Imaging Hadrons: Lattice QCD in the EIC Era

Brookhaven National Laboratory

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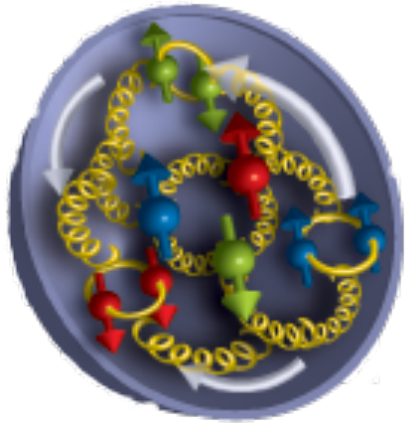
EIC big question 1

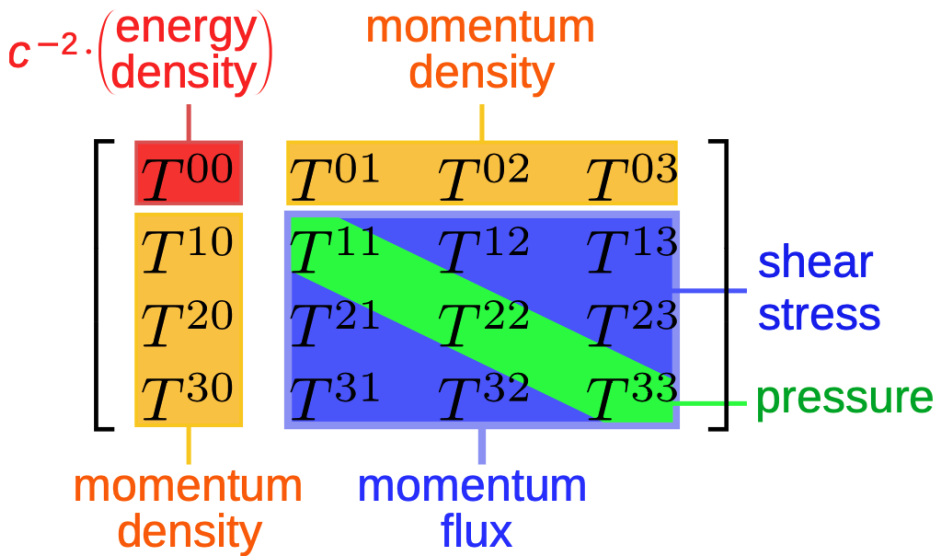
- how does proton spin arise from quarks & gluons ?

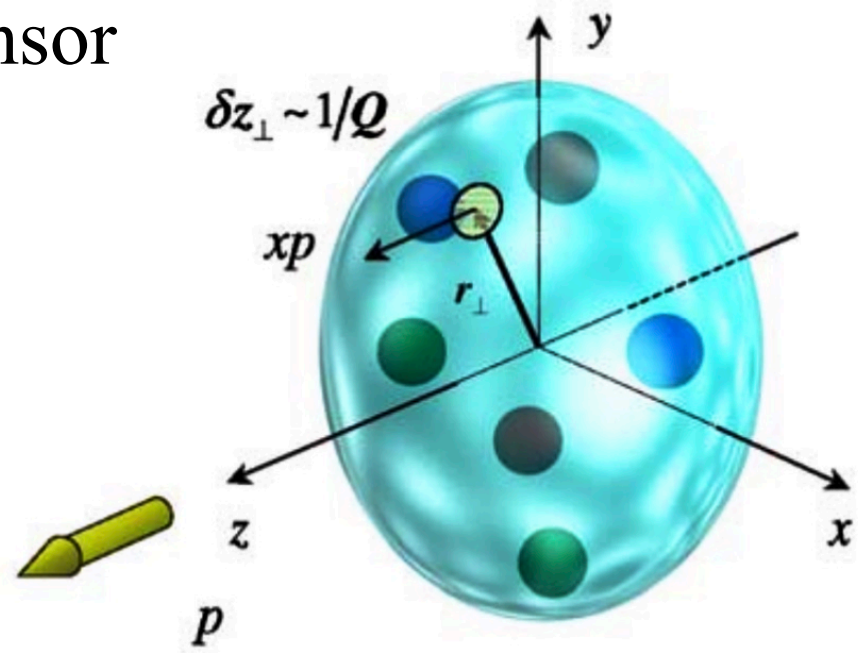
$$\frac{1}{2} = \text{  } + \text{  }$$

angular momentum of quarks: J_q

angular momentum of gluons: J_g

need 3-d distribution of energy-momentum tensor



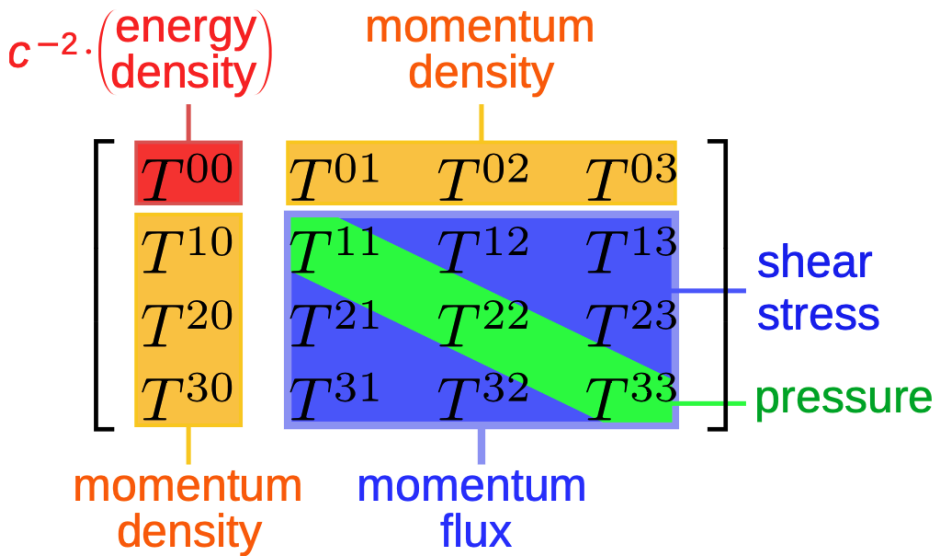


$$\langle p,s\,|\,T_{\mu\nu}\,|\,p',s'\rangle = \bar{U}(p,s)\left[M(t)\frac{P_\mu P_\nu}{m} + J(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2m} + D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m}\right]U(p',s')$$

EIC big questions 2

- how does proton mass arise from quarks & gluons ?

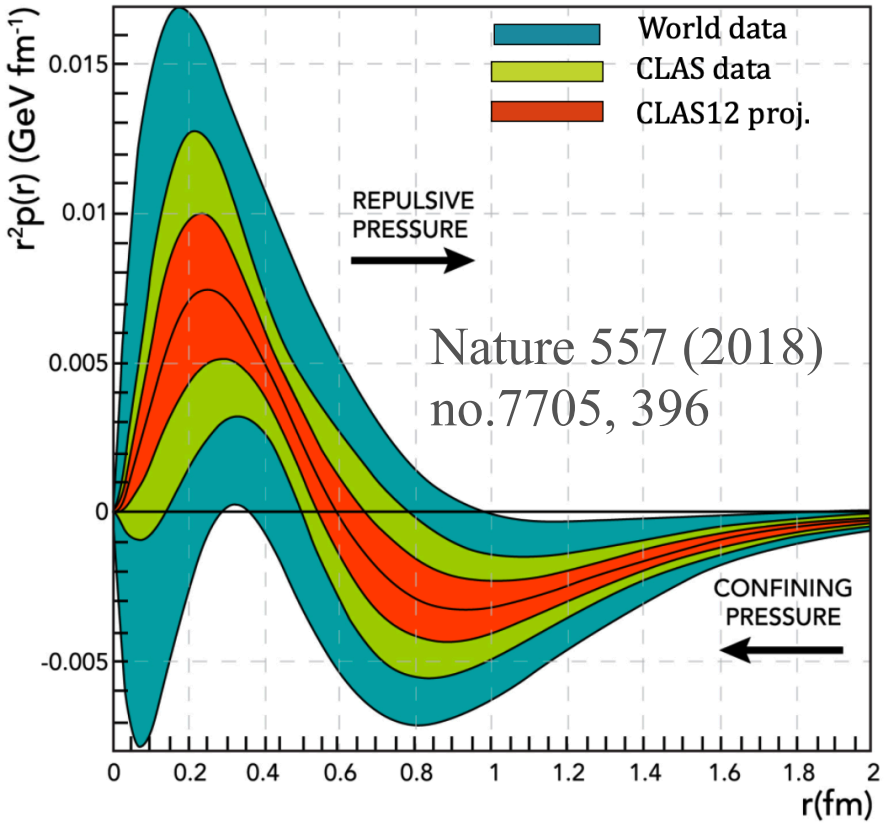
$$\langle p,s\,|\,T_{\mu\nu}\,|\,p',s'\rangle = \bar{U}(p,s)\left[M(t)\frac{P_\mu P_\nu}{m} + J(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2m} + D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m}\right]U(p',s')$$

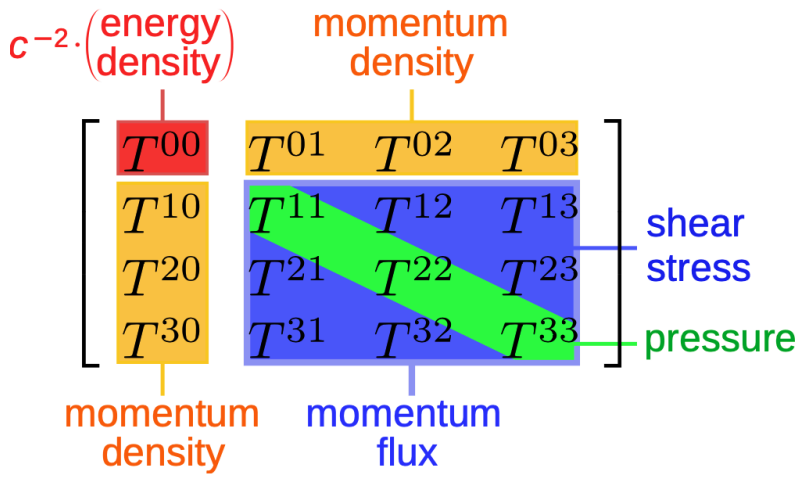


EIC big questions 3

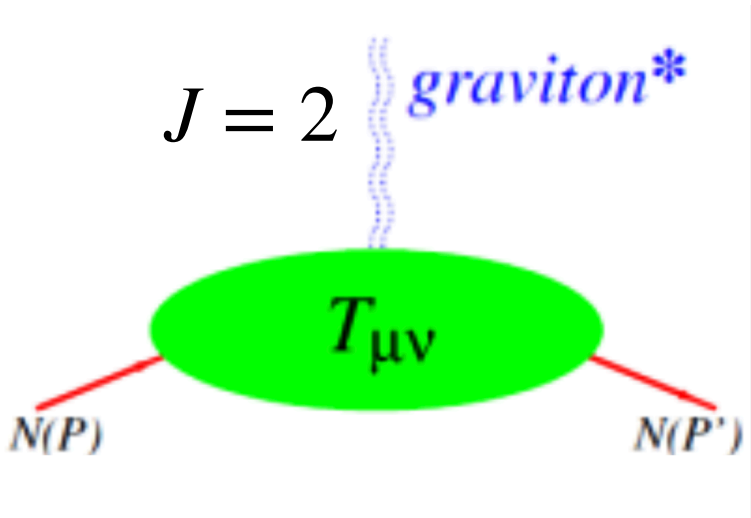
- how do quarks & gluons confine inside proton ?

$$\langle p,s|T_{\mu\nu}|p',s'\rangle = \bar{U}(p,s)\left[M(t)\frac{P_\mu P_\nu}{m} + J(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2m} + D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m}\right]U(p',s')$$

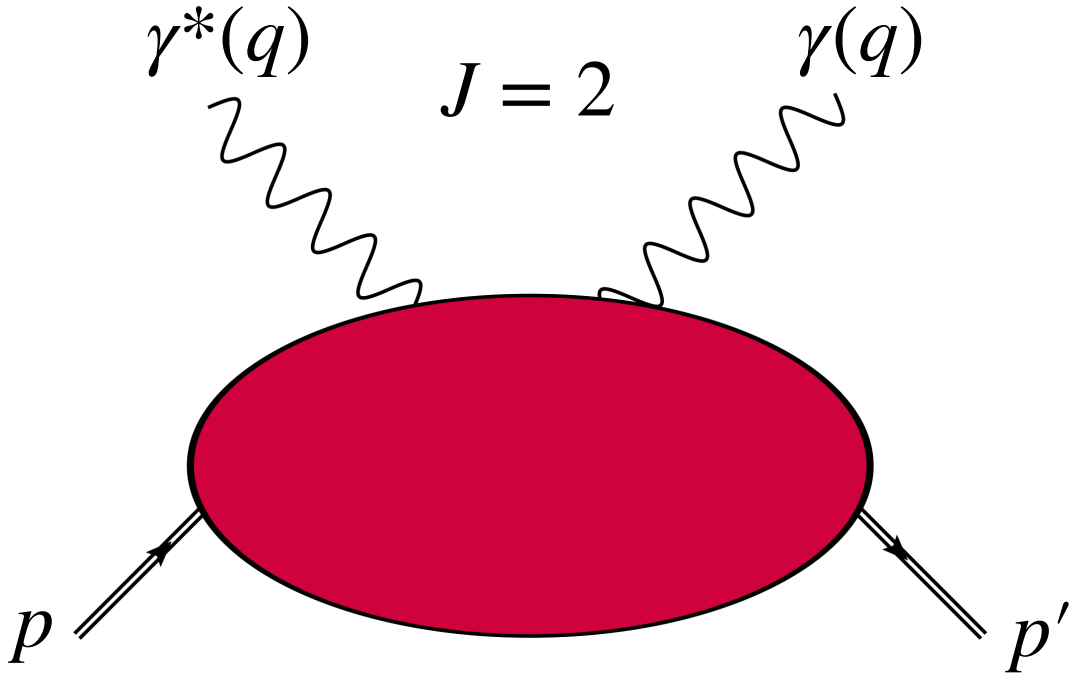




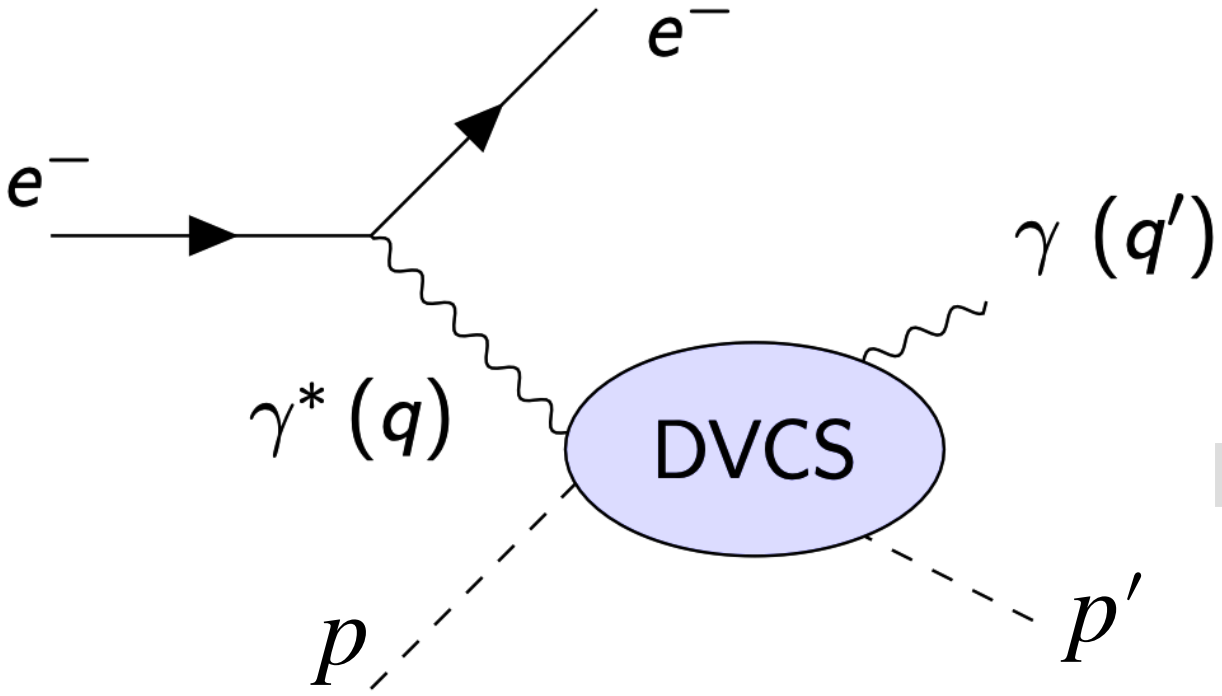
how do we access $T_{\mu\nu}$ at EIC ?

