Embedded Symmetry

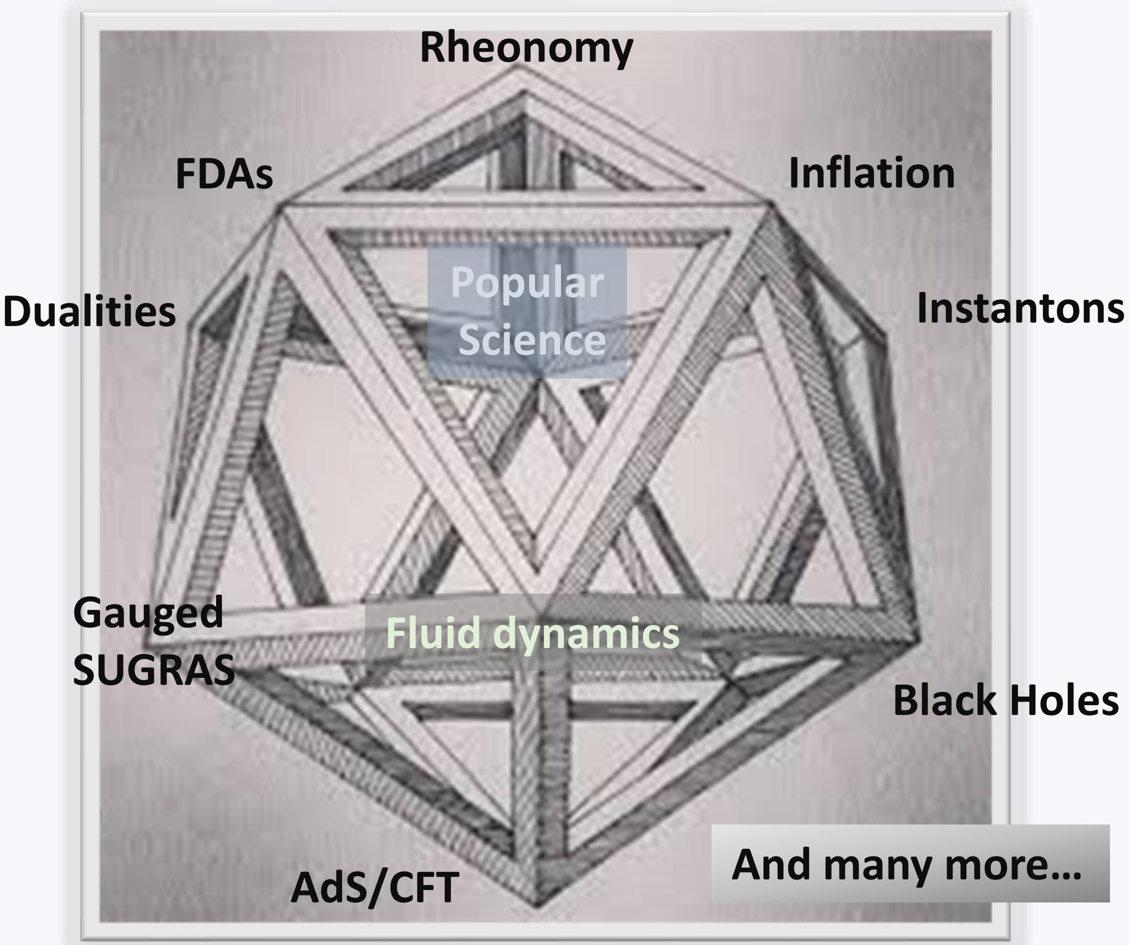
«Gravity, Geometry and Symmetry»

A celebration for Pietro Frè's 70's

October 7 2022

Mario Trigiante
(Politecnico di Torino)

Truly hard even to mention, in this short time, all Pietro Frè’s past and present contributions to theoretical physics and his multifaceted cultural interests



My work with Pietro

- Pietro was my PhD supervisor at SISSA-ISAS (Trieste, 1995-1997) on a project originally on T-algebras in supergravity

- *The sense of beauty* as his driving force to unveil symmetry

- A fruitful collaboration on several subjects (except T-algebras....)

- Focus only on one subject…

A Fruitful Idea…

- One of the topics we have been working on is gauged supergravities
- *Ungauged extended sugras: Characteristic global symmetry group G, no scalar potential* (physically uninteresting)
- Gauging a suitable subgroup G_g of G in order to introduce a scalar potential (*gauging*)
- Gain in physical content, loose global invariance under G (*superstring dualities*)

  - Can we formulate the conditions for a viable G_g in terms of scalar independent G-covariant conditions?
  - Can we formally restore global G-invariance in the gauged theory?

The embedding matrix

- In 1998 a *partial* solution was found for G =SL(8) in N=8 D=4: the ***embedding matrix***…

- Encode all information about the gauge algebra in a G-covariant tensor (**embedding matrix**)

$$\mathcal{D}_\mu \Phi = \left(\partial_\mu - g\, A_\mu^M\, X_M\right)\Phi$$

$$X_M = \Theta_M{}^\alpha t_\alpha \qquad t_\alpha \in \mathrm{Lie}(G)$$

ELSEVIER

Nuclear Physics B 532 (1998) 245–279

NUCLEAR PHYSICS B

$N = 8$ gaugings revisited: an exhaustive classification ★

Francesco Cordaro [a,b], Pietro Fré [a,b], Leonardo Gualtieri [a,b], Piet Termonia [a,b], Mario Trigiante [c]

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Received 21 April 1998; accepted 27 May 1998

Next let us introduce the fundamental item in the gauging construction. It is the 28×63 constant embedding matrix:

$$\mathcal{E} \equiv e^{\alpha}_{\Lambda\Sigma} \tag{36}$$

transforming under $SL(8,\mathbb{R})$ as its indices specify, namely in the tensor product of the adjoint with the antisymmetric **28** and that specifies which generators of $SL(8,\mathbb{R})$ are gauged and by means of which vector fields in the 28-dimensional stock. In particular,

➢ Reduce consistency constraints on G_g (closure of the gauge algebra and SUSY) to constant, G-covariant conditions on Θ

➢ Formally restore global G-invariance of the theory, provided Θ is transformed under G with all the other fields

