Polarized Gluon Distributions from HLFQCD

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Outline :

- Introduction
- Veneziano amplitudes and holographic QCD
- Quark distribution functions
- Gluon gravitational form factor
- Gluon distribution functions
- Conclusion

Introduction

- In early 1980s : proton spin $\Rightarrow$ three-valence quarks !

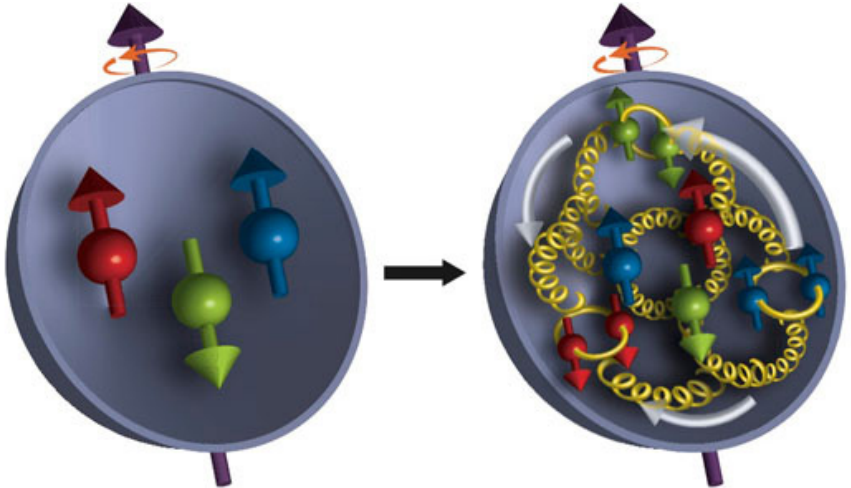
- In 1988 the European Muon Collaboration (EMC) at CERN $\Rightarrow \Delta\Sigma = 0.060(47)(69)$ at $Q^2 = 10$ GeV^2. **"proton spin puzzle"**, [European Muon Collaboration, Phys.Lett.B 206 (1988) 364]

- In 2008, A. Thomas et al. shown missing spin is produced by the valence and sea quarks orbital angular momentum. [Phys.Rev.Lett. 101 (2008) 102003]

- Recent Monte Carlo calculation shows that 50% of the proton spin come from gluon polarization. [Bass, Steven D., APS Physics 10 (2017) 23]

- LQCD $\Delta G(\mu^2 = 10GeV^2) = 0.251(47)(16)$, [Large-momentum effective theory, X. Ji, Phys. Rev. Lett. 110, 262002 (2013)],

- In 2022, Signature of the Gluon Orbital Angular Momentum, [S. Bhattacharya et al., PRL 128 (2022) 18, 182002]



Proton spin-decomposition

- There are two established approaches to look at the compositions of the proton spin :

Frame-independent spin sum rule (Ji)

$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$$

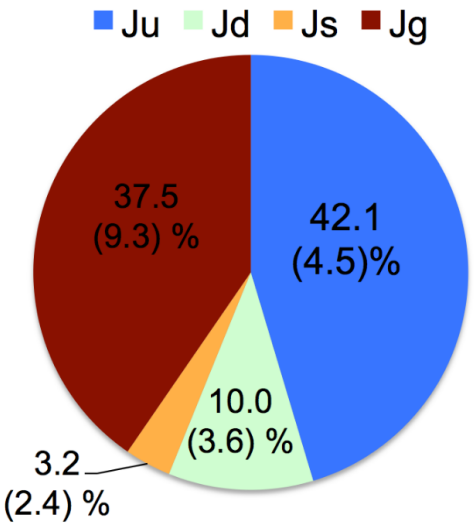
- $\Delta\Sigma/2$ and L_q^z (sum to J_q) are the quark helicity and OAM, respectively;
- Quark and gluon contributions J_q and J_g can be obtained from GPD moments;
- The sum rule also works for the transverse angular momentum in the IMF.

