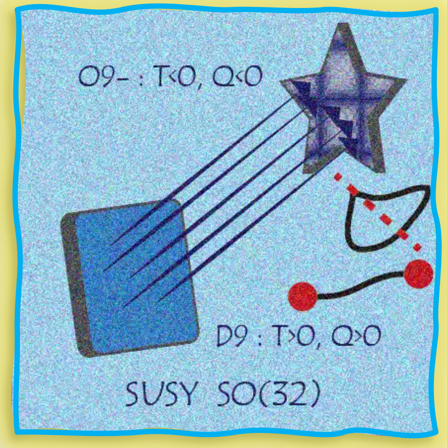
$$S = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

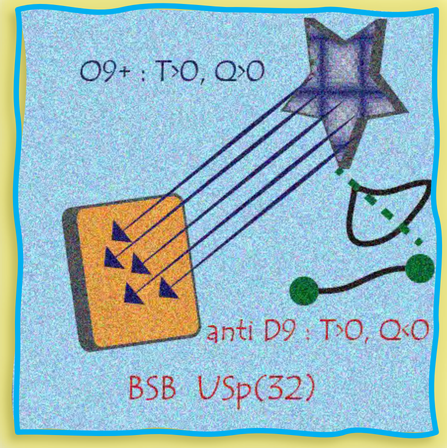
Brane SUSY Breaking

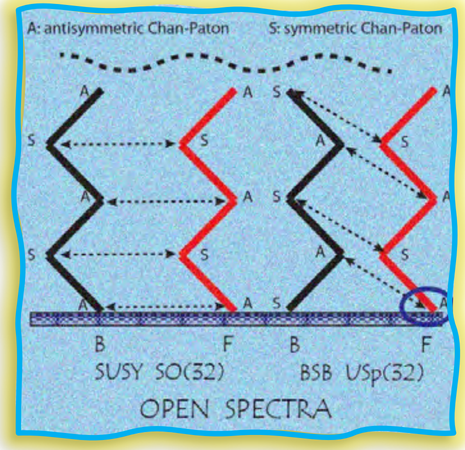
(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- Non-linear SUSY: ∃ goldstino!
- NO TACHYONS

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)







$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\, \mathcal{H}_3^2 - \frac{1}{4}\, e^{-\phi}\, \mathrm{tr}\, \mathcal{F}^2 - T\, e^{-\phi} + \ldots \right\}$$

VACUUM ENERGY → POTENTIAL

NOTE:
- Expansion in powers of $\alpha' R$
- Expansion in powers of $g_s = e^{\phi}$

Cosmological Potentials

What potentials lead to slow-roll, and where ?

- **If V does not vanish :** a convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\,dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\,d\mathbf{x}\cdot d\mathbf{x}$$
$$V e^{2\mathcal{B}} = V_0$$
$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$
$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

$$m\ddot{x} + b\dot{x} = f$$

- Driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ **Quadratic potential?** Far away from origin *(Linde, 1983)*

❖ **Exponential potential?** YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{3\gamma\phi/2}$: Climbing & Descending Scalars

- $\gamma < 1$? Both signs of speed

(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

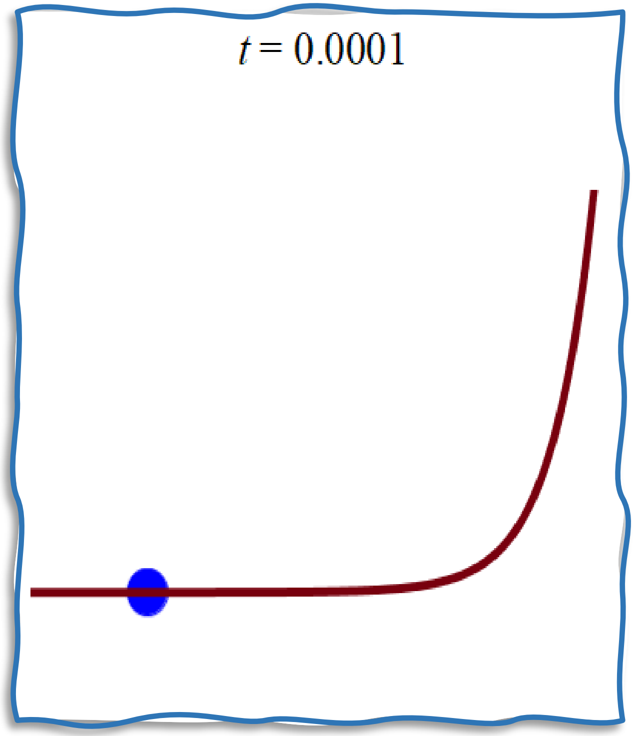
a. "Climbing" solution (φ climbs, then descends):

b. "Descending" solution (φ only descends):

