



भारतीय प्रौद्योगिकी संस्थान दिल्ली
Indian Institute of Technology Delhi



Diffraction Vector Meson Production Using Sartre With Machine Learning

QCD with Electron Ion Collider (QEIC) II

Jaswant Singh and Tobias Toll

December 19, 2022

Indian Institute of Technology, New Delhi

Contents

Electron-proton Scattering

Dipole Model

Electron-Ion Scattering

The Event Generator: *Sartre*

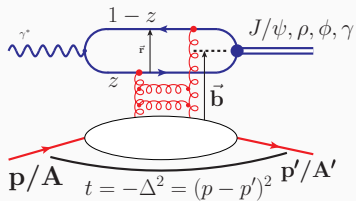
Machine Learning

Results

Conclusions

Electron-proton Scattering

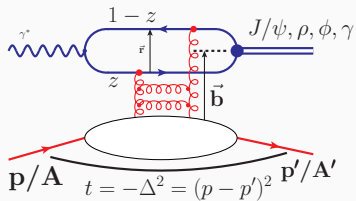
Diffractive Vector Meson Production



- Exclusive measurement :

$$e + p \rightarrow e + \text{VM}/\gamma + p$$

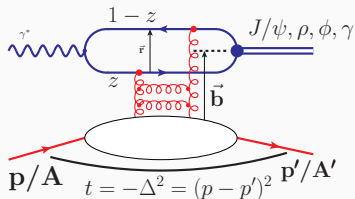
Diffraction Vector Meson Production



- Exclusive measurement :

$$e + p \rightarrow e + \text{VM}/\gamma + p$$
- Experimental Signature :
 Rapidity gap in final state particles

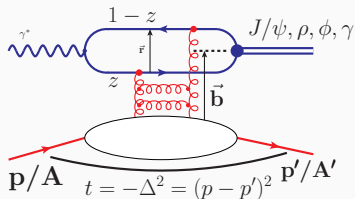
Diffractive Vector Meson Production



- Exclusive measurement :

$$e + p \rightarrow e + \text{VM}/\gamma + p$$
- Experimental Signature :
 Rapidity gap in final state particles
- Diffractive process :
 cross-section distribution resemble
 to that of diffraction in optics
 (color neutral exchange)

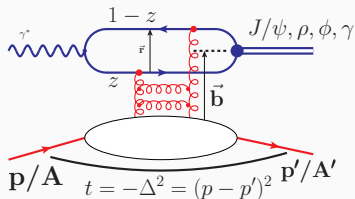
Diffractive Vector Meson Production



- Exclusive measurement :
 $e + p \rightarrow e + \text{VM}/\gamma + p$
- Experimental Signature :
Rapidity gap in final state particles
- Diffractive process :
cross-section distribution resemble
to that of diffraction in optics
(color neutral exchange)

- At HERA, 15% of total DIS events were diffractive events

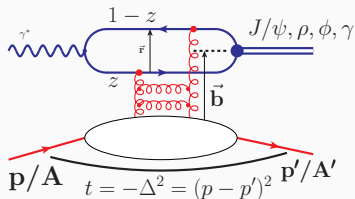
Diffractive Vector Meson Production



- Exclusive measurement :

$$e + p \rightarrow e + \text{VM}/\gamma + p$$
- Experimental Signature :
 Rapidity gap in final state particles
- Diffractive process :
 cross-section distribution resemble
 to that of diffraction in optics
 (color neutral exchange)
- At HERA, 15% of total DIS events were diffractive events
- Measure the momentum transfer $t = (p - p')^2$ and Fourier transform
 to get spatial structure

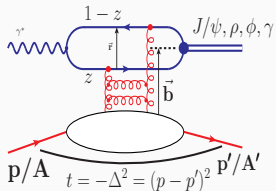
Diffractive Vector Meson Production



- Exclusive measurement :

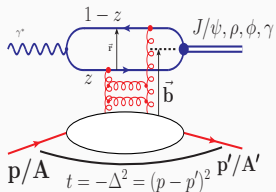
$$e + p \rightarrow e + \text{VM}/\gamma + p$$
 - Experimental Signature :
 Rapidity gap in final state particles
 - Diffractive process :
 cross-section distribution resemble
 to that of diffraction in optics
 (color neutral exchange)
-
- At HERA, 15% of total DIS events were diffractive events
 - Measure the momentum transfer $t = (p - p')^2$ and Fourier transform to get spatial structure
 - Sensitive to high gluon densities

Diffractive Vector Meson Production....



