

Accessing Linearly polarized gluon TMD in back to back J/ Ψ and jet production at EIC

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Plan of Talk

- Gluon TMDs
- Kinematics
- Azimuthal Asymmetry
- TMD parameterization
- Results and Discussion
- Conclusion

Gluon TMDs

- ❖ TMD PDFs $f(x, p_T, Q^2)$
- ❖ The Gluon Correlator

Field Strengths Gauge Links

$$\Phi^{\mu\nu}(x, q_T) = \int \frac{d\xi^- d^2 \xi_T}{M_p (2\pi)^3} e^{iq \cdot \xi} \left\langle P | Tr [F^{+\mu}(0) \boxed{U^{[C]}} F^{+\nu}(\xi) \boxed{U^{[C]}}] | P \right\rangle \Big|_{\xi^+ = 0}$$

- ❖ For unpolarized proton,

Mulders(2001)

$$\Phi_g^{\nu\nu'}(x, \mathbf{p}_T^2) = -\frac{1}{2x} \left\{ g_\perp^{\nu\nu'} \boxed{f_1^g(x, \mathbf{p}_T^2)} - \left(\frac{p_T^\nu p_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'} \frac{\mathbf{p}_T^2}{2M_p^2} \right) \boxed{h_1^{\perp g}(x, \mathbf{p}_T^2)} \right\}$$

Unpolarized G TMD

Linearly polarized G TMD

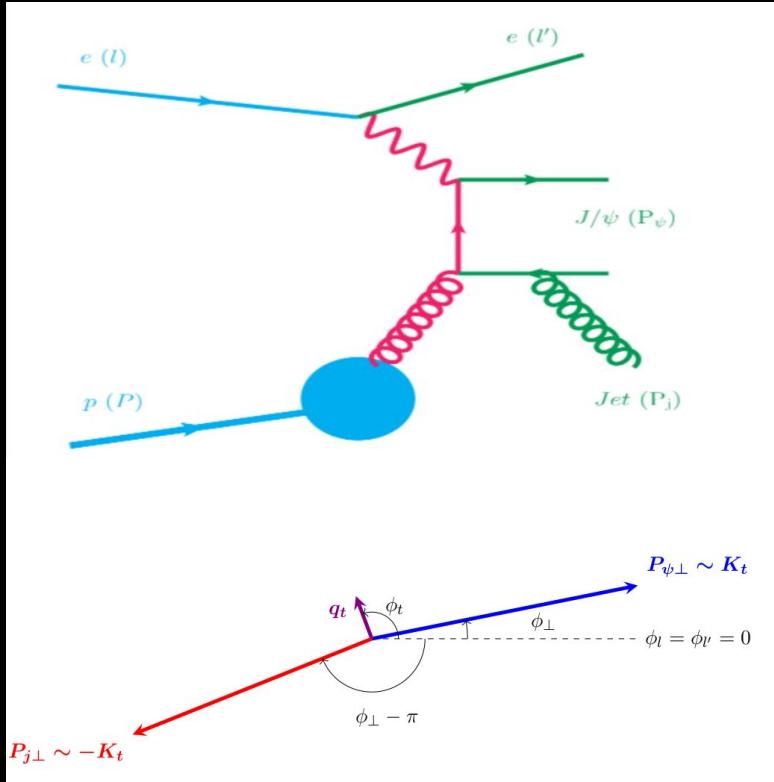
Linearly Polarized Gluon TMD $h_1^{\perp g}(x, \mathbf{p}_T^2)$

- Probed in SIDIS and Drell-Yan processes
- $\cos 2\phi_t$ Azimuthal asymmetries can help to extract
- Not extracted from data
- In small-x region it can be of Weizsäcker Williams type or Dipole type with gauge link ++ or -- and +- or -+ of both
- Positivity bound

$$\frac{q_t^2}{2M_p^2} |h_1^{\perp g}(x, q_t^2)| \leq f_1^g(x, q_t^2)$$

Kinematics

$$e^-(l) + p(P) \rightarrow e^-(l') + J/\psi(P_\psi) + Jet(P_j) + X$$



- $\gamma^* + g \rightarrow c\bar{c}(^{2S+1}L_J^{(1,8)}) + g$ D'Alesio (2019)
- Virtual photon and proton travels along $\pm z$ -axis
- Leptonic plane \Rightarrow measuring azimuthal angles

$$Q^2 = -q^2, \ s = (P + l)^2,$$

$$x_B = \frac{Q^2}{2P \cdot q}, \ y = \frac{P \cdot q}{P \cdot l}, \ z = \frac{P \cdot P_\psi}{P \cdot q}$$

$$\mathbf{q}_t \equiv \mathbf{P}_{\psi\perp} + \mathbf{P}_{j\perp}, \quad \mathbf{K}_t \equiv \frac{\mathbf{P}_{\psi\perp} - \mathbf{P}_{j\perp}}{2}$$

- In back to back scattering $\Rightarrow |\mathbf{q}_t|^2 \ll |\mathbf{K}_t|^2 \sim M_\psi^2$

\Rightarrow TMD factorization

- ϕ_t and ϕ_\perp

Scattering cross-section

TMD Factorization $\text{TMDs} \Rightarrow \text{Collinear} \otimes \text{Transverse}$
Observed states Delta function

$$d\sigma = \frac{1}{2s} \left[\frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_\psi}{2E_\psi (2\pi)^3} \frac{d^3 P_j}{2E_j (2\pi)^3} \right] \int dx d^2 p_T (2\pi)^4 \delta^4(q + p_g - P_j - P_\psi)$$

Diagram illustrating the decomposition of the scattering cross-section:

