

# Probing gluon **O**rbital **A**ngular **M**omentum through Exclusive dijet production at the EIC

---



**QCD with Electron Ion Collider (QEIC) 11**

**Shohini Bhattacharya**

Brookhaven National Laboratory

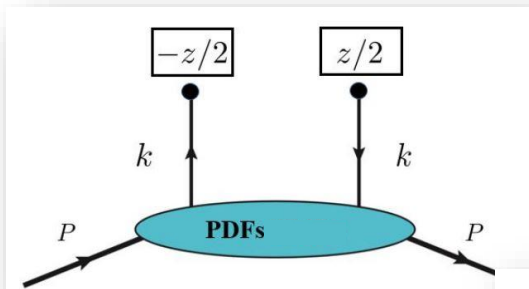
19 December 2022



# Quantum Chromodynamics (QCD): Non-perturbative functions

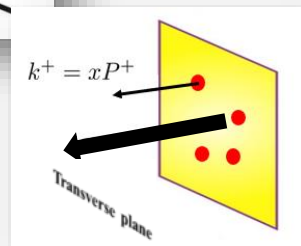
← **Parton motion**  
← **Nucleon motion**

**Snapshots of the nucleons**



**Parton Distribution Functions**

**PDFs** ( $x$ )

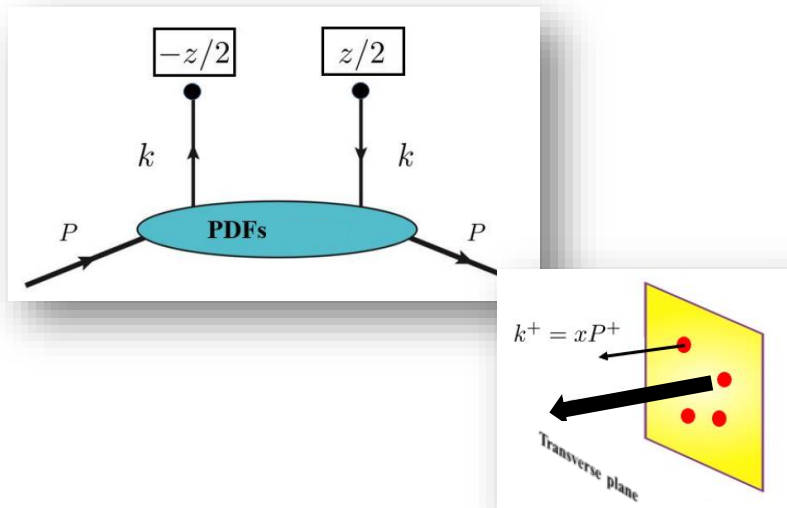