Limiting τ- speed (LM attractor):

$$v_{lim} = - \frac{\gamma}{\sqrt{1-\gamma^2}}$$

(Lucchin and Matarrese, 1985)



CLIMBING: INEVITABLE in ALL asymptotically exponential potentials with $\gamma \geq 1$!

- U(32) and Usp(32) 10D ORIENTIFOLDS : $\gamma = 1$
- SO(16)xSO(16) HETEROTIC : $\gamma = 3/2$

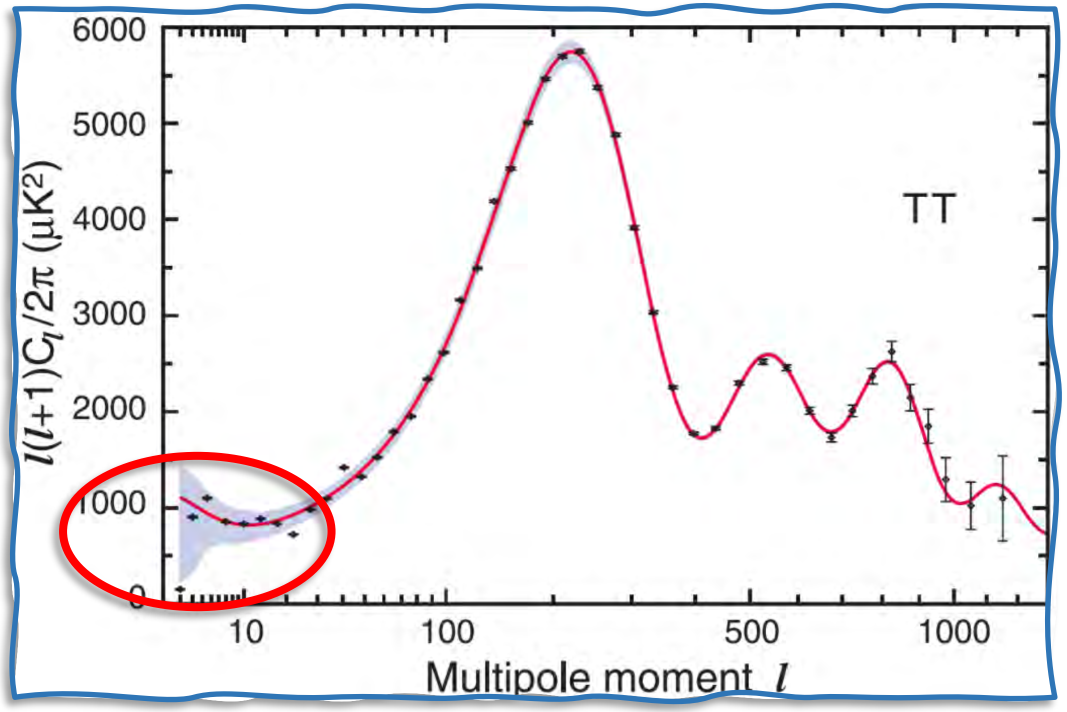
$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \frac{V_\varphi}{2\,V}\left(1 + \dot{\varphi}^2\right) = 0$$

$$\dot{\varphi} \sim \frac{C}{t} \quad \rightarrow \quad |C| = \frac{1}{1 + \gamma\,\mathrm{sign}(C)}$$

CLIMBING & DESCENT: mechanism to start inflation ?

Pre-Inflationary Relics in the CMB?