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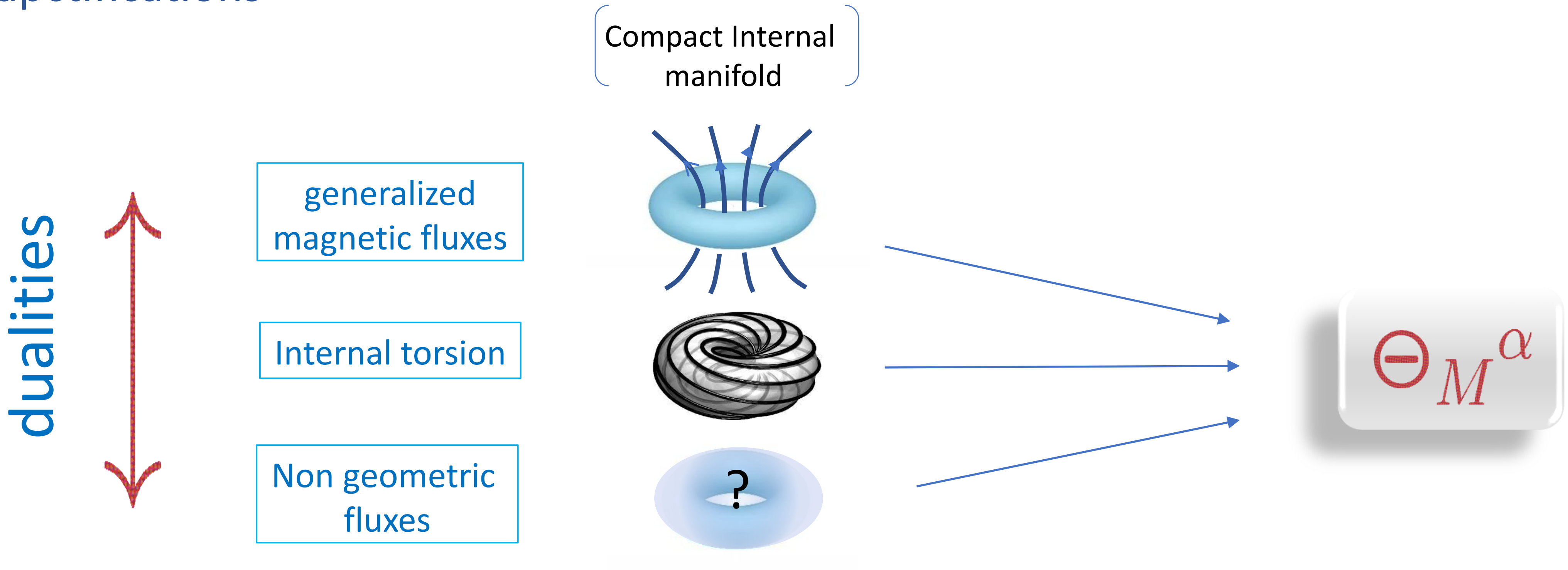
- Extension to a tensor with respect to the fully-fledged global symmetry group of the theory
  first done in D=3 maximal theory, G=$E_{8(8)}$ [H. Nicolai and H. Samtleben, Phys. Rev. Lett. 86 (2001), 1686-1689]

- Then in D=4 and D=5 theories, etc.

[B. de Wit, H. Samtleben and M.T., hep-th/0212239; hep-th/0412173; hep-th/0507289; 0705.2101]

- Physical meaning of the embedding tensor: background quantities in superstring/M-theory comapctifications



- Further related progress: *tensor hierarchy*

[B. de Wit, H. Samtleben, hep-th/0501243; B. de Wit, H. Nicolai, H. Samtleben, 0801.1294, 0805.4767]

- New «dyonic» gaugings

[G. Dall'Agata, G. Inverso, 1112.3345; G. Dall'Agata, G. Inverso, M.T. , 1209.0760 ; G. Dall'Agata, G. Inverso, A. Marrani, 1405.2437; H. Samtleben, A. Gallerati, M.T., 1410.0711; etc.]

Type IIB S-Folds from D=4 SUGRA

- Maximal SUGRA with gauge group

$$\mathcal{G} = [\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes N^{(6,2)}$$

embedded in Type IIB through ExFT [G. Inverso, H. Samtleben, M.T., 1612.05123]

- Its vacua uplifted to special S-fold solutions.

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S-fold solutions: non-geometric b.g.s featuring transition functions which involve duality transformation in $\mathrm{SL}(2,\mathbf{Z})_{\mathrm{IIB}}$ [C.Hull, A. Çatal-Özer, 0308133; C. Hull, 0406102]

- In our case S-folds have topology $\mathrm{AdS}_4 \times \tilde{S}^5 \times S^1$

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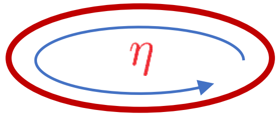

$$\eta \to \eta + T$$
$$\boldsymbol{\Psi} \to \mathfrak{M} \cdot \boldsymbol{\Psi}$$

- The monodromy $\mathfrak{M}$ is a hyperbolic element of SL(2,**Z**)$_{\mathrm{IIB}}$

$$\mathfrak{M} = J_n = -ST^n = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$$

$(n > 2)$ "**J-fold**"

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J-fold SUGRA sol.s $\leftrightarrow$ D=3 J-fold SCFT

[D.Gaiotto, E.Witten, 0807.3720;
N=4: B. Assel and A. Tomasiello, 1804.0641;
N=2; N. Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 2003.09154; E. Beratto, N. Mekareeya, M. Sacchi, 2009.10123; N. Bobev, F. Gautason, J. van Muiden, 2104.00977]

