

Sum rules for the Gravitational Form Factors in light-front dressed quark model

Jai D. More

Department of Physics, IIT Bombay, India

A. Mukherjee (IITB, India), S. Nair (IMP,CAS), S. Saha(IITB, India)



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JM, Mukherjee, Nair, Saha in preparation

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- 1 Re-Introduce
- 2 EMT and Dressed quark model
- 3 GFFs of quark and gluon
- 4 Plots of GFFs
- 5 D-term and pressure distributions
- 6 Conclusions

Notation [Harindranath 1996]

$$x^\mu = (x^+, x^-, \mathbf{x}^\perp)$$

$$\text{where } x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad \mathbf{x}^\perp = (x^1, x^2)$$

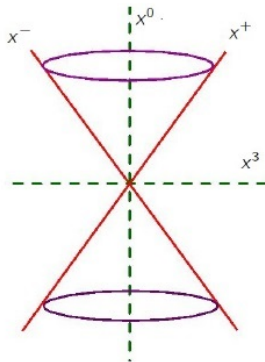
$$\text{Momentum: } p^\mu = (p^+, p^-, \mathbf{p}^\perp)$$

long. momentum \leftarrow \downarrow \rightarrow transverse momentum
 LF energy

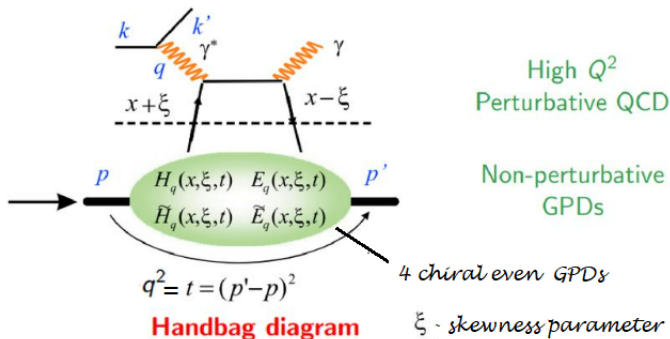
The metric tensor:

$$g^{\mu\nu} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

$$\text{Mass shell condition } p^- = \frac{\mathbf{p}_\perp^2 + m^2}{2p^+}$$



Deeply Virtual Compton Scattering:: $ep \rightarrow e'p'\gamma$



Bjorken limit:

$$Q^2 = \left. \begin{array}{l} -q^2 \rightarrow \infty \\ \nu \rightarrow \infty \end{array} \right\} x_B = \frac{Q^2}{2M\nu} \text{ fixed}$$

$\xi = 0$, momentum transfer is purely transverse



GPD \Longleftrightarrow GFFs [Shohini's Talk]

The second Mellin's moment of GPDs:

$$\int_{-1}^1 dx x H^a(x, \xi, t) = A^a(t) + \xi^2 D^a(t)$$

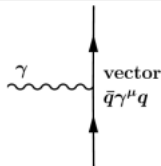
$$\int_{-1}^1 dx x E^a(x, \xi, t) = B^a(t) - \xi^2 D^a(t)$$

Ji sum rule: [Ji, PRL 78, (1997)]

coming up....

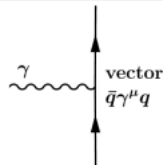
Gravitational Form Factors:

Recall: The electromagnetic interaction of a nucleon with an external EM field :: $\langle p' | J^\mu | p \rangle A_\mu$

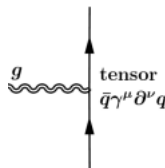


Gravitational Form Factors:

Recall: The electromagnetic interaction of a nucleon with an external EM field :: $\langle p' | J^\mu | p \rangle A_\mu$



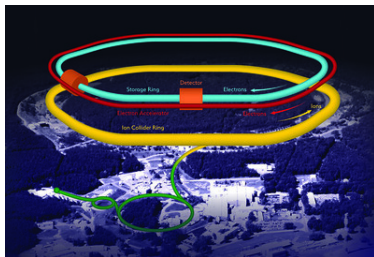
- Fundamentally one may think the gravitons interacting with the quarks and gluons
- Gravitons not feasible in collider yet. This can be thought of as a pair of vector bosons interacting with quarks and gluons.
- If one calculates the amplitude of such a process in the quantum field theory framework it appears to be dependent on the square of the momentum transfer q^2 .
- Moments of generalized parton distribution constrained by hard scattering process.



Figs. [Kumano 2018]

Where do you find GFFs?

- DVCS: quark structure @ JLAB proton D term: [Burkert 2018]
- DVMP: gluon structure @ Belle pion GFFs extracted [Kumano 2018]
- @Future EIC aims to extract gluon D-term



P.C. google

Energy momentum tensor

The QCD Lagrangian

$$\mathcal{L}_{QCD} = \frac{1}{2} \bar{\psi} (i \gamma_{\mu} D^{\mu} - m) \psi - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a,$$

where the covariant derivative $iD^{\mu} = i \overleftrightarrow{\partial}^{\mu} + g A^{\mu}$.