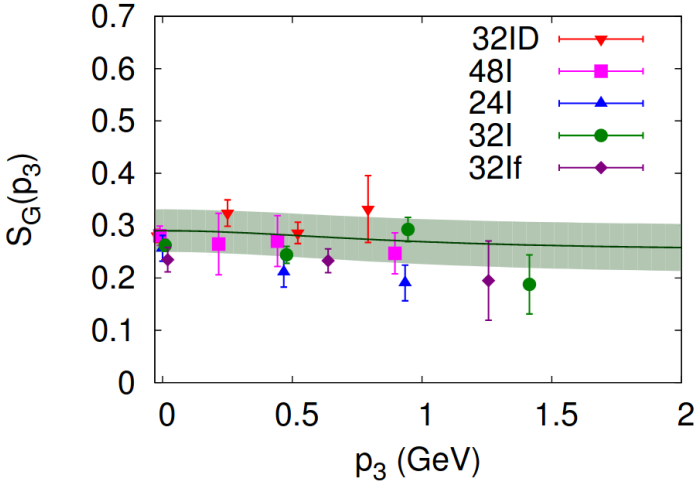
Infinite-momentum frame spin sum rule (Jaffe-Manohar)

$$\frac{1}{2}\Delta\Sigma + \Delta G + \ell_q + \ell_g = \frac{\hbar}{2}$$

- ΔG is the gluon helicity, ℓ_q and ℓ_g are canonical OAM;
- All terms have partonic interpretations, ℓ_q and ℓ_g are twist-three quantities;
- ΔG is measurable from e.g. RHIC-spin and EIC; ℓ_q and ℓ_g can be extracted from GPDs.

- State-of-the-art lattice study on the proton spin :





[-Y. Zhao, Nature Rev.Phys. 3 (2021) 1, 27-38]

Veneziano amplitudes and holographic QCD

- In LFHQCD the analytic structure of FFs and GPDs leads to a connection with the Veneziano amplitude, which incorporates the ρ Regge trajectory.

- Veneziano amplitude (G. VENEZIANO, 1968).

$$A(s,t) \sim B(1-\alpha(s), 1-\alpha(t)) \tag{1}$$

where $\alpha(t) = \alpha_0 + \alpha' t$ is the linear Regge trajectory.

- The duality beteen quark and hadron can be given through Veneziano model.

- For fixed t and large s the amplitude, $A(s,t) \sim s^{\alpha(t)-1}$.

Unpolarized quark distributions

- FF in HLFQCD

$$F_\tau(t) = \frac{1}{N_\tau} B\left(\tau - 1, \frac{1}{2} - \frac{t}{4\lambda}\right) \tag{2}$$

- Beta fun can be written as $B(\tau - 1, 1 - \alpha(t))$ with Regge trajectory

$$\alpha(t) = \frac{1}{2} + \frac{t}{4\lambda} \tag{3}$$

with $\sqrt{\lambda} = \kappa = m_\rho/\sqrt{2} = 0.548$ GeV.
- Using the integral rep of the Beta fun

$$F_\tau(t) = \frac{1}{N_\tau} \int_0^1 dx w'(x) w(x)^{-\alpha(t)} [1 - w(x)]^{\tau - 2} \tag{4}$$

with $w(0) = 0$, $w(1) = 1$ and $w'(x) \geq 0$

$$w(x) = x^{1-x} \exp[-a(1-x)^2] \tag{5}$$

- Flavor FF can be written in terms of the valence GPD at zero skewness, $\xi = 0$

$$F_\tau^q(t) = \int_0^1 dx H_\tau^q(x, \xi = 0, t) = \int_0^1 dx q_\tau(x) \exp[t f(x)] \tag{6}$$

