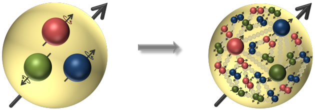
Quark and Gluon Spin and Orbital Angular Momentum in the Proton : A Light-Front Hamiltonian Approach

Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China



in collaboration with

Siqi Xu (IMP, China), Xingbo Zhao (IMP, China), Yang Li (USTC, China) and James P. Vary (ISU, USA)

QCD with Electron Ion Collider (QEIC) II, December 19, 2022

Overview

Introduction

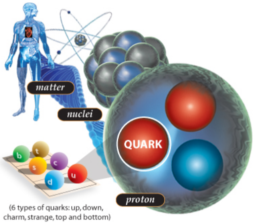
Basis Light-Front Quantization (BLFQ) Approach to

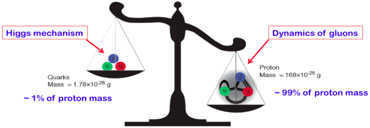
- Nucleon: $|qqq\rangle$ (Brief review)
- Nucleon: $|qqq\rangle + |qqqg\rangle$ (Based on arXiv:2209.08584)

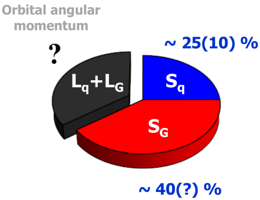
Conclusions

Fundamental Properties: Mass and Spin

- About 99% of the visible mass is contained within nuclei
- Nucleon: composite particles, built from nearly massless quarks (∼ 1% of the nucleon mass) and gluons
- *How does 99% of the nucleon mass emerge?*
- Quantitative decomposition of *nucleon spin* in terms of quark and gluon degrees of freedom is not yet fully understood.
- *To address these fundamental issues → nature of the subatomic force between quarks and gluons, and the internal landscape of nucleons.*

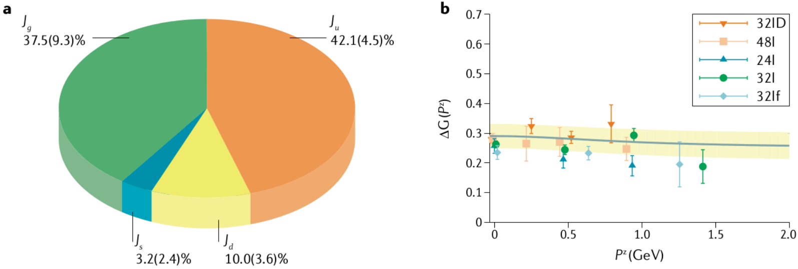






[1] Pictures (top to bottom) taken from A. Signori's talk, J. Qui talk, C. Lorce's talk

Spin sum rule	Formula	Terms	Characteristics
Frame independent (Ji)[30]	$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$	$\Delta\Sigma/2$ is the quark helicity L_q^z is the quark OAM J_g is the gluon contribution	The quark and gluon contributions, J_q and J_g, can be obtained from the GPD moments. A similar sum rule also works for the transverse angular momentum and has a simple parton interpretation
Infinite-momentum frame (Jaffe–Manohar)[31]	$\frac{1}{2}\Delta\Sigma + \Delta G + \ell_q + \ell_g = \frac{\hbar}{2}$	ΔG is the gluon helicity ℓ_q and ℓ_g are the quark and gluon canonical OAM, respectively	All terms have partonic interpretations; ℓ_q and ℓ_g are twist-three quantities. ΔG is measurable in experiments, including the RHIC spin and the EIC; ℓ_q and ℓ_g can be extracted from twist-three GPDs



[1] X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)
[2] Y.-B. Yang, R.S. Sufian, A. Alexandru et al., Phys. Rev. Lett. 118, 102001 (2017)

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PHYSICAL REVIEW LETTERS **128,** 182002 (2022)

Signature of the Gluon Orbital Angular Momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]
[1]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[2]*CPHT, CNRS, Ecole Polytechnique, Institut Polytechnique de Paris, 91128 Palaiseau, France*
[3]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

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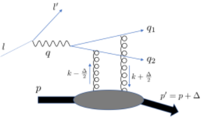
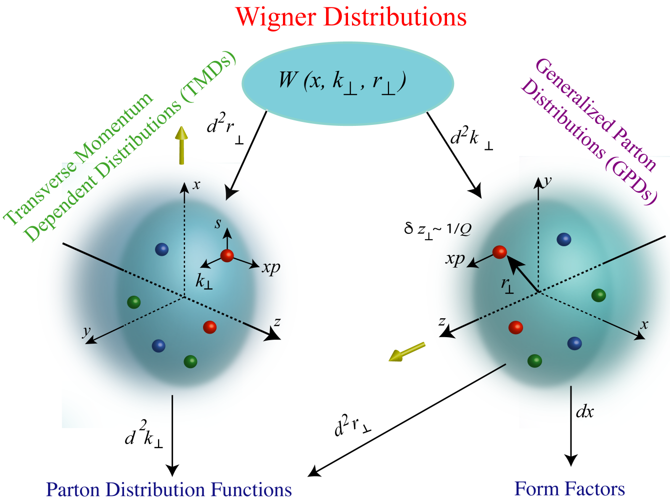
We propose a novel observable for the experimental detection of the gluon orbital angular momentum (OAM) that constitutes the proton spin sum rule. We consider longitudinal double spin asymmetry in exclusive dijet production in electron-proton scattering and demonstrate that the $\cos\phi$ azimuthal angle correlation between the scattered electron and proton is a sensitive probe of the gluon OAM at small x and its interplay with the gluon helicity. We also present a numerical estimate of the cross section for the kinematics of the Electron-Ion Collider.