The field strength tensor

$$F_a^{\mu\nu} = \partial^{\mu} A_a^{\nu} - \partial^{\nu} A_a^{\mu} + g f^{abc} A_b^{\mu} A_c^{\nu}.$$

ψ and $A^{\mu} :=$ the fermion and boson field respectively.

Energy momentum tensor

The symmetric QCD EMT

$$\theta^{\mu\nu} = \theta_q^{\mu\nu} + \theta_g^{\mu\nu}$$

where

$$\theta_q^{\mu\nu} = \frac{1}{2} \bar{\psi} i [\gamma^\mu D^\nu + \gamma^\nu D^\mu] \psi - g^{\mu\nu} \bar{\psi} (i \gamma^\lambda D_\lambda - m) \psi$$

$$\theta_g^{\mu\nu} = -F^{\mu\lambda a} F_{\lambda a}^\nu + \frac{1}{4} g^{\mu\nu} (F_{\lambda\sigma a})^2$$

Energy momentum tensor

The symmetric QCD EMT

$$\theta^{\mu\nu} = \theta_q^{\mu\nu} + \theta_g^{\mu\nu}$$

where

$$\theta_q^{\mu\nu} = \frac{1}{2} \bar{\psi} i [\gamma^\mu D^\nu + \gamma^\nu D^\mu] \psi - \underbrace{g^{\mu\nu} \bar{\psi} (i \gamma^\lambda D_\lambda - m) \psi}_{=0(EOM)}$$

$$\theta_g^{\mu\nu} = -F^{\mu\lambda a} F_{\lambda a}^\nu + \frac{1}{4} g^{\mu\nu} (F_{\lambda\sigma a})^2$$

Parametrization of matrix element in terms of GFFs for a spin -1/2 system

$$\begin{aligned}
 \langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle &= \bar{U}(p', s') \left[-B_i(q^2) \frac{P^\mu P^\nu}{M} \right. \\
 &+ (A_i(q^2) + B_i(q^2)) \frac{1}{2} (\gamma^\mu P^\nu + \gamma^\nu P^\mu) \\
 &+ C_i(q^2) \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} \\
 &\left. + \bar{C}_i(q^2) M g^{\mu\nu} \right] U(p, s),
 \end{aligned}$$

where $\bar{U}(p', s')$, $U(p, s) :=$ Dirac spinors $P^\mu := \frac{1}{2}(p' + p)^\mu$

$M :=$ mass of the target state, $q^\mu := (p' - p)^\mu$

A_i , B_i , C_i and $\bar{C}_i :=$ quark or gluon GFFs and $i \equiv (Q, G)$

[Harindranath, Kundu, Mukherjee PLB, 728 2014]

Equivalent decomposition:[Harindranath, Kundu, Mukherjee, PLB 728 (2014)]

$$\begin{aligned}
 \langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle &= \bar{U}(p', s') \left[A_i(q^2) \frac{P^\mu P^\nu}{M} + J_i(q^2) \frac{i(P^\mu \sigma^{\nu\rho} + P^\nu \sigma^{\mu\rho}) q_\rho}{2M} \right. \\
 &+ \left. D_i(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + M \bar{C}_i(q^2) g^{\mu\nu} \right] U(p, s),
 \end{aligned}$$

Momentum Conservation

$$\sum_i A_i(0) = 1$$

Mass

δg^{++}

$$\begin{aligned} \langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle &= \bar{U}(p', s') \left[A_i(q^2) \frac{P^\mu P^\nu}{M} + B_i(q^2) \frac{i(P^\mu \sigma^{\nu\rho} + P^\nu \sigma^{\rho\mu})}{2M} \right. \\ &\quad \left. + C_i(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + M \bar{C}_i(q^2) g^{\mu\nu} \right] U(p, s), \end{aligned}$$

[Polyakov and Schweitzer, (2018),
Pagels Phys. Rev., (1966).]

Total angular momentum conservation

$$\sum_i B_i(0) = 0$$

Spin

δg^{+i}

$$\langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle = \bar{U}(p', s') \left[\frac{q^\mu q^\nu}{4M} + J_i(q^2) \frac{i(P^\mu \sigma^{\nu\rho} + P^\nu \sigma^{\mu\rho}) q_\rho}{2M} \right. \\ \left. - \left[\frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + M C_i(q^2) g^{\mu\nu} \right] U(p, s) \right]$$

[Ji, PRL, 78:610–613,(1997)].

Unconstrained D term¹

$$\langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle = \bar{U}(p', s') \left[A_i(q^2) \frac{P^\mu P^\nu}{M} + B_i(q^2) \frac{i(P^\mu \sigma^{\nu\rho} + P^\nu \sigma^{\mu\rho})}{2M} \right. \\ \left. + D_i(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + \bar{C}_i(q^2) g^{\mu\nu} \right] U(p, s),$$

δg^{ij}

Mechanical properties
Pressure & shear force

Related to stress tensor and internal forces

¹determined from experiment

Conservation of EMT

$$\begin{aligned} \langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle &= \bar{U}(p', s') \left[A_i(q^2) \frac{p^\mu p^\nu}{M} + J_i(q^2) \frac{i(p^\mu \sigma^{\nu\rho} + p^\nu \sigma^{\mu\rho})}{2M} \right. \\ &\quad \left. + D_i(q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} + M \bar{C}_i(q^2) g^{\mu\nu} \right] U(p, s), \end{aligned}$$