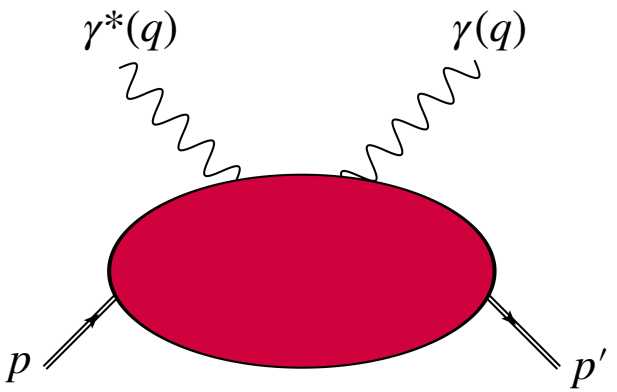


not by scattering a graviton, obviously!!

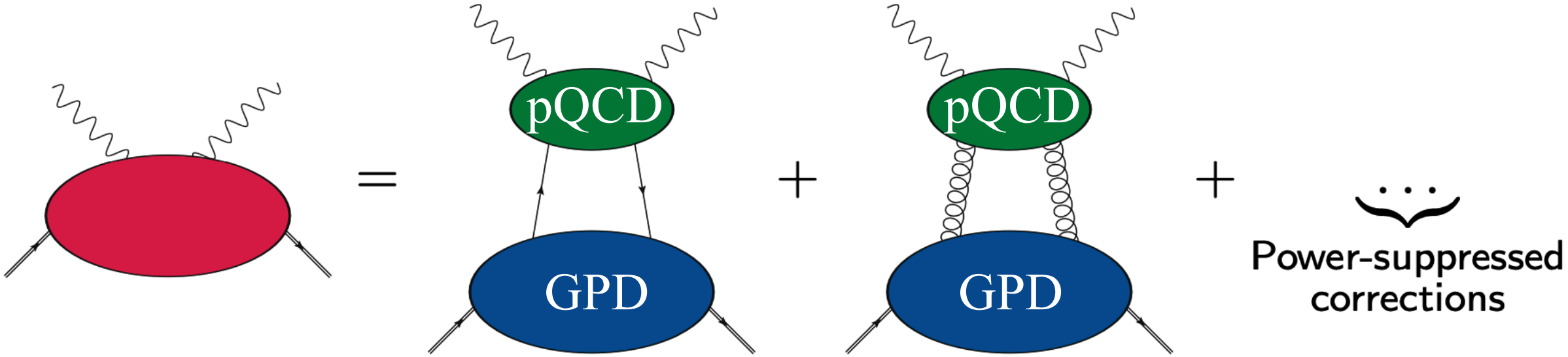


by scattering virtual photon



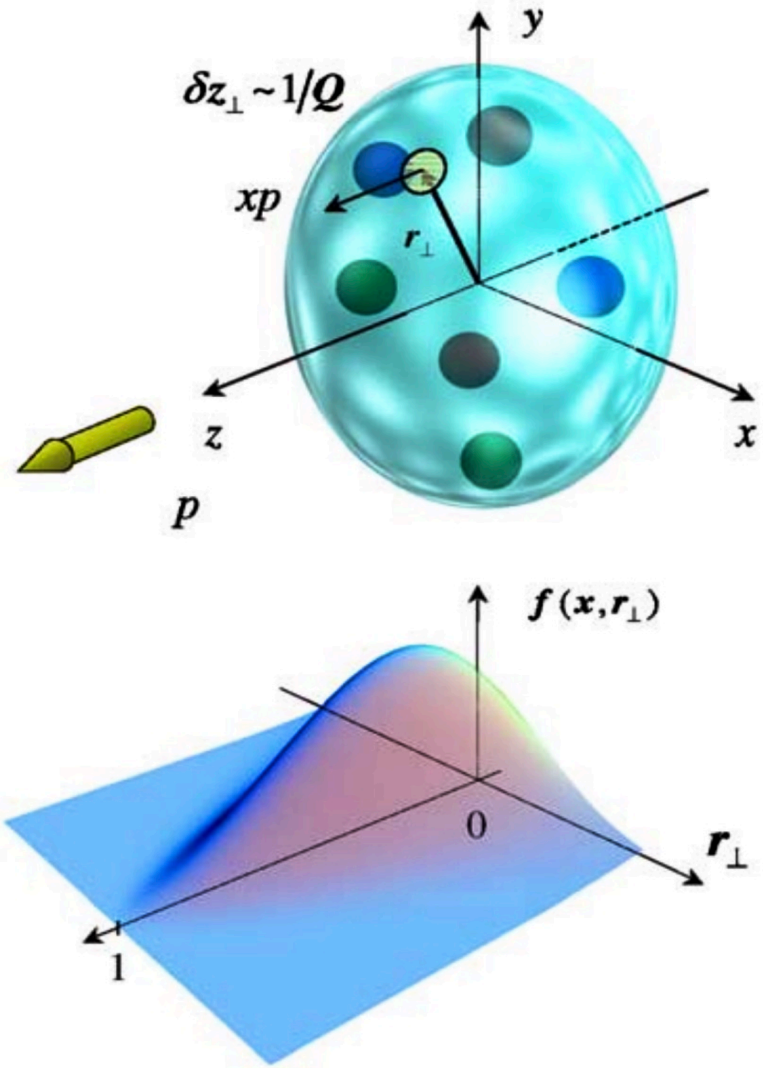


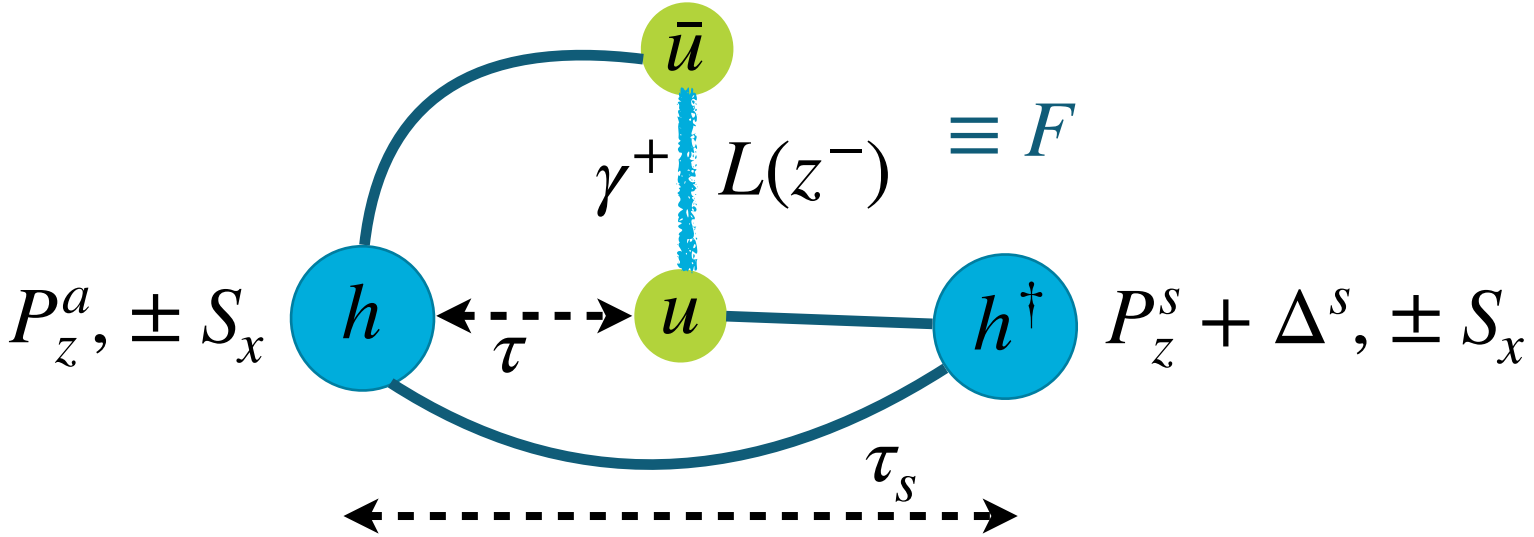
deeply virtual Compton scattering (DVCS)



generalized parton distribution (GPD)

GPD



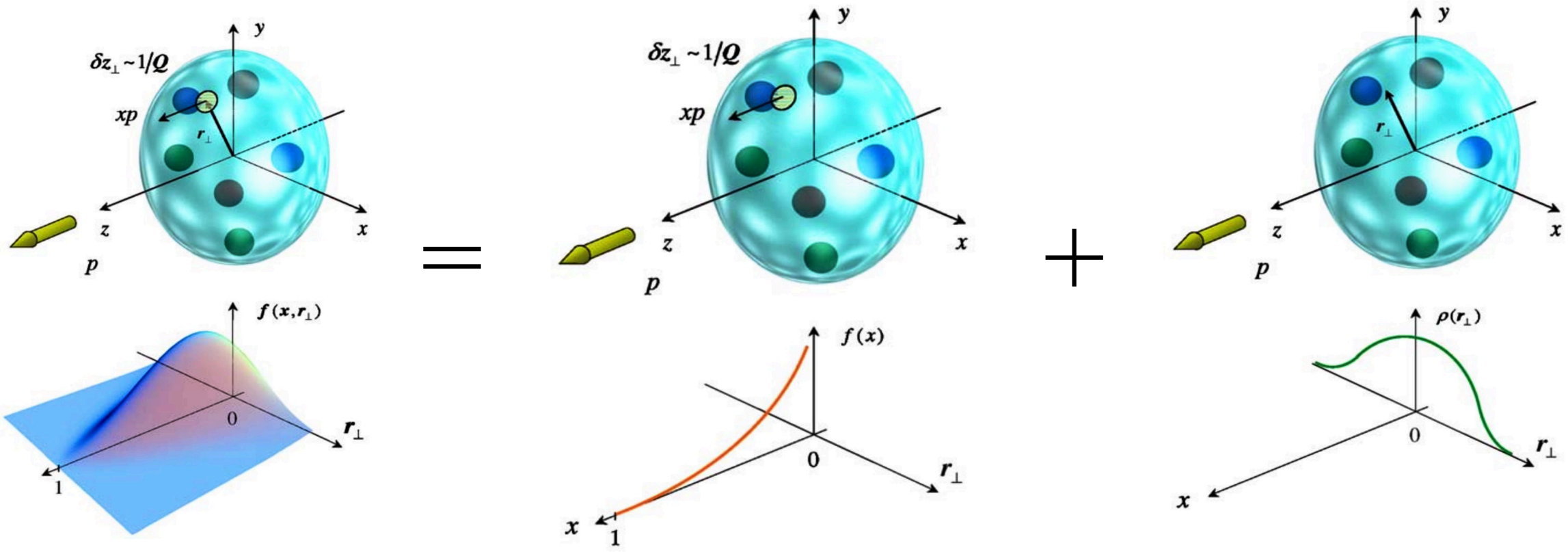


$$F = \bar{u}\left[\gamma^+ H + \frac{i\sigma^{+\mu}\Delta_\mu}{2m}E\right]u$$

H, E: GPD

N / q	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T $\tilde{H}_T$

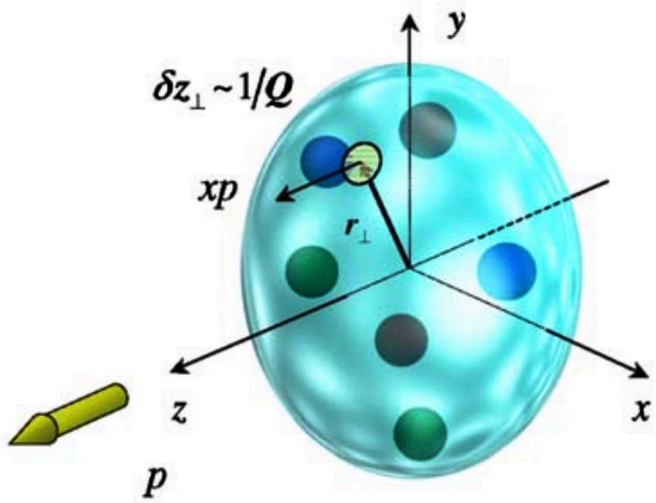
GPD = parton distribution function (PDF) + form factor (FF)



$$GPD(x,\xi,t) \qquad PDF(x) = GPD(x,0,0) \qquad FF(t) = \int dx\, GPD(x,\xi,t)$$

angular momentum from GPD

quark/gluon angular momentum contributions to proton spin:



$$J_{q/g}(t \to 0) = \frac{1}{2}\int dx\ x\left[H_{q/g}(x,\xi \to 0,t) + E_{q/g}(x,\xi \to 0,t)\right]$$

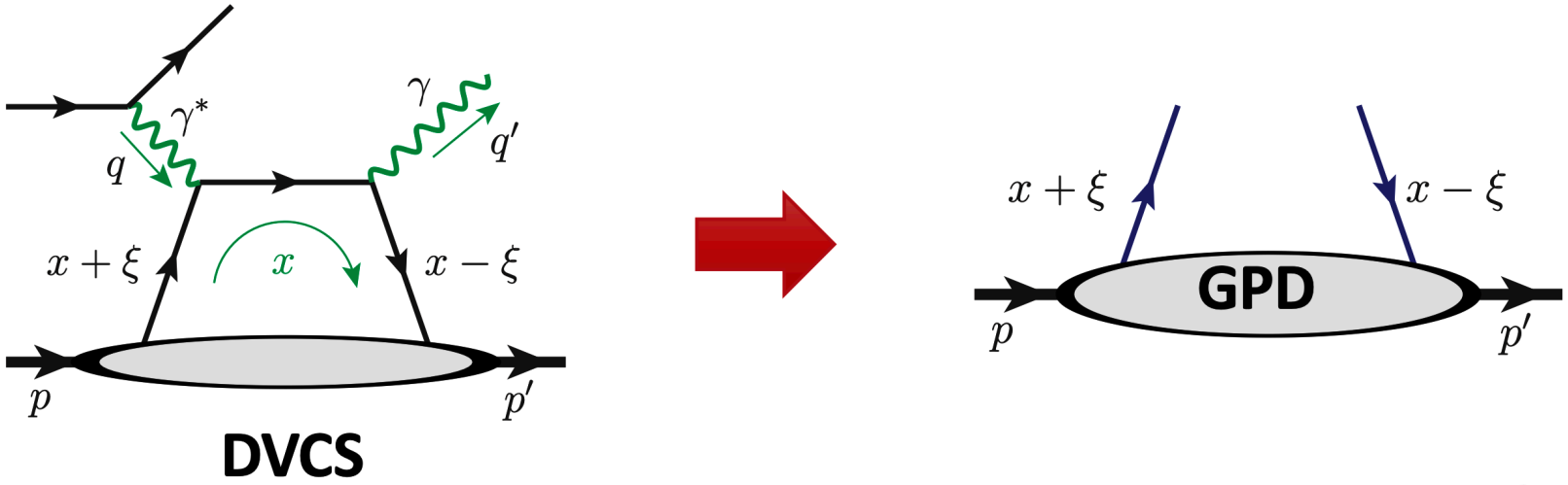
- need x-dependence of H, E
- need H, E at $\xi, t \to 0$

‘energy’ and ‘pressure’ distribution from GPD

$$M(t) + \xi^2 D(t) = \int dx \; xH(x, \xi, t)$$

- need x-dependence of H
- need H for large range of t for a each ξ

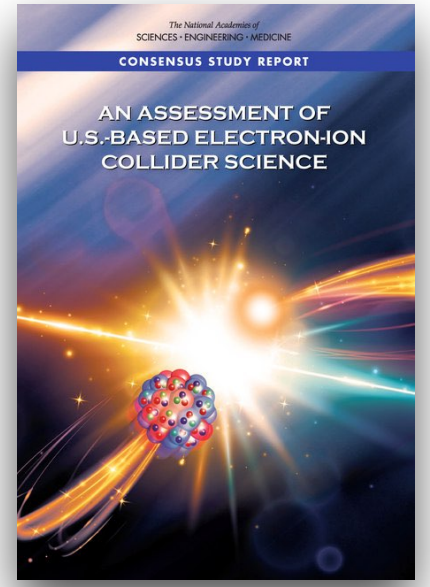
from DVCS to GPD



$$A_{DVCS}(\xi,t) = \int dx\; C_{pQCD}(x,\xi) \otimes H(x,\xi,t)$$

- very difficult to access x-dependence of GPD
- extrapolations to $\xi \to 0$ is model dependent
- not accessible for all values of t

the big science questions of EIC cannot be answered without supplementing experiments with QCD knowledge.