1 X. Ji, F. Yuan and Y. Zhao, Nature Reviews Physics 3, 65 (2021)

Nucleon Tomography



- $x \to$ longitudinal momentum fraction; $k_\perp \to$ parton transverse momentum; $r_\perp \to$ transverse distance from the center.

Basis Light-Front Quantization (BLFQ)

A computational framework for solving relativistic many-body bound state problems in quantum field theories [1]

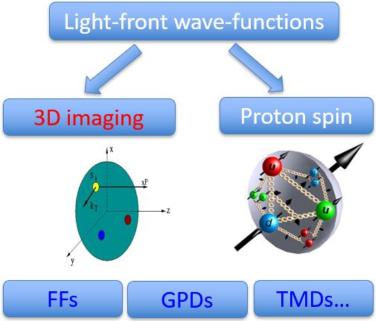
$$P^- P^+ |\Psi\rangle = M^2 |\Psi\rangle$$

- $P^- \equiv P^0 - P^3$: light-front Hamiltonian
- $P^+ \equiv P^0 + P^3$: longitudinal momentum
- $|\Psi\rangle$ mass eigenstate
- M^2 : mass squared eigenvalue for eigenstate $|\Psi\rangle$
- First-principle / effective Hamiltonian as input
- Evaluate observables

$$O \sim \langle\Psi|\hat{O}|\Psi\rangle$$

- direct access to light-front wavefunction of bound states

GOAL



[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Solution proposed by BLFQ

Discrete basis and their direct product	Truncation
2D HO $\phi_{nm}(p^\perp)$ in the transverse plane	$\sum_i (2n_i + \lvert m_i \rvert + 1) \leq N_{\max}$
Plane-wave in the longitudinal direction	$\sum_i k_i = K, \quad x_i = \frac{k_i}{K}$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$	Fock sector truncation

- Fock expansion of hadronic bound states:

$$|\text{Meson}\rangle = \psi_{(q\bar{q})}|q\bar{q}\rangle + \psi_{(q\bar{q}+1g)}|q\bar{q}g\rangle + \psi_{(q\bar{q}+q\bar{q})}|q\bar{q}q\bar{q}\rangle + \dots\,,$$

$$|\text{Baryon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle + \psi_{(3q+q\bar{q})}|qqqq\bar{q}\rangle + \dots\,,$$

[1] Vary, Honkanen, Li, Maris, Brodsky, Harindranath, *et. al.*, Phys. Rev. C 81, 035205 (2010).

Applications of BLFQ

QCD systems

- Heavy mesons: spectrum, decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, PDFs, GPDs

—Li, Chen, Zhao, Maris, Vary, Adhikari, M Li, Tang, A El-Hady, Lan, Wu, CM (2016 - 2022)

- Light mesons: spectrum, decay constant, elastic form factor, radii, distribution amplitude, PDFs, GPDs, TMDs

—Jia, Vary, Lan, Zhao, Qian, Li, Fu, J. Chen, Wu, CM (2018 - 2022)

- Baryons: EMFFs, axial form factor, radii, PDFs, GPDs, TMDs, OAM

—Xu, Hu, Peng, Zhu, Zhao, Li, Chakrabarti, Vary, Lan, Liu, CM (2019-2022)

- Tetraquarks: Masses of all-charm tetraquarks

—Kuang, Serafin, Zhao, Vary (2022)

QED systems

- Electron: anomalous magnetic moments, GPDs
- positronium: wave function, spectroscopy, FFs, GPDs
- Photon: wave function, structure functions, GPDs, TMDs

—Zhao, Wiecki, Li, Honkanen, Maris, Vary, Brodsky, Fu, Hu, Nair, CM (2013 - 2022)

Nucleon within BLFQ

- The LF eigenvalue equation: $H_{\text{eff}}|\Psi\rangle = M^2|\Psi\rangle$

$$H_{\text{eff}} = \sum_a \frac{\vec{p}_{\perp a}^2 + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b} \kappa^4 \left[x_a x_b (\vec{r}_{\perp a} - \vec{r}_{\perp b})^2 - \frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a+m_b)^2} \right]$$
$$+ \frac{1}{2}\sum_{a\neq b} \frac{C_F 4\pi\alpha_s}{Q_{ab}^2} \bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b) g_{\mu\nu}$$

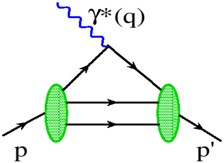
- For the first Fock sector:

$$|qqq\rangle = |n_{q1}, m_{q1}, k_{q1}, \lambda_{q1}\rangle \otimes |n_{q2}, m_{q2}, k_{q2}, \lambda_{q2}\rangle \otimes |n_{q3}, m_{q3}, k_{q3}, \lambda_{q3}\rangle$$