δg^{ij}

$$\sum_i \bar{C}_i(0) = 0$$

[Lorcé, Moutarde and Trawiński, EPJC 79(1), 89,(2019)]

Dressed quark model (DQM)

Instead of a proton state, we take a quark dressed with a gluon. This is a composite spin $1/2$ state. (relativistic)

- Due to the presence of gluon dressing, the model employs a gluonic degree of freedom
- The dressed quark state can be expanded in terms of light-front wave functions (LFWFs). Although the LFWF of a bound state, like a proton, cannot be calculated analytically, the LFWF for a dressed quark can be calculated analytically in perturbation theory
- LFWFs are boost invariant and can be written in terms of relative momenta that are frame independent.

Fock state expansion of quark state dressed with a gluon

$$\begin{aligned}
 |p^+, p_\perp, s\rangle &= \Phi^s(p) b_s^\dagger(p) |0\rangle + \sum_{s_1 s_2} \int \frac{dp_1^+ d^2 p_1^\perp}{\sqrt{16\pi^3 p_1^+}} \int \frac{dp_2^+ d^2 p_2^\perp}{\sqrt{16\pi^3 p_2^+}} \sqrt{16\pi^3 p^+} \\
 &\quad \times \delta^3(p - p_1 - p_2) \Phi_{s_1 s_2}^s(p; p_1, p_2) b_{s_1}^\dagger(p_1) a_{s_2}^\dagger(p_2) |0\rangle
 \end{aligned}$$

$\Phi^s(p)$: a normalized wavefunction;

$\Phi_{s_1 s_2}^s(p; p_1, p_2)$: two particle LFWF, related to the boost invariant wavefunction

$$\sqrt{P^+} \Phi_{s_1 s_2}^s(p; p_1, p_2) = \Psi_{s_1 s_2}^s(x_i, q_i^\perp)$$

[Harindranath and Kundu PRD 59 116013 (1999)]

The Jacobi momenta:

$$p_i^+ = x_i P^+ \text{ and } q_i^\perp = p_i^\perp + x_i P^\perp$$

such that

$$\sum_i x_i = 1, \quad \sum_i q_i^\perp = 0$$

The two particle LFWF ²

$$\begin{aligned} \Psi_{s_1 s_2}^{as}(x, q^\perp) &= \frac{1}{\left[m^2 - \frac{m^2 + (q^\perp)^2}{x} - \frac{(q^\perp)^2}{1-x} \right]} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi_{s_1}^\dagger \frac{1}{\sqrt{1-x}} \\ &\times \left[-2 \frac{q^\perp}{1-x} - \frac{(\sigma^\perp \cdot q^\perp) \sigma^\perp}{x} + \frac{im \sigma^\perp (1-x)}{x} \right] \chi_s (\epsilon_{s_2}^\perp)^* \end{aligned}$$

χ : two component spinor; m : dressed quark mass= bare quark mass

[Harindranath and Kundu PRD 59 116013 (1999);
Zhang and Harindranath, PRD 48, 4881 (1993)]

²Independent of the momentum of the bound state.

Two-component formalism: [Zhang and Harindranath, PRD 48, (1993)]

The quark field decomposition

$$\psi = \psi_+ + \psi_-$$

where $\psi_{\pm} = \Lambda_{\pm} \psi$ and Λ_{\pm} are the projection operators.

One uses the constraint equations in the light-cone gauge to eliminate the redundant degree of freedom and express the fields in terms of physically independent degrees of freedom.³

$A_a^{\perp}, \psi^+ \rightarrow$ "good" independent dof

$A_a^-, \psi^- \rightarrow$ "bad"

Fixing of gauge simplifies the relativistic fermion structure

³Terminology: Gell-Mann and Fritzsche [Harindranath hep-ph/9410390v1]

Drell-Yan Frame $q^+ = 0$

$$\text{Initial momentum: } p^\mu = \left(p^+, \mathbf{0}^\perp, \frac{M^2}{p^+} \right),$$

$$\text{Final momentum: } p'^\mu = \left(p^+, \mathbf{q}^\perp, \frac{\mathbf{q}^{\perp 2} + M^2}{p^+} \right),$$

$$\text{Invariant momentum transfer: } q^\mu = (p' - p)^\mu = \left(0, \mathbf{q}^\perp, \frac{\mathbf{q}^{\perp 2}}{p^+} \right).$$

Flag

$$\mathbf{p}^\perp = 0 \quad \implies \quad q^2 = -\mathbf{q}^{\perp 2}.$$

Recipe: To extract the four GFFs

$$\mathcal{M}_{ss'}^{\mu\nu} = \frac{1}{2} [\langle p', s' | \theta_i^{\mu\nu}(0) | p, s \rangle]$$

where the Lorentz indices $(\mu, \nu) \equiv \{+, -, 1, 2\}$, $(s, s') \equiv \{\uparrow, \downarrow\}$ is the helicity of the initial and final state. $\uparrow (\downarrow)$ positive (negative) spin projection along z -axis.