- The total cross-section $d\sigma$ is shown in red.
- The integral over phase space and momenta is shown in blue.
- The delta function constraint is shown in green.
- Below the integral:
 - The gluon correlator $\Phi_g^{\nu\nu'}(x, p_T)$ is shown in orange.
 - The lepton tensor $L^{\mu\mu'}(l, q)$ is shown in cyan.
 - The partonic matrix elements $\mathcal{M}_{\mu\nu}^{g\gamma^*\rightarrow J/\psi g}$ and $\mathcal{M}_{\mu'\nu'}^{*g\gamma^*\rightarrow J/\psi g}$ are shown in purple.
- Arrows indicate the flow from the observed states and delta function to the components below:

 - A red arrow points from "Gluon Correlator" to the gluon correlator term.
 - A cyan arrow points from "Leptonic Tensor" to the lepton tensor term.
 - A purple arrow points from "Partonic matrix elements" to the partonic matrix elements terms.

NRQCD factorization

- ★ $k^2 \ll M_c^2 \rightarrow$ Non-relativistic approx to QCD
- ★ matrix element in NRQCD

$$\mathcal{M}^{ab \rightarrow J/\psi} = \boxed{\sum_n \mathcal{M}[ab \rightarrow c\bar{c} \left({}^{2S+1} L_J^{(1,8)} \right)] \langle 0 | \mathcal{O}^{J/\psi} \left({}^{2S+1} L_J^{(1,8)} \right) | 0 \rangle}$$

Perturbative part

Non-perturbative part
(LDMes)

Bodwin, Braaten,
Lepege (1994)

J/ Ψ and Jet formation using NRQCD

Angular momentum
wave function

$$\begin{aligned} \mathcal{M}\left(\gamma^* g \rightarrow c\bar{c}[^{2S+1}L_J^{(1,8)}]g\right) &= \sum_{L_z S_z} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \boxed{\Psi_{LL_z}(\mathbf{k})} \langle LL_z; SS_z | JJ_z \rangle \\ &\times \textcolor{red}{Tr} \boxed{O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)} \end{aligned}$$

Feynman Diagram
contribution

Spin projection
operator for J/ Ψ

J/ Ψ formation using NRQCD

$k^2 \ll M_c^2$ limit, Matrix element \Rightarrow Taylor series $|_{k=0}$

LDMEs

S-Wave scattering amp

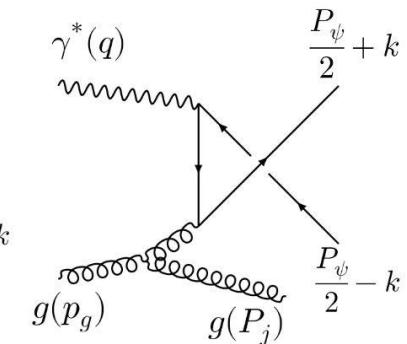
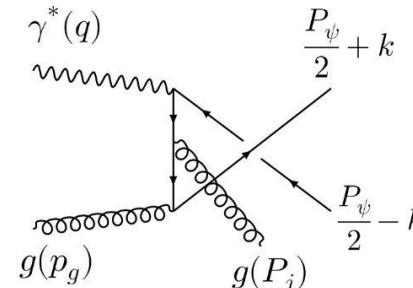
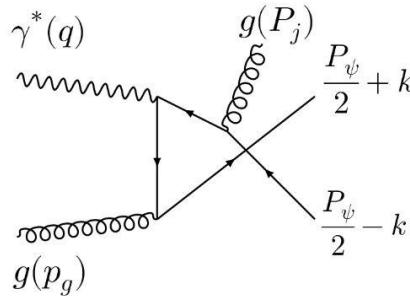
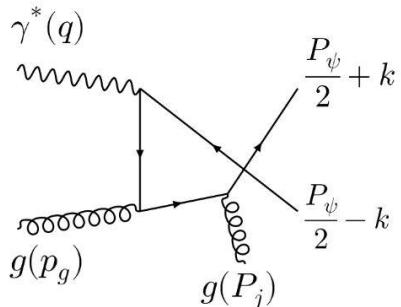
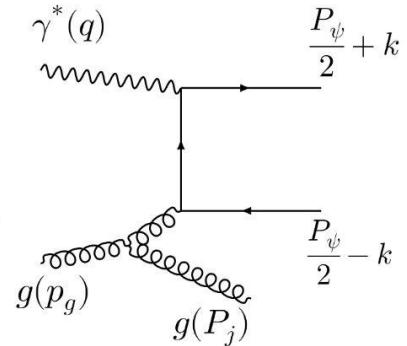
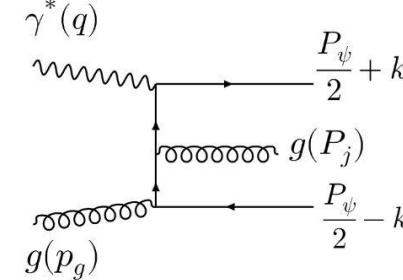
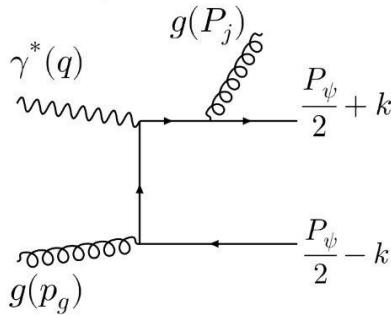
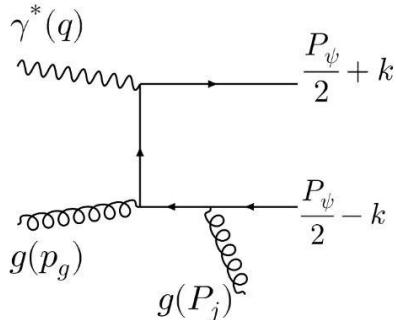
$$\begin{aligned} & \rightarrow \boxed{\mathcal{M}[^{2S+1}S_J^{(1,8)}]} \\ & \rightarrow \boxed{\mathcal{M}[^{2S+1}P_J^{(8)}]} \end{aligned}$$