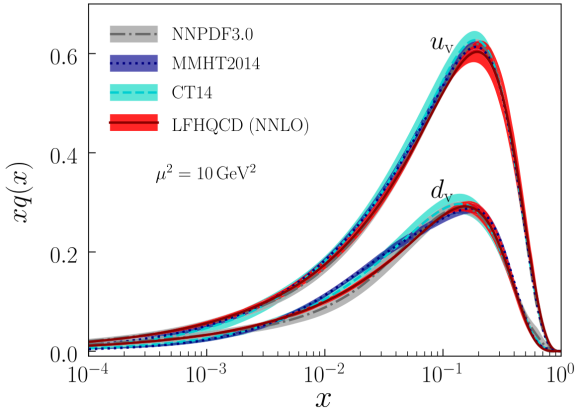
[- G. F. de Teramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, and A. Deur (HLFHS), Phys. Rev. Lett. 120, 182001 (2018)]

where $f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right)$ is the profile function and $q_\tau(x)$ is the unpolarized quark pdf for arbitrary twist-τ,

$$q_\tau(x) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{\alpha(0)} w'(x) \tag{7}$$

• The valence u and d quark pdfs can be calculated as,

$$\begin{aligned} u_v(x) &= (2 - \frac{r}{3}) q_{\tau=3}(x) + \frac{r}{3} q_{\tau=4}(x), \\ d_v(x) &= (1 - \frac{2r}{3}) q_{\tau=3}(x) + \frac{2r}{3} q_{\tau=4}(x) \end{aligned} \tag{8}$$

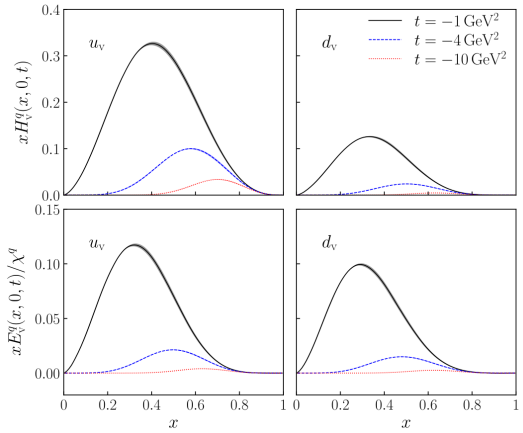


with the normalization conditions : $\int_0^1 dx u_\nu = 2$ and $\int_0^1 dx d_\nu = 1 \Longrightarrow r \leq 3/2$.

• The valence u and d quark GPDs are given as,

$$\begin{aligned} H_v^q(x,t) &= q_v(x) \exp[t f(x)] \\ E_v^q(x,t) &= e_v^q(x) \exp[t f(x)] \end{aligned} \tag{9}$$

$$e_v^q(x) = \chi_q[(1 - \gamma_q) q_{\tau=4}(x) + \gamma_q q_{\tau=6}(x)] \tag{10}$$



[- G. F. de Teramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, and A. Deur (HLFHS), Phys. Rev. Lett. 120, 182001 (2018)]

Polarized quark distributions

• Spin-dependent quark distributions can be uniquely determined in terms of the unpolarized distributions by chirality separation without the introduction of additional free parameters.
• Polarized distributions, for which the coupling of an axial current,

$$F_{A,\tau}(t) = \frac{1}{N_\tau} B\left(\tau - 1, 1 - \frac{t}{4\lambda}\right) \tag{11}$$

• $F_{A,\tau}$ has the same structure as $F_{V,\tau}$, but with the Regge trajectory replaced by the axial one,

$$\alpha_A(t) = \frac{t}{4\lambda} \tag{12}$$

• The polarized quark distribution for arbitrary twist-τ can be obtained as,

$$\Delta q_\tau(x) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w'(x) \tag{13}$$