Example: Diagonal component of EMT

$$[\mathcal{M}_{\sigma'\sigma}^{++}]_{2,D} = 2P^{+2} \sum_{\lambda_2, \lambda_2', \sigma_1} \int [x\kappa^\perp] \phi_{2\sigma'}^{*\sigma_1, \lambda_2'}((1-x), -\kappa'^\perp) \left[x \epsilon_{\lambda_2'}^{i*} \epsilon_{\lambda_2}^i \right] \phi_{2\sigma}^{\sigma_1, \lambda_2}((1-x), -\kappa^\perp)$$

Extraction of $A_i(q^2)$ and $B_i(q^2)$

$$\mathcal{M}_{\uparrow\uparrow}^{\boxed{++}} + \mathcal{M}_{\downarrow\downarrow}^{\boxed{++}} = 2 (P^+)^2 A_i(q^2),$$


$$\mathcal{M}_{\uparrow\downarrow}^{\boxed{++}} + \mathcal{M}_{\downarrow\uparrow}^{\boxed{++}} = \frac{iq^{(2)}}{M} (P^+)^2 B_i(q^2).$$

Extraction of $D_i(q^2)$ and $\overline{C}_i(q^2)$

$$\begin{aligned}
 & [\mathcal{M}_{\sigma'\sigma}^{11} + \mathcal{M}_{\sigma'\sigma}^{22}]_{2,D} \\
 &= \sum_{\lambda_2, \lambda'_2, \sigma_1} \int [x\boldsymbol{\kappa}^\perp] \phi_{2\sigma'}^{*\sigma_1\lambda'_2}((1-x), -\boldsymbol{\kappa}'^\perp; x, \boldsymbol{\kappa}'^\perp) [\mathcal{O}^{ii}] \phi_{2\sigma}^{\sigma_1\lambda_2}((1-x), -\boldsymbol{\kappa}^\perp; x, \boldsymbol{\kappa}^\perp)
 \end{aligned}$$

Extraction of $D_i(q^2)$ and $\overline{C}_i(q^2)$

$$\begin{aligned}
 & [\mathcal{M}_{\sigma'\sigma}^{11} + \mathcal{M}_{\sigma'\sigma}^{22}]_{2,D} \\
 &= \sum_{\lambda_2, \lambda'_2, \sigma_1} \int [x \kappa^\perp] \phi_{2\sigma'}^{*\sigma_1 \lambda'_2}((1-x), -\kappa'^\perp; x, \kappa'^\perp) [\mathcal{O}^{\text{ii}}] \phi_{2\sigma}^{\sigma_1, \lambda_2}((1-x),
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{O}^{\text{ii}} &= \frac{1}{x} \left[\left(\kappa^\perp \cdot \epsilon_{\lambda_2}^\perp \right) \left(\kappa^\perp \cdot \epsilon_{\lambda'_2}^{\perp*} + \mathbf{q}^\perp \cdot \epsilon_{\lambda'_2}^{\perp*} \right) \right. \\
 &+ \left(\kappa^{(1)} \epsilon_{\lambda_2}^{(2)} \right) \left(\kappa^{(1)} + \mathbf{q}^{(1)} \right) \epsilon_{\lambda'_2}^{(2)*} + \left(\kappa^{(2)} \epsilon_{\lambda_2}^{(1)} \right) \left(\kappa^{(2)} + \mathbf{q}^{(2)} \right) \epsilon_{\lambda'_2}^{(1)*} \\
 &\left. - \left(\kappa^{(1)} \epsilon_{\lambda_2}^{(2)} \right) \left(\kappa^{(2)} + \mathbf{q}^{(2)} \right) \epsilon_{\lambda'_2}^{(1)*} - \left(\kappa^{(1)} + \mathbf{q}^{(1)} \right) \epsilon_{\lambda'_2}^{(2)*} \left(\kappa^{(2)} \epsilon_{\lambda_2}^{(1)} \right) \right]
 \end{aligned}$$

$\bar{C}_i(q^2)$ is responsible for the non-conservation of the EMT

$$\langle p', s' | \partial_\mu \theta_i^{\mu\nu}(0) | p, s \rangle = i q^\nu M_n \bar{C}_i(q^2) \bar{U}(p', s') U(p, s).$$

Extraction of $D(q^2)$

$$\mathcal{M}_{\uparrow\downarrow}^{11} + \mathcal{M}_{\downarrow\uparrow}^{11} + \mathcal{M}_{\uparrow\downarrow}^{22} + \mathcal{M}_{\downarrow\uparrow}^{22} = i \left[\frac{q^2}{4M} B_i(q^2) - \frac{3q^2}{M} C_i(q^2) + 2M \bar{C}_i(q^2) \right] q^{(2)}.$$