# Quantum Chromodynamics (QCD): Non-perturbative functions

← **Parton motion**  
← **Nucleon motion**

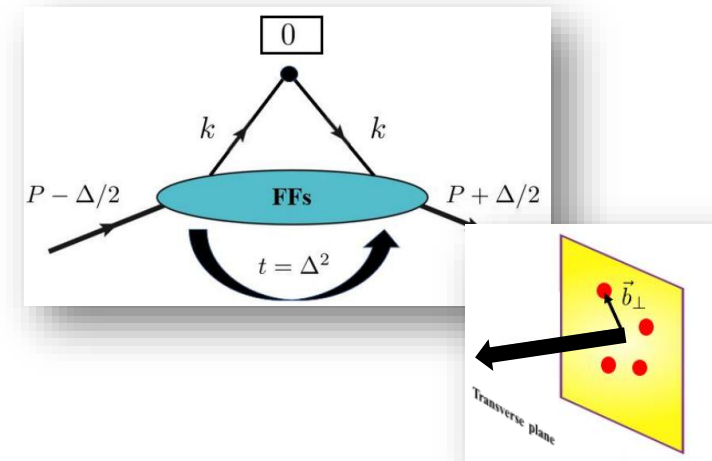
**Snapshots of the nucleons**



**PDFs** ( $x$ )

**Form Factors**

**FFs** ( $\Delta$ )



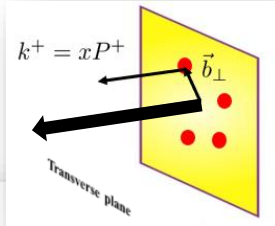


# Quantum Chromodynamics (QCD): Non-perturbative functions

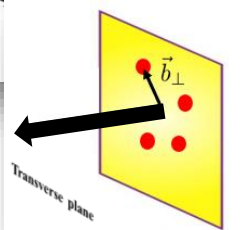
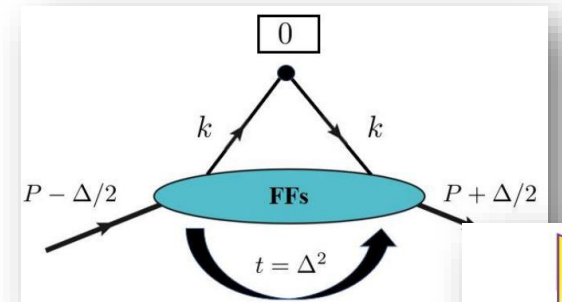
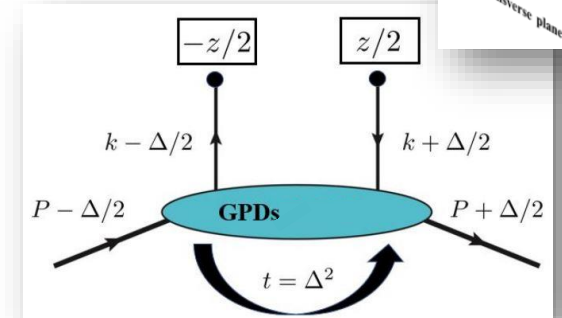
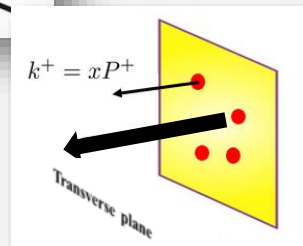
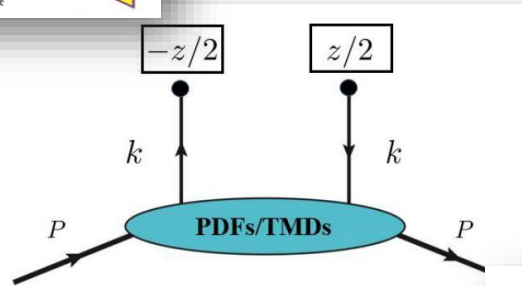
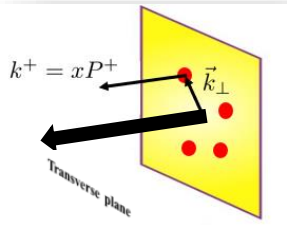
← **Parton motion**  
← **Nucleon motion**