- Transverse : 2D harmonic oscillator basis $\phi_{nm}(\vec{p}_\perp)$;
  Plane wave basis in longitudinal direction.
- The valence wavefunction in momentum space:

$$\Psi^{M_J}_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}} = \sum_{n_i, m_i} \left[\psi(\alpha_i) \prod_{i=1}^{3} \phi_{n_i m_i}(\vec{p}_{\perp i}) \right]$$

Nucleon Form Factors



- EM current: $J^\mu = \bar{\psi}\gamma^\mu\psi$
- $\langle p'; \uparrow | J^+(0) | p; \uparrow (\downarrow) \rangle \sim F_{1(2)}(q^2)$
- Two FFs: $F_{1(2)}(q^2 = -Q^2)$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2),$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2).$$

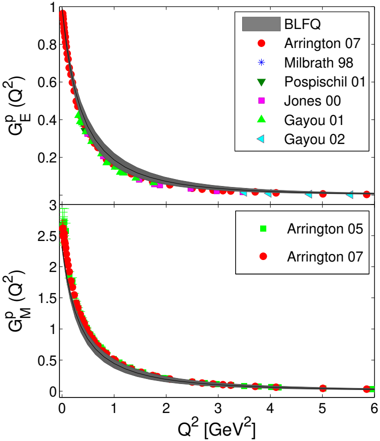
Proton Radii

$$\langle r_{\mathrm{E}}^2 \rangle = -6 \frac{\mathrm{d}G_{\mathrm{E}}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0},$$

$$\langle r_{\mathrm{M}}^2 \rangle = -\frac{6}{G_{\mathrm{M}}(0)} \frac{\mathrm{d}G_{\mathrm{M}}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}.$$

$$\sqrt{\langle r_E^2 \rangle} = 0.80 \pm 0.04\,(0.840^{+0.003}_{-0.002})\ \mathrm{fm}$$

$$\sqrt{\langle r_M^2 \rangle} = 0.83 \pm 0.03\,(0.849^{+0.003}_{-0.003})\ \mathrm{fm}$$



[1] CM, Siqi Xu, *et. al.*, Phys. Rev. D **102**, 016008 (2020)

Axial Form Factor

$$\langle N(p)|A^\mu|N(p')\rangle = \bar{u}(p')\left[\gamma^\mu G_A(q^2) + \frac{(p'-p)^\mu}{2m} G_p(q^2)\right]\gamma_5 u(p)$$

- Axial vector current: $A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$
- Experimentally measured by

$$\mu^-(l) + p(r) \to \nu_\mu(l') + n(r')$$

- Information on spin distributions

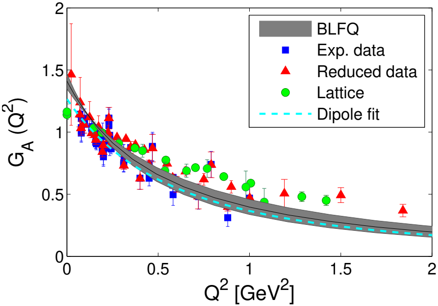
Proton axial charge and Radius

$$g_A = G_{\rm A}(0)\ ,$$

$$\langle r_{\rm A}^2\rangle = \frac{6}{g_{\rm A}}\frac{{\rm d}G_{\rm A}(Q^2)}{{\rm d}Q^2}\bigg|_{Q^2=0}$$

$$g_A = 1.41 \pm 0.06\,(1.2723 \pm 0.0023)$$

$$\sqrt{\langle r_A^2\rangle} = 0.68 \pm 0.07\,(0.667 \pm 0.12)\ \text{fm}$$

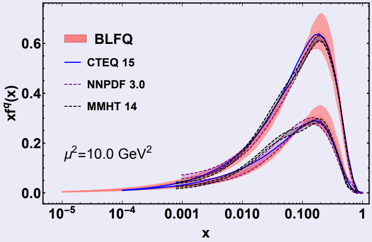


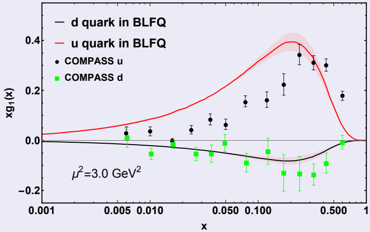
$$G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$

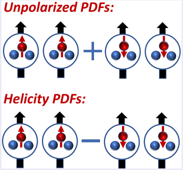
[1] CM, Siqi Xu *et. al.*, Phys. Rev. D **102**, 016008 (2020)

Parton Distribution Functions

Xu, CM, Lan, Zhao, Li, and Vary, PRD 104, 094036 (2021)





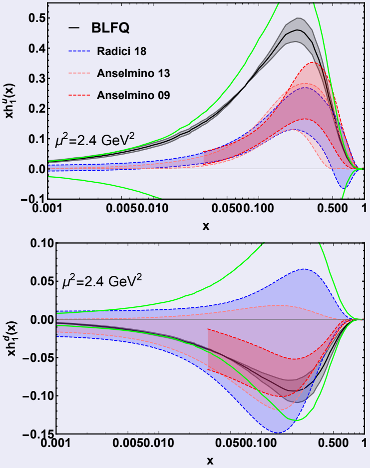


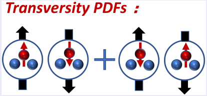
- Unpolarized PDFs $f_1(x)$: longitudinal momentum distribution of unpol. quark in unpol. proton.
- Helicity PDFs $g_1(x)$: longitudinal momentum distribution of the polarized quark
- Results correspond to leading Fock sector only.