(Gruppuso, Kitazawa, Mandolesi, Natoli, AS, 2015)



- **CLIMBING: Extend ΛCDM** to allow for low-ℓ suppression:

$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\rightarrow\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$

- **A new scale Δ.** Preferred value? **Depends on Galactic masking.**

$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\text{Mpc}^{-1}$$

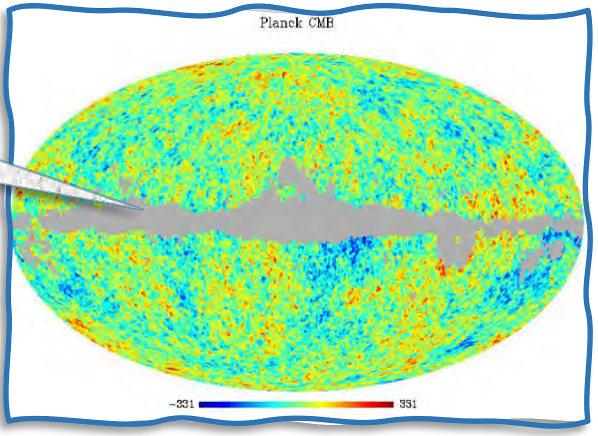
3 σ (30−degree extended mask)

$$\Delta^{Infl} \;\sim\; 10^{12} - 10^{14}\text{GeV} \;\;\text{for}\;\; \text{N} \;\sim\; 60-65$$

+ : $A_\ell \;\sim\; \ell(\ell+1)\int \frac{dk}{k}\; P_R(k)\; j_\ell(k\Delta\eta)^2 \;\sim\; P_R\left(k = \frac{\ell}{\Delta\eta}\right)$

− : Cosmic Variance

Masked region



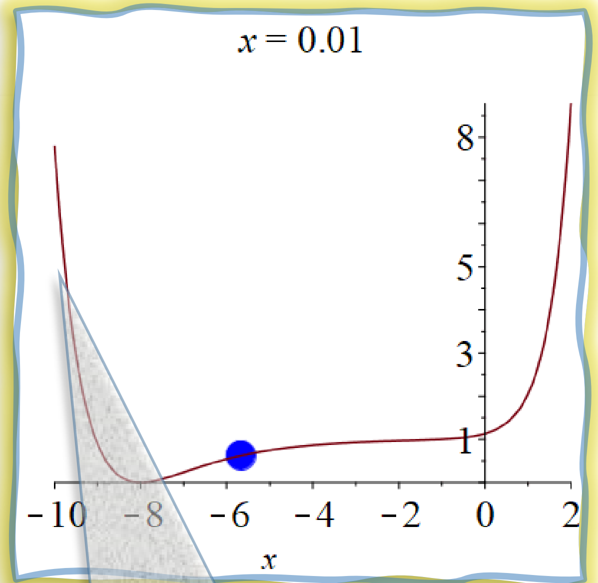
How to Sustain Inflation

INFLATION: $V = e^{\frac{3}{2}\gamma\phi} + V_{\rm flat}(\phi) \qquad \gamma \geq 1$

- HARD exponential potential + usual FLAT potentials
  - HARD: forces CLIMBING
  - FLAT: drives inflation during subsequent DESCENT

FAST-ROLL → SLOW-ROLL :

  - DEPRESSION of first multipoles
  - PREDICTION: tensor-to-scalar ratio enhanced at transition



E.g.: Starobinsky potential with «climbing» Exponential

- WITH PIETRO (and SASHA SORIN):

INTEGRABLE MODELS WITH NEAT ANALYTIC SOLUTIONS FOR BOTH PHASES

Integrable Exp-Like Potentials

(Fré, AS, Sorin, 2013)

$$V \;=\; e^{\frac{3}{2}\gamma\phi} \;+\; V_{\rm flat}(\phi) \qquad \gamma \;\geq\; 1$$