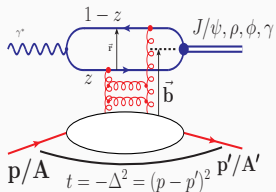
- Different stages of the Vector meson production:

Diffraction Vector Meson Production....



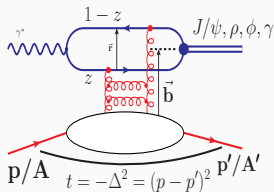
- Different stages of the Vector meson production:
 - $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$

Diffractive Vector Meson Production....



- Different stages of the Vector meson production:
 - $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$
 - $q\bar{q}$ dipole elastically scatters from the target

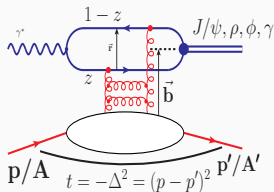
Diffractive Vector Meson Production....



■ Different stages of the Vector meson production:

- $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$
- $q\bar{q}$ dipole elastically scatters from the target
- $\psi_V^*(Q^2, r, z)$ is wavefunction for J/ψ

Diffractive Vector Meson Production....



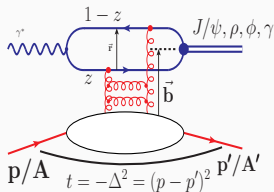
- Different stages of the Vector meson production:

- $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$
- $q\bar{q}$ dipole elastically scatters from the target
- $\psi_V^*(Q^2, r, z)$ is wavefunction for J/ψ

- Scattering amplitude is given by:

$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) = i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i \mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}$$

Diffractive Vector Meson Production....



■ Different stages of the Vector meson production:

- $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$
- $q\bar{q}$ dipole elastically scatters from the target
- $\psi_V^*(Q^2, r, z)$ is wavefunction for J/ψ

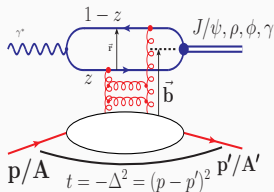
■ Scattering amplitude is given by:

$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) = i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i \mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}$$

■ in pQCD the diffraction formula is given by:

$$\frac{d\sigma^{\gamma^* p \rightarrow V p}}{dt} \sim [xg(x, Q^2)]^2$$

Diffractive Vector Meson Production....



- Different stages of the Vector meson production:

- $\psi(Q^2, z, r)$ is wavefunction for the $\gamma^* \rightarrow q\bar{q}$
- $q\bar{q}$ dipole elastically scatters from the target
- $\psi_V^*(Q^2, r, z)$ is wavefunction for J/ψ

- Scattering amplitude is given by:

$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) = i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i \mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}$$

- in pQCD the diffraction formula is given by:

$$\frac{d\sigma^{\gamma^* p \rightarrow V p}}{dt} \sim [xg(x, Q^2)]^2$$

- a sensitive probe to high gluon density and transverse spatial profile.

Dipole Model

Dipole Model

Golec-Biernat, Wusthoff 1999,
Kowalski, Teaney 2000

- The bSat dipole model:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})\right) \right]$$

Where: $T_p(\mathbf{b}) = \frac{1}{2\pi B_p} \exp(-\mathbf{b}^2/2B_p)$, $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$ and
 $\mu^2 = \mu_0^2 + \frac{C}{r^2}$

Dipole Model

Golec-Biernat, Wusthoff 1999,
Kowalski, Teaney 2000

- The bSat dipole model:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})\right) \right]$$

- The bNonSat:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})$$

Where: $T_p(\mathbf{b}) = \frac{1}{2\pi B_p} \exp(-\mathbf{b}^2/2B_p)$, $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$ and $\mu^2 = \mu_0^2 + \frac{C}{r^2}$

- The bSat dipole model:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})\right) \right]$$