$$\begin{aligned} & = \frac{1}{\sqrt{4\pi}} \boxed{R_0(0)} \overline{\langle O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k) \rangle} |_{k=0} \\ & = -i \sqrt{\frac{3}{4\pi}} \boxed{R'_1(0)} \sum_{L_z S_z} \varepsilon_{L_z}^\alpha(P_\psi) \langle LL_z; SS_z | JJ_z \rangle \\ & \quad \times \frac{\partial}{\partial k^\alpha} Tr[O(q, p_g, P_\psi, k) \mathcal{P}_{SS_z}(P_\psi, k)] |_{k=0} \\ & = \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1; \frac{1}{2} s_2 \middle| SS_z \right\rangle_V \left(\frac{P_\psi}{2} - k, s_1 \right) \bar{u} \left(\frac{P_\psi}{2} + k, s_2 \right) \\ & = \frac{1}{4M_\psi^{3/2}} (-\not{P}_\psi + 2\not{k} + M_\psi) \Pi_{SS_z}(\not{P}_\psi + 2\not{k} + M_\psi) + \mathcal{O}(k^2) \end{aligned}$$

P-Wave scattering amp

$$\mathcal{P}_{SS_z}(P_\psi, k)$$

Feynman Diagrams



Azimuthal Asymmetry

$$\frac{d\sigma}{dz dy dx_B d^2 q_t d^2 K_t} = d\sigma^U + d\sigma^L$$

$$d\sigma^U = \frac{1}{(2\pi)^4} \frac{1}{16sz(1-z)Q^4} (\mathbb{A}_0 + \mathbb{A}_1 \cos \phi_\perp + \mathbb{A}_2 \cos 2\phi_\perp) f_1^g(x, q_t^2)$$

$$d\sigma^L = \frac{1}{(2\pi)^4} \frac{1}{16sz(1-z)Q^4} (\mathbb{B}_0 \cos 2\phi_t + \mathbb{B}_1 \cos(2\phi_t - \phi_\perp) + \mathbb{B}_2 \cos 2(\phi_t - \phi_\perp) + \mathbb{B}_3 \cos(2\phi_t - 3\phi_\perp) + \mathbb{B}_4 \cos(2\phi_t - 4\phi_\perp)) \frac{q_t^2}{M_p^2} h_1^{\perp g}(x, q_t^2).$$

D'Alesio (2019)

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = 2 \frac{\int d\phi_t d\phi_\perp \cos 2\phi_t d\sigma(\phi_t, \phi_\perp)}{\int d\phi_t d\phi_\perp d\sigma(\phi_t, \phi_\perp)}$$

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = \frac{\int q_t dq_t \frac{q_t^2}{M_p^2} \mathbb{B}_0 h_1^{\perp g}(x, q_t^2)}{\int q_t dq_t \mathbb{A}_0 f_1^g(x, q_t^2)}$$

$\frac{q_t^2}{2M_p^2} |h_1^{\perp g}(x, q_t^2)| = f_1^g(x, q_t^2)$

$A^{\cos 2\phi_t} \rightarrow U = \frac{2*\mathbb{B}_0}{\mathbb{A}_0}$

Gaussian Parameterization

- ★ Drell-Yan and SIDIS \Rightarrow transverse momentum spectra \rightarrow roughly Gaussian in nature. [Stefano Melis \(2014\)](#)

PDFs

TMDs

$$f_1^g(x, \mathbf{q}_t^2) = f_1^g(x, Q) \frac{1}{\pi \langle \mathbf{q}_t^2 \rangle} e^{-\mathbf{q}_t^2 / \langle \mathbf{q}_t^2 \rangle}$$
$$h_1^{\perp g}(x, \mathbf{q}_t^2) = \frac{M_p^2 f_1^g(x, Q)}{\pi \langle \mathbf{q}_t^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{\mathbf{q}_t^2}{r \langle \mathbf{q}_t^2 \rangle}}$$

Transverse momentum dependent part

$$\frac{\mathbf{q}_t^2}{2M_p^2} |h_1^{\perp g}(x, \mathbf{q}_t^2)| \leq f_1^g(x, \mathbf{q}_t^2).$$

[D. Boer and C. Pisano \(2012\)](#)

Spectator Model

- In this model proton $\Rightarrow g+X$
- Remaining particle treated as spectator particle
- spectator particle \Rightarrow On-Shell $\Rightarrow M_x \Rightarrow$ continuous values
- P-g-X coupling is given by vertex \Rightarrow 2 form factors
- Model parameters fixed by fitting with collinear gluon distribution.