Potential function $\mathcal{V}$	$\mathcal{A}, \varphi, \mathcal{B}$	$\mathcal{L}$, Hamilt. Constr., dt_c
(1) $C_{11}e^{\varphi} + 2C_{12} + C_{22}e^{-\varphi}$	$\mathcal{A} = \log(xy)$ $\varphi = \log(\frac{x}{y})$ $\mathcal{B} = 0$	$\mathcal{L} = -2\dot{x}\dot{y} - C_{11}x^2 - 2C_{12}xy - C_{22}y^2$ $2\dot{x}\dot{y} = C_{11}x^2 + 2C_{12}xy + C_{22}y^2$ $dt_c = dt$
(2) $C_1 e^{2\gamma\varphi} + C_2 e^{(\gamma+1)\varphi}$ $(\gamma^2 \neq 1)$	$\mathcal{A} = \log(x^{\frac{1}{1+\gamma}}\, y^{\frac{1}{1-\gamma}})$ $\varphi = \log(x^{\frac{1}{1+\gamma}}\, y^{\frac{-1}{1-\gamma}})$ $\mathcal{B} = \log(x^{\frac{-\gamma}{1+\gamma}}\, y^{\frac{\gamma}{1-\gamma}})$	$\mathcal{L} = -4\dot{x}\dot{y} - 2(1-\gamma^2)[C_1 xy + C_2 x^{\frac{2}{1+\gamma}}]$ $2\dot{x}\dot{y} = (1-\gamma^2)[C_1 xy + C_2 x^{\frac{2}{1+\gamma}}]$ $dt_c = x^{-\frac{\gamma}{1+\gamma}}\, y^{\frac{\gamma}{1-\gamma}}\, dt$
(3) $C_1 e^{2\varphi} + C_2$	$\mathcal{A} = \frac{1}{2}\log x + v$ $\varphi = \frac{1}{2}\log x - v$ $\mathcal{B} = -\frac{1}{2}\log x + v$	$\mathcal{L} = -2\dot{x}\dot{v} - 2C_1 x - 2C_2 e^{2v}$ $\dot{x}\dot{v} = C_1 e^{2v} + C_2 x$ $dt_c = e^{v} x^{-\frac{1}{2}}\, dt$
(4) $C\varphi e^{2\varphi}$	$\mathcal{A} = \frac{1}{4}\log x + v$ $\varphi = \frac{1}{4}\log x - v$ $\mathcal{B} = -\frac{3}{4}\log x + v$	$\mathcal{L} = -\frac{1}{2}\dot{x}\dot{v} - C(\frac{1}{4}\log x - v)$ $\dot{x}\dot{v} = C(\frac{1}{2}\log x - 2v)$ $dt_c = x^{-\frac{3}{4}} e^{v}\, dt$
(5) $C\log(\coth\varphi) + D$	$\mathcal{A} = \frac{1}{2}\log(\frac{\xi^2-\eta^2}{2})$ $\varphi = \frac{1}{2}\log(\frac{\xi+\eta}{\xi-\eta})$ $\mathcal{B} = -\frac{1}{2}\log(\frac{\xi^2-\eta^2}{2})$	$\mathcal{L} = -\dot{\xi}^2 + \dot{\eta}^2 - 8C\log(\frac{\xi}{\eta}) - 8D$ $\dot{\xi}^2 - \dot{\eta}^2 = 8C\log(\frac{\xi}{\eta}) + 8D$ $dt_c = \frac{2\,dt}{\sqrt{\xi^2-\eta^2}}$