- The bNonSat:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})$$

Where: $T_p(\mathbf{b}) = \frac{1}{2\pi B_p} \exp(-\mathbf{b}^2/2B_p)$, $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$ and $\mu^2 = \mu_0^2 + \frac{C}{r^2}$

- Momentum transfer ($-t$) distribution is a Fourier Transform to the spatial distribution of the target

Dipole Model

Golec-Biernat, Wusthoff 1999,
Kowalski, Teaney 2000

- The bSat dipole model:

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2 \left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})\right) \right]$$

- The bNonSat:

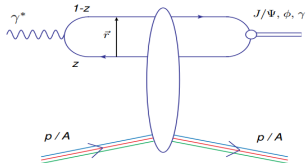
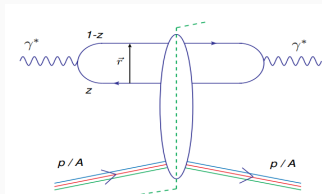
$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = \frac{\pi^2}{N_c} r^2 \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b})$$

Where: $T_p(\mathbf{b}) = \frac{1}{2\pi B_p} \exp(-\mathbf{b}^2/2B_p)$, $xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$ and $\mu^2 = \mu_0^2 + \frac{C}{r^2}$

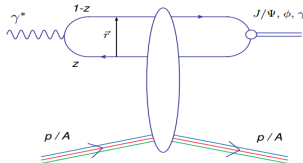
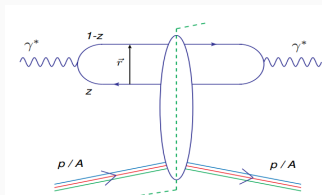
- Momentum transfer (-t) distribution is a Fourier Transform to the spatial distribution of the target
- Fundamental understanding of nucleon structure at high energy

Electron-Ion Scattering

Electron-Ion Scattering



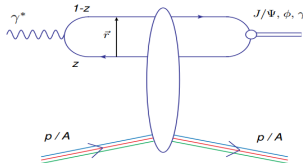
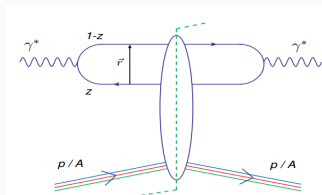
Electron-Ion Scattering



Good-Walker picture:

$$\begin{aligned}
 \sigma_{\text{inc}} &\propto \sum_{f \neq i} \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle \\
 &= \sum_f \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle - \langle i | \mathcal{A} | i \rangle^\dagger \langle i | \mathcal{A} | i \rangle \\
 &= \langle i | |\mathcal{A}|^2 | i \rangle - |\langle i | \mathcal{A} | i \rangle|^2 \\
 &= \langle |\mathcal{A}|^2 \rangle - |\langle \mathcal{A} \rangle|^2
 \end{aligned}$$

Electron-Ion Scattering

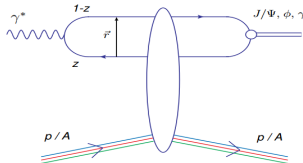
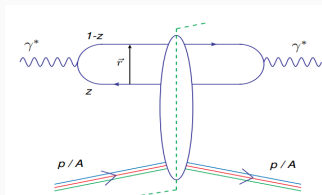


Good-Walker picture:

$$\mathcal{A}(\Omega_j) = \int dr \frac{dz}{4\pi} d^2\mathbf{b} (\Psi_V^* \Psi)(r, z) 2\pi r b J_0([1-z]r\Delta) \times e^{-i\mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(x, r, b, \Omega_j)$$

$$\begin{aligned} \sigma_{\text{inc}} &\propto \sum_{f \neq i} \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle \\ &= \sum_f \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle - \langle i | \mathcal{A} | i \rangle^\dagger \langle i | \mathcal{A} | i \rangle \\ &= \langle i | |\mathcal{A}|^2 | i \rangle - |\langle i | \mathcal{A} | i \rangle|^2 \\ &= \langle |\mathcal{A}|^2 \rangle - |\langle \mathcal{A} \rangle|^2 \end{aligned}$$