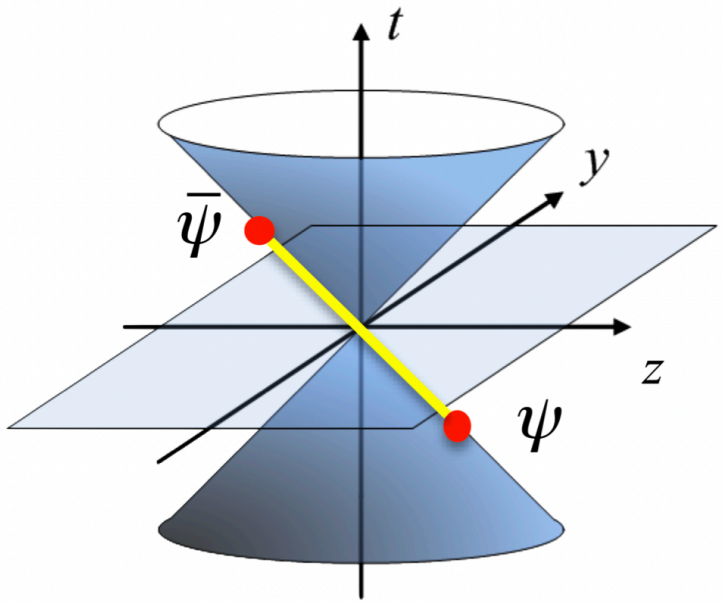
Summary of the National Academy of Science report

"The scientific challenges that would unfold with EIC require a robust theory program, not simply to design and interpret experiments, but also to develop the broad implications in an understanding of the quantum world, both through analytic theory as well as through lattice QCD simulations on large-scale computers."

‘partonic picture’ from lattice QCD

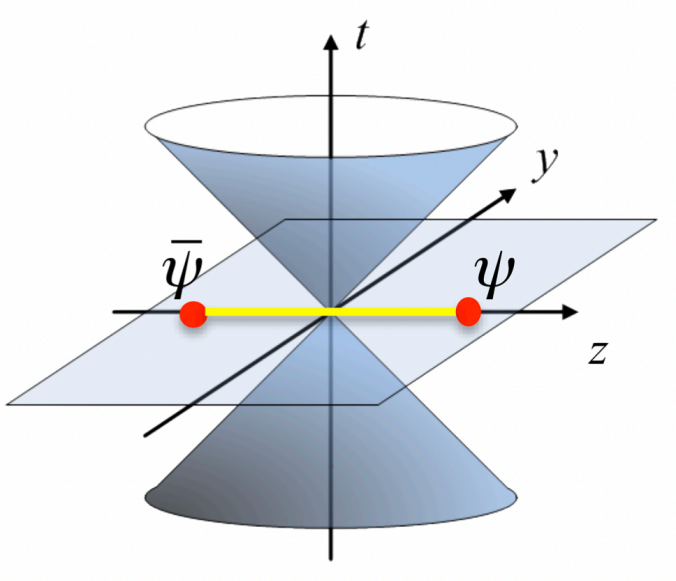
‘partonic picture’

- effective description of QCD as observed from an infinite momentum frame
  - infinite momentum limit first, regularize the theory later

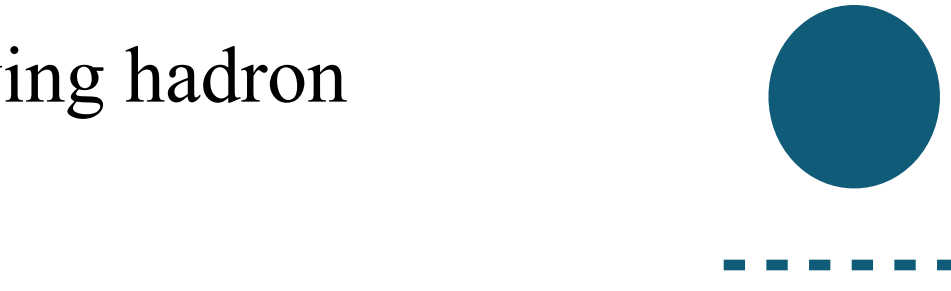


how to be light-like from lattice QCD with Euclidean time?

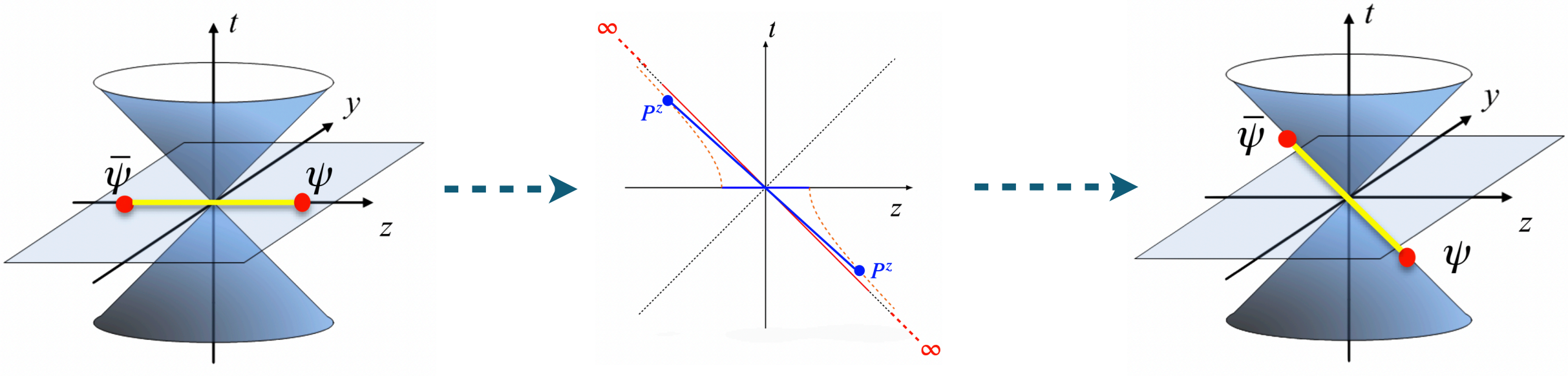
hadron at rest

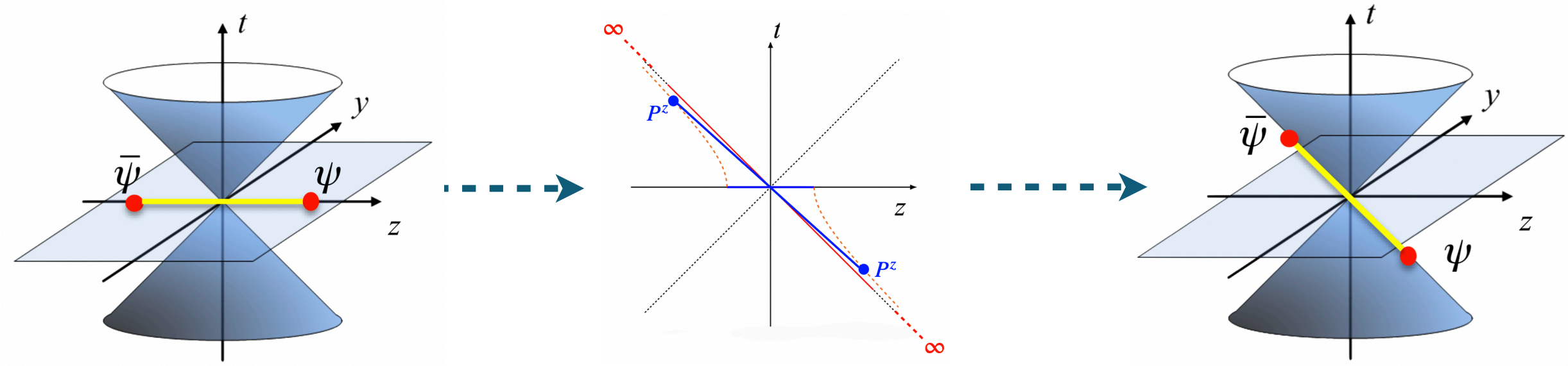


fast-moving hadron

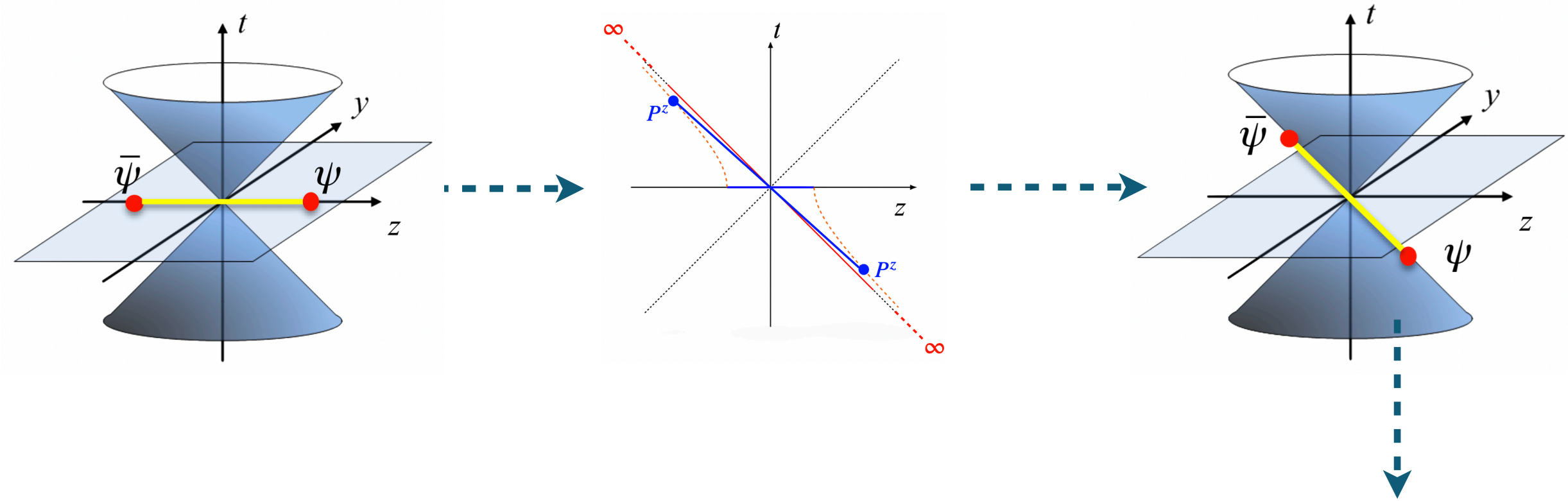


$P_z \approx E$





- regularize QCD first, infinite momentum limit later — opposite order of limits while seeing the ‘partonic picture’
- these 2 limits don’t commute; but it’s UV physics and can be ‘corrected’ through pQCD



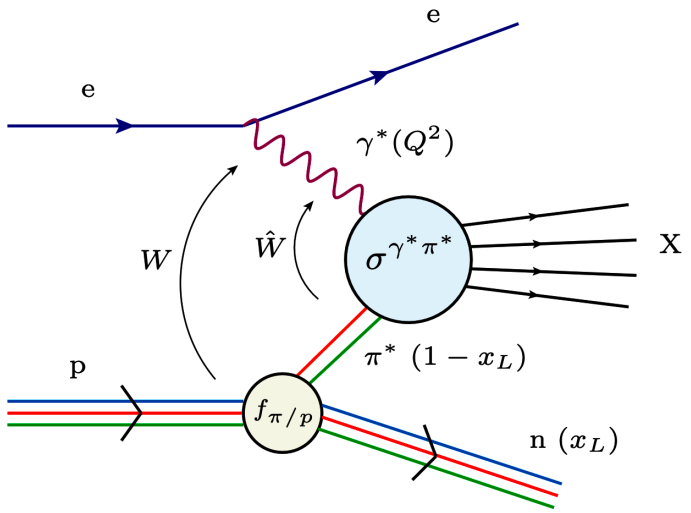
‘partonic picture’ ⟵ pQCD ⟵ renormalize

F.T. wrt z

‘simple cases’ : pion PDF

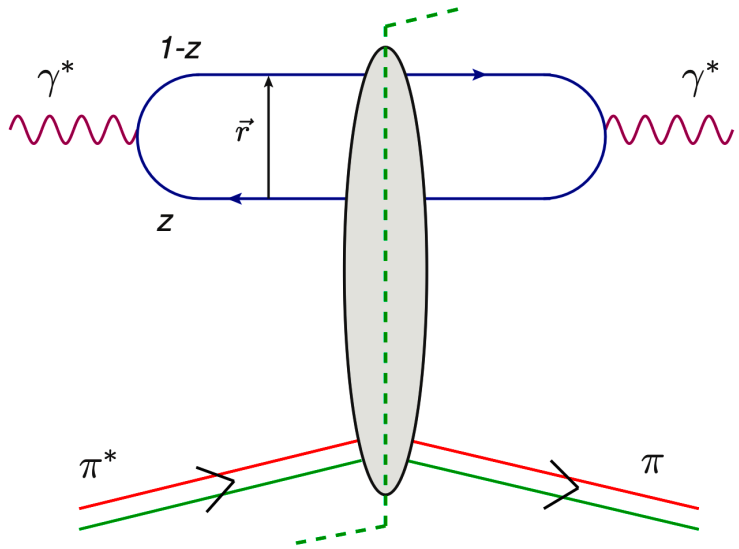
EIC big question 4

- can we observe saturated gluon regime ?