[1] NNPDF, EPJC 77, 663 (2017); HMMT, EPJC 75, 204 (2015); CTEQ, PRD 93, 033006 (2016).
[2] COMPASS Collaboration, Phys. Lett. B 693, 227 (2010).

Transversity Distribution

Xu, CM, Lan, Zhao, Li, and Vary, PRD 104, 094036 (2021)





- Transversity PDFs describe correlation between the transverse polarization of the nucleon and the transverse polarization of the parton.
- Satisfy Soffer Bound:

$$|h_1(x)| \leq \frac{1}{2}|f_1(x) + g_1(x)|$$

- Results correspond to leading Fock sector only, missing higher Fock sectors.

[1] M. Radici and A. Bacchetta, Phys. Rev. Lett. 120, 192001 (2018).
[2] M. Anselmino, *et. al.*, Phys. Rev. D 87, 094019 (2013).

GPDs for Spin-1/2 Target

$$\frac{P^+}{2\pi}\int dy^- e^{ixP^+y^-}\langle p'|\bar{\psi}_q(-y/2)\gamma^+\psi_q(y/2)|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$
$$= H^q(x,\xi,t)\,\bar{u}(p')\gamma^+u(p) + E^q(x,\xi,t)\,\bar{u}(p')i\sigma^{+\nu}\frac{\Delta_\nu}{2M_n}u(p)\,,$$

Off-forward matrix elements

In momentum space no probabilistic interpretation

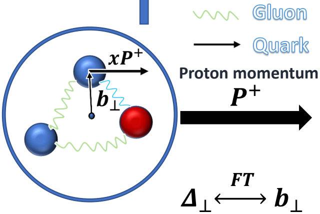
➢GPDs in impact parameter space: $\mathcal{X}(x,b) = \frac{1}{2\pi}\int d^2\Delta e^{-i\Delta^\perp\cdot b^\perp}\mathcal{X}(x,t).$

At t=0, 2nd moment of GPDs: angular momentum

$$J^q = \frac{1}{2}[A^q(0) + B^q(0)]$$

Second moment of GPDs give gravitational FFs

$$\int_0^1 dx\ xH_v^q(x,t) = A^q(t), \quad \int_0^1 dx\ xE_v^q(x,t) = B^q(t)$$



[1] Ji, Phys. Rev. Lett. 78, 610 (1997); Burkardt, Int. J. Mod. Phys. A 18, 173-208 (2003)

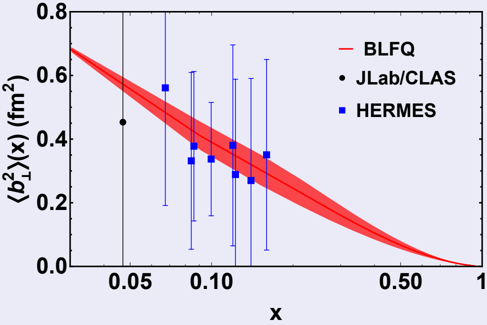
x-Dependent Squared Radius

$$\langle b_\perp^2 \rangle^q(x) = \frac{\int d^2\vec{b}_\perp b_\perp^2 H^q(x, b_\perp)}{\int d^2\vec{b}_\perp H^q(x, b_\perp)},$$

- Transverse squared radius:

$$\langle b_\perp^2 \rangle = \sum_q e_q \int_0^1 \mathrm{d}x f^q(x) \langle b_\perp^2 \rangle^q(x)$$

- BLFQ result: $\langle b_\perp^2 \rangle = 0.40 \pm 0.04$ fm^2
- Experimental data [2]: $\langle b_\perp^2 \rangle_{\text{exp}} = 0.43 \pm 0.01$ fm^2



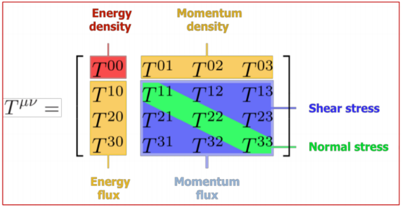
[1] Xu, CM, Lan, Zhao, Li, and Vary, PRD 104, 094036 (2021)
[2] R. Dupre, M. Guidal and M. Vanderhaeghen, PRD 95, 011501 (2017).

Nucleon Gravitational Form Factors

Nucleon scattering by the classical gravitational field is described by the gravitational (energy momentum tensor) form factors (GFFs).

$$\langle P'|T_i^{\mu\nu}(0)|P\rangle = \bar{U}'\Big[-B_i(q^2)\frac{\bar{P}^\mu\bar{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\bar{P}^\nu+\gamma^\nu\bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2)Mg^{\mu\nu}\Big]U$$

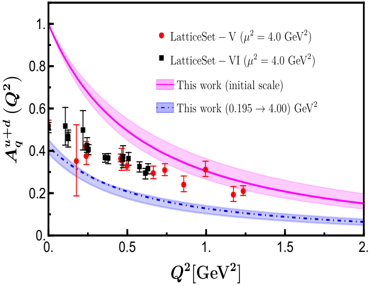
- Matrix elements of the energy momentum tensor (EMT) contain fundamental information about various mechanical properties.