Spectator Model

$$F^g(x, q_t^2) = \int_M^\infty dM_X \rho_X(M_X) \boxed{\hat{F}^g(x, q_t^2; M_X)}$$

TMDs

$$\boxed{\rho_X(M_X)} = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

Model dependent form factors

Spectral function

$$\boxed{\hat{f}_1^g(x, q_t^2; M_X)} = -\frac{1}{2} g^{ij} [\Phi^{ij}(x, q_t, S) + \Phi^{ij}(x, q_t, -S)]$$

$$= [(2Mxg_1 - x(M + M_X)g_2)^2 [(M_X - M(1-x))^2 + q_t^2]$$

$$+ 2q_t^2 (q_t^2 + xM_X^2) g_2^2 + 2q_t^2 M^2 (1-x) (4g_1^2 - xg_2^2)]$$

$$\times [(2\pi)^3 4xM^2 (L_X^2(0) + q_t^2)^2]^{-1}$$

$$\boxed{\hat{h}_1^{\perp g}(x, q_t^2; M_X)} = \frac{M^2}{\varepsilon_t^{ij} \delta^{jm} (q_t^j q_t^m + g^{jm} q_t^2)} \varepsilon_t^{ln} \delta^{nr} [\Phi^{nr}(x, q_t, S) + \Phi^{nr}(x, q_t, -S)]$$

$$= [4M^2 (1-x) g_1^2 + (L_X^2(0) + q_t^2) g_2^2] \times [(2\pi)^3 x (L_X^2(0) + q_t^2)^2]^{-1}$$

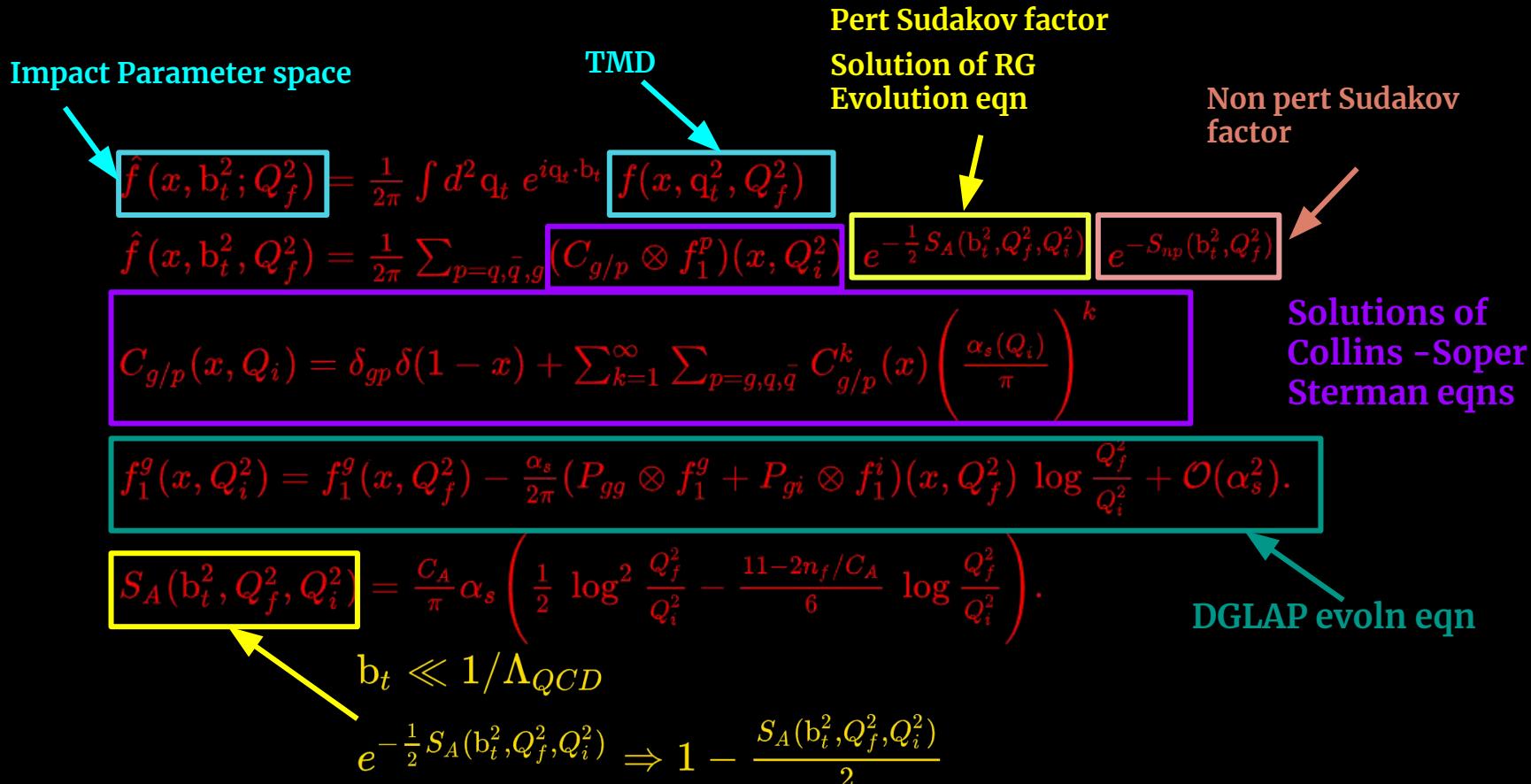
Spectator Model

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2} = \kappa_{1,2} \frac{p^2 (1-x)^2}{(q_t^2 + L_X^2(\Lambda_X^2))^2}$$

Diagram illustrating the components of the gluon momentum function:

- gluon momentum** (cyan text) points to the p^2 term.
- Normalization constants** (green text) points to the $\kappa_{1,2}$ term.
- Cut-off parameters** (blue text) points to the Λ_X^2 term.
- Off-shellness of gluon** (magenta text) points to the $L_X^2(\Lambda_X^2)$ term.