(6) $C\,\mathrm{Im}[\log(\frac{e^{2\varphi}+i}{e^{2\varphi}-i}) + D]$	$\mathcal{A} = \frac{1}{2}\log(\frac{\xi^2-\eta^2}{2})$ $\varphi = \frac{1}{2}\log(\frac{\xi+\eta}{\xi-\eta})$ $\mathcal{B} = -\frac{1}{2}\log(\frac{\xi^2-\eta^2}{2})$ $z = \frac{1}{\sqrt{2}}(\xi e^{\frac{i\pi}{4}} + \eta e^{-\frac{i\pi}{4}})$	$\mathcal{L} = 2\,\mathrm{Im}[-\dot{z}^2 - 8C\log z - 8D]$ $\mathrm{Im}[\dot{z}^2 - 8C\log z - 8D] = 0$ $dt_c = \frac{2\,dt}{\sqrt{\xi^2-\eta^2}}$
(7) $C_1(\cosh\gamma\varphi)^{\frac{2}{\gamma}-2} + C_2(\sinh\gamma\varphi)^{\frac{2}{\gamma}-2}$	$\mathcal{A} = \frac{1}{2\gamma}\log(\xi^2-\eta^2)$ $\varphi = \frac{1}{2\gamma}\log(\frac{\xi+\eta}{\xi-\eta})$ $\mathcal{B} = (\frac{1}{2\gamma}-1)\log(\xi^2-\eta^2)$	$\mathcal{L} = -\frac{\dot{\xi}^2-\dot{\eta}^2}{2\gamma^2} - C_1\xi^{\frac{2}{\gamma}-2} - C_2\eta^{\frac{2}{\gamma}-2}$ $\dot{\xi}^2 - \dot{\eta}^2 = 2\gamma^2[C_1\xi^{\frac{2}{\gamma}-2} + C_2\eta^{\frac{2}{\gamma}-2}]$ $dt_c = (\xi^2-\eta^2)^{\frac{1}{2\gamma}-1}\, dt$
(8) $\mathrm{Im}[C(i+\sinh 2\gamma\varphi)^{\frac{1}{\gamma}-1}]$	$\mathcal{A} = \frac{1}{2\gamma}\log(\xi^2-\eta^2)$ $\varphi = \frac{1}{2\gamma}\log(\frac{\xi+\eta}{\xi-\eta})$ $\mathcal{B} = (\frac{1}{2\gamma}-1)\log(\xi^2-\eta^2)$ $z = \frac{1}{\sqrt{2}}(\xi e^{\frac{i\pi}{4}} + \eta e^{-\frac{i\pi}{4}})$	$\mathcal{L} = \mathrm{Im}[-\frac{1}{2\gamma^2}\dot{z}^2 - \frac{C}{2}z^{\frac{2}{\gamma}-2}]$ $\mathrm{Im}[\frac{1}{2\gamma^2}\dot{z}^2 - \frac{C}{2}z^{\frac{2}{\gamma}-2}] = 0$ $dt_c = (\xi^2-\eta^2)^{\frac{1}{2\gamma}-1}\, dt$
(9) $C_1 e^{2\gamma\varphi} + C_2 e^{\frac{2}{\gamma}\varphi}$ $(\gamma^2 \neq 1)$	$\mathcal{A} = \frac{1}{\sqrt{1-\gamma^2}}(\hat{\mathcal{A}} - \gamma\hat{\varphi})$ $\varphi = \frac{1}{\sqrt{1-\gamma^2}}(\hat{\varphi} - \gamma\hat{\mathcal{A}})$ $\mathcal{B} = \mathcal{A}$	$\mathcal{L} = \frac{\dot{\hat{\varphi}}^2 - \dot{\hat{\mathcal{A}}}^2}{2} - C_1 e^{2\hat{\mathcal{A}}\sqrt{1-\gamma^2}} - C_2 e^{\frac{2\hat{\varphi}\sqrt{1-\gamma^2}}{\gamma}}$ $\frac{\dot{\hat{\varphi}}^2 - \dot{\hat{\mathcal{A}}}^2}{2} = -C_1 e^{2\hat{\mathcal{A}}\sqrt{1-\gamma^2}} - C_2 e^{\frac{2\hat{\varphi}\sqrt{1-\gamma^2}}{\gamma}}$ $dt_c = \exp[\frac{\hat{\mathcal{A}} - \gamma\hat{\varphi}}{\sqrt{1-\gamma^2}}]\, dt$