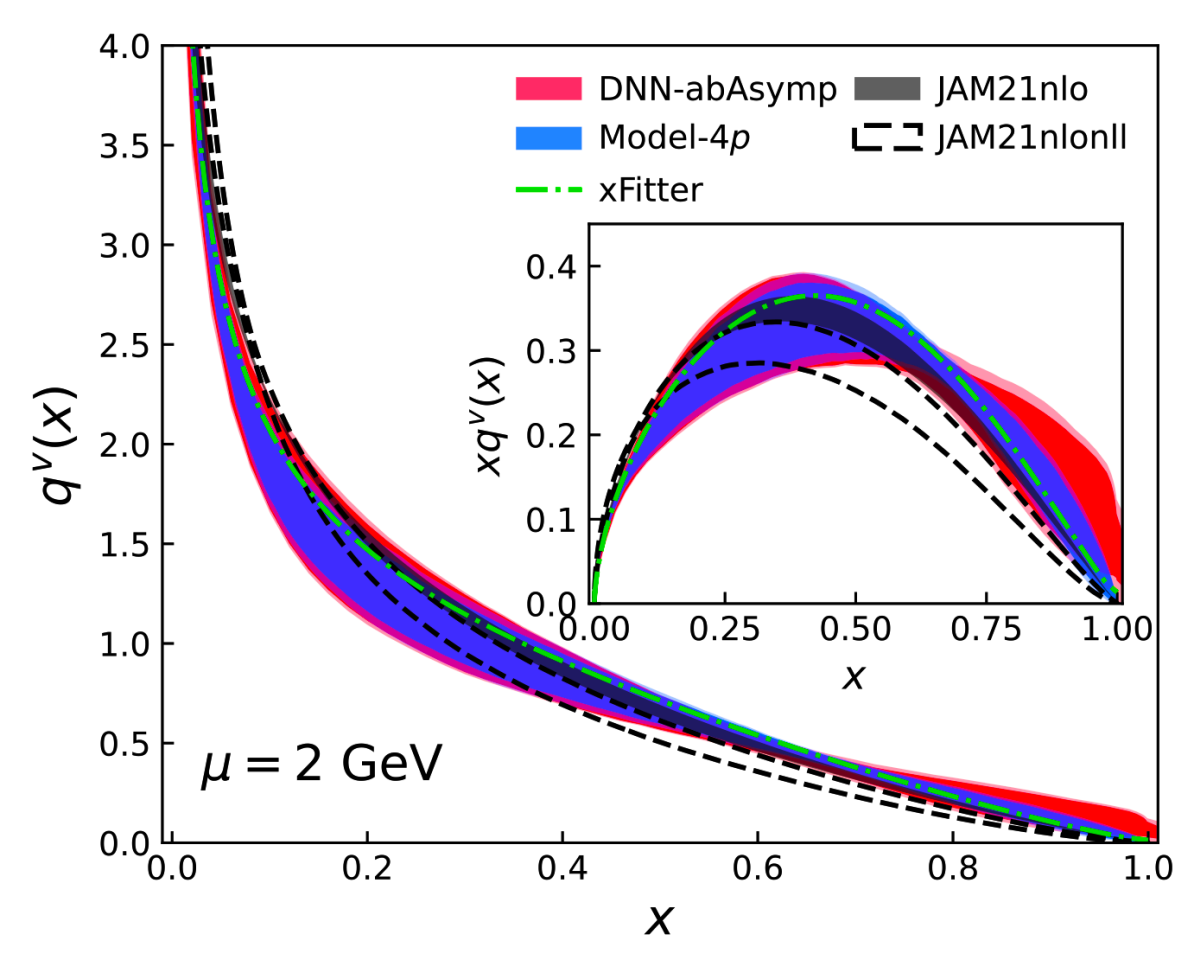
universality (hadron independence) of gluon situation: proton vs. pion

Kumar, Toll: Talks tomorrow



needs pion PDF

‘simple cases’ : NNLO valance pion PDF



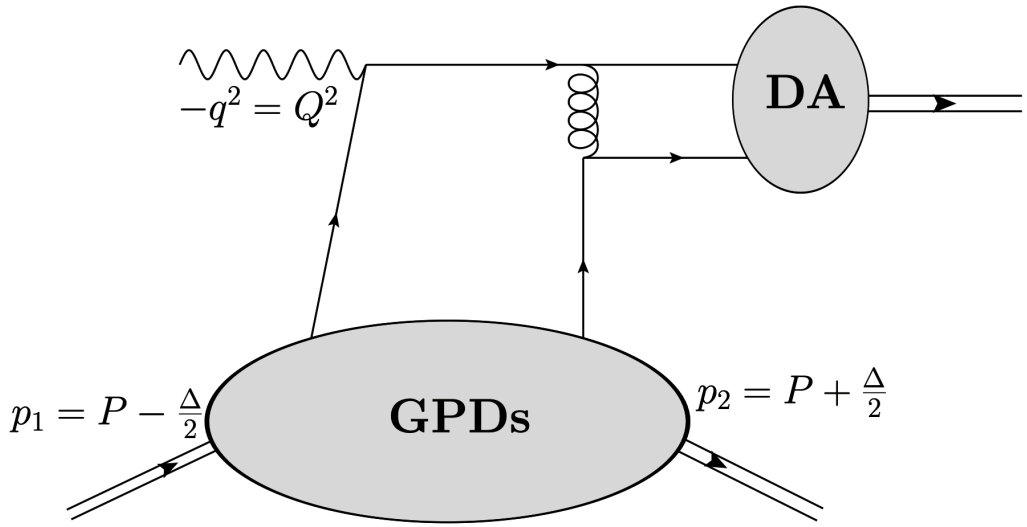
Xiang Gao et.al., Phys. Rev. D, to appear (arXiv: 2208.02297)

‘simple cases’ : pion distribution aptitude (DA)

$\pi, \rho, J/\Psi$

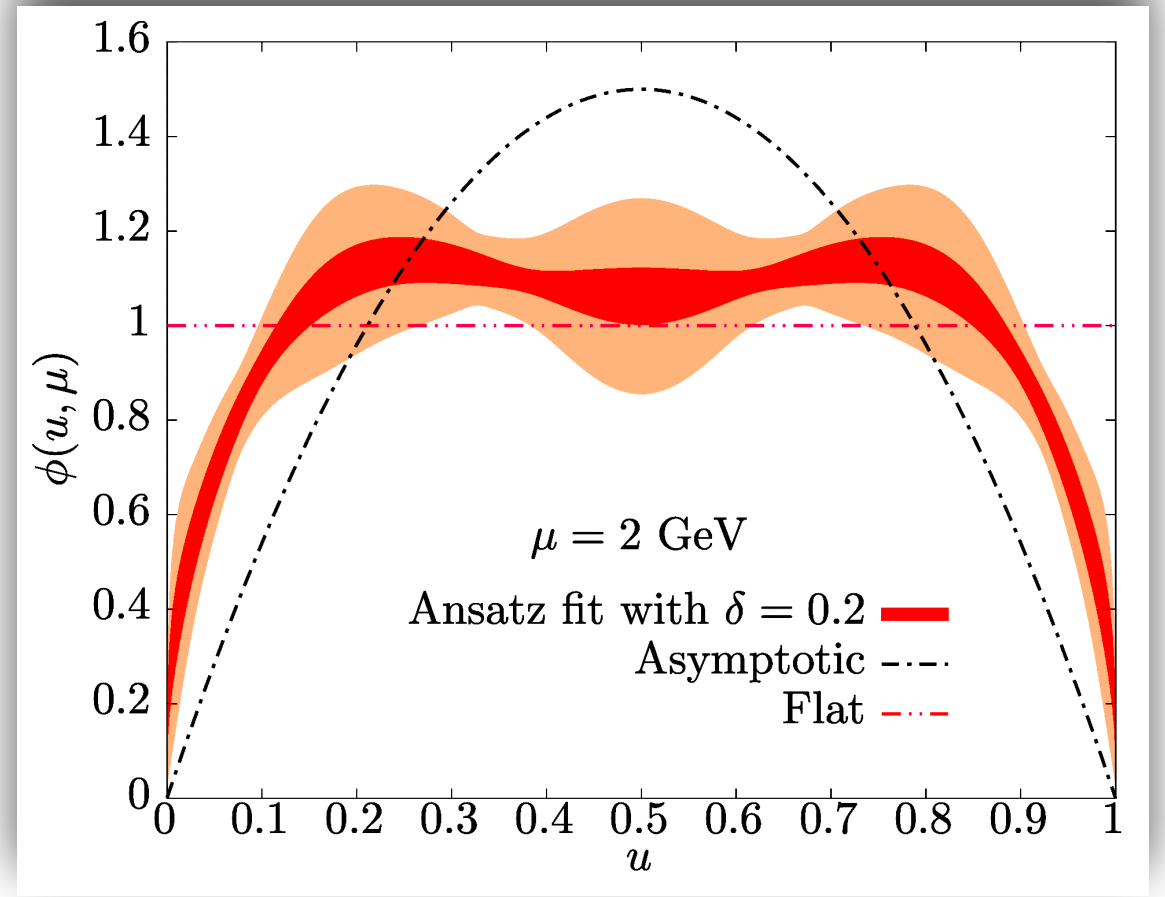
accessing GPD through DVMP

needs meson DA



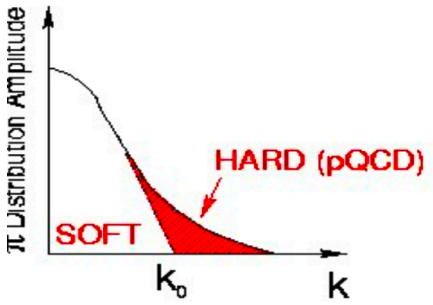
deeply virtual meson production (DVMP)

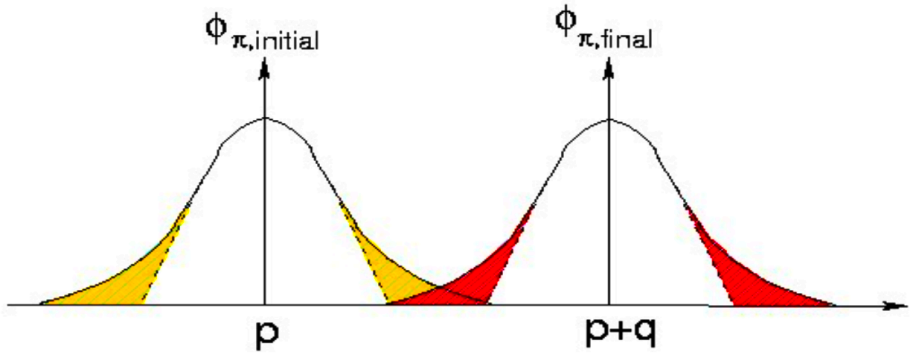
‘simple cases’ : pion distribution aptitude (DA)

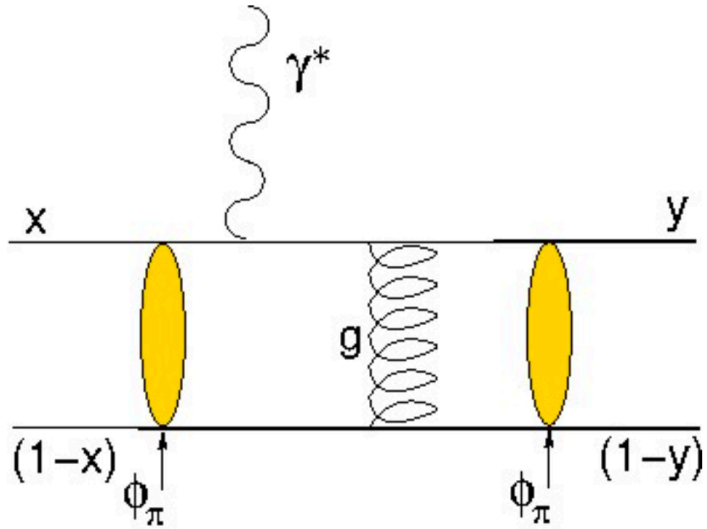


Nikhil Karthik et.al., Phys. Rev. D106, 074505 (2022)

‘simple cases’ : pion FF at large momentum transfer







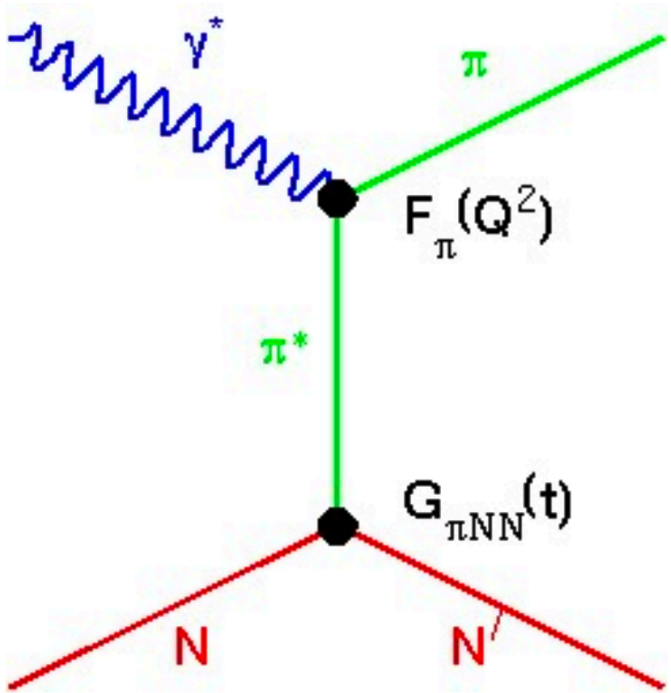
- at what energy strong interaction is perturbative/partonic ?
  - transition between non-perturbative & perturbative QCD regime

‘simple cases’ : pion FF at large momentum transfer

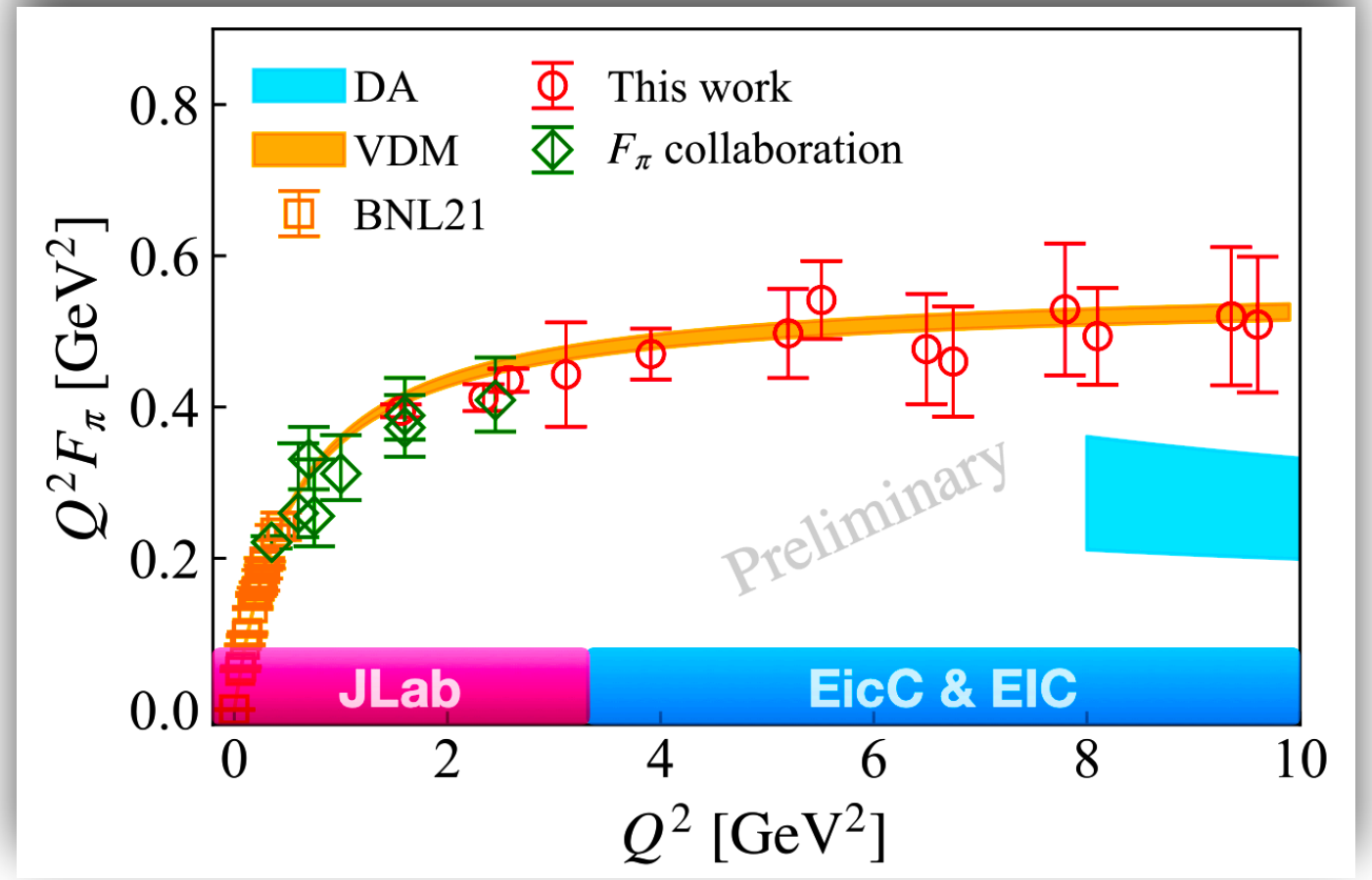
experimental measurements at JLAB & EIC is model-sensitive