Type IIB S-Folds from D=4 SUGRA

- $\mathcal{N}$=4 with symmetry $SO(4)_R$ J-fold [vacuum found in H. Samtleben, A. Gallerati, M.T., 1410.0711; D=10 uplift in: G. Inverso, H. Samtleben, M.T., 1612.05123]
- $\mathcal{N}$=0& SO(6) ; $\mathcal{N}$=1&SU(3) J-fold [A. Guarino, C. Sterckx, 1907.04177]
- $\mathcal{N}$=2& SU(2) x $U(1)_R$ J-fold [A. Guarino, C. Sterckx, M.T., 2002.03692]
- $\mathcal{N}$=2& U(1) x $U(1)_R$ J-fold 1-parameter, KK spectrum [vacua found in 2002.03692 ; D=10 uplift in: A. Giambrone, E.Malek, H. Samtleben, M.T., 2103.10797]
- $\mathcal{N}$=0& $U(1)^3$ (3-param.s) ; $\mathcal{N}$=1&$U(1)^2$ (2- param.s) J-folds and DWs [vacua found in 2002.03692 ; D=10 uplift in: A. Guarino, C. Sterckx, 2103.12652]
- $\mathcal{N}$=2& U(1) x $U(1)_R$ J-fold 2-parameters (D=4 vacuum, SUGRA and KK spectrum) [N. Bobev, F. Gautason, J. van Muiden, 2104.00977; M. Cesaro, G. Larios, O. Varela, 2109.11608]
- $\mathcal{N}$=0& U(1) x $U(1)_R$ stable J-fold, 2-parameters (D=4 vacuum and KK spectrum) [A. Giambrone, A. Guarino , E.Malek, H. Samtleben, C. Sterckx, M.T., 2112.11966]

Type IIB S-Folds from D=5 SUGRA

- **N=1, N=2&U(2):** Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 1907.11132, 2003.09154;
- **N=4 and N=2&U(1)2 (1- param.s) J-folds and DWs**: I. Arav, J.Gauntlett, M.Roberts, C.Rosen, 2103.15201

N=2 S-Folds

Solution with U(1) x U(1)$_R$ symmetry and 1 parameter χ (flat dir. of V $\longrightarrow$ exactly marginal deformation)

- *Background geometry:* $\mathrm{AdS}_4 \times \tilde{S}^5 \times S^1 = \mathrm{AdS}_4 \times S^2 \times \tilde{S}^3 \times S^1$

$$\boxed{ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left(ds^2_{\mathrm{AdS}_4} + ds^2_{\mathrm{S}^2} + \cos^2(\theta)\,ds^2_{\mathrm{S}^3} + d\eta^2\right)}$$

$$\Delta \equiv (6 - 2\cos(2\theta))^{-\frac{1}{4}}$$

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Solution with U(1) x U(1)$_R$ symmetry and 1 parameter χ

Isometry U(1) x U(1)$_R$

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$$\Delta \equiv (6 - 2\cos(2\theta))^{-\frac{1}{4}}$$

$$ds^2_{\mathrm{S}^2} = d\theta^2 + \sin^2(\theta)\, d\varphi^2$$

$$ds^2_{\mathrm{S}^3} = \hat{\sigma}_2^2 + 8\,\Delta^4\,(\hat{\sigma}_1^2 + \hat{\sigma}_3^2)$$

U(1)$_R$

$$\hat{\sigma}^1 \equiv \sigma^1 + \chi\,(-\cos(\alpha)\cos(\beta)\cos(\gamma) + \sin(\alpha)\sin(\gamma))d\eta\,,$$
$$\hat{\sigma}^2 \equiv \sigma^2 - \chi\,(\sin(\alpha)\cos(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma))d\eta\,,$$
$$\hat{\sigma}^3 \equiv \sigma^3 + \chi\,\cos(\gamma)\sin(\beta)d\eta\,.$$

$$d\sigma^x - \epsilon^{xyz}\,\sigma^y \wedge \sigma^z = 0$$

Internal coord.s $$y^a = (y^i, \eta) = (\underbrace{\theta, \phi}_{S^2}, \underbrace{\alpha, \beta, \gamma}_{\tilde{S}^3}, \underbrace{\eta}_{S^1})$$

N=2 S-Folds

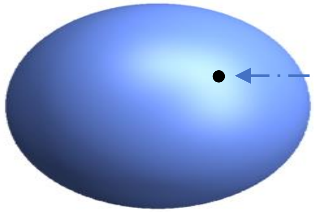
Solution with U(1) x $U(1)_R$ symmetry and 1 parameter χ

$\tilde{S}^3$ isometry

The parameter χ induces a second twist: $\mathbf{h}(\eta) = e^{2\chi \mathbf{H}\eta} \in \mathrm{U}(1) \in \mathrm{SU}(2)$

defining a fibration of the 3-sphere over S^1

$\chi = 0$

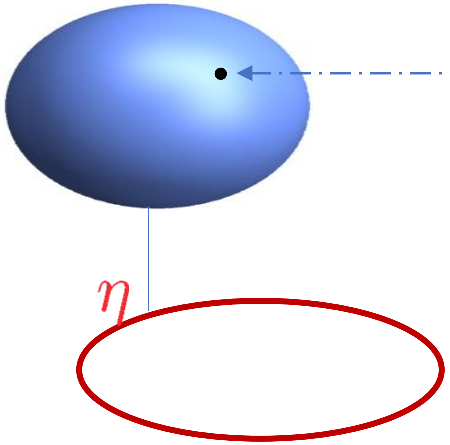


$\tilde{S}^3$

$g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

$\mathrm{SU}(2) \rightarrow g[\alpha, \beta, \gamma] \leftarrow \mathrm{U}(1)_\mathrm{R}$

$\chi \neq 0$



$\tilde{S}^3$

S^1

$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

Locally: $\hat{g}[\alpha, \beta, \gamma, \eta] = g[\alpha', \beta', \gamma']$

χ affects the *global geometry* of the background

N=2 S-Folds

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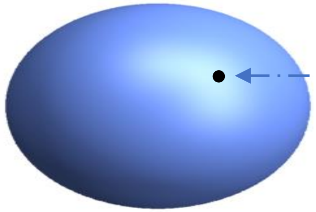

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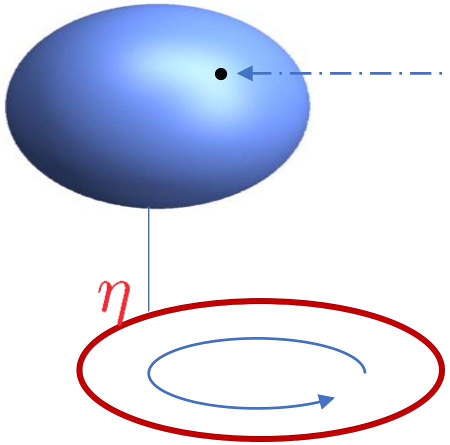