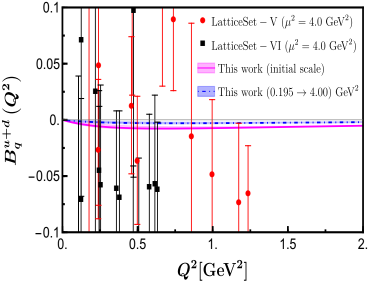


[1] Swagato Mukherjee's talk.

Preliminary Results : $A_q(Q^2)$ and $B_q(Q^2)$

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle$$





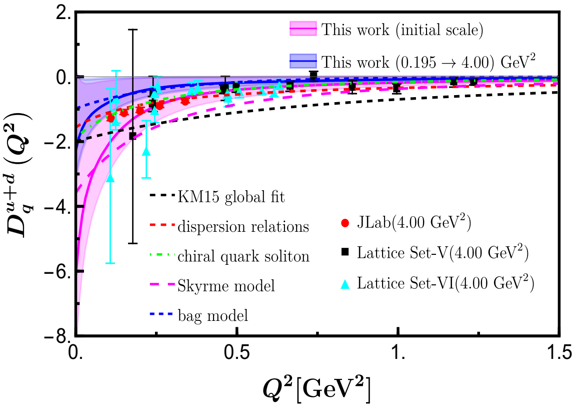
- $A(Q^2)$ and $B(Q^2)$ are extracted from the T^{++} component.
- Spin sum rule: $J^i = \frac{1}{2}\left(A^i(0) + B^i(0)\right)$

$$\sum_q A^q(0) = 1 \text{ and } \sum_q B^q(0) = 0$$

[1] S. Nair *et. al.*, in preparation.

Preliminary Results : $D_q(Q^2) = 4C_q(Q^2)$

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle$$



- Qualitative behavior of $D_q(Q^2)$ is compatible with lattice and experimental data.
- Provides mechanical properties.

Angular Momentum Distributions in Transverse Plane

- Kinetic OAM and spin distributions

Lorcé et al., PLB 776 (2018)

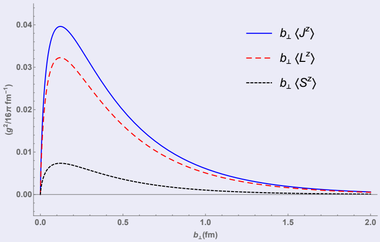
$$\langle L^z \rangle (b_\perp) = -\iota\varepsilon^{3jk} \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, \frac{\partial\langle T^{+k}\rangle}{\partial\Delta_\perp^j}\Big|_{\mathrm{DY}}$$

$$= \Lambda^z \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp} \left[L(t) + t\,\frac{dL(t)}{dt}\right]$$

$$\langle S^z\rangle(b_\perp) = \frac{1}{2}\,\varepsilon^{3jk} \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, \langle S^{+jk}\rangle\Big|_{\mathrm{DY}}$$

$$= \frac{\Lambda^z}{2} \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\, e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, G_A(t)$$

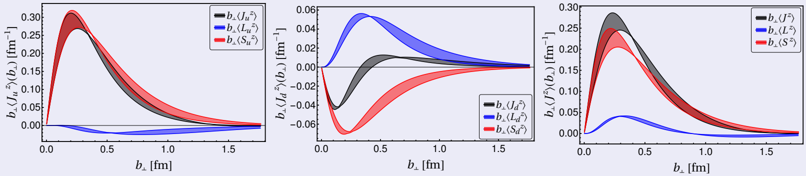
where $L(t) = \frac{1}{2}\left(A(t) + B(t) - G_A(t)\right)$



$$\langle J^z \rangle (b_\perp) = \langle L^z \rangle (b_\perp) + \langle S^z \rangle (b_\perp)$$

Cédric Lorcé et al., Phys. Lett. B 776 (2018) 38-47

Flavor contributions: [*Y. Liu, S. Xu, CM, X. Zhao, J. P. Vary, Phys. Rev. D* ***105****, 094018 (2022)*]

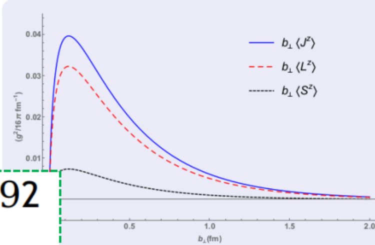


Angular Momentum Distributions in Transverse Plane

- The $b_\perp$ dependent distributions of kinetic OAM and spin in light-front:

$$\langle L^z\rangle(b_\perp) = -\iota\varepsilon^{3jk}\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\,e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,\frac{\partial\langle T^{+k}\rangle}{\partial\Delta^j_\perp}\Big|_{\mathrm{DY}}$$

$$= \Lambda^z\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\,e^{-\iota\vec{\Delta}_\perp\cdot\vec{b}_\perp}\left[L(t) + t\,\frac{dL(t)}{\ldots}\right]$$



$$\Delta\Sigma_q \approx 0.92$$

$$\Delta\Sigma_u \approx 1.18$$

$$\Delta\Sigma_d \approx -0.26$$

Quark Helicity > 90%

No Gluon contribution

$$\langle S^z\rangle(b_\perp) = \frac{1}{2}\varepsilon^{3jk}\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\,e^{-\ldots}$$