model-dependent extrapolation $t \rightarrow m_\pi^2$

independent QCD confirmation is essential

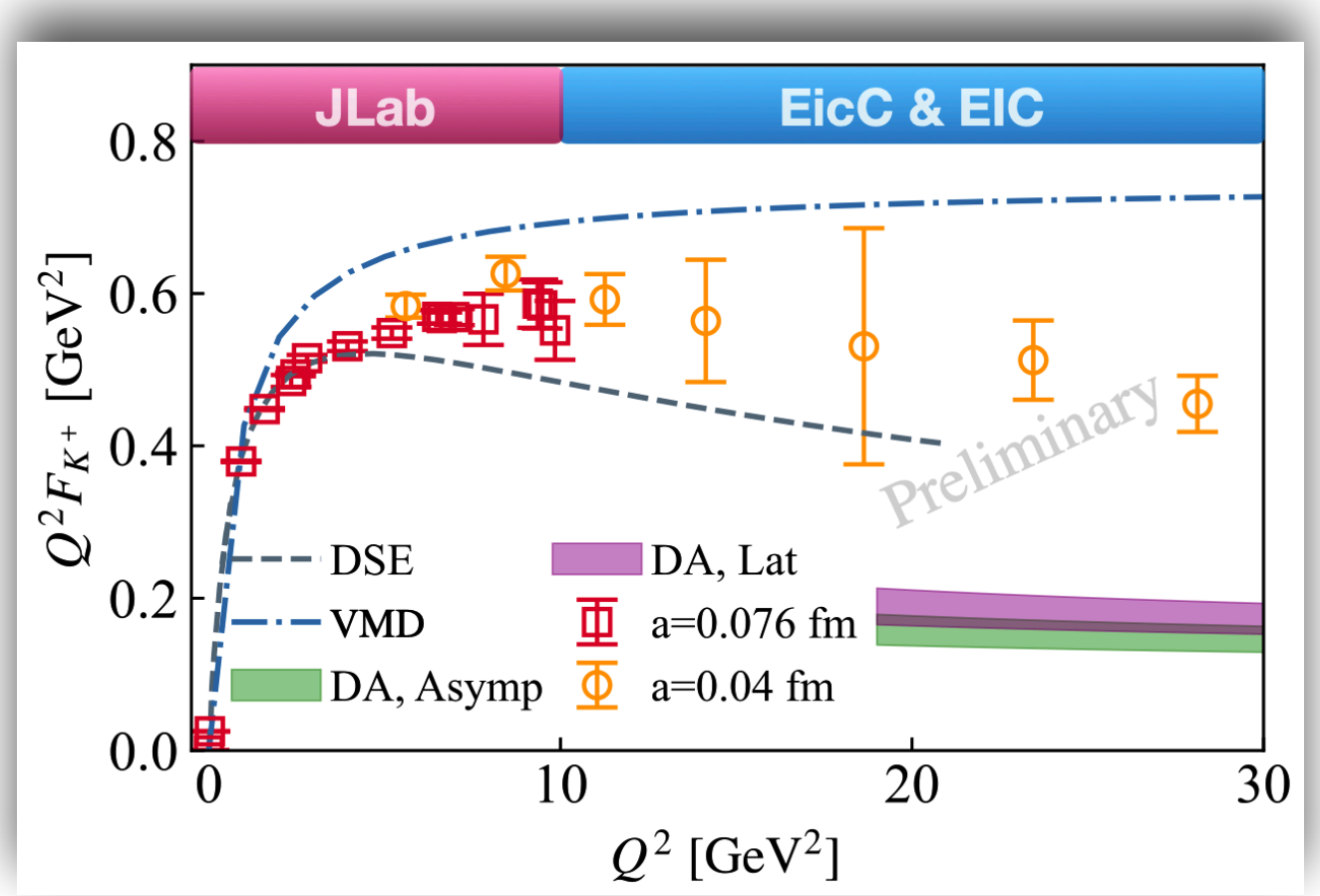


‘simple cases’ : pion FF at large momentum transfer



Qi Shi et.al., in preparation

‘simple cases’ : kaon FF at very large momentum transfer



Qi Shi et.al., in preparation

proton GPD from lattice QCD: fast and accurate

GPDs from lattice QCD:

- x dependence
- t dependence
- at $\xi = 0$

a novel Lorentz invariant formalism for lattice calculations of GPD

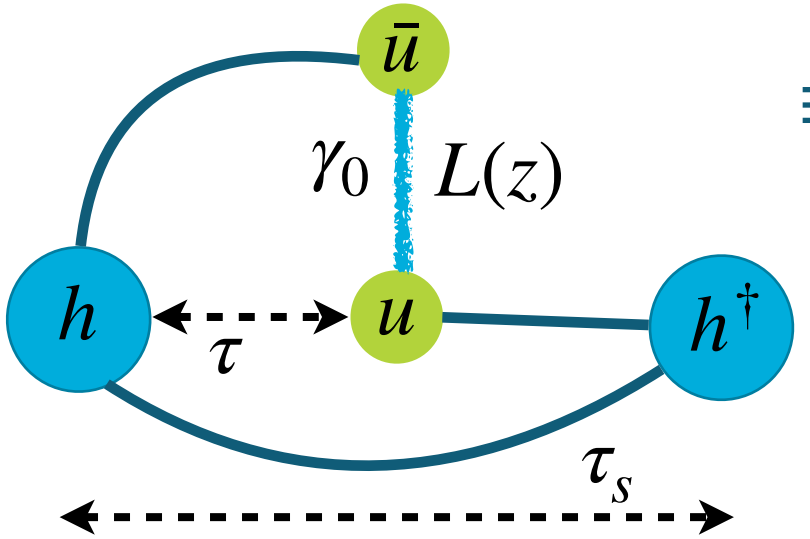
- faster: $\gtrsim 5\times$ faster access to t-dependence of GPD
- accurate: reduces frame-dependent power corrections

Shohini Bhattacharya et al., Phys. Rev. D, to appear (arXiv:2209.05373)

the way it was …

momenta transfer: $P_z^s - \frac{\Delta^s}{2}, \pm S_x$

symmetric



$\equiv F_0^s$

$P_z^s + \frac{\Delta^s}{2}, \pm S_x$

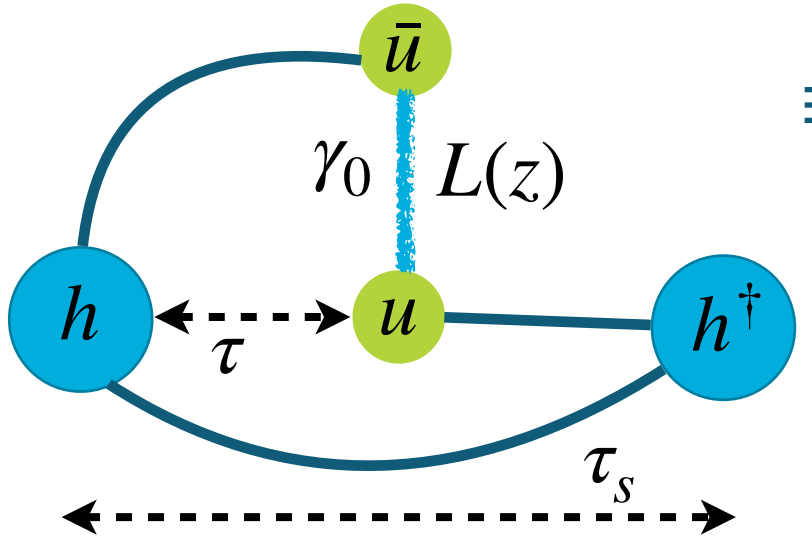
$\mathscr{H}_0^s, \mathscr{E}_0^s$: pseudo-/quasi-GPD

$$F_0^s = \bar{u}\left[\gamma_0 \mathscr{H}_0^s + \frac{i\sigma^{0\mu}\Delta_\mu^s}{2m}\mathscr{E}_0^s\right] u$$

$\mathscr{H}_0^s, \mathscr{E}_0^s$ + pQCD matching + $z^2 \to 0 / P_z \to \infty$

H, E: light-cone GPD

the way it was

momenta transfer: $P_z^s - \frac{\Delta^s}{2}, \pm S_x$  $P_z^s + \frac{\Delta^s}{2}, \pm S_x$

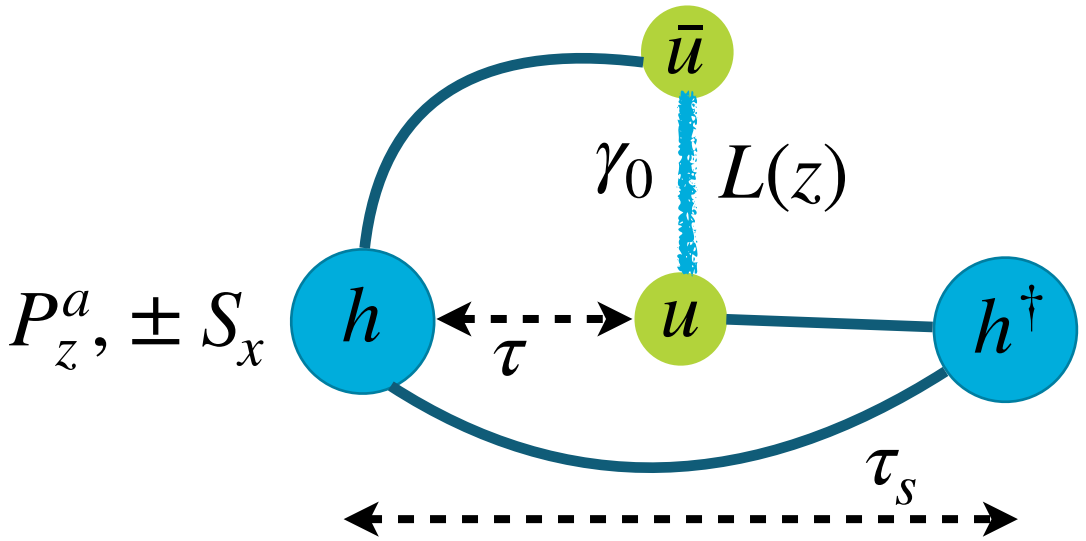
$\equiv F_0^s$

symmetric

- need a separate calculation for each $\Delta^2 = -t$
- each calculation is $2\times$ costlier than asymmetric momenta transfer

the way we wanted …

momenta transfer:
asymmetric



$\mathcal{H}_0^a, \mathcal{E}_0^a$: pseudo-/quasi-GPD

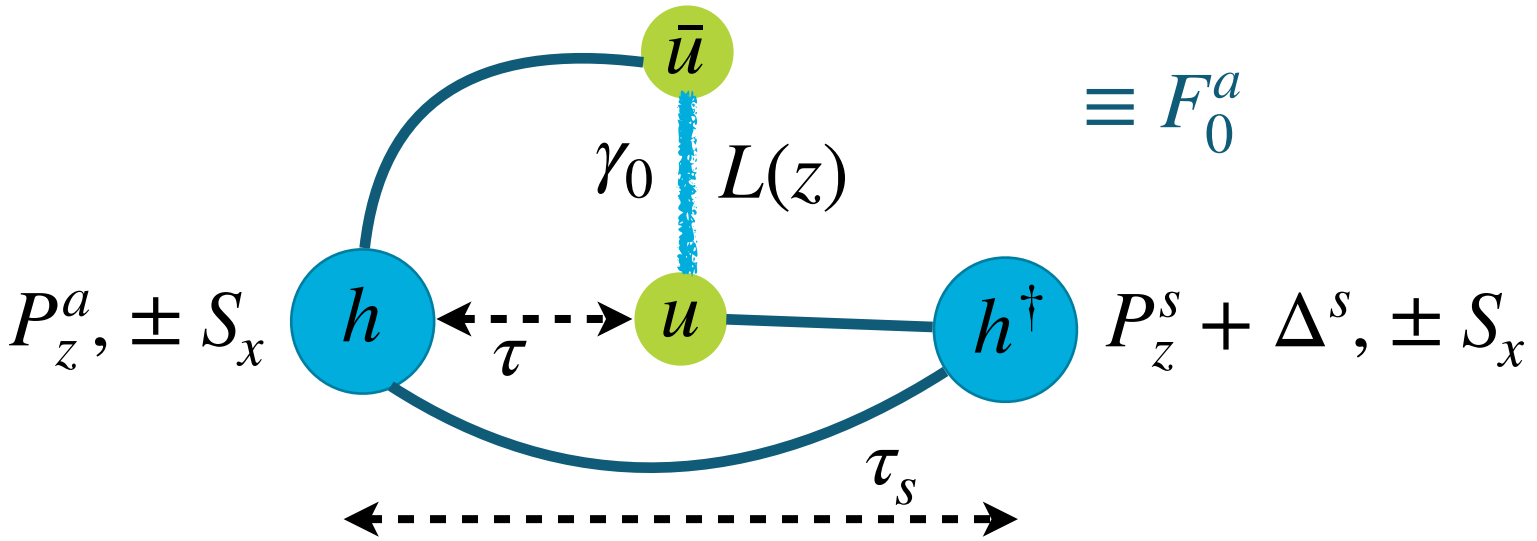
$$F_0^a = \bar{u}\left[\gamma_0 \mathcal{H}_0^a + \frac{i\sigma^{0\mu}\Delta_\mu^a}{2m}\mathcal{E}_0^a\right] u$$

$\mathcal{H}_0^a, \mathcal{E}_0^a$ + pQCD matching + $z^2 \to 0 / P_z \to \infty$