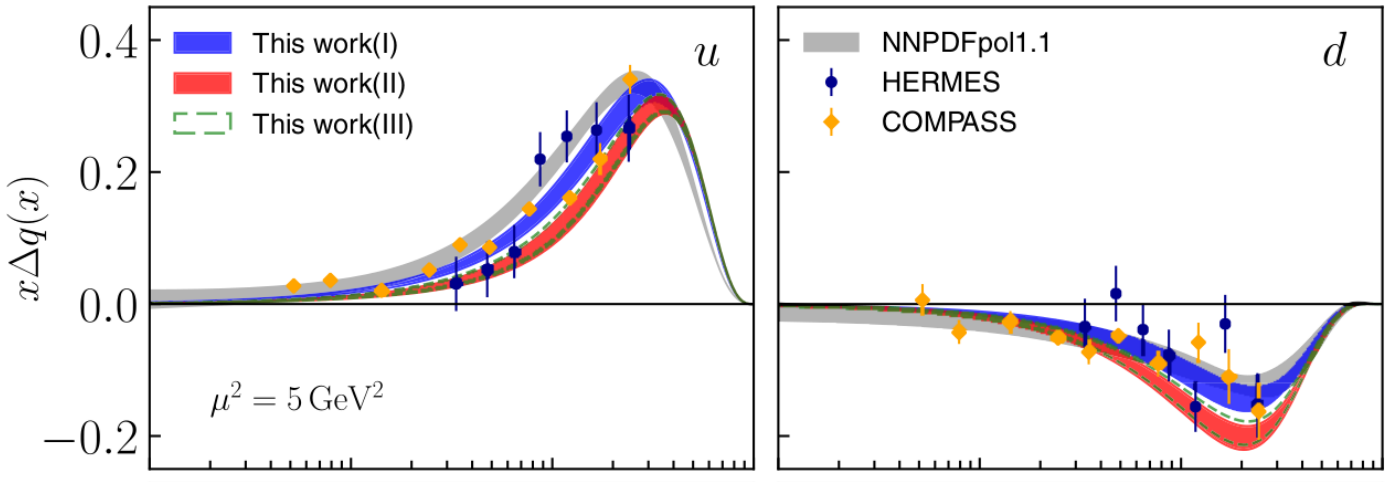
• The polarized quark distributions for the proton,

$$\Delta q(x) = c_\tau \Delta q_\tau(x) - c_{\tau+1} \Delta q_{\tau+1} \tag{14}$$

• Spin-aligned and spin-antialigned distributions are the linear combitations of the polarized and unpolarized distributions,

$$q_\uparrow(x) = \frac{1}{2}[q(x) + \Delta q(x)], \qquad q_\downarrow(x) = \frac{1}{2}[q(x) - \Delta q(x)] \tag{15}$$

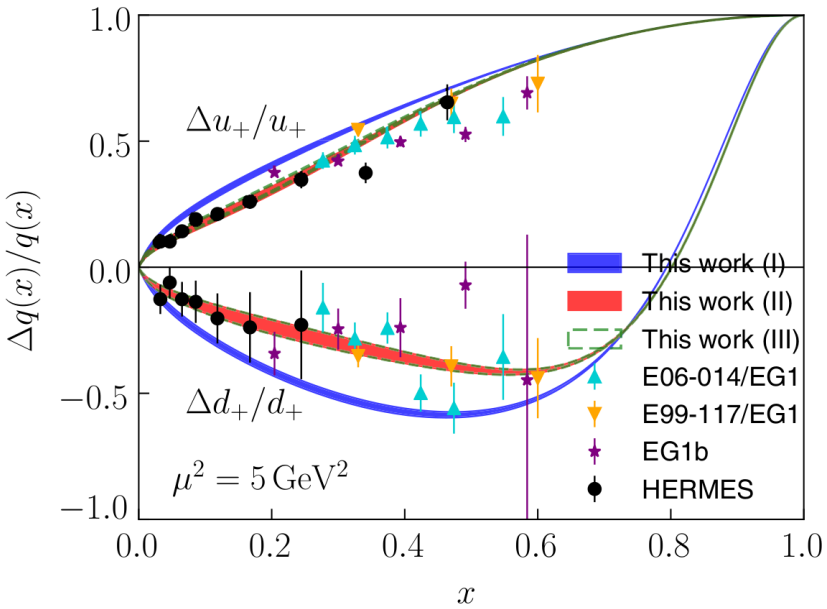
[- T. Liu, R. S. Sufian, G. F. de Téramond, H. G. Dosch, S. J. Brodsky, and A. Deur, Phys. Rev. Lett. 124, 082003 (2020)]