TMD Evolution



TMD Evolution

non perturbative region

functional form ([Scarpa\(2020\)](#))

**bt range (0.0-1.5 GeV²) ([C. P. Yuan \(2003\)](#),
Scarpa (2020))**

$$f_1^g(x, q_t^2) = \frac{1}{2\pi} \int_0^\infty b_t db_t J_0(b_t q_t) \left\{ f_1^g(x, Q_f^2) - \frac{\alpha_s}{2\pi} \left[\left(\frac{C_A}{2} \log^2 \frac{Q_f^2}{Q_i^2} - \frac{11C_A - 2n_f}{6} \log \frac{Q_f^2}{Q_i^2} \right) f_1^g(x, Q_f^2) + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x, Q_f^2) \log \frac{Q_f^2}{Q_i^2} - 2f_1^g(x, Q_f^2) \right] \right\} \times e^{-S_{np}(b_t^2)}.$$

$$\frac{q_t^2}{M_p^2} h_1^{\perp g(2)}(x, q_t^2) = \frac{\alpha_s}{\pi^2} \int_0^\infty db_t b_t J_2(q_t b_t) \left[C_A \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^g(\hat{x}, Q_f^2) + C_F \sum_{p=q,\bar{q}} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_1^p(\hat{x}, Q_f^2) \right] \times e^{-S_{np}(b_t^2)}.$$

$$S_{np} = \frac{A}{2} \log \left(\frac{Q_f}{Q_{np}} \right) b_c^2,$$

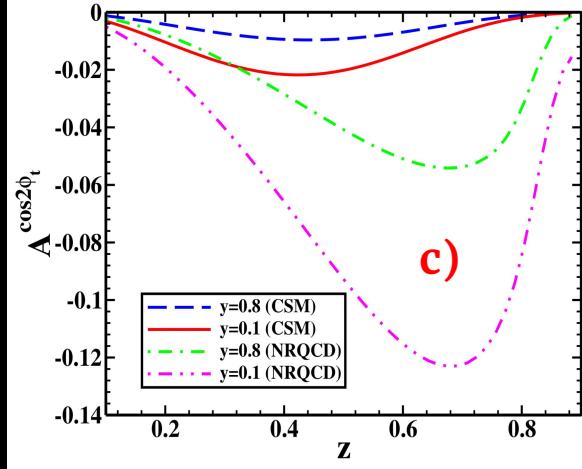
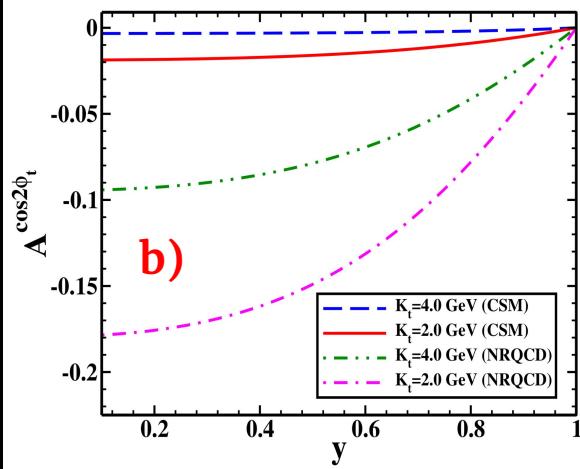
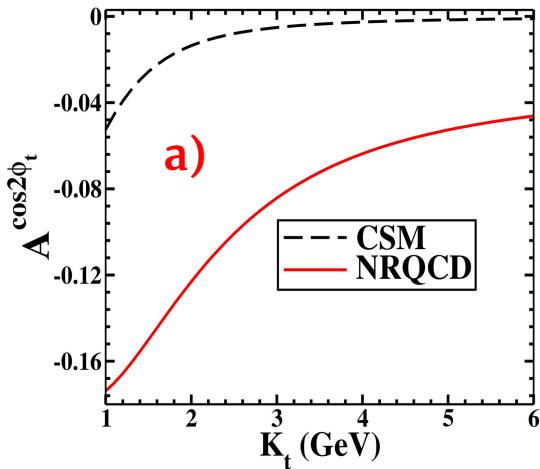
$$Q_{np} = 1 \text{ GeV},$$

$$b_c = \sqrt{b_t^2 + \left(\frac{2e^{-\gamma_E}}{Q_f} \right)^2}$$

Results and Discussion

Gaussian Parameterization

$\sqrt{s} = 140 \text{ GeV}$



$y \in (0,1)$

$z = 0.7$

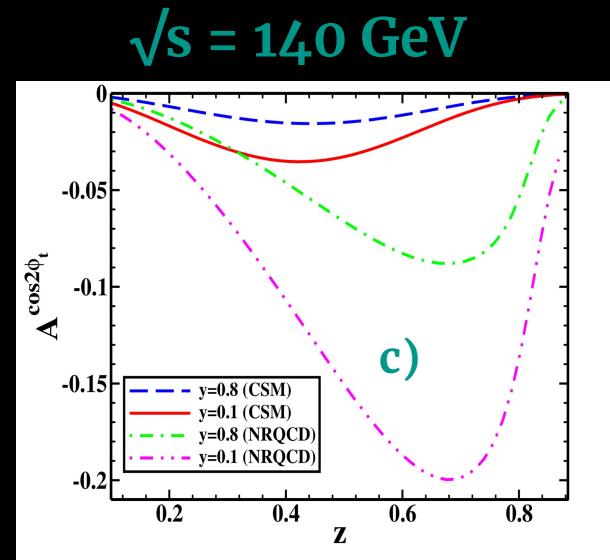
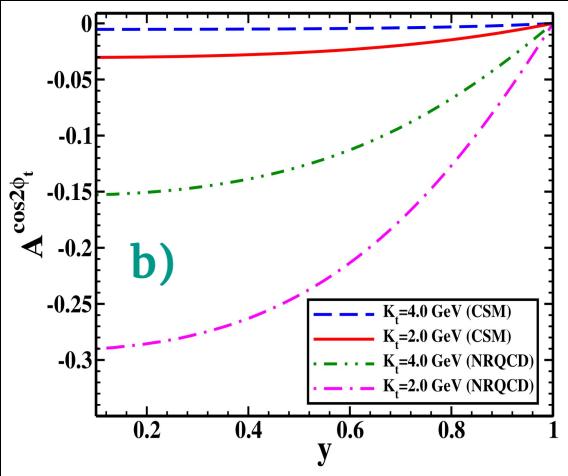
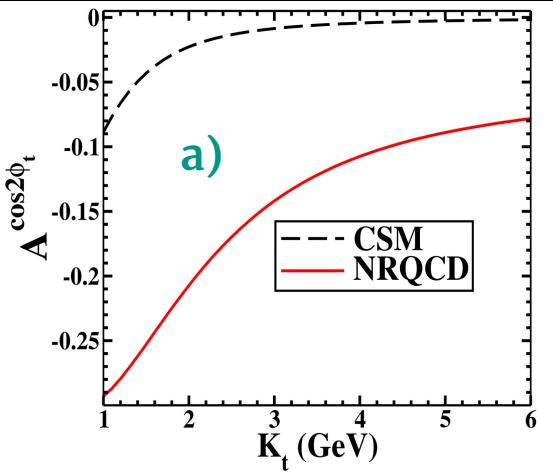
$K_t = 3.0 \text{ GeV}$