$\tilde{S}^3$

η

S^1

$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

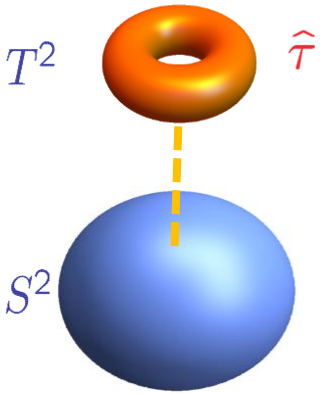
$\tilde{S}^3 \ltimes S^1 \equiv \tilde{S}^3 \times [0, T]/\sim$

$$[g\,(\alpha, \beta, \gamma)\,,\, \eta = 0] \sim [\underbrace{h(T)}_{\text{new monodromy}} \cdot g\,(\alpha, \beta, \gamma)\,,\, \eta = T]$$

N=2 S-Folds

Global properties of the parameter χ: $\chi \in \left[0, \frac{2\pi}{T}\right)$

- Write the 3-sphere as a Hopf fibration over S^2. The Hopf-fiber and S^1 combine in a torus so that $\tilde{S}^3 \ltimes S^1$ can be written as an elliptic fibration over S^2



$\hat{\tau} = \frac{i}{4\pi} - \frac{1}{2\pi}\chi T$ (χ part of a complex structure parameter) $z = \frac{\psi}{4\pi} + \hat{\tau}\frac{\eta}{T}\,, \quad 0 \leq \psi < 4\pi$

$\chi \to \chi + \frac{2\pi}{T} \Rightarrow \hat{\tau} \to \hat{\tau} - 1$ (Dehn twist)

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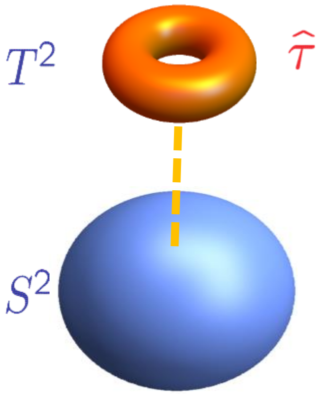

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- Same global property of conformal manifold found from KK spectrum computed within the framework of ExFT [E. Malek, H. Samtleben, 1911.12640]. Grouped KK states in OSp(2|4) supermultiplets.

N=2 S-Folds

KK spectrum

- Conformal dimension for a spin-J state in a representation [**k**] of SU(2)

$$\Delta = \frac{1}{2} + \sqrt{\frac{17}{4} + \frac{1}{2}R^2 - J(J+1) - 2k(k+1) + \ell(\ell+4) + 4\left(\frac{\pi n}{T} - j\chi\right)^2}$$

$\ell = \tilde{S}^5$-level $\quad n = S^1$-level

$j = -k, -k+1, \ldots, k-1, k$: U(1) $\subset$ SU(2)-charge

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- New χ –twist $\mathbf{h}(\eta)$ introduces extra dependence of the KK states on η

$$\Phi^{(k,j)}_{(n)}(y^i, \eta) = \hat{\Phi}^{(k,j)}_{(n)}(y^i)\, \underbrace{e^{-2i\,j\,\chi\,\eta}}_{\mathbf{h}(\eta)}\, e^{\frac{2i\,\pi\,n\,\eta}{T}} \qquad \frac{\partial^2}{\partial\eta^2}\Phi^{(k,j)}_{(n)} = -4\left(j\chi - \frac{\pi n}{T}\right)^2 \Phi^{(k,j)}_{(n)}$$

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- $\chi = \frac{p\,\pi}{T}$ (p=1,2) two real states with $n = pj$ become massless when $pj \in \mathbb{Z}$

SPACE INVADERS

[Duff,Nilsson,Pope, '86]

N=2 S-Folds

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- $\chi = \frac{p\,\pi}{T}$ (p=1,2) two vectors in Adj(SU(2)) (| j |=1) become massless