An Integrable Model for Climbing & Inflation

(Fré, AS, Sorin, 2013)

MANY exactly solvable (exp-like) potentials can be identified with techniques drawn from the theory of integrable systems. SIMPLEST INSTANCE :

$$ds^2 = - e^{2\mathcal{A}} d\tau^2 + e^{\frac{2\mathcal{A}}{d-1}} d\mathbf{x} \cdot d\mathbf{x}$$

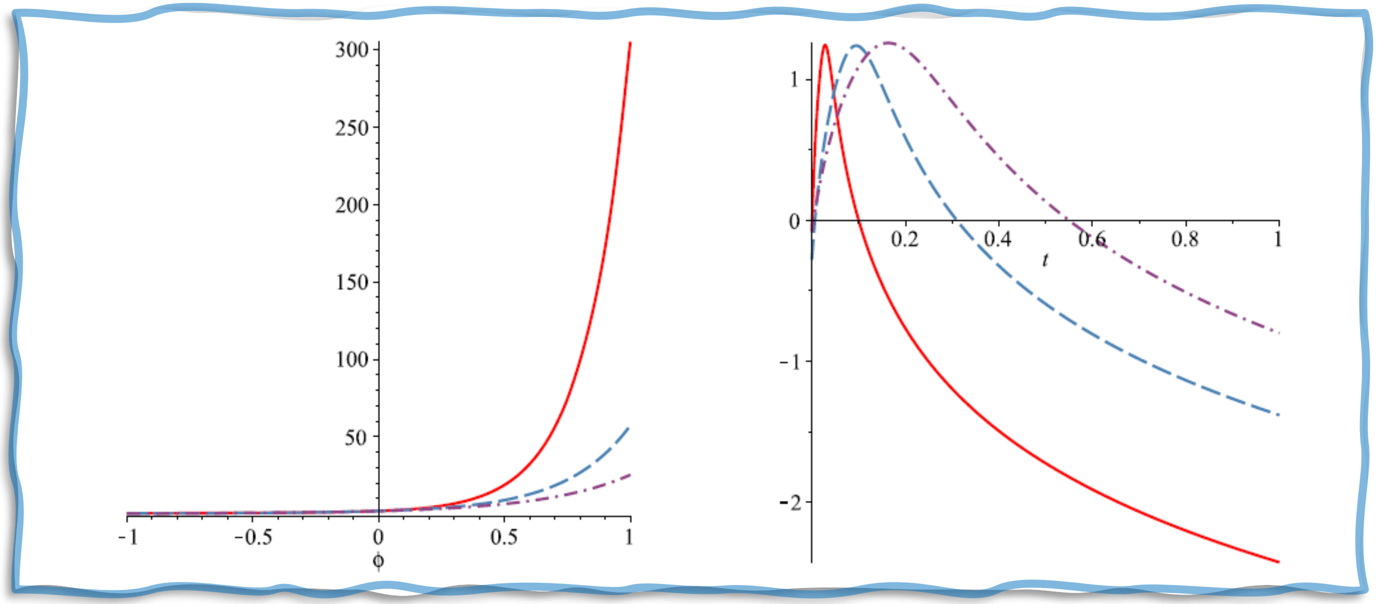
$$V(\phi) = \lambda \left(e^{\frac{2}{\gamma}\varphi} + e^{2\gamma\varphi} \right) \qquad (\gamma < 1)$$

$$\mathcal{L} = \frac{1}{2}\left(\dot{\varphi}^2 - \dot{\mathcal{A}}^2\right) - e^{2\mathcal{A}}\left(C_1 e^{2\gamma\varphi} + C_2 e^{\frac{2}{\gamma}\varphi}\right)$$

$$\hat{\mathcal{A}} = \frac{1}{\sqrt{1-\gamma^2}} (\mathcal{A} + \gamma\varphi)$$

$$\hat{\varphi} = \frac{1}{\sqrt{1-\gamma^2}} (\varphi + \gamma\mathcal{A})$$

$$\omega^2 = \frac{\lambda}{\gamma} \sqrt{1-\gamma^2}\, e^{2\mathcal{A}_0\sqrt{1-\gamma^2}}$$



$$e^{\varphi} = e^{\varphi_0}\left[\frac{\sinh(\omega\gamma\tau)}{\cosh\omega(\tau-\tau_0)}\right]^{\frac{\gamma}{1-\gamma^2}} \qquad e^{\mathcal{A}} = e^{\mathcal{A}_0}\left[\frac{\cosh^{\gamma^2}\omega(\tau-\tau_0)}{\sinh(\omega\gamma\tau)}\right]^{\frac{1}{1-\gamma^2}} \qquad dt_c = - e^{\mathcal{A}} d\tau$$

Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

STRING EXPONENTIAL POTENTIALS → CAN DRIVE spontaneous compactification

SIMPLEST CASE (10→9) :

$$e^{\phi} = e^{u+\phi_0}\, u^{\frac{1}{3}}$$
$$ds^2 = e^{-\frac{u}{6}}\, u^{\frac{1}{18}}\, dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\, e^{-\frac{3}{2}(u+\phi_0)}\, du^2$$

- ❖ SPONTANEOUS COMPACTIFICATIONS: interval of FINITE length $\sim \frac{1}{\sqrt{T}}$
- ❖ FINITE 9D Planck mass and gauge coupling
- ❖ BUT : g_s diverges at one end & curvature at the other

BOUNDS on g_s & CURVATURE ?