The quark helicity asymmetry is consistent with the pQCD constraint,

$$\lim_{x\to 1} \frac{\Delta q(x)}{q(x)} = 1, \qquad \lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = x^{1/2} \quad (16)$$

here exponant $1/2$ is the difference between the intercepts of the vector and axial Regge trajectories



[- T. Liu, R. S. Sufian, G. F. de Téramond, H. G. Dosch, S. J. Brodsky, and A. Deur, Phys. Rev. Lett. 124, 082003 (2020)]

Unpolarized gluon distributions

• Gluonic distributions can be described from the coupling of the metric fluctuations induced by the spin-two Pomeron with the energy momentum tensor in anti–de Sitter space, $\int d^5x\sqrt{g}h_{MN}T^{MN}$.
• By performing a small deformation of the AdS metric about its AdS background $g_{MN} \to g_{MN} + h_{MN}$,

$$S_g[h] = -\frac{1}{4}\int d^5x\sqrt{g}e^{\varphi_g(z)}\left(\partial_L h^{MN}\partial^L h_{MN} - \frac{1}{2}\partial_L h\partial^L h\right) \tag{17}$$

• With the additional gauge condition $\partial_M h = 0$, The linearized Einstein equations can be obtained as,

$$-\frac{z^3}{e^{\varphi_g(z)}}\partial_z\left(\frac{e^{\varphi_g(z)}}{z^3}\partial_z h^\nu_\mu\right) + \partial_\rho\partial^\rho h^\nu_\mu = 0, \tag{18}$$

• The plane wave solution with the polarization indices of the above Eq. with $q^2 = -Q^2 < 0$ is given by, $h^\nu_\mu(x,z) = \epsilon^\nu_\mu e^{-iq.x}H(q^2,z)$.
• For a soft-wall profile $\varphi_g(z) = -\lambda_g z^2$, with $a = Q^2/4\lambda_g$, $\xi = \lambda_g z^2$

$$H(a,\xi) = a(2+a)\int_0^1 dx x^{a-1}(1-x)e^{-\xi x(1-x)} \tag{19}$$

[-G. F. de Téramond, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, and A. Deur (HLFHS), Phys. Rev. D 104, 114005 (2021)]

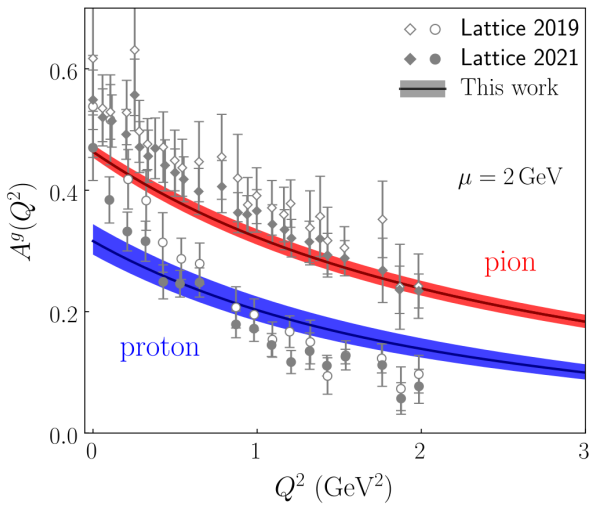
• The gluon GFF $A_\tau^g(Q^2)$ can be written in terms of bulk field propagator as,

$$A_\tau^g(Q^2) = \int_0^\infty \frac{dz}{z^3} H(Q^2, z)\Phi_\tau^2(z) \tag{20}$$

with $\Phi_\tau^g \sim z^\tau e^{-\lambda_g z^2/2}$, the twist-$\tau$ hadron bound-state solution and normalization $A_\tau^g(0) = 1$.