Electron-Ion Scattering



Good-Walker picture:

$$\mathcal{A}(\Omega_j) = \int dr \frac{dz}{4\pi} d^2\mathbf{b} (\Psi_V^* \Psi)(r, z) 2\pi r b J_0([1-z]r\Delta) \times e^{-i\mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(x, r, b, \Omega_j)$$

$$\sigma_{\text{inc}} \propto \sum_{f \neq i} \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle$$

$$= \sum_f \langle i | \mathcal{A} | f \rangle^\dagger \langle f | \mathcal{A} | i \rangle - \langle i | \mathcal{A} | i \rangle^\dagger \langle i | \mathcal{A} | i \rangle$$

$$= \langle i | |\mathcal{A}|^2 | i \rangle - |\langle i | \mathcal{A} | i \rangle|^2$$

$$= \langle |\mathcal{A}|^2 \rangle - |\langle \mathcal{A} \rangle|^2$$

$$\frac{d\sigma_{\text{tot}}}{dt} = \frac{1}{16\pi} \langle |\mathcal{A}|^2 \rangle, \quad \frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle \mathcal{A} \rangle|^2$$

Electron-Ion Scattering ...

ep:

$$1 - \mathcal{N}^p(x, \mathbf{r}, \mathbf{b}) = 1 - \frac{d\sigma_{q\bar{q}}^p(x, \mathbf{r}, \mathbf{b})}{2 d^2\mathbf{b}}$$

Using Independent Scattering

Electron-Ion Scattering ...

ep:

$$1 - \mathcal{N}^p(x, \mathbf{r}, \mathbf{b}) = 1 - \frac{d\sigma_{q\bar{q}}^p(x, \mathbf{r}, \mathbf{b})}{2 d^2\mathbf{b}}$$

eA:

Using Independent Scattering

$$1 - \mathcal{N}^A(x, \mathbf{r}, \mathbf{b}) = \prod_{i=1}^A \left(1 - \mathcal{N}^p(x, \mathbf{r}, |\mathbf{b} - \mathbf{b}_i|) \right)$$

Electron-Ion Scattering ...

ep:

$$1 - \mathcal{N}^p(x, \mathbf{r}, \mathbf{b}) = 1 - \frac{d\sigma_{q\bar{q}}^p(x, \mathbf{r}, \mathbf{b})}{2 d^2\mathbf{b}}$$

eA: Using Independent Scattering

$$1 - \mathcal{N}^A(x, \mathbf{r}, \mathbf{b}) = \prod_{i=1}^A \left(1 - \mathcal{N}^p(x, \mathbf{r}, |\mathbf{b} - \mathbf{b}_i|) \right)$$

bSat:

$$\frac{d\sigma_{q\bar{q}}^{(A)}}{d^2\mathbf{b}}(x, r, \mathbf{b}, \Omega_j) = 2 \left[1 - \exp \left(- \frac{\pi^2}{2N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^A T(|\mathbf{b} - \mathbf{b}_i|) \right) \right]$$

Coherent:

$$\left\langle \frac{d\sigma_{q\bar{q}}}{d^2b} \right\rangle_{\Omega} = 2 \left[1 - \left(1 - \frac{T_A(b)}{2} \sigma_{q\bar{q}}^p \right)^A \right]$$

Electron-Ion Scattering ...

Coherent:

$$\left\langle \frac{d\sigma_{q\bar{q}}}{d^2b} \right\rangle_{\Omega} = 2 \left[1 - \left(1 - \frac{T_A(b)}{2} \sigma_{q\bar{q}}^p \right)^A \right]$$

Incoherent:

- We don't have analytical solution
- Numerical Methods

$$\langle \mathcal{A} \rangle_{\Omega} \approx \frac{1}{C_{max}} \sum_{j=1}^{C_{max}} \mathcal{A}(\Omega_j)$$

Electron-Ion Scattering ...