H, E: light-cone GPD

the way we wanted

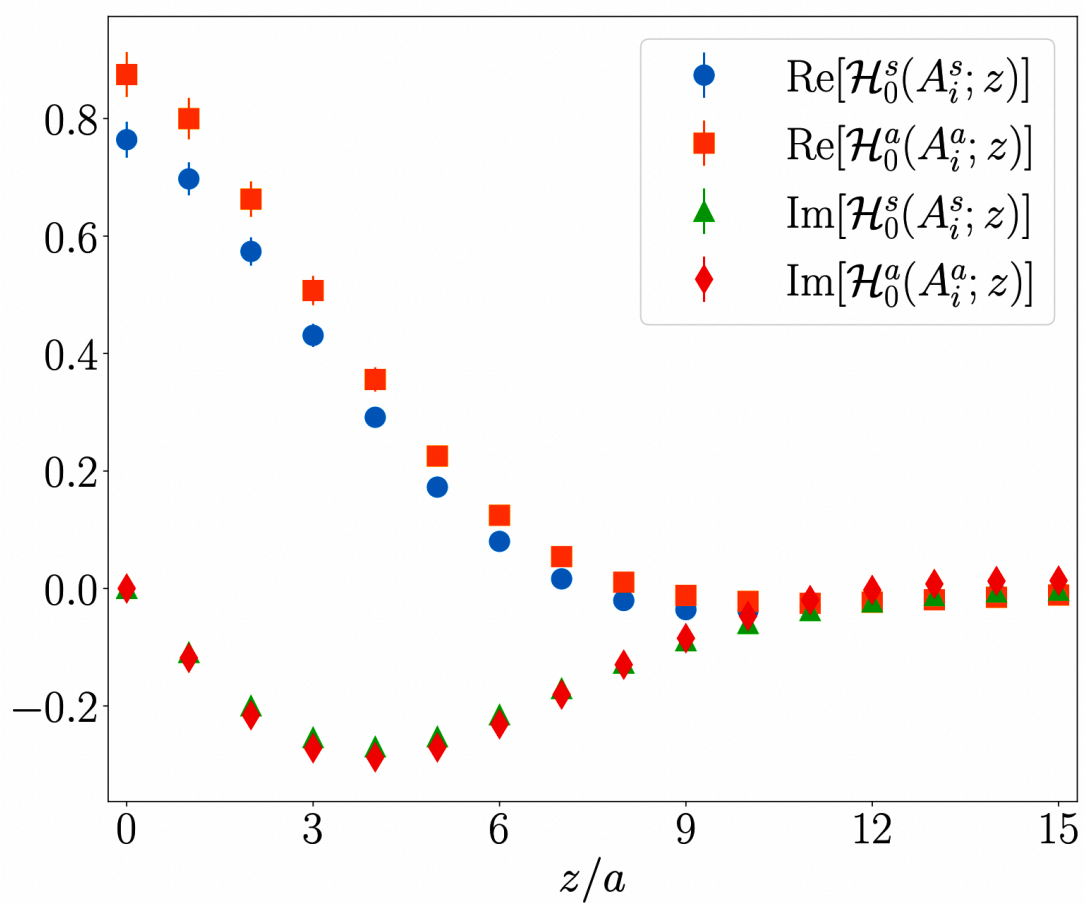
momenta transfer:
asymmetric

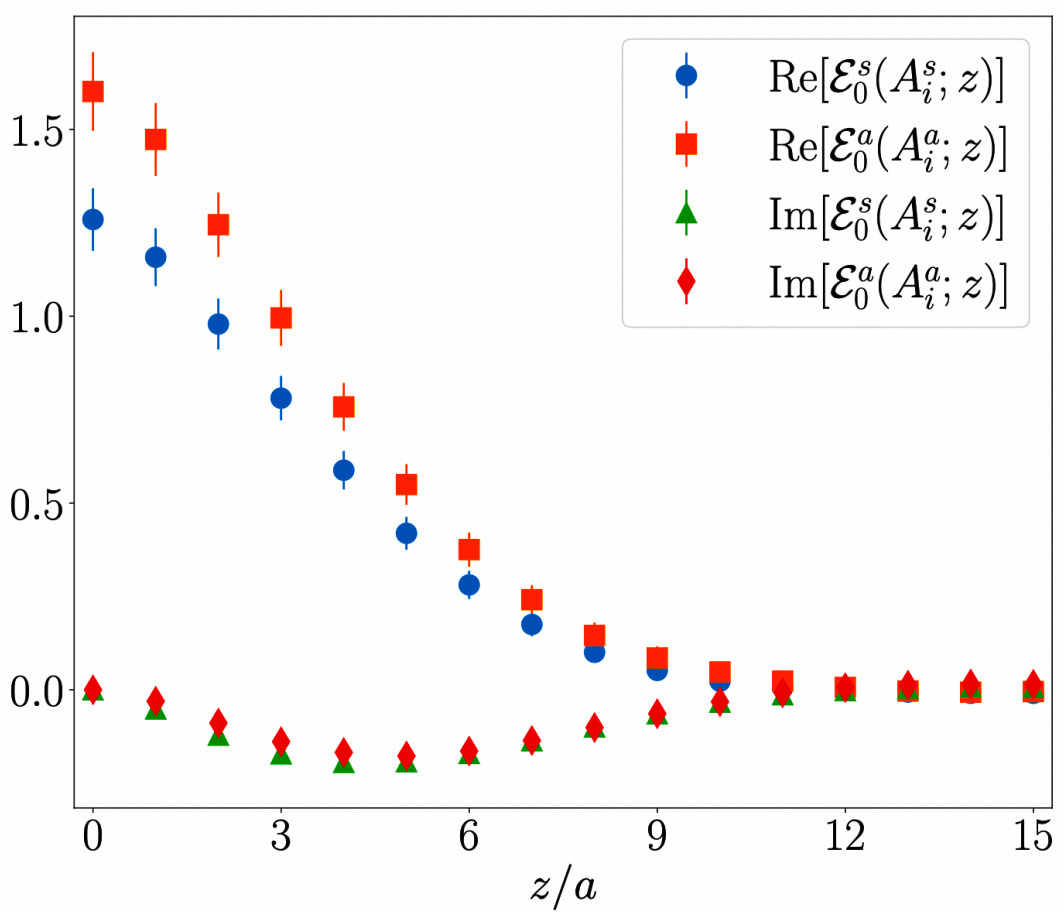


- multiple Δ^2 within a single calculation
- each calculation is $2\times$ faster than symmetric frame

$\gtrsim 5\times$ faster access to t-dependence of GPD

the way it went …

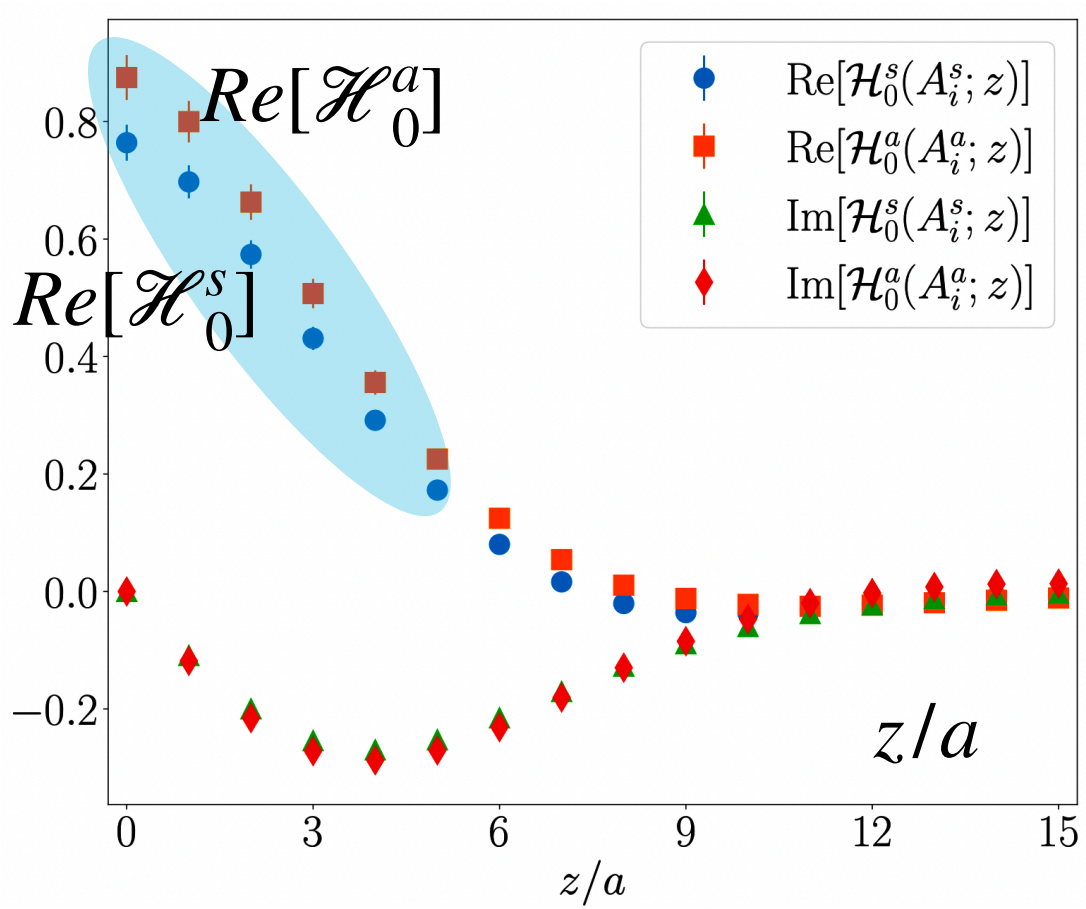


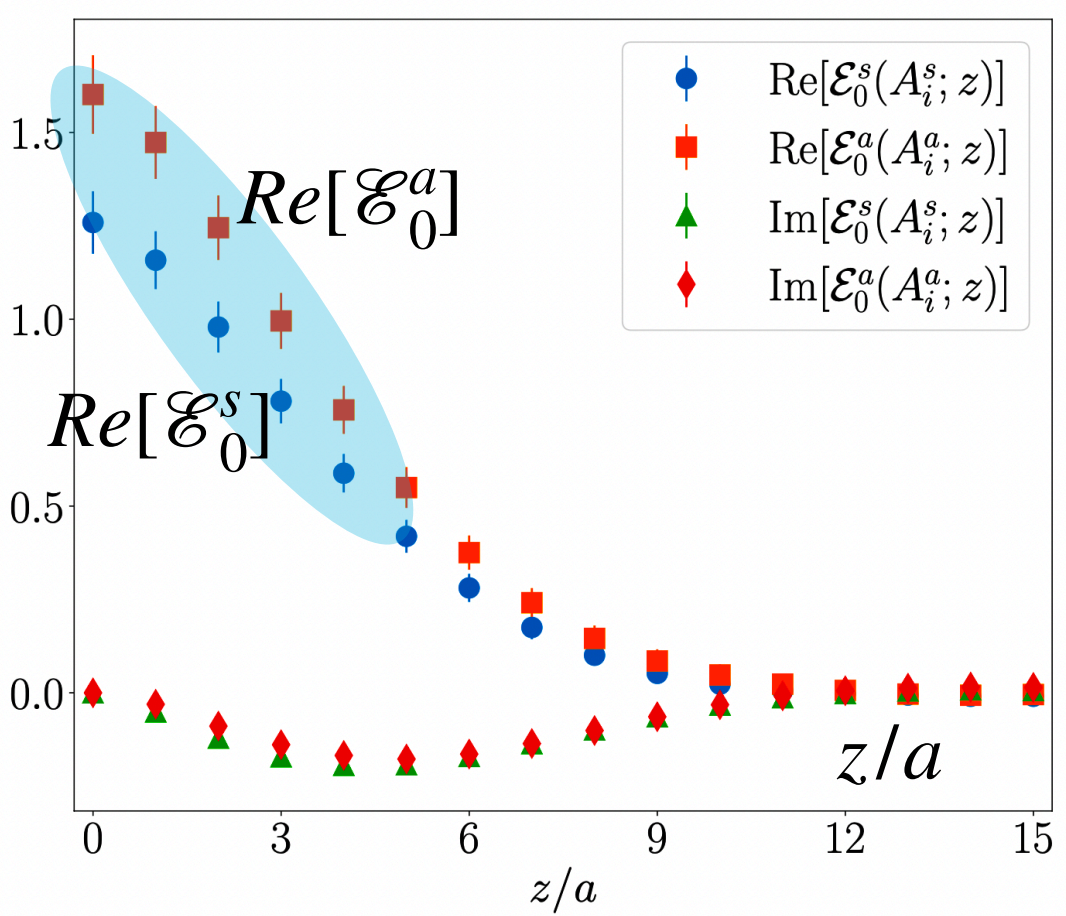


$$P_z = 1.25 \text{ GeV}, t \simeq -0.67 \text{ GeV}, \xi = 0$$

$$m_\pi = 260 \text{ MeV}, a = 0.093 \text{ fm}, 32^3 \times 64, N_f = 2 + 1 + 1 \text{twisted mass fermions}$$

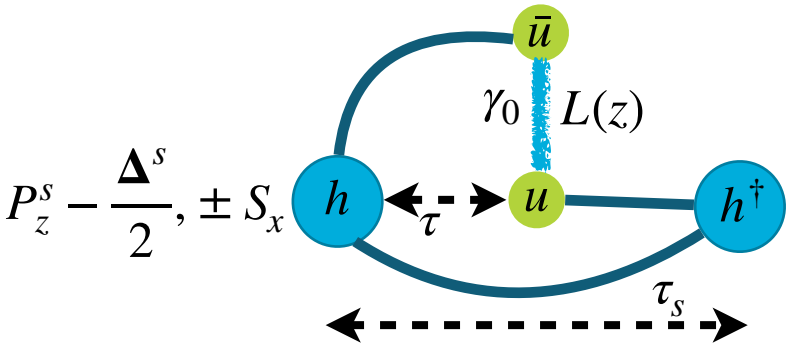
the way it went





frame-dependent power corrections

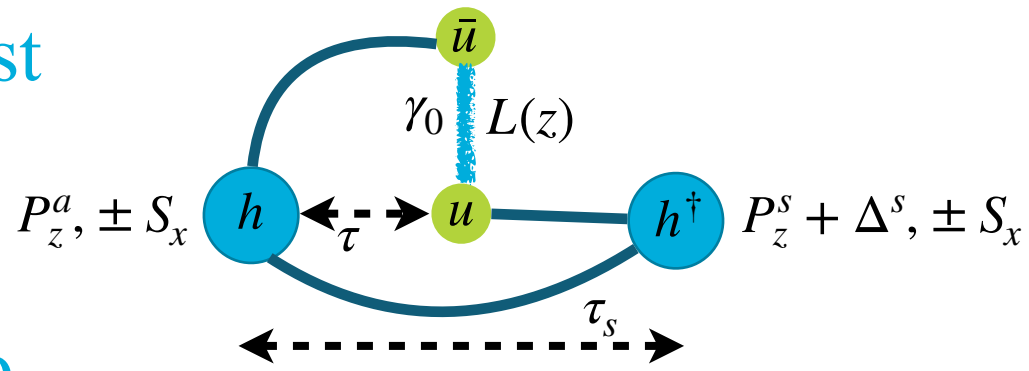
the way it went wrong



symmetric

Lorentz boost

transverse to Wilson line*



asymmetric

$$F_0^s \leftrightarrow \gamma F_0^a - \gamma\beta F_\perp^a$$

$$\perp \equiv x, y$$

frame-dependent power corrections

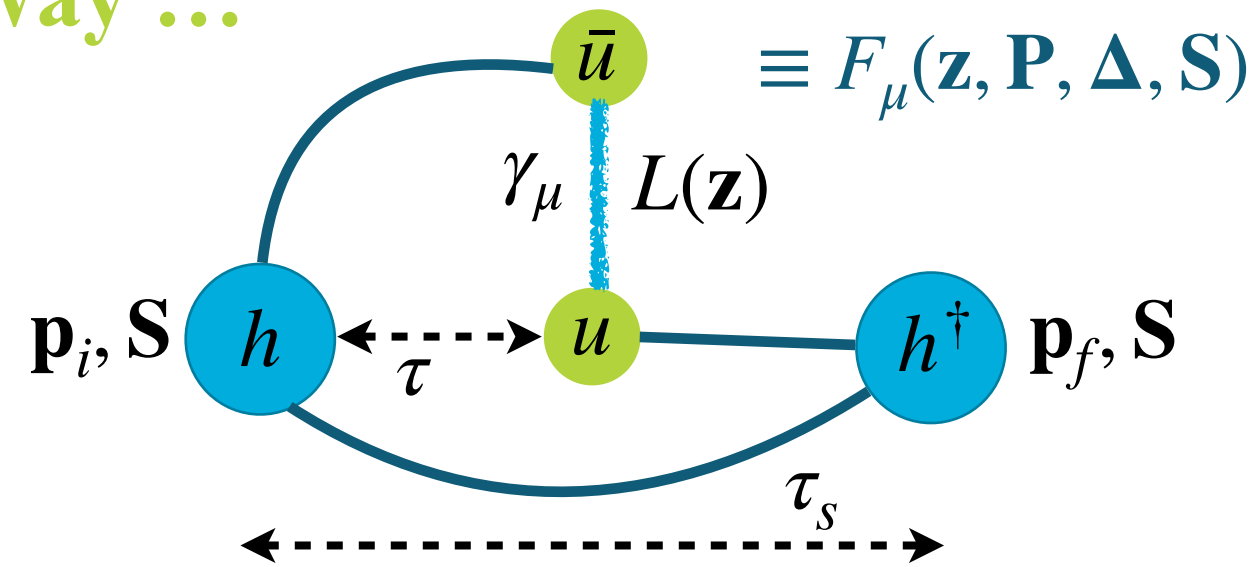
$$P_z \to \infty \qquad F_0^s \leftrightarrow F_0^a$$