• The gluon GFF, $A_\tau^g(t)$ can be express in terms of the generalized Veneziano model for a spin-two current as,

$$A_\tau^g(t) = \frac{1}{N_\tau} B\Big(\tau - 1, 2 - \alpha_P(t)\Big) \tag{21}$$



where the Donnachie and Landshoff Regge trajectory for soft Pomeron is

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t, \tag{22}$$

with intercept $\alpha_P(0) = 1.08$ and slope $\alpha'_P = 0.25$ GeV^{-2}.

• For large momentum transfer $-t = Q^2$, the gluon GFF reproduces the hard-scattering power behavior,

$$A_\tau(Q^2) \sim \Big(\frac{1}{Q^2}\Big)^{\tau-1} \tag{23}$$

[-G. F. de Téramond, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, and A. Deur (HLFHS), Phys. Rev. D 104, 114005 (2021)]

- The gluon GFF $A^g_\tau(t)$ can be written in the reparametrization form,

$$A^g_\tau(t) = \frac{1}{N_\tau}\int_0^1 dx w'(x) w(x)^{1-\alpha_P(t)}[1-w(x)]^{\tau-2} \tag{24}$$

- The gluon GFF can also be expressed in terms of gluon GPDs at zero skewness

$$A^g_\tau(t) = \int_0^1 x dx H^g_\tau(x,\xi=0,t) = \int_0^1 x dx g_\tau(x) e^{tf(x)} \tag{25}$$

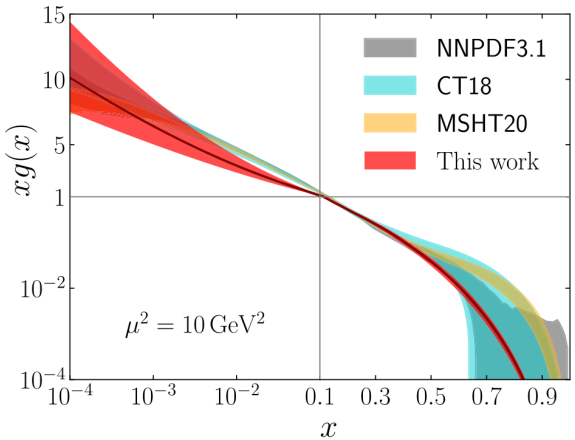
$$g_\tau(x) = \frac{1}{N_\tau}\frac{w'(x)}{x}[1-w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)} \tag{26}$$

with the normalization $\int_0^1 dx x g_\tau(x) = 1$.

- The unpolarized gluon pdf, which can be obtained as,

$$g(x) = \sum_\tau c_\tau g_\tau(x)$$

$$\int_0^1 x dx \left[g(x) + \sum_q q(x)\right] = 1 \tag{27}$$



[-G. F. de Téramond, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, and A. Deur (HLFHS), Phys. Rev. D 104, 114005 (2021)]

Polarized gluon distributions

• For the polarized gluon distribution one needs to replace the soft Pomeron Regge trajectory as,

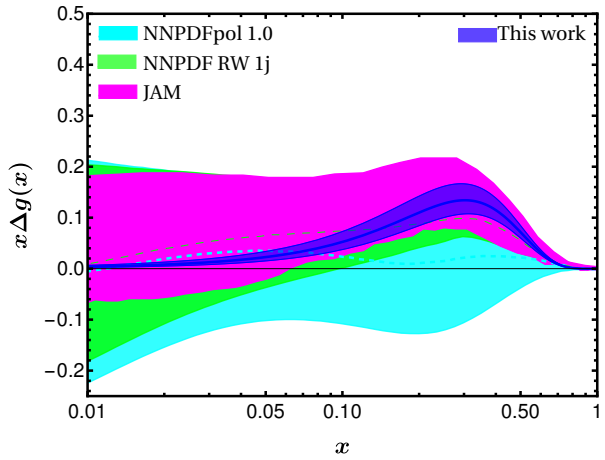
$$\tilde{\alpha}_P(t) = \tilde{\alpha}_P(0) + \alpha'_P t, \tag{28}$$

• Polarized gluon pdf for any arbitrary twist-τ can be found as,

$$\Delta g_\tau(x) = \frac{1}{N_\tau}\frac{w'(x)}{x}[1-w(x)]^{\tau-2}w(x)^{1-\tilde{\alpha}_P(0)}$$

• Gluon helicity pdf is given as,

$$\Delta g(x) = \sum_\tau c_\tau \Delta g_\tau(x) \tag{29}$$



• $\Delta G = \int_0^1 dx \Delta g(x) = 0.22^{+0.056}_{-0.044}$ [-B. Gurjar, C. Mondal, D. Chakrabarti, e-Print : 2209.14285 [hep-ph]].