$\chi = \frac{\pi}{T}$ at $n = 1$

$\chi = \frac{2\pi}{T}$ at $n = 2$

N=2 S-Folds

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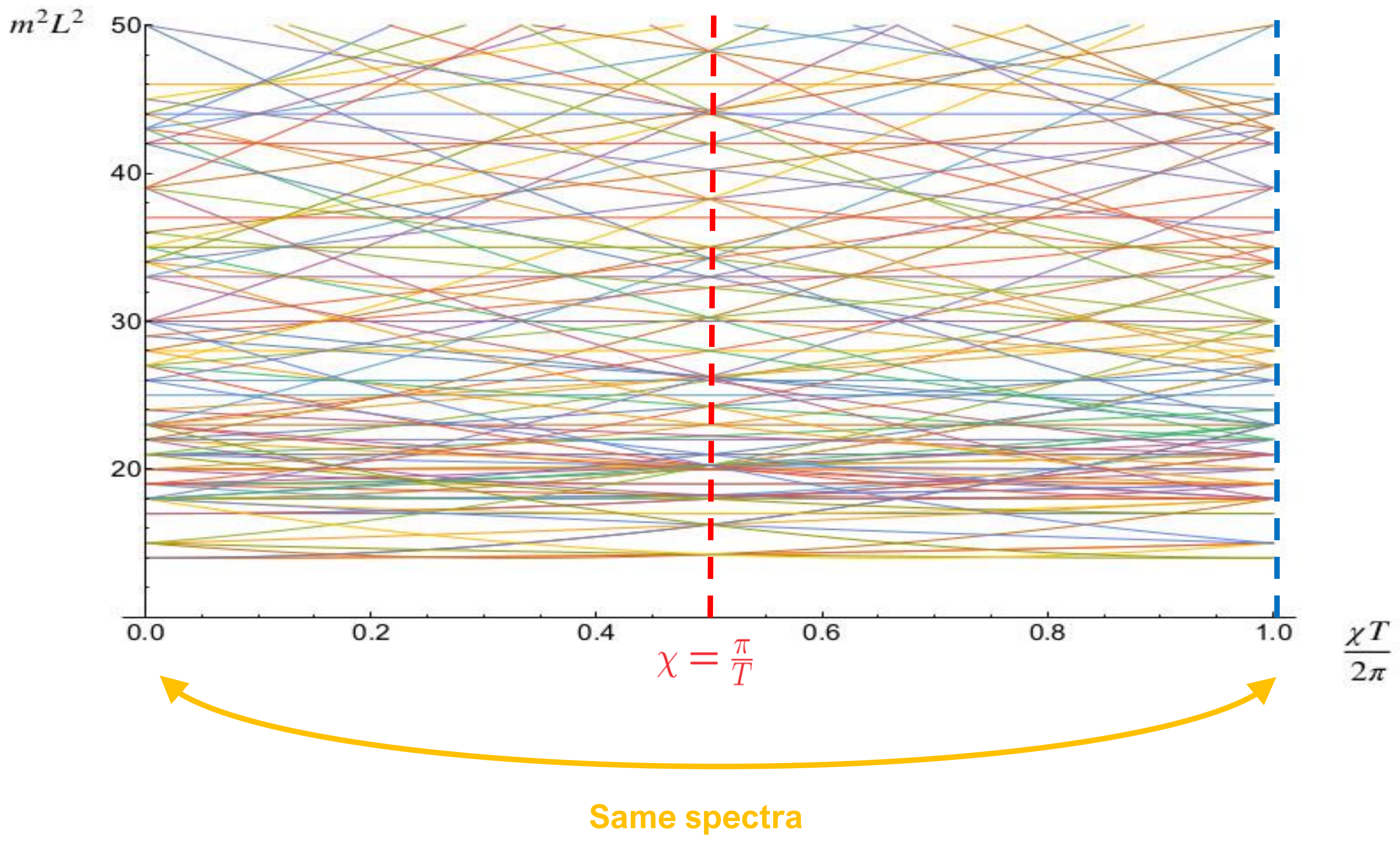
- $\chi = \frac{p\,\pi}{T}$ two vectors in Adj(SU(2)) (| j |=1) become massless $\boxed{\mathrm{U}(1) \to \mathrm{SU}(2)}$

Example: spin-2 at level $\ell = 3$



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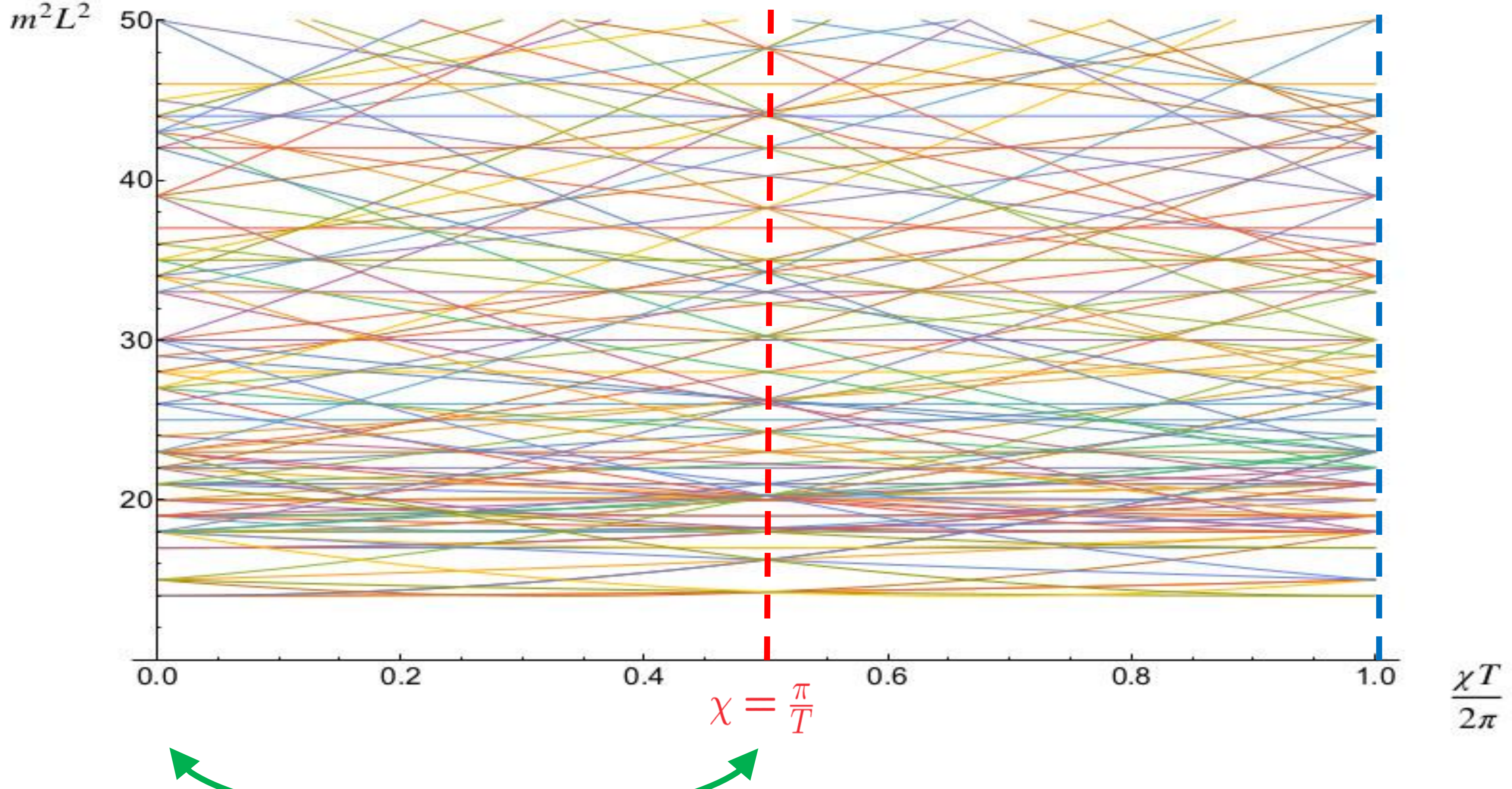