Coherent:

$$\left\langle \frac{d\sigma_{q\bar{q}}}{d^2b} \right\rangle_{\Omega} = 2 \left[1 - \left(1 - \frac{T_A(b)}{2} \sigma_{q\bar{q}}^P \right)^A \right]$$

Incoherent:

- We don't have analytical solution
- Numerical Methods

$$\langle \mathcal{A} \rangle_{\Omega} \approx \frac{1}{C_{max}} \sum_{j=1}^{C_{max}} \mathcal{A}(\Omega_j)$$

Amplitude:

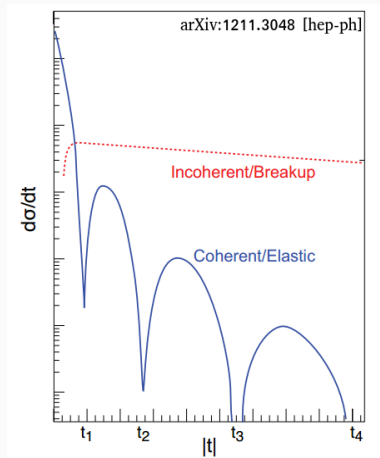
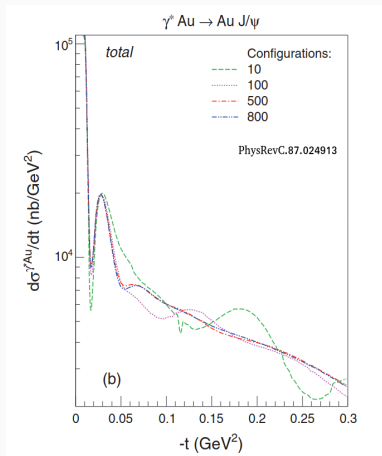
$$\mathcal{A}(\Omega_j) = \int dr \frac{dz}{4\pi} d^2\mathbf{b} (\Psi_V^* \Psi)(r, z) 2\pi r b J_0([1-z]r\Delta) e^{-i\mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}(x, r, b, \Omega_j)$$

Coherent, Incoherent and Total Cross-section

$$\left\langle \frac{d\sigma_{q\bar{q}}}{d^2b} \right\rangle_{\Omega} = 2 \left[1 - \left(1 - \frac{T_A(b)}{2} \sigma_{q\bar{q}}^p \right)^A \right]$$

$$\sigma_{incoherent} = \langle |\mathcal{A}|^2 \rangle - |\langle \mathcal{A} \rangle|^2$$

$$\frac{d\sigma_{\text{Total}}}{dt} = \frac{d\sigma_{\text{Coherent}}}{dt} + \frac{d\sigma_{\text{Incoherent}}}{dt}$$



The Event Generator: *Sartre*

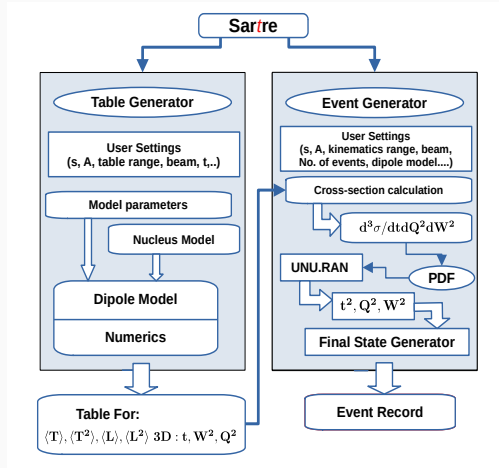
Sartre

- Dedicated to the Exclusive diffractive vector meson production
- Sartre: Event Generator for **ep** and **eA**
- bSat, bNonSat is basis of Sartre
- Use 3D lookup tables in $(\mathbf{Q}^2, \mathbf{W}^2, \mathbf{t})$ and use the powerful computer to produce the tables
- Hosted at sartre.hepforge.org
- Developer/maintainer : Tobias Toll, Thomas Ullrich

Comput.Phys.Commun. 185 (2014) 1835-1853

Event generator: Exclusive diffractive vector meson production and DVCS

- For total need ~ 500 configuration
- For each kinematic point (4D integral) we have to average over all number of configurations
- Look-up table: Amplitudes stored in the tables corresponding to (Q^2, W^2, t)



Look-up Table: Sartre

- Look-up Tables: $\langle |\mathcal{A}_T|^2 \rangle, |\langle \mathcal{A}_T \rangle|, \langle |\mathcal{A}_L|^2 \rangle, |\langle \mathcal{A}_L \rangle|$