• $\Delta G = \int_{0.02}^{0.3} \Delta g(x) = 0.2$, with $-0.7 < \Delta G < 0.5$, [PHENIX measurements, PRL 103, 012003(2009)].

• $\Delta G = \int_{0.05}^{0.2} \Delta g(x) = 0.23(6)$, [NNPDF, Nucl. Phys. B 887, 276 (2014)].

• $\Delta G = \int_{0.05}^{1} dx \Delta g(x) = 0.19(6)$, [Marco Stratmann et al. PRL 113, 012001 (2014)].

• $\Delta G(\mu^2 = 10 GeV^2) = 0.251(47)(16)$, [LQCD at physical pion mass, PRL 110, 262002 (2013)].

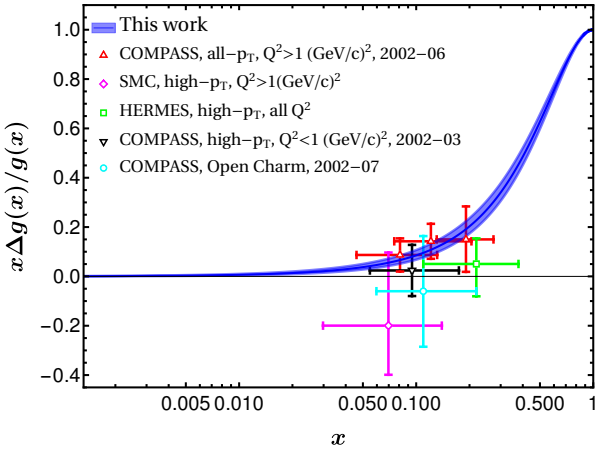
Gluon GPDs

- The helicity asymmetry for the gluon is given as,

$$\frac{\Delta g(x)}{g(x)} = w(x)^{\alpha_P(0)-\tilde{\alpha}_P(0)} \qquad (30)$$

which satisfy the pQCD constarints at the endpoints, i.e.,

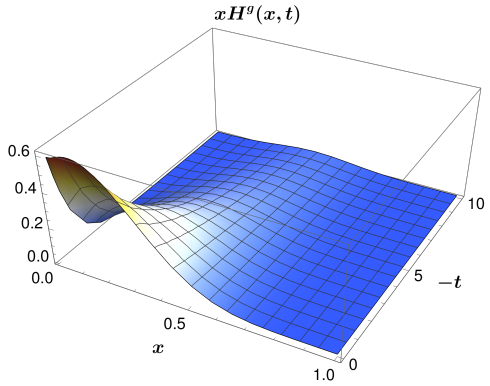
$$\lim_{x\to 0}\frac{\Delta g(x)}{g(x)} = 0, \quad \& \quad \lim_{x\to 1}\frac{\Delta g(x)}{g(x)} = 1. \quad (31)$$

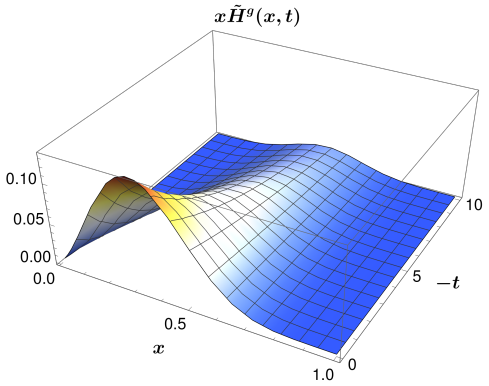


- The unpolarized and polarized gluon GPDs at zero skewness, $\xi = 0$ can be written as,

$$\begin{aligned} H^g_\tau(x) &= g_\tau(x)e^{tf(x)} \\ \tilde{H}^g_\tau(x) &= \Delta g_\tau(x)e^{tf(x)} \end{aligned} \qquad (32)$$