CMSWZ set of LDMES

$$Q^2 = M_\psi^2 + K_t^2$$

Results and Discussion

Spectator Model



$y \in (0,1)$

$z = 0.7$

$K_t = 3.0 \text{ GeV}$

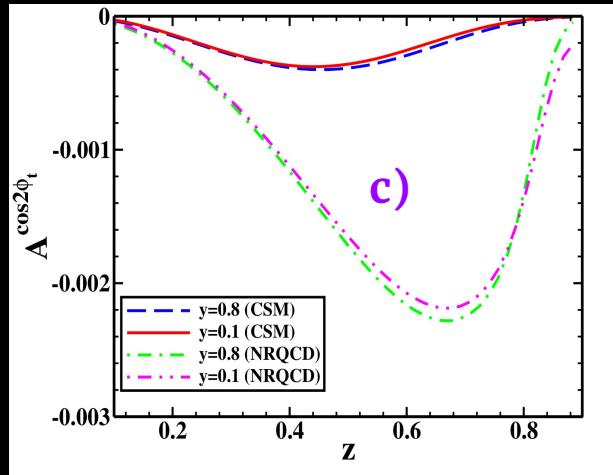
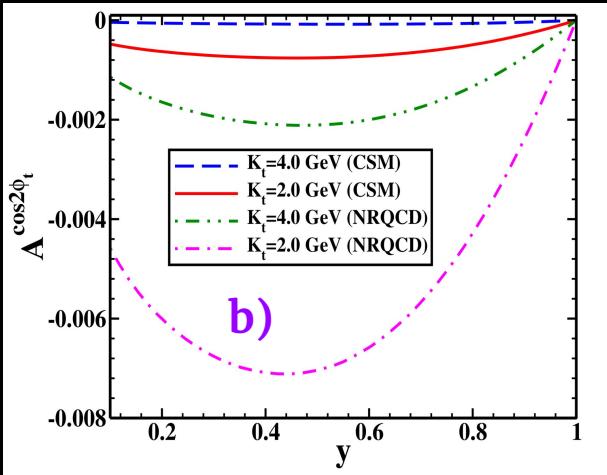
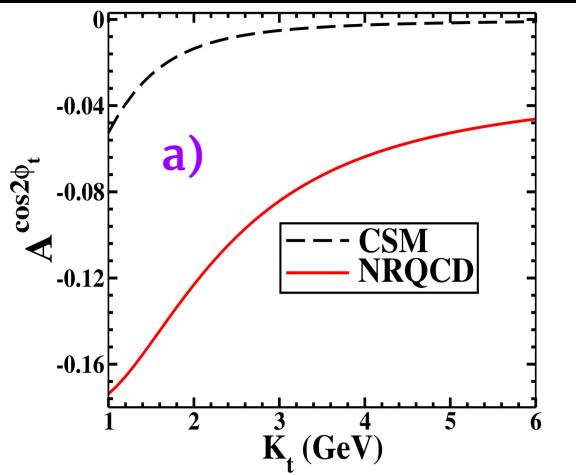
CMSWZ set of LDMES

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Results and Discussion

TMD Evolution

$\sqrt{s} = 140 \text{ GeV}$



$y \in (0,1)$

$z=0.7$

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CMSWZ set of LDMEs

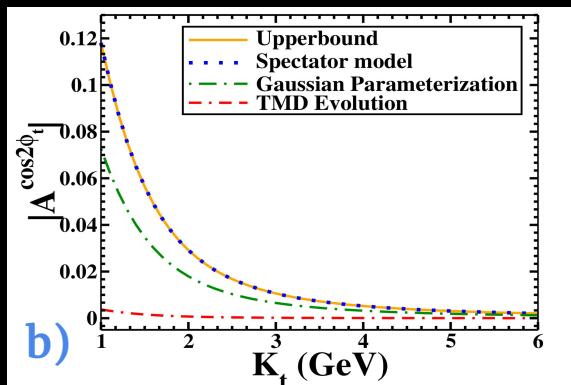
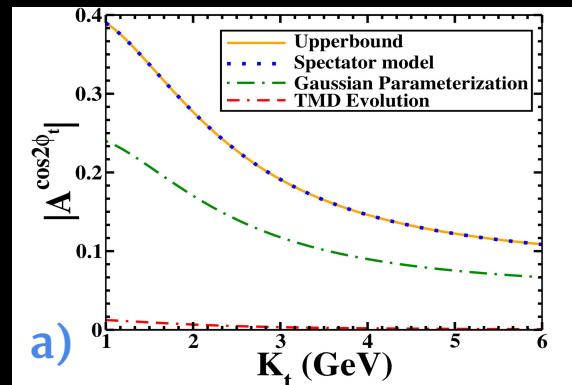
$$Q^2 = M_\psi^2 + K_t^2$$