Different spectra
same only for integer j

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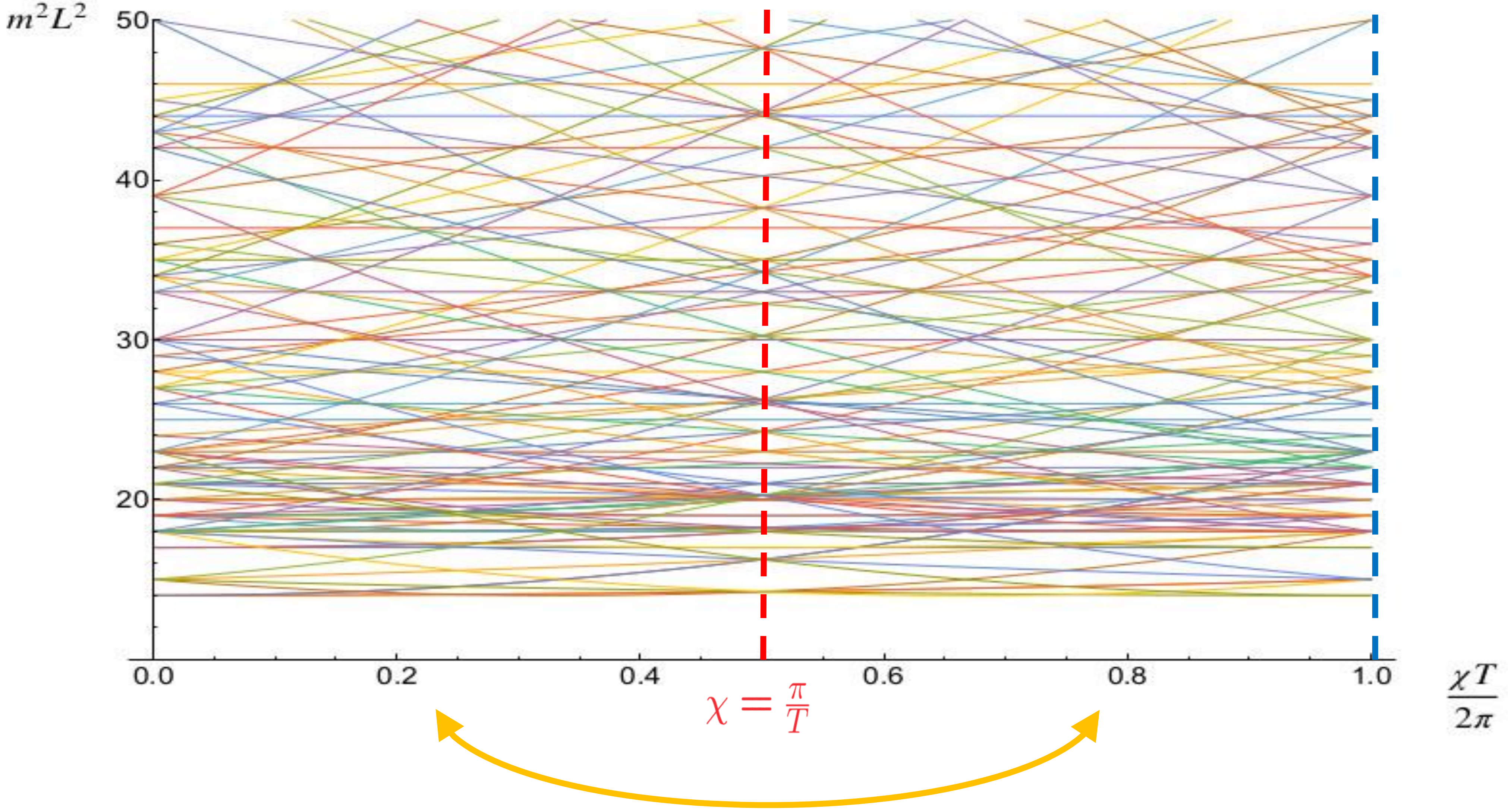


Symmetry from periodicity of χ and parity: $\chi \to -\chi$

- We found that the N=2 vacua are points in a web of marginally connected 3-parameter N=0,2,4 vacua. Proof of perturbative stability of the N=0 ones, evidence for their non-pert. stability: possible N=0 conformal manifold of CFTs.

[A. Giambrone, A. Guarino , E.Malek, H. Samtleben, C. Sterckx, M.T., 2112.11966]

- Recently found in N=3 gauged sugra with a web of marginally connected N = 3, 2, 1 and 0 vacua. Pert. stable in SUGRA

[A,Giambrone, P.Frè, D.Ruggeri, P.Vasko, M.T. 2206.09971]

To be done…

- ➢ Embed vacua in M-theory
- ➢ Study holographic duality and RG flow
- ➢ Etc.

It all started from a good idea….

Many other great ideas you’ve had, and many more are to come,
while pursuing the beauty of symmetry.

I wish to thank you Pietro for your friendship, your collaboration …
…and your constant inspiration

to your new start…

Cheers!

Gravitino masses as functions of δ, χ_1, χ_2

$$m^2L^2 : \frac{2\delta\left(\delta^3 \pm \sqrt{2}\sqrt{2\delta^4 + 8\delta^2 + (\chi_1 - \chi_2)^2 + 8} + 5\delta\right) + (\chi_1 - \chi_2)^2 + 8}{4\left(\delta^2 + 1\right)} \times 2$$

$$\frac{\pm 2\sqrt{\left(\delta^2 + 1\right)\left(9\delta^2 + (\chi_1 + \chi_2)^2 + 9\right)} + 10(1 + \delta^2) + (\chi_1 + \chi_2)^2}{4\left(\delta^2 + 1\right)} \times 2$$

Previously discussed N=2 vacua: $\delta = 0, \chi_1 = -\chi_2 = \chi$

Type IIB S-Folds from D=4 SUGRA

- They locally coincide with (singular) Janus solutions

[**N=0:** D.Bak, M. Gutperle,S. Hirano, 0304129;
N=1:E. D'Hoker, J. Estes,M. Gutperle, 0603012;
N=4: E. D'Hoker, J. Estes,M. Gutperle, 0705.0022]

$$\mathrm{AdS}_4 \times \tilde{S}^5 \times \mathbb{R}$$

- Expected dual SCFT:

Janus sol.s $\longleftrightarrow$ D=3 conformal Janus interfaces in N=4 D=4 SYM;

[**N=0:** A.B.Clark, D.Z.Freedman, A.Karch, M.Schnabl, 0407073;
N=0,1,2,4: D'Hoker, J. Estes, M. Gutperle, 0603013;
N=4: D.Gaiotto, E.Witten, 0804.2907]

J-fold SUGRA sol.s $\longleftrightarrow$ D=3 J-fold SCFT

[D.Gaiotto, E.Witten, 0807.3720;
N=4: B. Assel and A. Tomasiello, 1804.0641;
N=2; N. Bobev, F. Gautason, K. Pilch, M.Suh,J. van Muiden, 2003.09154; E. Beratto, N. Mekareeya, M. Sacchi, 2009.10123; N. Bobev, F. Gautason, J. van Muiden, 2104.00977]

N=2 S-Folds

Solution with U(2)-symmetry: the SL(2,R)$_{\text{IIB}}$-invariant sector

Isometry SU(2) x U(1)$_R$

- *Background geometry:* $\text{AdS}_4 \times \tilde{S}^5 \times S^1 = \text{AdS}_4 \times S^2 \times \tilde{S}^3 \times S^1$

$$\boxed{ds^2 = \frac{1}{2}\Delta^{-1}\left(ds^2_{\text{AdS}_4} + ds^2_{\text{S}^2} + \cos^2(\theta)\, ds^2_{\text{S}^3} + d\eta^2\right)}$$

$$\Delta \equiv (6 - 2\cos(2\theta))^{-\frac{1}{4}}$$

- *5-form field strength:*

$$\tilde{F}_5 \equiv dC_{(4)} + \frac{1}{2}\epsilon_{\alpha\beta}B^{\alpha}_{(2)} \wedge H^{\beta}_{(3)} = (1+\star)4\Delta^4 \sin(\theta)\cos^3(\theta)\left[3\, d\theta \wedge d\phi \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3 - d\eta \wedge \left(\cos(2\theta)\, d\theta - \frac{1}{2}\sin(2\theta)\sin(2\phi)\, d\phi\right) \wedge \sigma_1 \wedge \sigma_2 \wedge \sigma_3\right]$$

$$B^{\alpha}_{(2)} = (B_{(2)}, C_{(2)})$$

$$H^{\alpha}_{(3)} = \mathrm{d}B^{\alpha}_{(2)}$$

N=2 S-Folds

Solution with U(2)-symmetry: the SL(2,R)$_{\text{IIB}}$-covariant sector

- *2-form fields:*

$$B^{\alpha}_{(2)} = (B_{(2)}, C_{(2)}) = A(\eta)^{\alpha}{}_{\beta}\, \mathfrak{b}^{\beta}_{(2)}$$

$$\mathfrak{b}^{1}_{(2)} = \frac{1}{\sqrt{2}} \cos(\theta) \left[\left(\cos(\phi)\, d\theta + \frac{1}{2} \sin(2\theta)\, d(\cos(\phi)) \right) \wedge \sigma_2 + \cos(\phi) \frac{4 \sin(2\theta)}{6 - 2\cos(2\theta)} \sigma_1 \wedge \sigma_3 \right]$$

$$\mathfrak{b}^{2}_{(2)} = -\frac{1}{\sqrt{2}} \cos(\theta) \left[\left(\sin(\phi)\, d\theta + \frac{1}{2} \sin(2\theta)\, d(\sin(\phi)) \right) \wedge \sigma_2 + \sin(\phi) \frac{4 \sin(2\theta)}{6 - 2\cos(2\theta)} \sigma_1 \wedge \sigma_3 \right]$$

- *Axion-dilaton system* $\tau = C_{(0)} + i\, e^{-\varphi}$ *:*

$$m_{\alpha\beta} = \frac{1}{\text{Im}(\tau)} \begin{pmatrix} |\tau|^2 & -\text{Re}(\tau) \\ -\text{Re}(\tau) & 1 \end{pmatrix} = (A(\eta)^{-1})^{\sigma}{}_{\alpha}\, (A(\eta)^{-1})^{\gamma}{}_{\beta}\, \mathfrak{m}_{\sigma\gamma}$$

$$\mathfrak{m}_{\sigma\gamma} = 2\,\Delta^2 \begin{pmatrix} \sin^2(\theta)\cos^2(\phi) + 1 & -\frac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\frac{1}{2}\sin^2(\theta)\sin(2\phi) & \sin^2(\theta)\sin^2(\phi) + 1 \end{pmatrix}$$

N=2 S-Folds

Solution with U(2)-symmetry: the $SL(2,R)_{IIB}$-covariant sector

- *2-form fields:*

$$B^{\alpha}_{(2)} = (B_{(2)}, C_{(2)}) = A(\eta)^{\alpha}{}_{\beta}\, \mathfrak{b}^{\beta}_{(2)}$$

$$\mathfrak{b}^{1}_{(2)} = \frac{1}{\sqrt{2}}\cos(\theta)\left[\left(\cos(\phi)\, d\theta + \frac{1}{2}\sin(2\theta)\, d(\cos(\phi))\right)\wedge\sigma_2 + \cos(\phi)\frac{4\sin(2\theta)}{6-2\cos(2\theta)}\sigma_1\wedge\sigma_3\right]$$

$$\mathfrak{b}^{2}_{(2)} = -\frac{1}{\sqrt{2}}\cos(\theta)\left[\left(\sin(\phi)\, d\theta + \frac{1}{2}\sin(2\theta)\, d(\sin(\phi))\right)\wedge\sigma_2 + \sin(\phi)\frac{4\sin(2\theta)}{6-2\cos(2\theta)}\sigma_1\wedge\sigma_3\right]$$

- *Axion-dilaton system* $\tau = C_{(0)} + i\, e^{-\varphi}$ *:*

$$m_{\alpha\beta} = \frac{1}{\text{Im}(\tau)}\begin{pmatrix} |\tau|^2 & -\text{Re}(\tau) \\ -\text{Re}(\tau) & 1 \end{pmatrix} = (A(\eta)^{-1})^{\sigma}{}_{\alpha}\,(A(\eta)^{-1})^{\gamma}{}_{\beta}\,\mathfrak{m}_{\sigma\gamma}$$