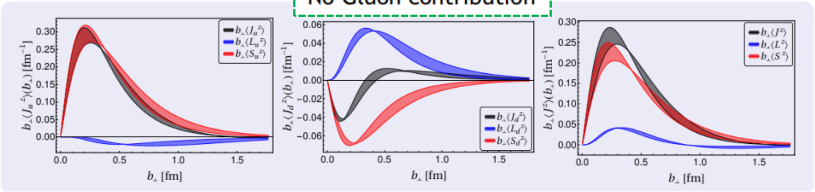
$$= \frac{\Lambda^z}{2}\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\,e^{-\iota\vec{\Delta}_\perp \ldots}$$

$\ldots(b_\perp) = \langle L^z\rangle(b_\perp) + \langle S^z\rangle(b_\perp)$

Lorcé et al., Phys. Lett. B 776 (2018) 38-47

where $L(t) == \frac{1}{2}(A(t) + B(t\ldots$

Flavor contributions: [*Y. Liu, S… Rev. D* ***105****, 094018 (2022)*]



Effective Hamiltonian with One Dynamical Gluon

$$|\,\text{Baryon}\rangle = a\,|\,qqq\rangle + b\,|\,qqqg\rangle + c\,|\,qqqq\bar{q}\rangle + \ldots$$

kinetic energy

transverse confining potential [2]

$$H_{\text{eff}} = \sum_a \frac{\vec{p}^{\,2}_{\perp a} + m_a^2}{x_a} + \frac{1}{2}\sum_{a\neq b}\kappa^4\left[x_a x_b(\vec{r}_{\perp a} - \vec{r}_{\perp b})^2\right]$$

$$- \frac{1}{2}\sum_{a\neq b}\kappa^4\left[\frac{\partial_{x_a}(x_a x_b \partial_{x_b})}{(m_a+m_b)^2}\right] + H_{\text{vertex}} + H_{\text{inst}}$$

longitudinal confining potential [3]

QCD interactions [4]

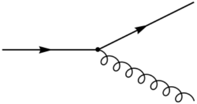
[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[2] Brodsky, Teramond, Dosch and Erlich, Phys. Rep. 584, 1 (2015).
[3] Li, Maris, Zhao and Vary, Phys. Lett. B (2016).
[4] Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998).

Light-Front QCD Hamiltonian

[Brodsky et al, 1998]

$$P_{-,LFQCD} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\,\psi - \frac{1}{2}\int d^3x\, A_a^i (i\partial^\perp)^2 A_a^i$$

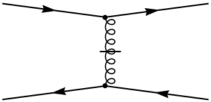
$$+g\int d^3x\, \bar{\psi}\gamma_\mu A^\mu\, \psi$$



$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+}\gamma_\nu A^\nu \psi$$

$$-ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^+ T^c\psi \frac{1}{(i\partial^+)^2}(i\partial^+ A_a^\mu A_{\mu b})$$

$$+\frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^+ T^a\psi \frac{1}{(i\partial^+)^2}\bar{\psi}\gamma^+ T^a\psi$$



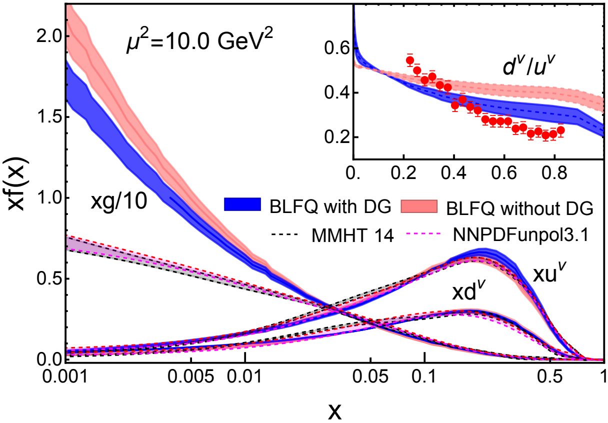
$$+ig\int d^3x\, f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$-\frac{1}{2}g^2\int d^3x\, f^{abc} f^{ade}\, i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2}(i\partial^+ A_d^+ A_{\nu e})$$

$$+\frac{1}{4}g^2\int d^3x\, f^{abc} f^{ade}\, A_b^\mu A_c^\nu A_{\mu d} A_{\nu e}.$$