The quark GFFs: [JM, Mukherjee, Nair, Saha, PRD 105, (2022)]

$$A_Q(q^2) = 1 + \frac{g^2 C_F}{2\pi^2} \left[\frac{11}{10} - \frac{4}{5} \left(1 + \frac{2m^2}{q^2} \right) \frac{f_2}{f_1} - \frac{1}{3} \log \left(\frac{\Lambda^2}{m^2} \right) \right]$$

$$B_Q(q^2) = \frac{g^2 C_F}{12\pi^2} \frac{m^2}{q^2} \frac{f_2}{f_1},$$

$$D_Q(q^2) = \frac{5g^2 C_F}{6\pi^2} \frac{m^2}{q^2} \left(1 - f_1 f_2 \right) = 4 C_Q(q^2),$$

$$\overline{C}_Q(q^2) = \frac{g^2 C_F}{72\pi^2} \left(29 - 30 f_1 f_2 + 3 \log \left(\frac{\Lambda^2}{m^2} \right) \right),$$

where

$$f_1 := \frac{1}{2} \sqrt{1 + \frac{4m^2}{q^2}},$$

$$f_2 := \log \left(1 + \frac{q^2 (1 + 2f_1)}{2m^2} \right).$$

The gluon GFFs

$$A_G(q^2) = \frac{g^2 C_F}{8\pi^2} \left[\frac{29}{9} + \frac{4}{3} \ln \left(\frac{\Lambda^2}{m^2} \right) - \int dx \left(\left(1 + (1-x)^2 \right) + \frac{4m^2 x^2}{q^2 (1-x)} \right) \frac{\tilde{f}_2}{\tilde{f}_1} \right]$$

$$B_G(q^2) = -\frac{g^2 C_F}{2\pi^2} \int dx \frac{m^2 x^2}{q^2} \frac{\tilde{f}_2}{\tilde{f}_1}$$

$$D_G(q^2) = \frac{g^2 C_F}{6\pi^2} \left[\frac{2m^2}{3q^2} + \int dx \frac{m^2}{q^4} (x ((2-x)q^2 - 4m^2 x)) \right] \frac{\tilde{f}_2}{\tilde{f}_1}$$

$$\overline{C}_G(q^2) = \frac{g^2 C_F}{72\pi^2} \left[10 + 9 \int dx \left(x - \frac{4m^2 x^2}{q^2 (1-x)} \right) \frac{\tilde{f}_2}{\tilde{f}_1} - 3 \ln \left(\frac{\Lambda^2}{m^2} \right) \right]$$

where,

$$\tilde{f}_1 := \sqrt{1 + \frac{4m^2 x^2}{q^2 (1-x)^2}}$$

$$\tilde{f}_2 := \ln \left(\frac{1 + \tilde{f}_1}{-1 + \tilde{f}_1} \right)$$

Plots of GFFs

Plot of $A(q^2)$:

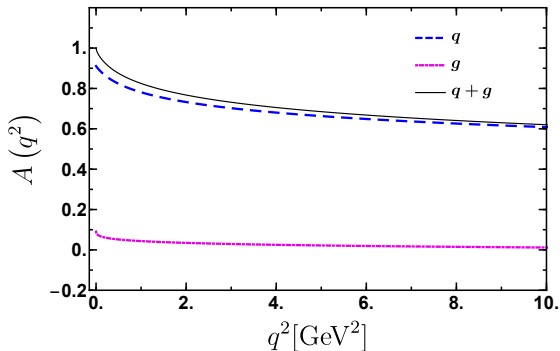
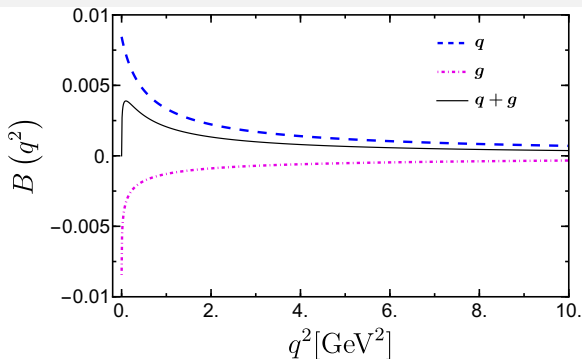


Figure: Plot of the GFFs $A_i(q^2)$ as a function of q^2 , with $m = 0.3$ GeV and $g = 1$.

Infer: Conservation of momentum

$$\sum_i A_i(0) = 1$$

Plot of $B(q^2)$:



Infer: anomalous gravitomagnetic moment

$$\sum_i B_i(0) = 0$$

Conservation of total angular momentum:

$$J(0) = \frac{1}{2}[(A(0) + B(0))]$$

Plot of $D(q^2)$:

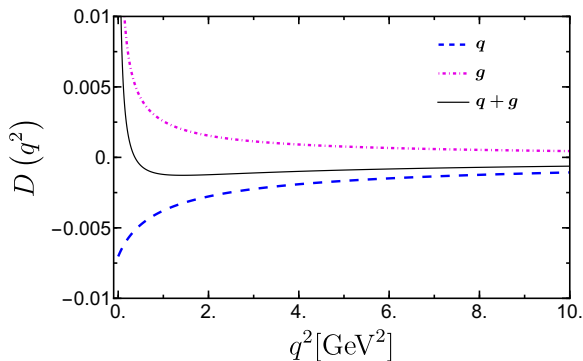


Figure: Plot of the GFFs $D_i(q^2)$ as a function of q^2 , with $m = 0.3 \text{ GeV}$ and $g = 1$.