Look-up Table: Sartre

- Look-up Tables: $\langle |\mathcal{A}_T|^2 \rangle, |\langle \mathcal{A}_T \rangle|, \langle |\mathcal{A}_L|^2 \rangle, |\langle \mathcal{A}_L \rangle|$
- lots of CPU power to generate them (\sim months)

Look-up Table: Sartre

- Look-up Tables: $\langle |\mathcal{A}_T|^2 \rangle, |\langle \mathcal{A}_T \rangle|, \langle |\mathcal{A}_L|^2 \rangle, |\langle \mathcal{A}_L \rangle|$
- lots of CPU power to generate them (\sim months)
- Computationally difficult for standard computer

Look-up Table: Sartre

- Look-up Tables: $\langle |\mathcal{A}_T|^2 \rangle, |\langle \mathcal{A}_T \rangle|, \langle |\mathcal{A}_L|^2 \rangle, |\langle \mathcal{A}_L \rangle|$
- lots of CPU power to generate them (\sim months)
- Computationally difficult for standard computer
- Table range:

Look-up Table: Sartre

- Look-up Tables: $\langle |\mathcal{A}_T|^2 \rangle, |\langle \mathcal{A}_T \rangle|, \langle |\mathcal{A}_L|^2 \rangle, |\langle \mathcal{A}_L \rangle|$
- lots of CPU power to generate them (\sim months)
- Computationally difficult for standard computer
- Table range:
 - $0 \leq |t| \leq 0.5, 384 \leq W^2 \leq 20164, 0.001 \leq Q^2 \leq 20.0 \text{ GeV}^2$

Machine Learning

Machine Learning in Sartre:Neural Network

- Fit to the data (3D tables) generated using the dipole model

Machine Learning in Sartre:Neural Network

- Fit to the data (3D tables) generated using the dipole model
- Though Neural Network is black-box
 - Fit to dipole model helps us to have some handle of the physics

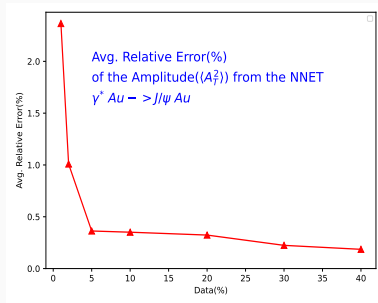
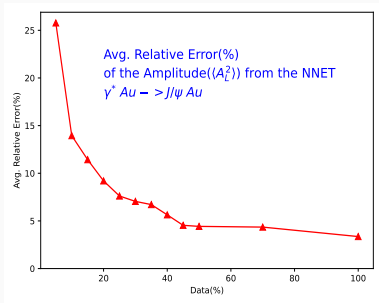
Machine Learning in Sartre: Neural Network

- Fit to the data (3D tables) generated using the dipole model
- Though Neural Network is black-box
 - Fit to dipole model helps us to have some handle of the physics
- A nice computation tool to reduce CPU hours
 - With current framework in Sartre it take months to generate tables in all kinematic region
 - Any new physics input, produce all tables again from beginning

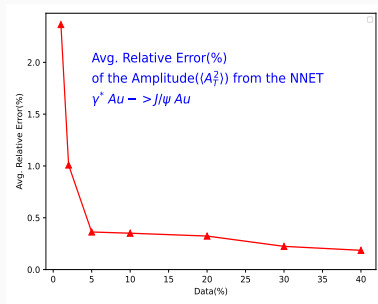
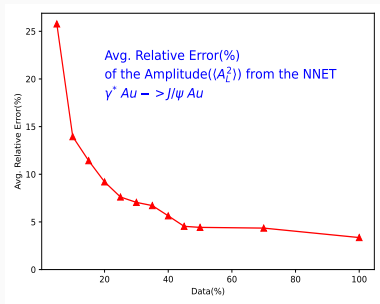
Machine Learning in Sartre: Neural Network

- Fit to the data (3D tables) generated using the dipole model
- Though Neural Network is black-box
 - Fit to dipole model helps us to have some handle of the physics
- A nice computation tool to reduce CPU hours
 - With current framework in Sartre it take months to generate tables in all kinematic region
 - Any new physics input, produce all tables again from beginning
- Way out : Produce tables with few bins and train NN to produce to tables in all kinematic region
 - Long-term, Use NN architecture with all parameters of dipole model as input