6

Unpolarized PDFs



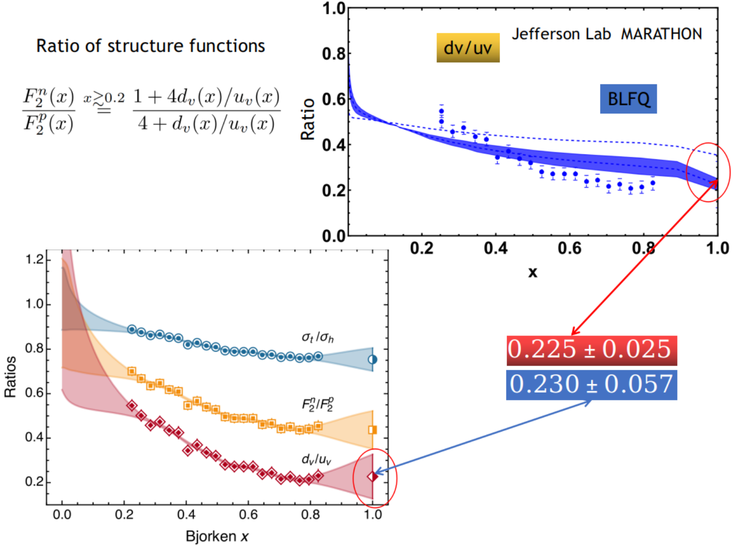
- Within $|qqq\rangle$, model scale $\mu_0^2 = 0.195 \pm 0.020$ GeV^2
- Gluon is generated dynamically from the QCD evolution.
- Including dynamical gluon (DG), model scale $\mu_0^2 = 0.23 - 0.25$ GeV^2
- Including DG, gluon distribution is closer to global fits.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].

Valence Quark Ratio in Proton

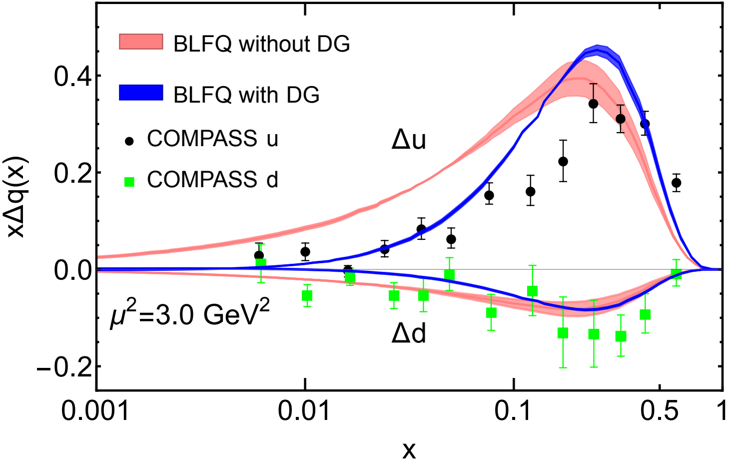
Ratio of structure functions

$$\frac{F_2^n(x)}{F_2^p(x)} \overset{x \gtrsim 0.2}{=} \frac{1+4d_v(x)/u_v(x)}{4+d_v(x)/u_v(x)}$$



[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[2] Cui, Gao, Binosi, Chang, Roberts and Schmidt, Chin. Phys. Lett. **39**, 041401 (2022).

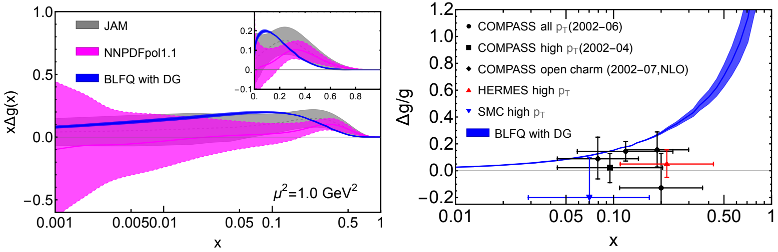
Quark Helicity PDFs



- Distributions improve at small x and $x > 0.5$.
- $\frac{1}{2}\Sigma_u = 0.438 \pm 0.004$, strongly dominates over $\frac{1}{2}\Delta\Sigma_d = -0.080 \pm 0.002$.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[2] M. G. Alekseev et al. (COMPASS Collaboration), Phys. Lett. B 693, 227 (2010).

Gluon Helicity Distriution

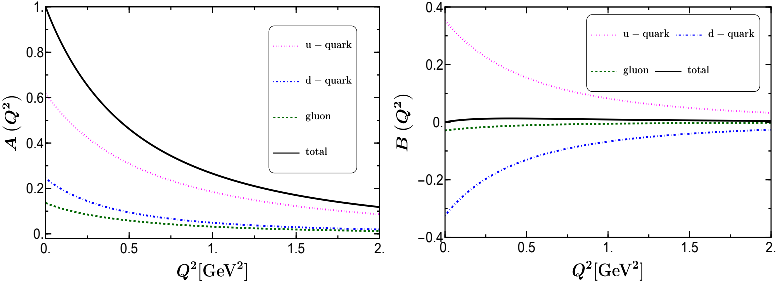


- $\Delta G = \int_0^1 \mathrm{d}x \Delta g(x) = 0.131 \pm 0.003$, is sizeable to the proton spin.
- PHENIX Collaboration: $\Delta G^{[0.02,0.3]} = 0.2 \pm 0.1$ PRL 103 (2009) 012003
- Resolving small-x issue is one of the major goals of the future EICs.

[1] S. Xu, CM, X. Zhao, Y. Li, J. P. Vary, 2209.08584 [hep-ph].
[2] N. Sato *et al.* [JAM], PRD**93** (2016); E. R. Nocera *et al.* [NNPDF], NPB 887 (2014).

