Supergravity, Geometry, AdS/CFT ...and Pietro Fré

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Plan

A very biased recollection of some of Pietro's contributions to supergravity, geometry, the AdS/CFT correspondence...

...and his influence on my research and career

The beginning (for me)

- Fall 1997: I start my PhD at SISSA, missing by a couple of years to cross paths with Pietro... meanwhile Maldacena invents the AdS/CFT correspondence
- Claim: type IIB string theory on $AdS_5 \times S^5$ is $\mathcal{N} = 4$ SYM in a limit
- One of the first spectacular tests passed by this bold claim is that the conformal dimensions of an infinite set of chiral operators in $\mathcal{N} = 4$ SYM match with the spectrum of KK harmonics in $AdS_5 \times S^5$
- This was calculated in 1985 by [Kim,Romans,van Nieuwenhuizen]
- The AdS/CFT correspondence had revived interest in supergravity, Anti de Sitter and Kaluza-Klein spectroscopy

First attempts at extending the AdS/CFT correspondence

- 1998: Klebanov and Witten write a seminal paper which generalises Maldacena's first proposal in a highly non-trivial way
- Type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ is dual to a $\mathcal{N} = 1$ quiver gauge theory
- Relation to interesting geometry:
  1. $T^{1,1}$ is a homogeneous Sasaki-Einstein manifold $\leftrightarrow$ the cone $C(T^{1,1})$ is a singular algebraic variety $\{z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0 \subset \mathbb{C}^4\}$, admitting a Ricci-flat Kähler metric $\rightarrow$ a Calabi-Yau cone
  2. The moduli space of the Klebanov-Witten gauge theory reproduces exactly the same singular algebraic variety
  3. Both $T^{1,1}$ and $C(T^{1,1})$ are "toric" $\rightarrow$ toric geometry
- A non-trivial test of this proposal was performed a year later by [Ceresole,Dall'Agata,D'Auria,Ferrara]: worked out the complete KK spectrum on $T^{1,1}$ and matched this to dimensions of operators constructed with the fields of the Klebanov-Witten model

Extensions to M theory*

- KW suggest that any supersymmetric compactification of $D = 11$ supergravity, of the type $\mathrm{AdS}_4 \times Y_7$ where Y_7 is a Sasaki-Einstein manifold must be dual to some $d = 3$ SCFT with $\mathcal{N} = 2$

- In particular, the last sentence of the KW paper reads

> nontrivially on the canonical line bundle of the conifold. We hope it will be possible to construct a three-dimensional field theory corresponding to M2-branes on (38).

where

$$\{z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2 = 0 \subset \mathbb{C}^5\} \qquad (38)$$

- The base of this cone is a Sasaki-Einstein 7-manifold called $V_{5,2}$

- However, the situation was awkward: how were we supposed to figure out the SCFT dual of $V_{5,2}$ or other Sasaki-Einstein manifolds, if we didn't even understand the dual of S^7?!

*Title of section 4 of the Klebanov-Witten paper

M theory compactifications

- List of all the known (in 1999) $AdS_4 \times \boldsymbol{Y_7}$ solutions, where $\boldsymbol{Y_7}$ is weak $\boldsymbol{G_2}$, Sasaki-Einstein, or 3-Sasakian: table from [Duff,Nilsson,Pope] (1986)