Avg. Relative Error(%) vs Data

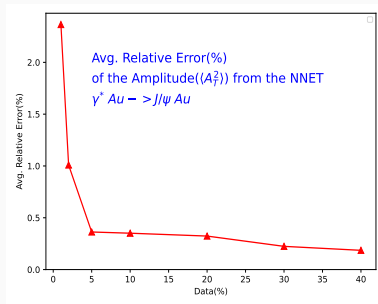
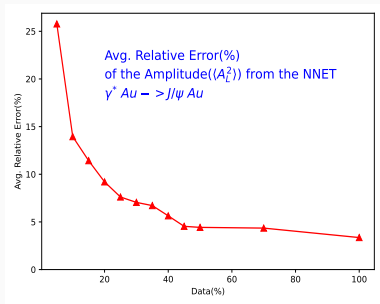


Avg. Relative Error(%) vs Data



- Needs $\sim 40\%$ (of ~ 20000) data to train the neural network with in the error(5%) cut-off for Longitudinal case

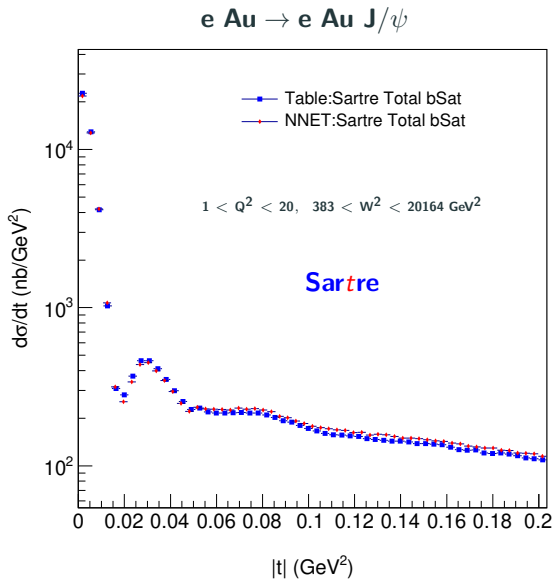
Avg. Relative Error(%) vs Data



- Needs $\sim 40\%$ (of ~ 20000) data to train the neural network with in the error(5%) cut-off for Longitudinal case
- Needs $\sim 5\%$ (of ~ 20000) data to train the neural network with in the error(5%) cut-off for Transverse case

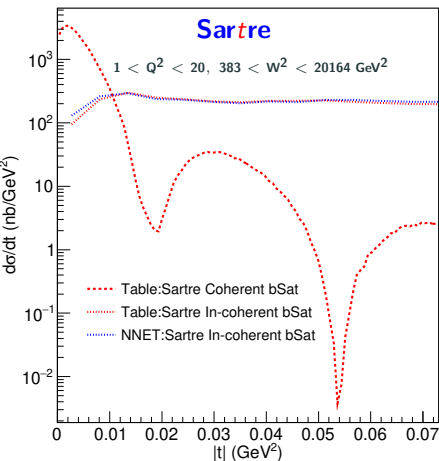
Results

Results

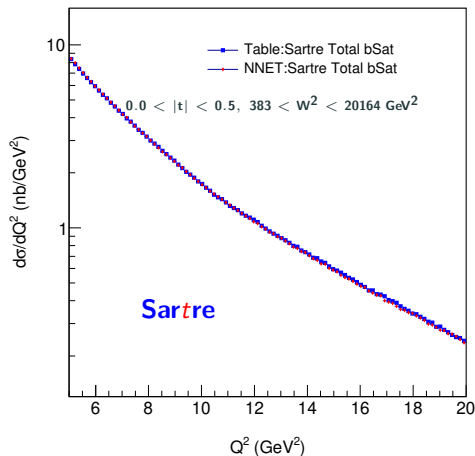


Results...