- Both the GPDs are differ only with their collinear pdfs.





x-Dependent Squared Radius

- x-Dependent Squared Radius

$$\langle b_\perp^2 \rangle^{q/g}(x) = \frac{\int d^2 b_\perp b_\perp^2 H^{q/g}(x, b_\perp)}{\int d^2 b_\perp H^{q/g}(x, b_\perp)} = \frac{1}{\lambda_{q/g}} \log\left(\frac{1}{w(x)}\right) \tag{33}$$

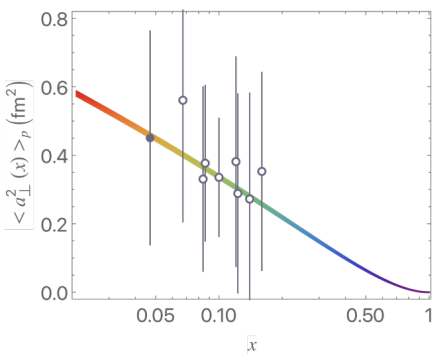
- Transverse squared radius : $b_\perp \propto 1/Q$

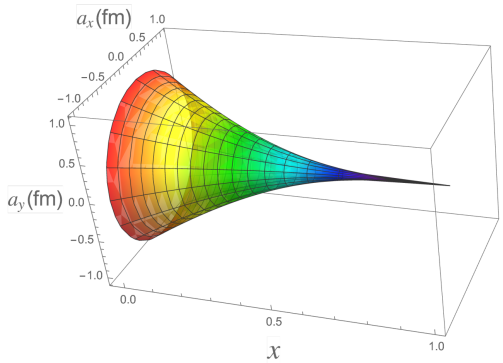
$$\langle b_\perp^2 \rangle^q = \sum_q e_q \int_0^1 dx f^q(x) \langle b_\perp^2 \rangle^q(x) \tag{34}$$

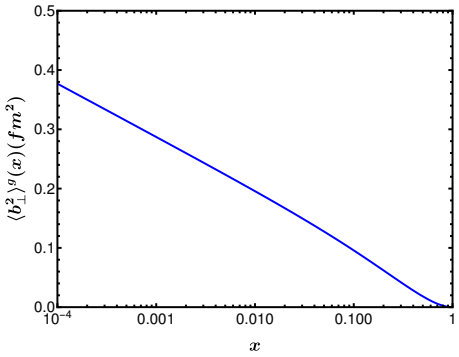
HLFQCD results : $\langle b_\perp^2 \rangle^q = 0.36$ fm^2, Experimental data : $\langle b_\perp^2 \rangle^q = 0.43 \pm 0.01$ fm^2

- Transverse squared radius :

$$\langle b_\perp^2 \rangle^g = \int_0^1 dx g(x) \langle b_\perp^2 \rangle^g(x) \simeq 0.44 \text{fm}^2 \tag{35}$$







Conclusion

- Spin can be decomposed in several ways : (1) Frame-independent (Ji), (2) IMF (Jaffe-Manohar).
- Correspondence between HLFQCD & Veneziano model.
- Unpolarized quark pdfs $\Rightarrow$ Polarized quark pdfs : "By changing the linear Reggon Regge trajectories ! !"
- Gluon GFFs & unpolarized pdfs for $|qqqg\rangle$.
- Unpolarized gluon pdfs $\Rightarrow$ polarized/helicity gluon pdfs : "By changing the linear Pomeron Regge trajectories ! !"
- pQCD constraints on helicity asymmetry.
- gluon GPDs and color-transperency for the quarks and gluons.

Thanks for your attentation !