* Euclidean lattice: the operator must remain space-like $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \sqrt{\frac{E_i^a + E_f^a}{2E_f^a}} \quad \beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$

the new Lorentz invariant way …

$$\mathbf{P} = (\mathbf{p}_i + \mathbf{p}_f)/2$$

$$\boldsymbol{\Delta} = \mathbf{p}_f - \mathbf{p}_i$$



Lorentz covariant parameterization:

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

8 Lorentz invariant amplitudes $A_i\left(\mathbf{z}\cdot\mathbf{P}, \mathbf{z}\cdot\boldsymbol{\Delta}, \boldsymbol{\Delta}^2, \mathbf{z}^2\right)$

the new Lorentz invariant way …

From A's to GPD, Lorentz invariant mapping:

$$F^{+} = \bar{u}\left[\gamma^{+}\mathcal{H} + \frac{i\sigma^{+\mu}\Delta_{\mu}}{2m}\mathcal{E}\right]u$$

$$\mathcal{H} = A_1 + \left(\frac{\boldsymbol{\Delta}\cdot\mathbf{z}}{\mathbf{P}\cdot\mathbf{z}}\right)A_3$$

$$\mathcal{E} = -A_1 - \left(\frac{\boldsymbol{\Delta}\cdot\mathbf{z}}{\boldsymbol{\Delta}\cdot\mathbf{z}}\right)A_3 + 2A_5 + 2\left(\mathbf{P}\cdot\mathbf{z}\right)A_6 + 2\left(\boldsymbol{\Delta}\cdot\mathbf{z}\right)A_8$$

frame-dependent mapping:

$$\mathcal{H}_0^{s/a} = \sum h_i^{s/a} A_i \qquad \mathcal{E}_0^{s/a} = \sum e_i^{s/a} A_i$$

frame-dependent kinematic factors

the new Lorentz invariant way

$$\mathcal{H} = A_1 + \left(\frac{\boldsymbol{\Delta}\cdot\mathbf{z}}{\mathbf{P}\cdot\mathbf{z}}\right) A_3$$

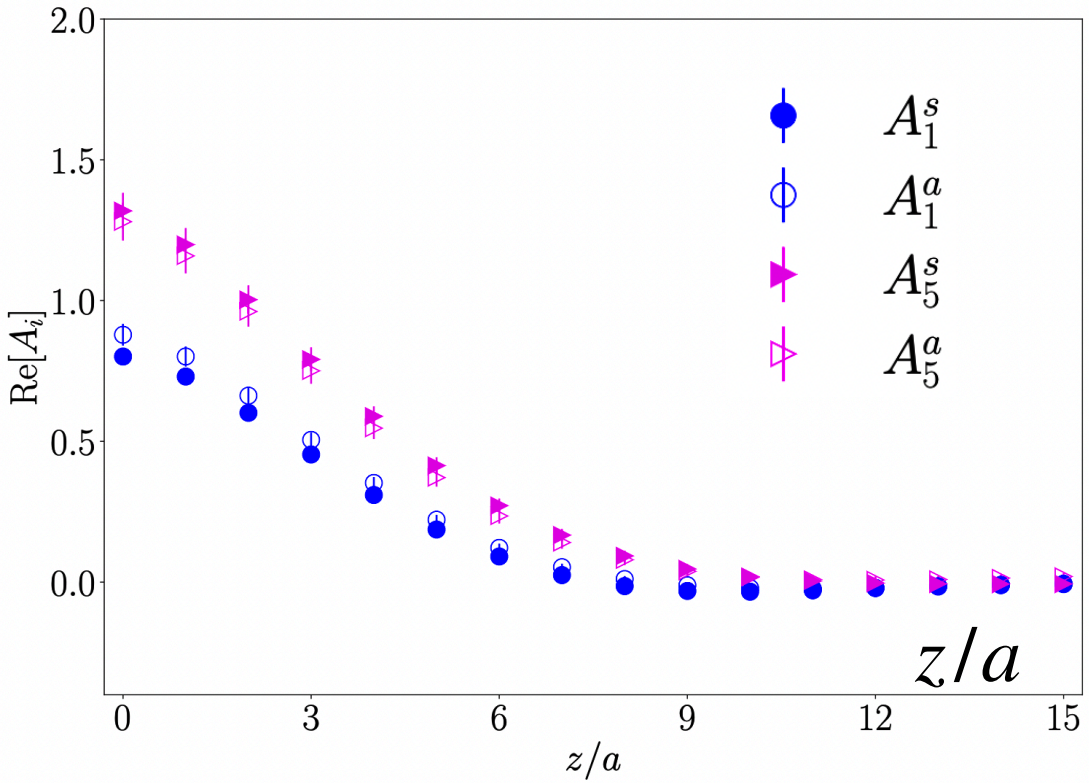
frame-dependent power corrections

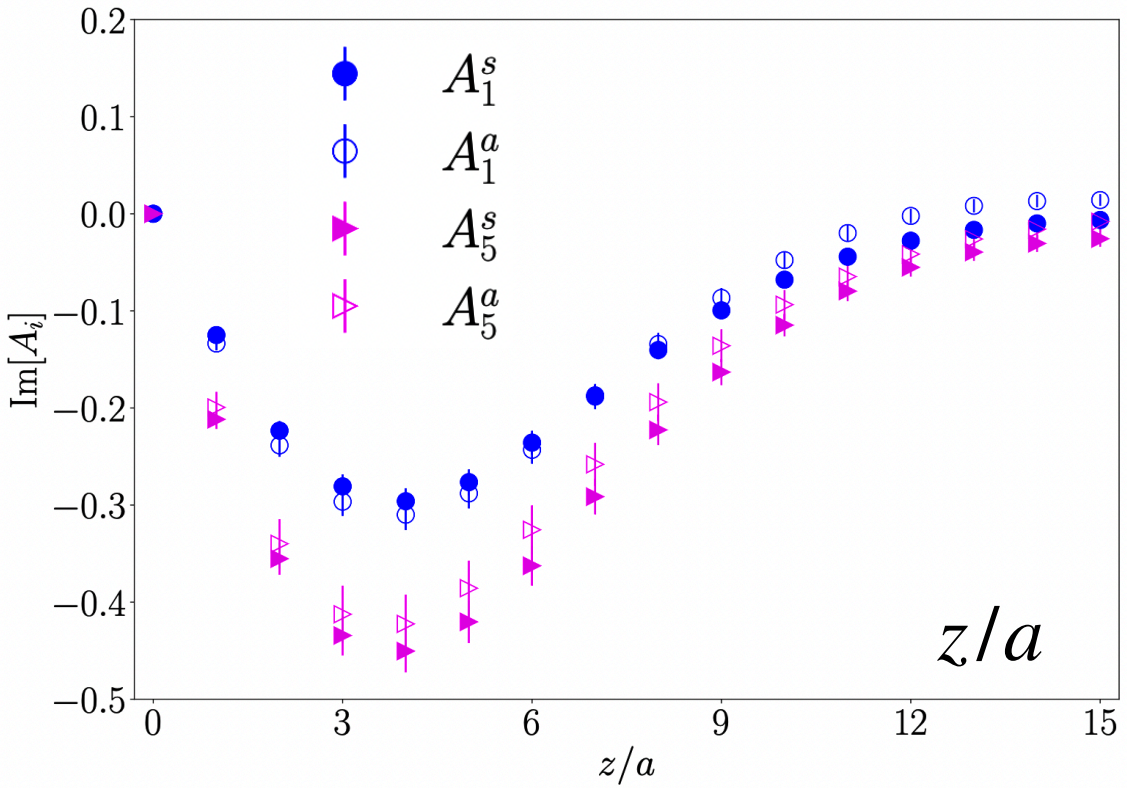
$$\mathcal{E} = -A_1 - \left(\frac{\boldsymbol{\Delta}\cdot\mathbf{z}}{\boldsymbol{\Delta}\cdot\mathbf{z}}\right) A_3 + 2A_5 + 2\left(\mathbf{P}\cdot\mathbf{z}\right) A_6 + 2\left(\boldsymbol{\Delta}\cdot\mathbf{z}\right) A_8$$

$$\mathcal{H}_0^{s/a}, \mathcal{E}_0^{s/a} \xrightarrow{z^2 \to 0/P_z \to \infty} \mathcal{H}, \mathcal{E}$$

the way it works …

A_i are frame independent*

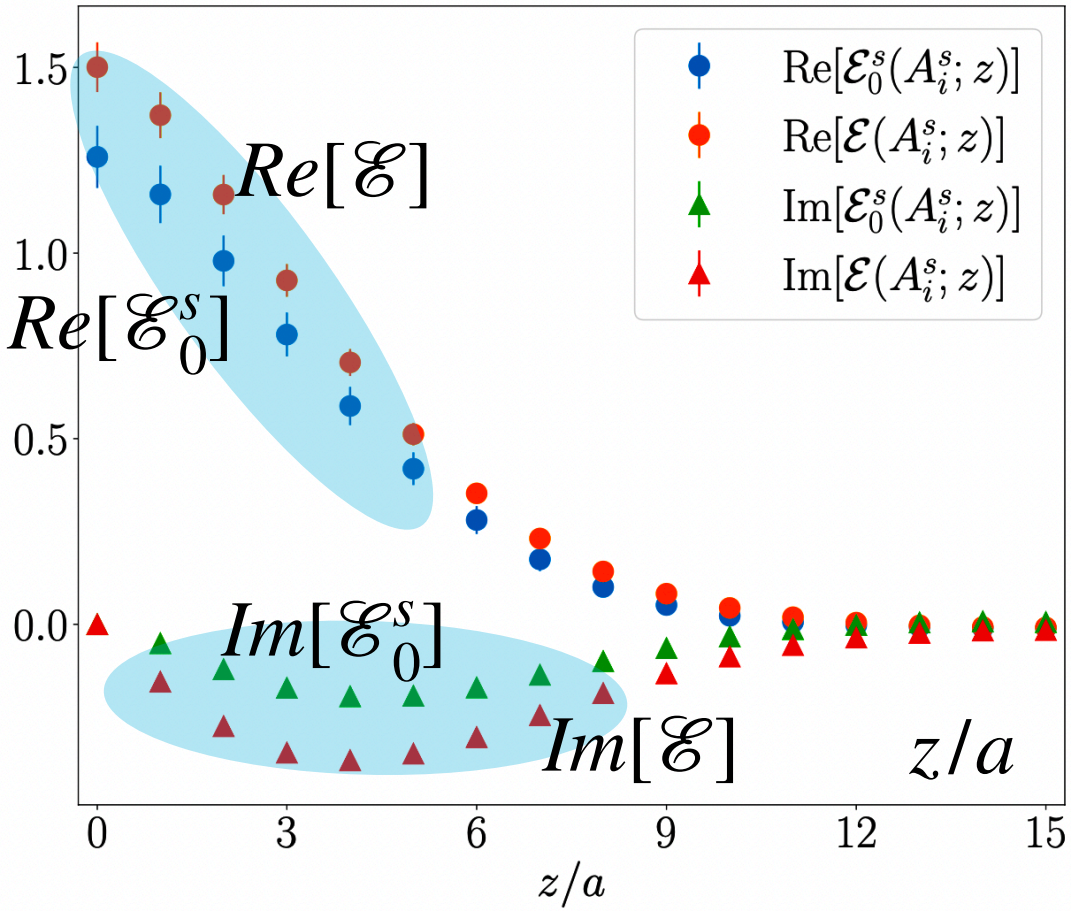




* A_i can be obtained in any frame from linear combinations of F_μ's with different proton polarizations