Dependence on η *through*

$$A(\eta)^{\alpha}{}_{\beta} \equiv \begin{pmatrix} \cosh(\eta) & \sinh(\eta) \\ \sinh(\eta) & \cosh(\eta) \end{pmatrix} \in SL(2,\mathbb{R})_{\text{IIB}}$$

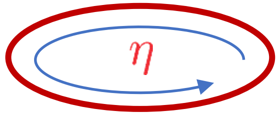
$$\boxed{\mathfrak{m}_{\sigma\gamma} = 2\,\Delta^2\begin{pmatrix} \sin^2(\theta)\cos^2(\phi)+1 & -\frac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\frac{1}{2}\sin^2(\theta)\sin(2\phi) & \sin^2(\theta)\sin^2(\phi)+1 \end{pmatrix}}$$

N=2 S-Folds

The $SL(2,R)_{IIB}$-twist **A(η)** induces a monodromy

$$\mathfrak{M} = A(\eta)^{-1} \cdot A(\eta + T) = \begin{pmatrix} \cosh(T) & \sinh(T) \\ \sinh(T) & \cosh(T) \end{pmatrix}$$

$$\eta \to \eta + T$$



$$\boxed{\begin{array}{c} \mathbf{B}_{(2)} \to \mathfrak{M} \cdot \mathbf{B}_{(2)} \\ \tau \to \mathfrak{M} \cdot \tau \end{array}}$$

S-fold solution to Type IIB superstring theory:

[G. Inverso, H. Samtleben, M.T., 1612.05123; B. Assel and A. Tomasiello, 1804.0641]

$$A(\eta) \to A(\eta) \cdot g_n \qquad e^T = \frac{1}{2}(n + \sqrt{n^2 - 4}) \qquad g_n \equiv \begin{pmatrix} \frac{(n^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{n}{\sqrt{2}(n^2-4)^{\frac{1}{4}}} & \frac{\sqrt{2}}{(n^2-4)^{\frac{1}{4}}} \end{pmatrix} \quad (n > 2)$$

$$\boxed{\mathfrak{M} \to g_n^{-1} \cdot \mathfrak{M} \cdot g_n = J_n = -S \cdot T^n \in SL(2, \mathbb{Z})_{IIB}}$$

N=2 S-Folds

Solution with U(1) x $U(1)_R$ symmetry and 1 parameter χ

Vacua found in [A. Guarino, C. Sterckx, M.T., 2002.03692] within an N=1 truncation of the maximal theory

Uplifted to Type IIB solution in [A. Giambrone, E.Malek, H. Samtleben, M.T., 2103.10797] where the KK spectrum was computed and the global properties of the parameter χ studied

χ is a flat direction of the scalar potential at the extremum

$$V_0 = -\frac{3}{|c|} = -\frac{3}{(L_{\mathrm{AdS}})^2}$$

χ expected to be exactly marginal deformation of dual SCFT, coordinate of its **Conformal Manifold**

N=2 S-Folds

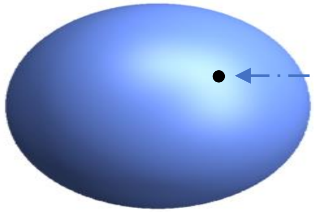
Solution with U(1) x U(1)$_R$ symmetry and 1 parameter χ

$\tilde{S}^3$ isometry

The parameter χ induces a second twist: $\mathbf{h}(\eta) = e^{2\chi \mathbf{H}\eta} \in \mathrm{U}(1) \in \mathrm{SU}(2)$

defining a fibration of the 3-sphere over S^1

$\chi = 0$

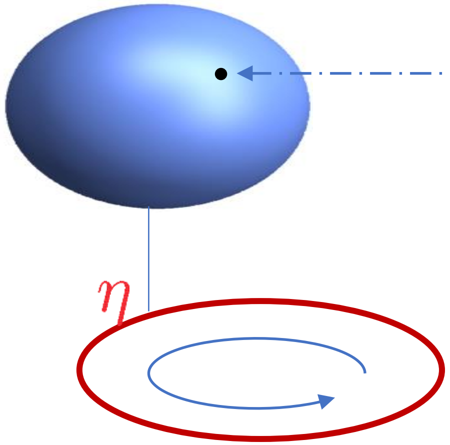


$\tilde{S}^3$

$g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

$\mathrm{SU}(2) \;\to\; g[\alpha, \beta, \gamma] \leftarrow\; \mathrm{U}(1)_R$

$\chi \neq 0$



$\tilde{S}^3$

η

S^1

$\hat{g}[\alpha, \beta, \gamma, \eta] = h(\eta) \cdot g[\alpha, \beta, \gamma] \in \mathrm{SU}(2)$

SU(2) broken to U(1) commuting with $h(T) = \begin{pmatrix} \cos(\chi T) & \sin(\chi T) \\ -\sin(\chi T) & \cos(\chi T) \end{pmatrix}$

$\mathrm{U}(1) \to \mathrm{SU}(2)$ enhancement: $\chi = \dfrac{\pi}{T}, \dfrac{2\pi}{T}$

N=2 S-Folds

KK spectrum

- Within the framework of ExFT we computed the KK spectrum on the solution and the OSp(2|4) supermultiplet structure (*see Henning's talk for a review of the general approach*).

[E. Malek, H. Samtleben, 1911.12640; M. Cesàro, O.Varela, 2012.05249]

- Only use S^5 x S^1 scalar harmonics.

- Generic pattern: at each level the KK states gather in long vector, gravitino and graviton multiplets

[see C. Cordova, T. Dumitrescu, K, Intriligator, 1602.01217 for notation]

$L\bar{L}[J]^{R}_{\Delta}$ Lorentz spin of HWS $J = 0,\ \frac{1}{2},\ 1$

$R = \mathrm{U}(1)_{\mathrm{R}}$-charge vector, gravitino, graviton

- Shortening if the unitarity bound $\Delta \geq 1 + |R| + J$ is saturated