A comparison: [Metz etal Phys. Lett. B, 820:136501(2021)]

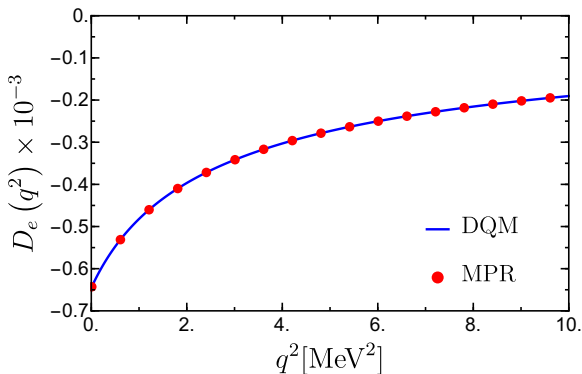


Figure: Plot of electron GFF $D_e(q^2)$ as function of q^2 , here we set $m = 0.511 \text{ MeV}$, $\alpha = \frac{1}{137}$.

A comparison: [Metz etal Phys. Lett. B, 820:136501(2021)]

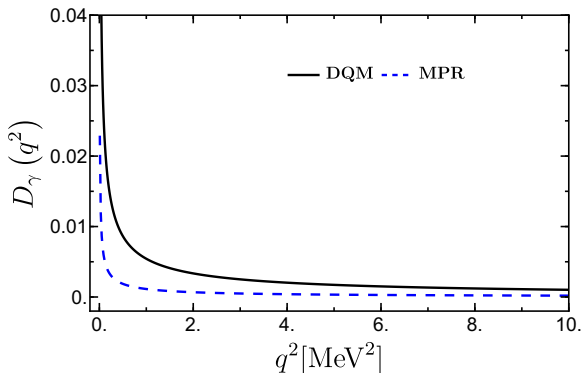


Figure: Plot of electron GFF $D_e(q^2)$ as function of q^2 , here we set $m = 0.511 \text{ MeV}$, $\alpha = \frac{1}{137}$.

Plot of $\overline{C}(q^2)$:

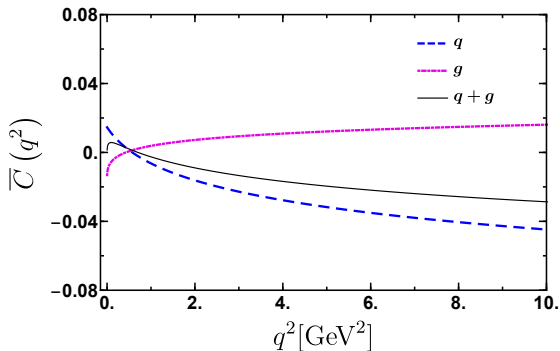


Figure: Plot of the GFFs $\overline{C}_i(q^2)$ as a function of q^2 , with $m = 0.3 \text{ GeV}$ and $g = 1$.

$$\text{Infer: } \sum_i \overline{C}_i(0) = 0$$

Wave packets [Chakrabarti and Mukherjee (2005), Diehl (2002)]

- Densities corresponds to probability hence two momentum integral
- Dependence of average momentum and momentum transfer
- These probabilities are preferably studied in impact parameter space and so is pressure distributions
- Not only yields the Fourier transformed pressure in the impact parameter space but also gives smooth plots for the distribution
- Use Gaussian wave function with a rational choice of width

Pressure and Shear force distributions

$$\theta_a^{ij}(r) = p_a(r) \delta^{ij} + s_a(r) \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) \quad [\text{Polyakov \& Schweitzer 2018}]$$

$$\frac{1}{2M} \frac{d}{d\mathbf{b}^\perp} \left[\mathbf{b}^\perp \frac{d}{d\mathbf{b}^\perp} \tilde{D}_a(\mathbf{b}^\perp) \right] - M \tilde{\tilde{C}}_a(\mathbf{b}^\perp) - \frac{\mathbf{b}^\perp}{M} \frac{d}{d\mathbf{b}^\perp} \left[\frac{1}{\mathbf{b}^\perp} \frac{d}{d\mathbf{b}^\perp} \tilde{D}_a(\mathbf{b}^\perp) \right]$$

[Freese, Miller (2021)]

$$\text{FT: } \boxed{\tilde{\mathcal{F}}_a(\mathbf{b}^\perp) = \frac{1}{2\pi} \int_0^\infty d\mathbf{q}^\perp{}^2 J_0(\mathbf{q}^\perp \mathbf{b}^\perp) \mathcal{F}_a(q^2)}$$

$\mathcal{F} := (A, B, D, \overline{C})$ $J_0 :=$ Bessel function of the zeroth order
 $\mathbf{b}^\perp :=$ Impact parameter $M :=$ mass of the dressed quark state.