- NO : for the curvature (with 2-derivative actions)
- YES : for g_s

(In)Stability

(Basile, Mourad, AS, 2018)

1. Dudas-Mourad vacua: STRONG COUPLING but STABLE !

- E.g.: Scalar perturbations:

$$ds^2 = e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu + (1-7A)\,dz^2\right] ,$$

$$A'' + A'\left(24\,\Omega' - \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) + A\left(m^2 - \frac{7}{4}\,e^{2\Omega}\,V - 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) = 0$$

❖ Schrödinger-like form:

$$m^2\,\Psi = \left(b + \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} = \frac{d}{dr} - \alpha(r)\,, \qquad \mathcal{A}^\dagger = -\frac{d}{dr} - \alpha(r)\,, \qquad b = \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1 + \frac{9}{4}\,\alpha_O\,y^2} > 0$$

NO tachyons : PERTUBATIVE STABILITY

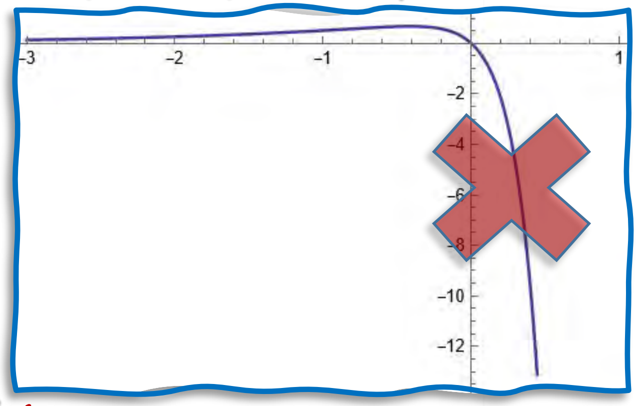
2. CLIMBING SCALAR : Instability of isotropy! ORIGIN of COMPACTIFICATION ?

INTEGRABLE Dudas-Mourad (-like) Vacua

(Pelliconi and AS, 2021)

COSMOLOGY → COMPACTIFICATIONS : the POTENTIAL is INVERTED

- How to attain a BOUNDED string coupling g_s ?
  1. potentials that are defined only for g_s SMALL ENOUGH
  2. (negative) overcritical exponential potentials
- BOTH SITUATIONS can be explored adapting the potentials examined with Pietro and Sacha
- E.g.: $$V(\phi) = \lambda \left(- e^{\frac{2}{\gamma}\,\varphi} + e^{2\gamma\varphi} \right) \qquad (\gamma < 1)$$



- SURPRISINGLY: stable vacuum with inverted potential !

OUTLOOK

- ∃ 3 10D strings with broken supersymmetry:
  - two orientifolds (U(32), Usp(32)), one heterotic (SO(16)xSO(16))

$$\Delta\,\mathcal{S} \;=\; -\,\frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\,T\,e^{-\gamma\,\phi}$$

- ∃ sharp predictions: $\gamma = 3/2$ (orientifolds) ; $\gamma = 5/2$ (heterotic) in EINSTEIN FRAME

- ∀ $\gamma \geq 3/2$ (& ∀ potentials with an (over) critical exponential) :
  - CLIMBING DYNAMICS IN COSMOLOGY for $\gamma \geq \gamma_c$ [& surprising asymptotics …]

- INTEGRABLE MODELS:
  a) many examples where this type of setting starts an inflationary phase
  b) many examples of Dudas-Mourad-like vacua with bounded g_s

THANK YOU, PIETRO, FOR A LONG FRIENDSHIP AND A VERY NICE COLLABORATION !

Thank You

Best Wishes, Pietro !