Results and Discussion

Upperbound

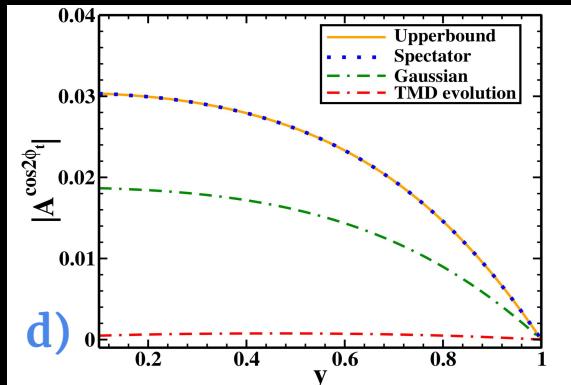
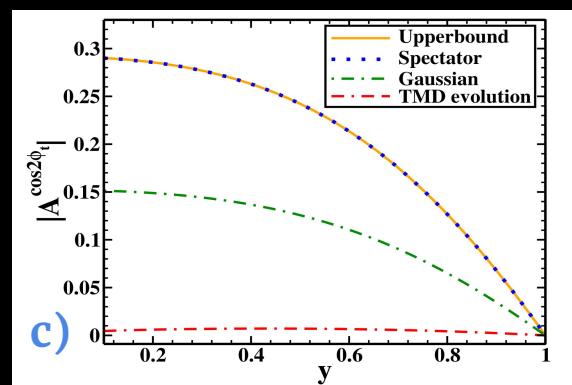
$\sqrt{s} = 140 \text{ GeV}$

$y=0.3$



$z=0.7$

$K_t=2.0 \text{ GeV}$



CMSWZ set of LDMEs

$$Q^2 = M_\psi^2 + K_t^2$$

Upperbound - Spectator model

$$\begin{aligned}
F^g(x, \mathbf{q}_t^2) &= \int_M^\infty dM_X \rho_X(M_X) \hat{F}^g(x, \mathbf{q}_t^2; M_X) & \frac{\mathbf{q}_t^2}{2M_p^2} |h_1^{\perp g}(x, \mathbf{q}_t^2)| &= f_1^g(x, \mathbf{q}_t^2) \\
\rho_X(M_X) &= \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right] & \langle \cos 2\phi_t \rangle &\equiv A^{\cos 2\phi_t} = 2 \frac{\int d\phi_t d\phi_\perp \cos 2\phi_t d\sigma(\phi_t, \phi_\perp)}{\int d\phi_t d\phi_\perp d\sigma(\phi_t, \phi_\perp)} \\
\hat{f}_1^g(x, \mathbf{q}_t^2; M_X) &= -\frac{1}{2} g^{ij} [\Phi^{ij}(x, \mathbf{q}_t, S) + \Phi^{ij}(x, \mathbf{q}_t, -S)] & \langle \cos 2\phi_t \rangle &\equiv A^{\cos 2\phi_t} = \frac{\int \mathbf{q}_t d\mathbf{q}_t \frac{\mathbf{q}_t^2}{M_p^2} \mathbb{B}_0 h_1^{\perp g}(x, \mathbf{q}_t^2)}{\int \mathbf{q}_t d\mathbf{q}_t \mathbb{A}_0 f_1^g(x, \mathbf{q}_t^2)} \\
&= [(2Mxg_1 - x(M + M_X)g_2)^2 [(M_X - M(1-x))^2 + \mathbf{q}_t^2] \\
&\quad + 2\mathbf{q}_t^2 (\mathbf{q}_t^2 + xM_X^2) g_2^2 + 2\mathbf{q}_t^2 M^2 (1-x) (4g_1^2 - xg_2^2)] \\
&\quad \times [(2\pi)^3 4xM^2 (L_X^2(0) + \mathbf{q}_t^2)^2]^{-1} \\
\hat{h}_1^{\perp g}(x, \mathbf{q}_t^2; M_X) &= \frac{M^2}{\varepsilon_t^{ij} \delta^{jm} (\mathbf{q}_t^j \mathbf{q}_t^m + g^{jm} \mathbf{q}_t^2)} \varepsilon_t^{ln} \delta^{nr} [\Phi^{nr}(x, \mathbf{q}_t, S) + \Phi^{nr}(x, \mathbf{q}_t, -S)] \\
&= [4M^2 (1-x) g_1^2 + (L_X^2(0) + \mathbf{q}_t^2) g_2^2] \times [(2\pi)^3 x (L_X^2(0) + \mathbf{q}_t^2)^2]^{-1}
\end{aligned}$$