Table 6

Solution	G	$\mathscr{H}$	N	b_2	Stable?
Round S^7	$SO(8)$	1	8	0	Yes
Squashed S^7	$SO(5)\times SU(2)$	G_2	1	0	Yes
$S^5\times S^2 = M(1,0)$	$SU(4)\times SU(2)$	$SO(7)$	0	1	No
$S^4\times S^3$	$SO(5)\times SU(2)\times SU(2)$	$SO(7)$	0	0	No
$S^2\times S^2\times S^3 = Q(0,0,1)$	$[SU(2)]^4$	$SO(7)$	0	2	No
$S^2\times T_1S^3 = Q(0,1,1)$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	No
Twisted $(S^2\times S^2)\times S^3$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	No
$CP^2\times S^3 = M(0,1)$	$SU(3)\times SU(2)\times SU(2)$	$SO(7)$	0	1	No
$\frac{SU(3)}{SO(3)_{max}}\times S^2$	$SU(3)\times SU(2)$	$SO(7)$	0	1	No
$\frac{SO(5)}{SO(3)_{max}}$	$SO(5)$	G_2	1	0	Yes
$V_{5,2}$	$SO(5)\times U(1)$	$SU(3)$	2	0	Yes
$M(3,2)$	$SU(3)\times SU(2)\times U(1)$	$SU(3)$	2	1	Yes
$M(m,n)$	$SU(3)\times SU(2)\times U(1)$	$SO(7)$	0	1	See below
$Q(1,1,1)$	$[SU(2)]^3\times U(1)$	$SU(3)$	2	2	Yes
$Q(p,q,r)$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	See below
$N(1,1)_I$	$SU(3)\times SU(2)$	$SU(2)$	3	1	Yes
$N(1,1)_{II}$	$SU(3)\times SU(2)$	G_2	1	1	Yes
$N(k,l)_{I,II}$	$SU(3)\times U(1)$	G_2	1	1	Yes
T^7	$[U(1)]^7$	1	8	21	Yes
$K3\times T^3$	$[U(1)]^3$	$SU(2)$	4	25	Yes

From KK spectroscopy to extensions of AdS/CFT

- For $AdS_4 \times \boldsymbol{Y_7}$ solutions, where $\boldsymbol{Y_7}$ is one of the three homogeneous Sasaki-Einstein manifolds, the KK spectra where worked out in Torino!
  - In 1985: [Castellani,D'Auria,Fré,Pilch,van Nieuwenhuizen] for $\boldsymbol{Y_7 = M^{3,2}}$
  - In 1999: a second [Ceresole,Dall'Agata,D'Auria,Ferrara] for $\boldsymbol{Y_7 = V_{5,2}}$
  - In 2000: [Merlatti] for $\boldsymbol{Y_7 = Q^{1,1,1}}$
- In 1999 Pietro and his collaborators write an important paper, that will have an enduring impact on the AdS/CFT correspondence, as well as on my research throughout my academic life!
- They proposed three-dimensional quiver gauge theories dual to the Sasaki-Einstein manifolds $\boldsymbol{M^{3,2}}$ and $\boldsymbol{Q^{1,1,1}}$
- Their paper anticipated several key ingredients that were crucial for the developments that occurred in the next decade

AdS/CFT, Sasakian geometry, etcetera

3D superconformal theories

from Sasakian seven-manifolds:

new nontrivial evidences for AdS_4/CFT_3 *

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Abstract

In this paper we discuss candidate superconformal $\mathcal{N} = 2$ gauge theories that realize the AdS/CFT correspondence with M–theory compactified on the homogeneous Sasakian 7-manifolds M^7 that were classified long ago. In particular we focus on the two cases $M^7 = Q^{1,1,1}$ and $M^7 = M^{1,1,1}$, for the latter the Kaluza Klein spectrum being completely known. We show how the toric description of M^7 suggests the gauge group and the supersingleton fields. The conformal dimensions of

An inspiration for my future research...

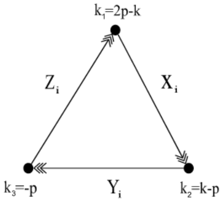
- 2000: I was in my final year of the PhD, learning about the AdS/CFT correspondence and trying to generalise Maldacena's proposal...
- Pietro's paper caught my attention and I tried to understand what they were saying.. with limited success..
- The same year [Billó,Fré,Merlatti,Zaffaroni] write another paper discussing a proposal for an $\mathcal{N} = 3$ quiver theory dual to the 3-Sasakian manifold $N^{1,1}$
- I remember spending time trying to match the baryonic operators of their theory to the geometry of $N^{1,1}$, getting a mismatch by a factor...