Forces: Normal and Tangential [Chakrabarti etal. (2020), Anikin (2019)]

The combination of pressure and shear defines the normal and the tangential forces experienced by a ring of radius b^\perp

$$F_n(\mathbf{b}^\perp) = 2\pi \mathbf{b}^\perp \left(p(\mathbf{b}^\perp) + \frac{1}{2} s(\mathbf{b}^\perp) \right),$$

$$F_t(\mathbf{b}^\perp) = 2\pi \mathbf{b}^\perp \left(p(\mathbf{b}^\perp) - \frac{1}{2} s(\mathbf{b}^\perp) \right).$$

The energy density and pressure distributions [Lorce et al. (2019)]

$$\begin{aligned}\mu_i(\mathbf{b}^\perp) &= M \left[\frac{1}{2} A_i(\mathbf{b}^\perp) + \overline{C}_i(\mathbf{b}^\perp) + \frac{1}{4M^2} \frac{1}{\mathbf{b}^\perp} \frac{d}{d\mathbf{b}^\perp} \right. \\ &\quad \times \left. \left(\mathbf{b}^\perp \frac{d}{d\mathbf{b}^\perp} \left[\frac{1}{2} B_i(\mathbf{b}^\perp) - 4C_i(\mathbf{b}^\perp) \right] \right) \right],\end{aligned}$$

$$\sigma_i^r(\mathbf{b}^\perp) = M \left[-\overline{C}_i(\mathbf{b}^\perp) + \frac{1}{M^2} \frac{1}{\mathbf{b}^\perp} \frac{dC_i(\mathbf{b}^\perp)}{d\mathbf{b}^\perp} \right],$$

$$\sigma_i^t(\mathbf{b}^\perp) = M \left[-\overline{C}_i(\mathbf{b}^\perp) + \frac{1}{M^2} \frac{d^2 C_i(\mathbf{b}^\perp)}{d\mathbf{b}^{\perp 2}} \right],$$

$$\sigma_i(\mathbf{b}^\perp) = M \left[-\overline{C}_i(\mathbf{b}^\perp) + \frac{1}{2} \frac{1}{M^2} \frac{1}{\mathbf{b}^\perp} \frac{d}{d\mathbf{b}^\perp} \left(\mathbf{b}^\perp \frac{dC_i(\mathbf{b}^\perp)}{d\mathbf{b}^\perp} \right) \right],$$

$$\Pi_i(\mathbf{b}^\perp) = M \left[-\frac{1}{M^2} \mathbf{b}^\perp \frac{d}{d\mathbf{b}^\perp} \left(\frac{1}{\mathbf{b}^\perp} \frac{dC_i(\mathbf{b}^\perp)}{d\mathbf{b}^\perp} \right) \right].$$

The energy density and pressure distributions [Lorce et al. (2019)]

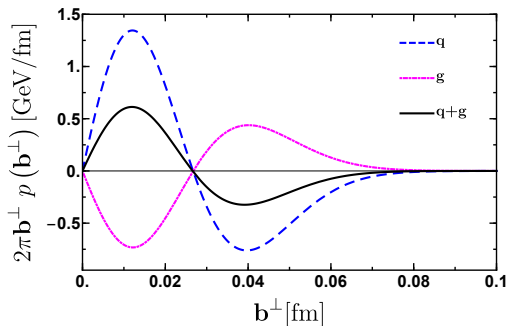
$$\mu_i(\mathbf{b}^\perp) = M \left[\frac{1}{2} A_i(\mathbf{b}^\perp) + \overline{C}_i(\mathbf{b}^\perp) + \frac{1}{4M^2} \frac{1}{\mathbf{b}^\perp} \frac{d}{d\mathbf{b}^\perp} \right. \\ \left. \times \left(\mathbf{b}^\perp \frac{d}{d\mathbf{b}^\perp} \left[\frac{1}{2} B_i(\mathbf{b}^\perp) - 4C_i(\mathbf{b}^\perp) \right] \right) \right],$$

$$\left. \begin{array}{l} \sigma_i^r(\mathbf{b}^\perp) \\ \sigma_i^t(\mathbf{b}^\perp) \\ \sigma_i(\mathbf{b}^\perp) \end{array} \right\}$$

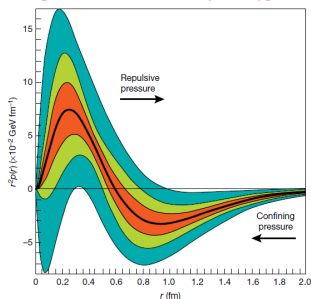
$$\sigma_i = \frac{(\sigma_i^r + \sigma_i^t)}{2}$$

$$\Pi_i = \sigma_i^r - \sigma_i^t.$$

Analysis of Pressure distributions



[Burkert, Nature (2018)]

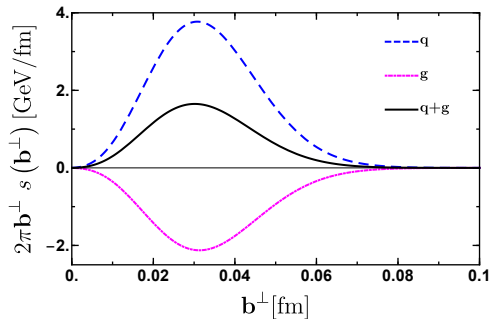


Infer: 1) The net repulsive force (inner region) and the attractive force (outer region) are balanced