Upperbound TMD evolution

$$f_1^g(x,{\mathbf q}_t^2) = \tfrac{1}{2\pi} \int_0^\infty {\mathbf b}_t d{\mathbf b}_t J_0({\mathbf b}_t {\mathbf q}_t) \Bigg\{ f_1^g(x,Q_f^2) \\ - \tfrac{\alpha_s}{2\pi} \Bigg[\left(\tfrac{C_A}{2} \; \log^2 \tfrac{Q_f^2}{Q_i^2} - \tfrac{11 C_A - 2 n_f}{6} \; \log \tfrac{Q_f^2}{Q_i^2} \right) f_1^g(x,Q_f^2) \\ + (P_{gg} \otimes f_1^g + P_{gi} \otimes f_1^i)(x,Q_f^2) \; \log \tfrac{Q_f^2}{Q_i^2} - 2 f_1^g(x,Q_f^2) \Bigg] \Bigg\} \times e^{-S_{np}({\mathbf b}_t^2)}.$$

$$\tfrac{{\mathbf q}_t^2}{M_p^2} h_1^{\perp g(2)}(x,{\mathbf q}_t^2) = \tfrac{\alpha_s}{\pi^2} \int_0^\infty d{\mathbf b}_{\mathsf t} {\mathbf b}_{\mathsf t} \; J_2({\mathbf q}_{\mathsf t} {\mathbf b}_{\mathsf t}) \; \Bigg[C_A \int_x^1 \tfrac{d\hat{x}}{\hat{x}} \left(\tfrac{\hat{x}}{x} - 1 \right) f_1^g(\hat{x},Q_f^2) \\ + C_F \sum_{p=q,\bar{q}} \int_x^1 \tfrac{d\hat{x}}{\hat{x}} \left(\tfrac{\hat{x}}{x} - 1 \right) f_1^p(\hat{x},Q_f^2) \Bigg] \times e^{-S_{np}({\mathbf b}_{\mathsf t}{}^2)}.$$

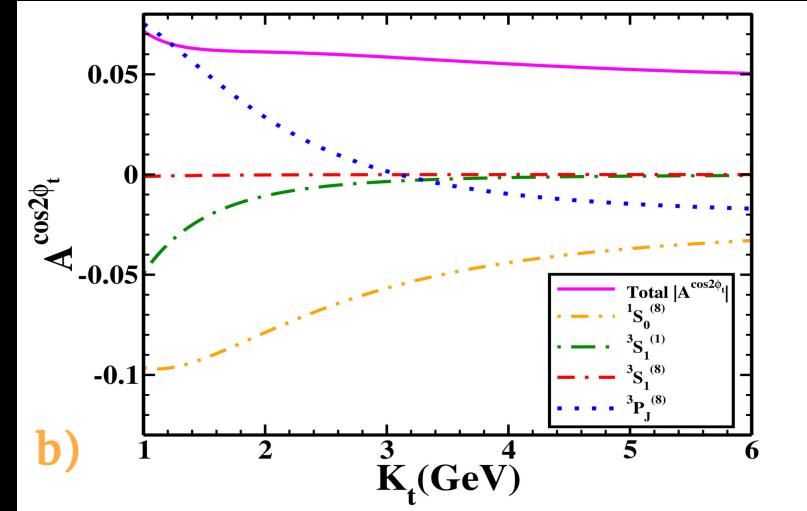
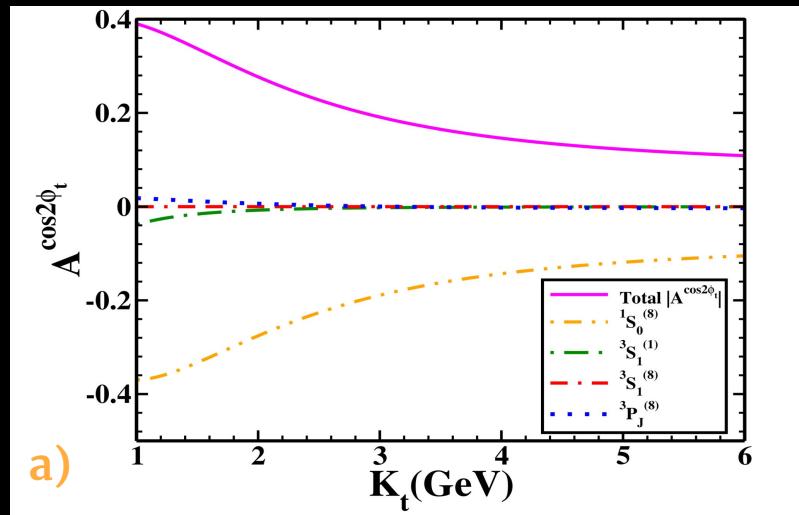
$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = 2 \frac{\int \mathrm{d}\phi_t \mathrm{d}\phi_\perp \cos 2\phi_t \mathrm{d}\sigma(\phi_t,\phi_\perp)}{\int \mathrm{d}\phi_t \mathrm{d}\phi_\perp \mathrm{d}\sigma(\phi_t,\phi_\perp)}$$

$$\langle \cos 2\phi_t \rangle \equiv A^{\cos 2\phi_t} = \frac{\int {\mathbf q}_t \; \mathrm{d}{\mathbf q}_t \; \tfrac{{\mathbf q}_t^2}{M_p^2} \; \mathbb{B}_0 \; h_1^{\perp g}(x,{\mathbf q}_t^2)}{\int {\mathbf q}_t \; \mathrm{d}{\mathbf q}_t \; \mathbb{A}_0 \; f_1^g(x,{\mathbf q}_t^2)}$$

Results and Discussion

LDMEs Comparison

$\sqrt{s} = 140 \text{ GeV}$



CMSWZ set of LDMEs

$z=0.7$

SV set of LDMEs

$y=0.3$

$$Q^2 = M_\psi^2 + K_t^2$$

Conclusion

- NRQCD \rightarrow J/ Ψ and Jet production rate
- parameterized gluon TMDs \rightarrow Gaussian, Spectator model and implemented TMD Evolution
- Asymmetry: TMD evolution < Gaussian < Spectator \leq Upperbound.
- These kinematical regions will be accessible at EIC.
- These results will help us to extract the Linearly polarized gluon TMD

Thank You