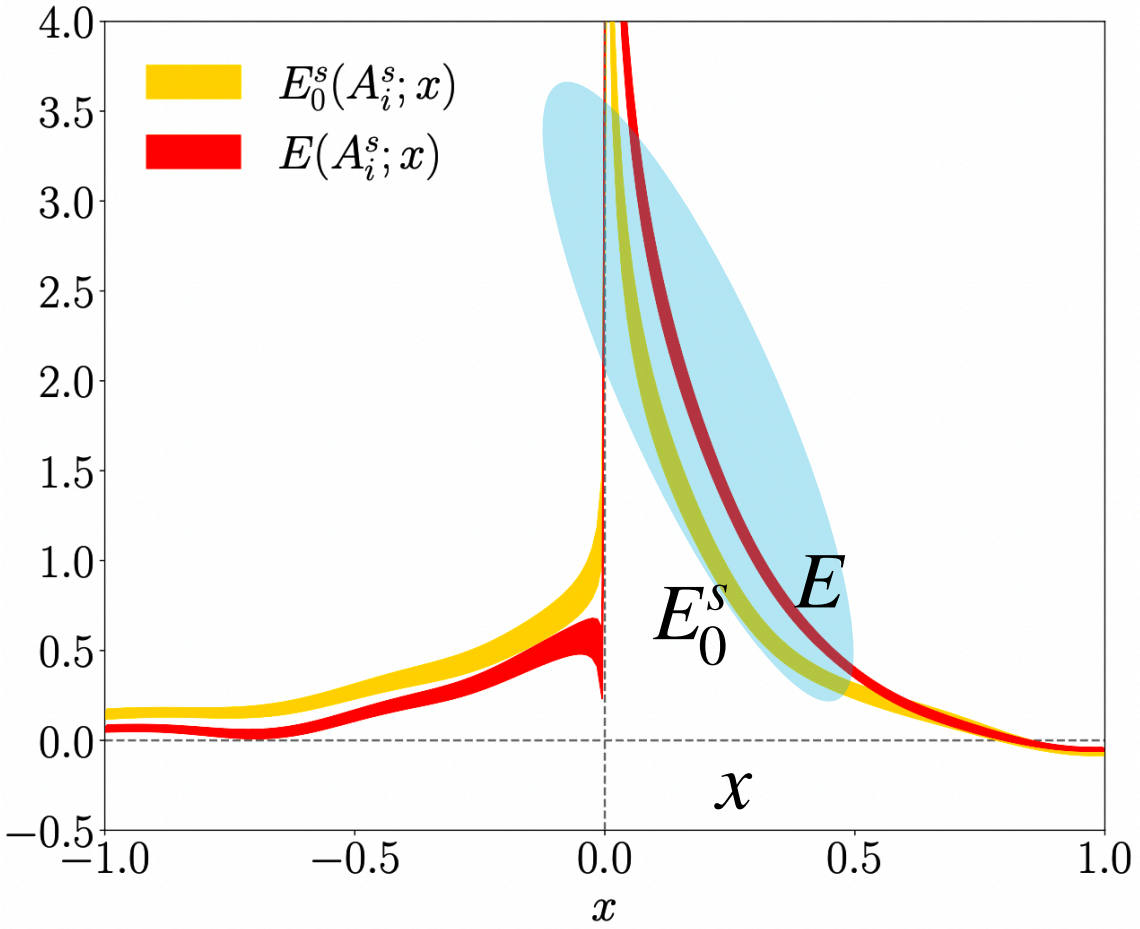
the way it works …

reduced frame-dependent power corrections in GPD matrix elements



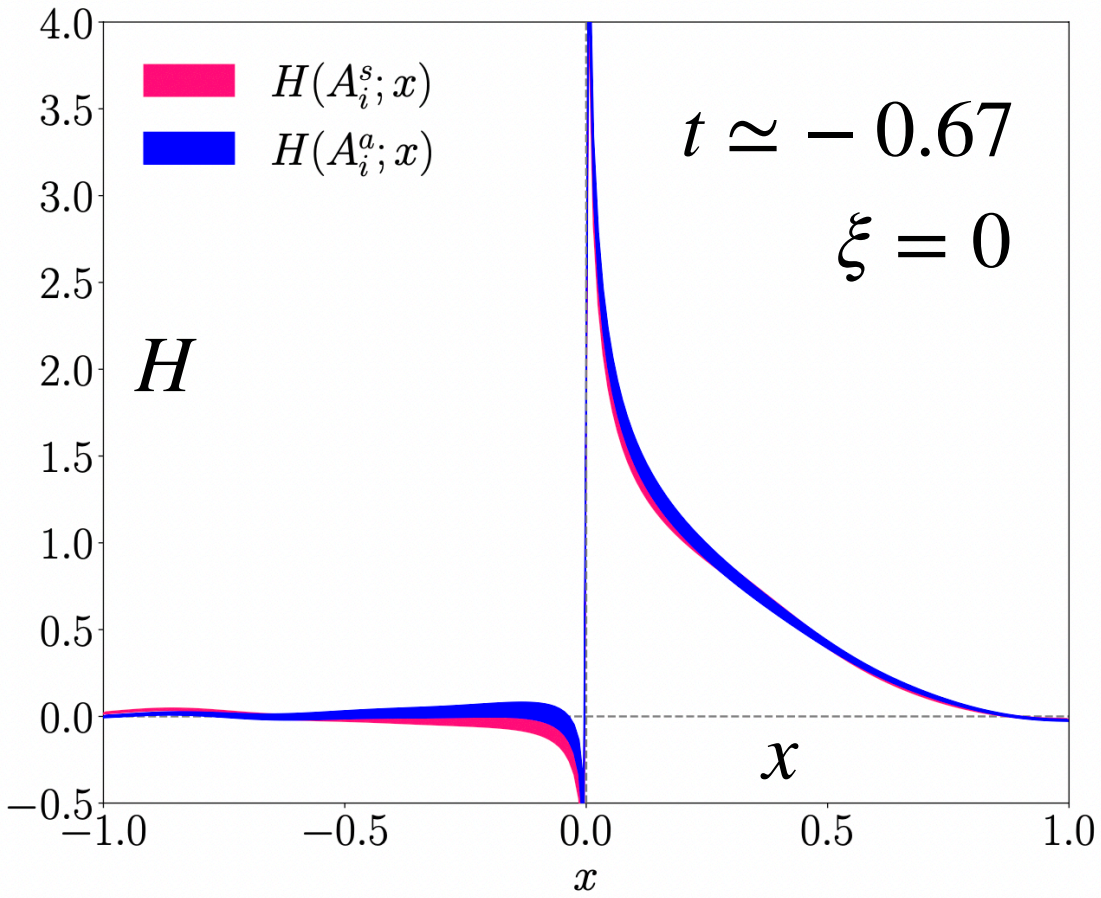
the way it works …

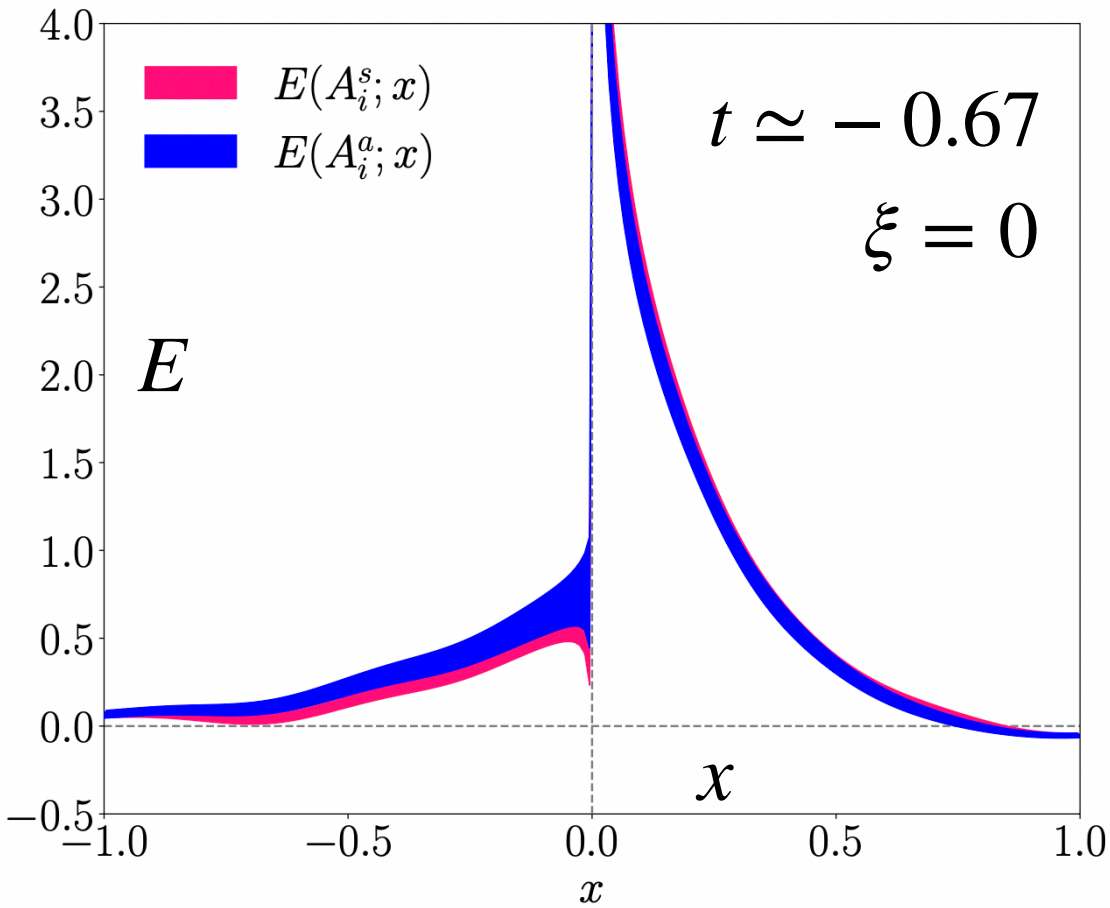
reduced frame-dependent power corrections in GPD



the way it works …

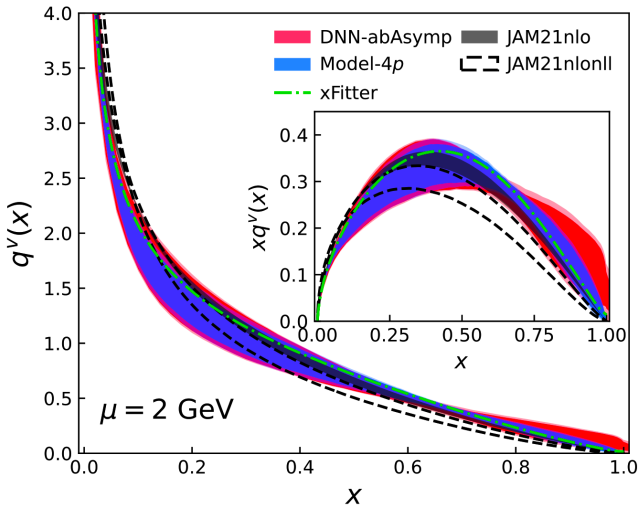
H and E GPD

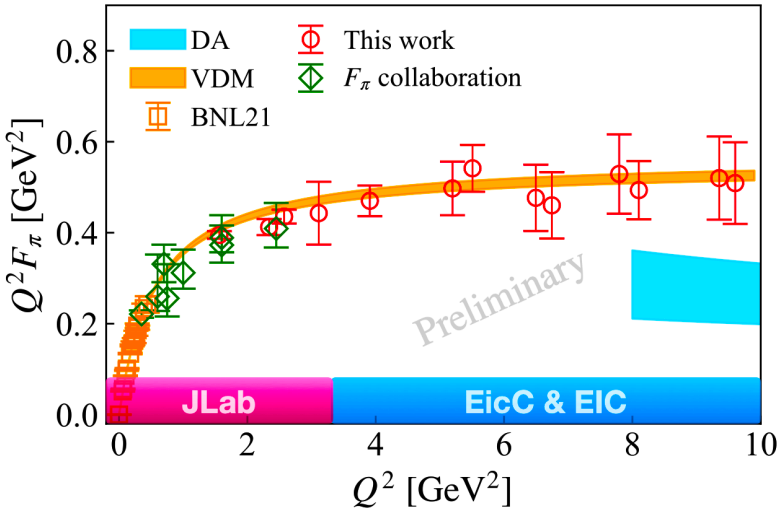


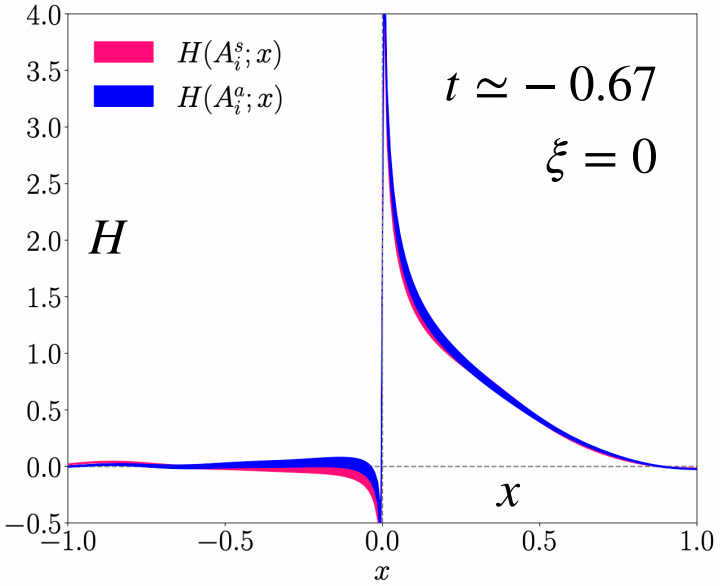


lattice QCD is essential for EIC's scientific success

rapid and robust progress





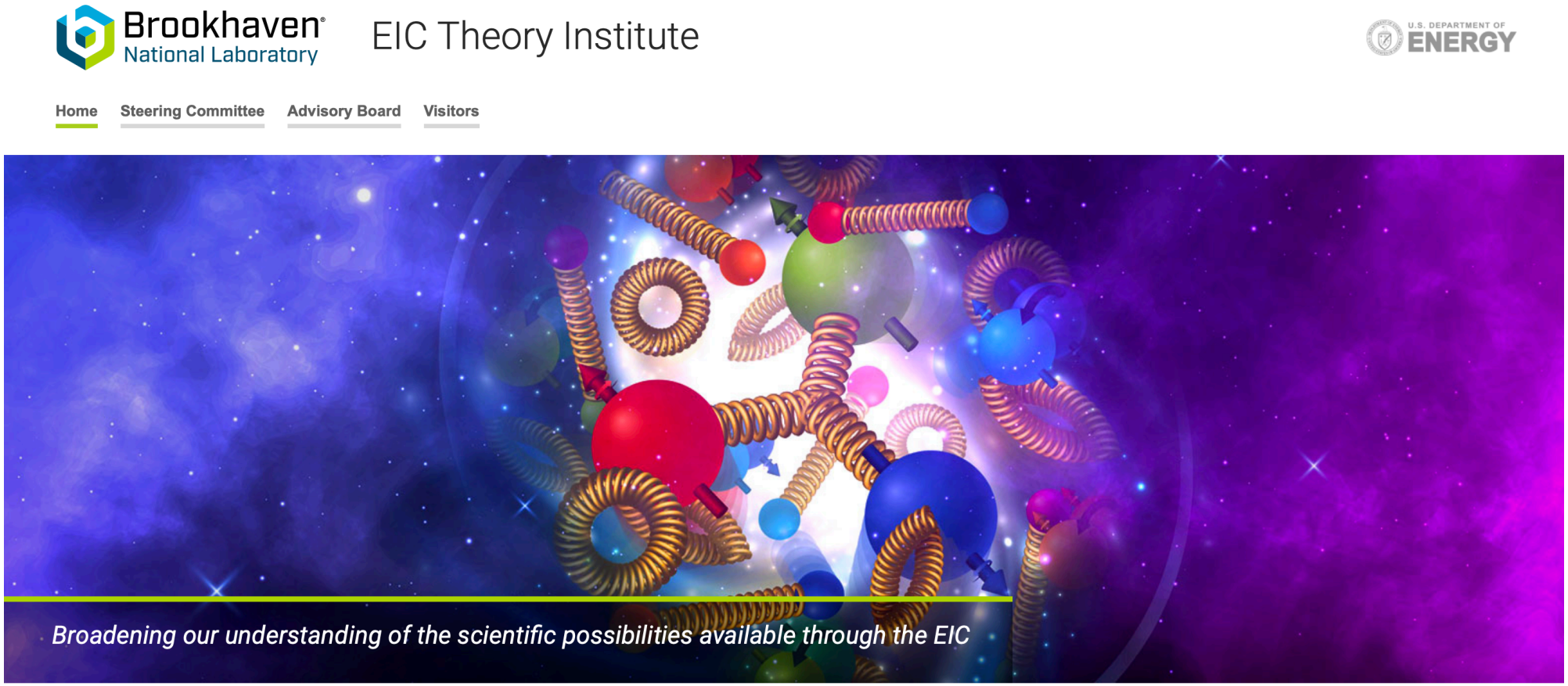


the journey has just began

need to train and engage young talents to sustain efforts for decades

great opportunity for Indian science community to play a vital role

BNL EIC Theory Institute



https://www.bnl.gov/eic-theory/

BNL EIC Theory Institute

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Visitor Program

The visitor program of the BNL EIC Theory Institute provides financial support for short-term (< 2 weeks), mid-term (2 weeks-1 month) and long-term (1-3 months) visits to BNL. Qualified applicants will be selected by the Steering Committee in consultation with an external Advisory Board. Applications for short and mid-term visits will be considered on a rolling basis.

Select your application

I am Faculty or a Staff Scientist

I am a Student or Post-doc

Long-term visitors, who will be considered Scholars-in-Residence, are encouraged to assist in organizing workshops, meetings, and schools, and to deliver lectures on their expertise either at BNL or in collaboration with CFNS. The Scholars-in-Residence must have a current faculty or staff affiliation. The number of such positions are very limited, and we request interested parties to submit their applications no later than January 31, 2023 for prompt consideration.

The BNL EIC Theory Institute will also host a dedicated program for students and post-doctoral fellows to visit and interact with scientists at BNL and Stony Brook, and to participate in workshops, meetings, and schools. The Theory Institute warmly welcomes international visitors. It is important that they satisfy BNL's visitor requirements and can secure visas if necessary.