Some of my later contributions...

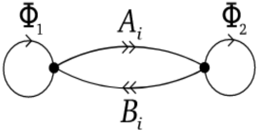
- Constructed new infinite families of Sasaki-Einstein manifolds (2004)
- Constructed infinite families of $d = 4$, $\mathcal{N} = 1$ (toric) quiver gauge theories dual to Sasaki-Einstein 5-manifolds (2004-2005)
- Sasakian geometry: volume extremization; interpreted as dual to a-maximization for $d = 4$, $\mathcal{N} = 1$ SCFTs $\to$ predicted F-extremization for $d = 3$, $\mathcal{N} = 2$ SCFTs (2005-2006)
- Constructed new infinite families of asymptotically conical Ricci-flat Kähler metrics on (partial) resolutions (2007)
- Studied the moduli space of $d = 3$, $\mathcal{N} = 2$, Chern-Simons matter theory $\to$ relation to Calabi-Yau four-folds $\to$ AdS_4/CFT_3 (2008)

Some of my later contributions...

- Constructed infinite families of $d = 3$, $\mathcal{N} = 2$ (toric) quiver gauge theories dual to Sasaki-Einstein 7-manifolds, including a new proposal for $Y_7 = M^{3,2}$ ($p = 2, k = 3$ in the figure below) (2008)

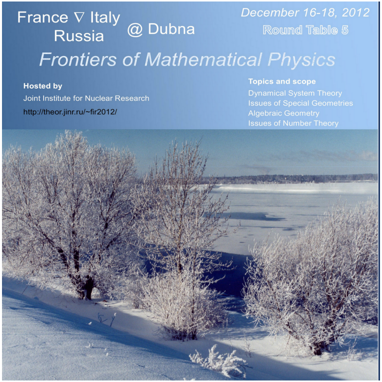


- Constructed the $d = 3$, $\mathcal{N} = 2$ quiver gauge theory dual to the Sasaki-Einstein 7-manifold $Y_7 = V_{5,2}$ (2009)



Crossing paths with Pietro...

- Pietro's work has been characterized by deep geometric insights [cf. other contributions at this conference]. For example, his well-known paper on "Geometric supergravity in $D = 11$" [D'Auria,Fré] (1982)
- Possibly the first time met Pietro it was in Dubna in December 2012

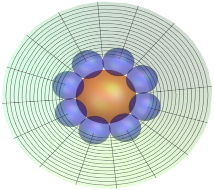


Crossing paths with Pietro...

- I also recall quite clearly my talk at the conference "Supergravity at 40" at the GGI, where I spoke about supergravity, Sasakian geometry, etcetera in front of Pietro and some other "fathers" of our field...
- In the following years Pietro continued to work, among other things, on Calabi-Yau singularities and their use in holography:
  - A new class of supersymmetric M2-branes solutions of $D = 11$ supergravity [Fré] (2016)
  - Study of Kähler quotient resolution of $\mathbb{C}^3/\Gamma$ Calabi-Yau singularities (generalized Kronheimer construction) in relation to $d = 3$, $\mathcal{N} = 2$ Chern–Simons theories [Bruzzo,Fino,Fré] (2017)
  - Crepant resolutions of the $\mathbb{C}^3/\mathbb{Z}_4$ singularity and application to the gauge/gravity duality [Bruzzo,Fino,Fré,Grassi,Markushevich] (2019)
  - ...

Back to the present

- In 2018 (while I was in sabbatical at GGI), Pietro asked me if I would consider moving to Torino, specifically in the maths department
- I thought it was a great opportunity and a year later I moved!
- We started* a fun collaboration with Pietro, Ugo and Massimo, that led to the construction of new asymptotically conical Ricci-flat Kähler metrics on a (partial) resolution of the $\mathbb{C}^3/\mathbb{Z}_4$ singularity



*Unfortunately, a few months after I moved to Torino, a global pandemic hit the planet!

Conclusions

Conclusions

Thank you, Pietro!