Preliminary Results : $A_q(Q^2)$ and $B_q(Q^2)$

$$|\text{Nucleon}\rangle = \psi_{(3q)}|qqq\rangle + \psi_{(3q+1g)}|qqqg\rangle$$

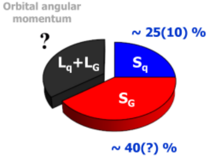


- $A(Q^2)$ and $B(Q^2)$ are extracted from the T^{++} component.
- Spin sum rule: $J^i = \frac{1}{2}\left(A^i(0) + B^i(0)\right)$

$$\boxed{\sum_i A^i(0) = 1 \text{ and } \sum_i B^i(0) = 0}$$

[1] S. Nair *et. al.*, in preparation.

GTMDs & OAM



Generalized Transverse-Momentum Parton Distribution functions

$$W^{[\gamma^+]}_{\lambda\lambda'}(P, x, \vec{k}_\perp, \Delta) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle p', \lambda' \middle| \bar{\psi}\left(-\frac{1}{2}z\right)\gamma^+\psi\left(\frac{1}{2}z\right) \middle| p, \lambda \right\rangle$$

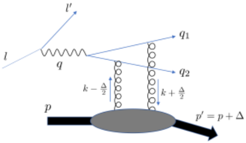
$$W^{[\delta^{ij}]}_{\lambda\lambda'}(P, x, \vec{k}_\perp, \Delta) = \frac{1}{xP^+}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle p', \lambda' \middle| G^{+i}\left(-\frac{1}{2}z\right) G^{+i}\left(\frac{1}{2}z\right) \middle| p, \lambda \right\rangle$$

Parameterization:

$$W^{[\gamma^+]}_{\lambda\lambda'}(P, x, \vec{k}_\perp, \Delta) = W^{[\delta^{ij}]}_{\lambda\lambda'}(P, x, \vec{k}_\perp, \Delta)$$
$$= \frac{1}{2M}\bar{u}(p', \lambda')\left[F_{1,1} + \frac{i\sigma^{j+}}{p^+}(k^j_\perp F_{1,2} + \Delta^j_\perp F_{1,3}) + i\frac{\sigma^{ij}k^i_\perp \Delta^j_\perp}{M^2} F_{1,4}\right] u(p, \lambda)$$

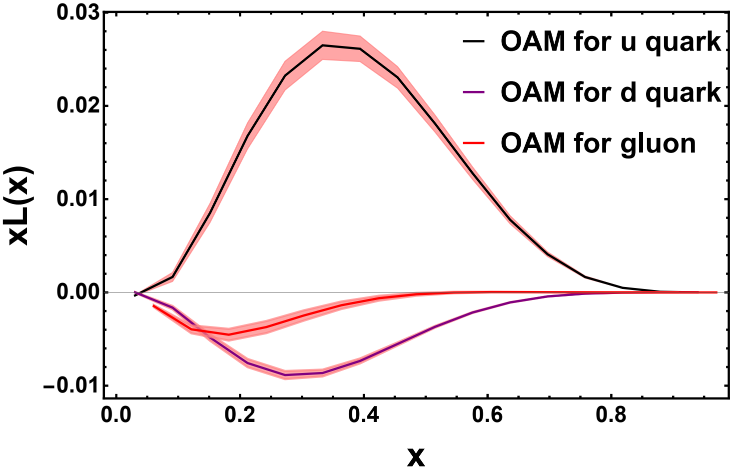
$F_{1,4}$ is related to the orbital angular momentum

$$L_{q,g}(x) = -\int d^2k_\perp \frac{k_\perp^2}{M^2} F_{1,4}(x, k_\perp, \Delta_\perp = 0)$$



[1] S. Bhattacharya, R. Boussarie and Y. Hatta, arXiv-hep:2201.08709 (2022)

Prelinary Results of OAM

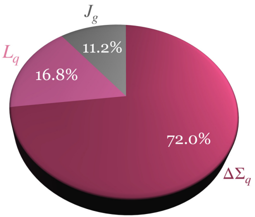


- Final remark : $\frac{1}{2}\Delta\Sigma_q = 0.36;\ \Delta\Sigma_g = 0.13;\ l_q = 0.02;\ l_g = -0.01$

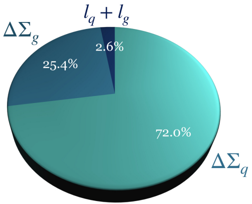
BLFQ Prediction on Spin Decomposition

With one dynamical gluon

(a) Kinetic



(b) Canonical



Conclusions

- Discussed structure of proton from eigenstates of LF effective Hamiltonians
- Considered $|qqq\rangle$ and $|qqqg\rangle$.
- LF Hamiltonian $\Rightarrow$ Wavefunctions $\Rightarrow$ Observables.
- Provides good description of data/global fits for various observables.
- Discussed gluon distributions of the nucleon.
- With one dynamical gluon, the quark spin contributes 70%; the gluon spin plays a substantial role (26%) in understanding the nucleon spin.
- *This is not a complete picture . . . long way to go*.

Enormous amount of possibilities with future EICs . . . . . . Thank You