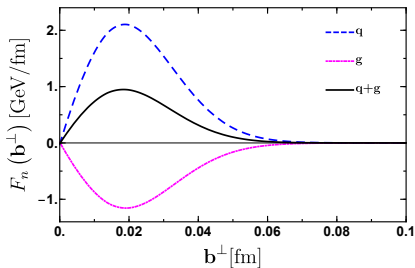
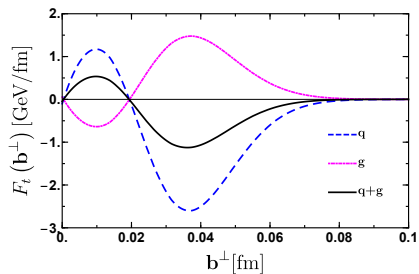
2) Satisfies Von-Laue condition:

$$\int_0^\infty d^2 b^\perp p(b^\perp) = 0$$

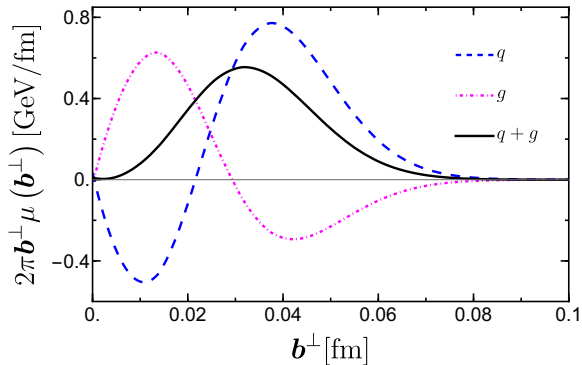
Analysis of Shear Force



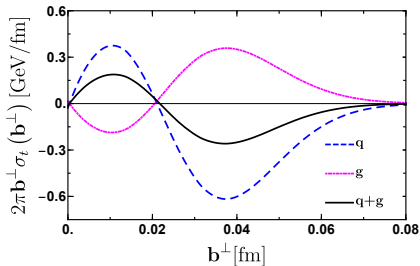
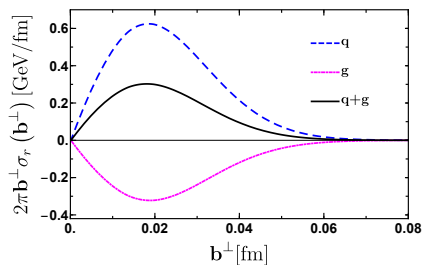
Analysis of Force



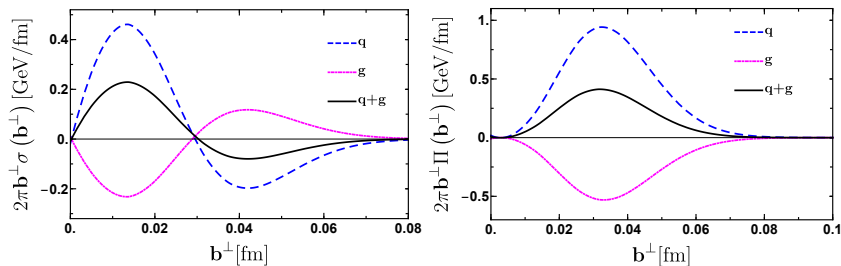
Energy density



Radial and tangential Pressure



Isotropic Pressure and Pressure Anisotropy



Note:

- Stress anisotropy in self-gravitating systems has been a subject of study
- It has been shown to affect the physical properties, stability and structure of stellar matter. [Dev, Gleiser (2002), Dev, Gleiser (2003), Silva et al. (2015)]
- Vanishes at the center and positive everywhere
- Ensures that radial pressure is always larger than tangential pressure

Concluding remarks

- EMT encapsulates the momentum, energy and pressure distributions.
- Though nucleon scattering by a gravitational field is not feasible, it's noteworthy that the GFFs can be extracted from experimental data.
- Analysis of quark and gluon GFFs in dressed quark model satisfies the sum rule.
- The pressure and shear distributions along with 2D distributions of quark and gluon are studied.
- The GFFs, pressure and shear force exhibit similar qualitative nature when compared to existing phenomenological models.



Backup

Gaussian wave packet

LFWF is replaced by the Gaussian wave packet

$$\frac{1}{16\pi^3} \int \frac{d^2\mathbf{p}^\perp dp^+}{p^+} \phi(p) |p^+, \mathbf{p}^\perp, \lambda\rangle$$

with $\phi(p) = p^+ \delta(p^+ - p_0^+) \phi(\mathbf{p}^\perp)$. We choose a Gaussian shape for $\phi(\mathbf{p}^\perp)$ in transverse momentum :

$$\phi(\mathbf{p}^\perp) = e^{-\frac{\mathbf{p}^\perp{}^2}{2\Delta^2}}$$

where Δ is the width of Gaussian.

[Chakrabarti, D. and Mukherjee, A. (2005). *Phys. Rev. D*, 72:034013]

[Diehl, M. (2002). *Eur. Phys. J. C*, 25:223–232.