$e \text{ Au} \rightarrow e \text{ Au } J/\psi$



$e \text{ Au} \rightarrow e \text{ Au } J/\psi$



Conclusions

Conclusions: Neural Network

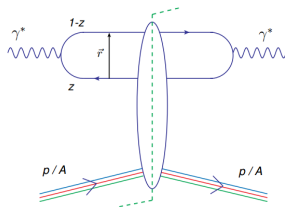
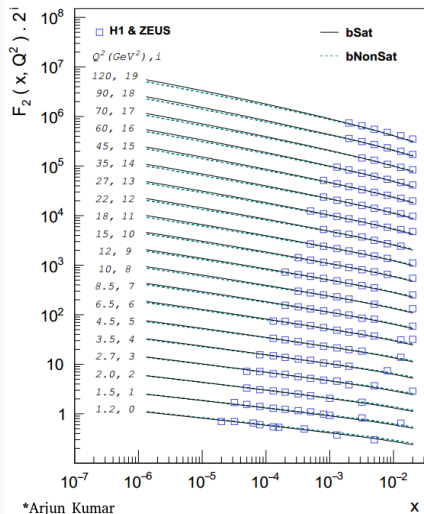
- We need 10% of the bins for Transverse and 40-50% for Longitudinal amplitude square tables.
- In total we need to generate $\sim 25\%$ of the table points.
- We can produce the tables(NN) much faster than using model.
- Work in progress to reduce the error for longitudinal amplitude

Outlook and Summary

- Need to reduce the error of outcome of the Neural Network(For longitudinal case)
- Produce the more tables for the other species(pb, Zr,...).
- We generated the events using the 3D table produced from the neural network in Sartre
- Dipole Scattering amplitude:
$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V^p}(x, Q, \Delta) = i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i \mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}}$$
- Scattering amplitude(NNET):
$$\mathcal{A}_{T,L}^{\gamma^* p \rightarrow V^p}(x, Q, \Delta) = i \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \int d^2\mathbf{b} (\Psi_V^* \Psi)_{T,L} e^{-i \mathbf{b} \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} (NNT)$$
- we are working on the inclusive diffractive deep inelastic scattering(DDIS)
- Upgrade Sartre to inclusive diffraction.
 - We need 4D tables
 - Need to include t-dependence

*Thank
You!*

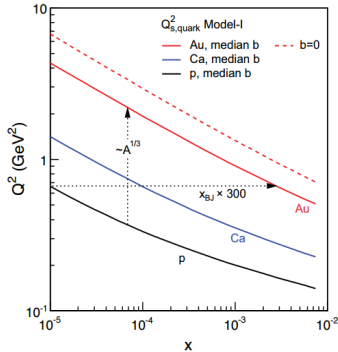
Back-up Slides



- ❖ No indication of saturation
- ❖ Well explained by bSat & bNonSat dipole model
- ❖ No saturation even in LHeC kinematic regime ($x = 10^{-7}$)

saturation scale

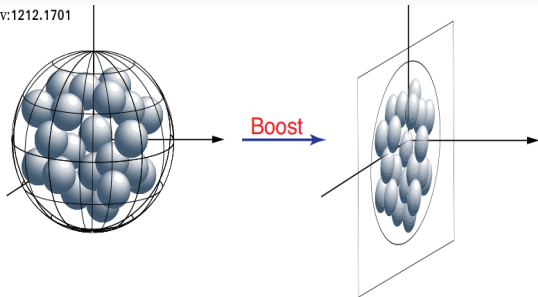
arXiv:hep-ph/0304189



Proton : $Q^2(x) = \left(\frac{x_0}{x}\right)^\lambda \quad \lambda \sim 0.2 - 0.3$

- ❖ Q_x^2 value large at low- x
 - treat as perturbative scale
- ❖ Two way to increase the saturation scale:
 - Increase the c.o.m energy in ep collisions to probe small x values ($x \sim \frac{Q^2}{W^2}$)
 - Change the target particle from proton to nucleus
- ❖ Use heavy ion geometry to enhance saturation scale

arXiv:1212.1701



Nucleus:

$$Q_s^2(x, A) \sim A^{\frac{1}{3}} \left(\frac{1}{x} \right)^{\lambda} \sim \left(\frac{A}{x} \right)^{1/3}$$

$$x \sim \frac{Q^2}{W^2} \quad W^2 = M_A^2 + Q^2 \frac{(1-x)}{x}$$