**Snapshots of the nucleons**

**Generalized Parton Distributions**

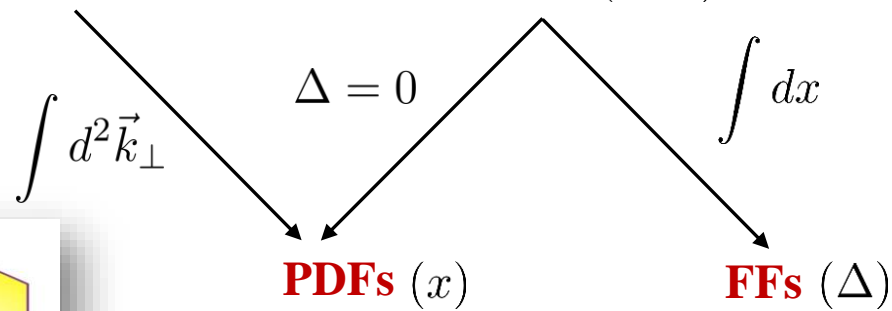


**Transverse Momentum-dependent Distributions**



**TMDs**  $(x, \vec{k}_\perp)$

**GPDs**  $(x, \Delta)$



**PDFs**  $(x)$

**FFs**  $(\Delta)$



# Quantum Chromodynamics (QCD): Non-perturbative functions

← **Parton motion**  
← **Nucleon motion**

**Snapshots of the nucleons**

## Generalized **T**ransverse **M**omentum-dependent **D**istributions

**GTMDs**  $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

$\int d^2 \vec{k}_\perp$

**TMDs**  $(x, \vec{k}_\perp)$

**GPDs**  $(x, \Delta)$

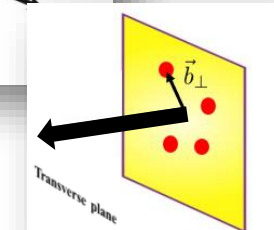
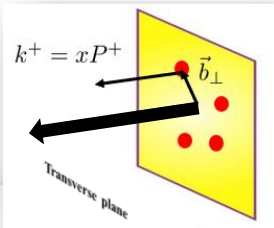
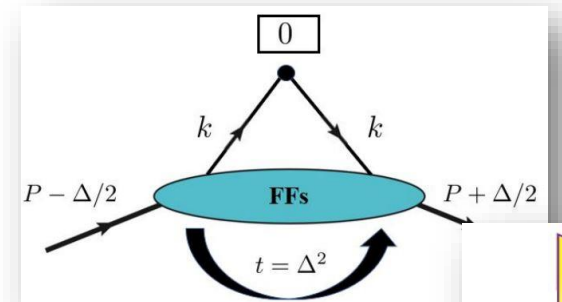
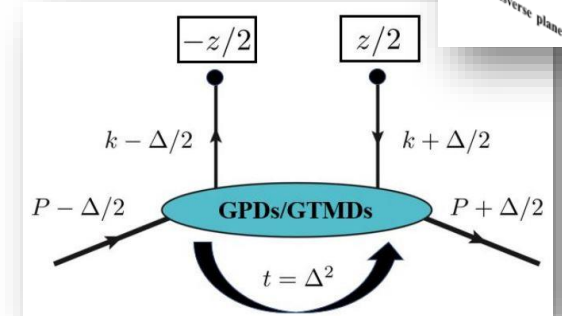
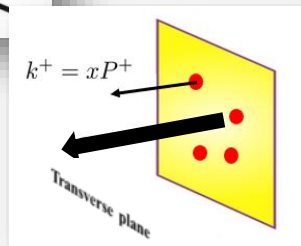
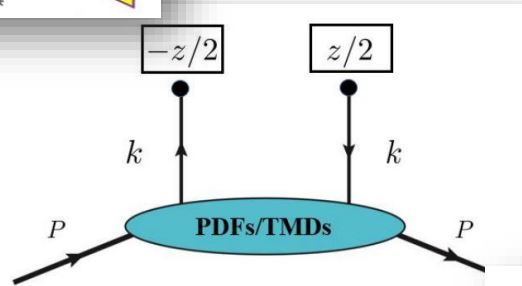
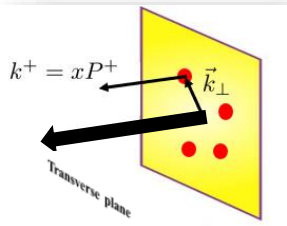
$\Delta = 0$

$\int d^2 \vec{k}_\perp$

$\int dx$

**PDFs**  $(x)$

**FFs**  $(\Delta)$



# Outline

## 1. Progress in the last 6 years

(Meissner, Metz, Schlegel, 2009)

**GTMDs**  $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

$\int d^2 \vec{k}_\perp$

**TMDs**  $(x, \vec{k}_\perp)$

**GPDs**  $(x, \Delta)$

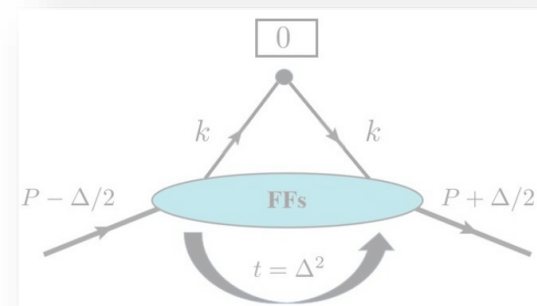
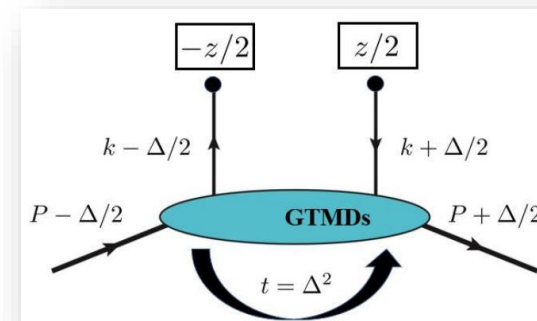
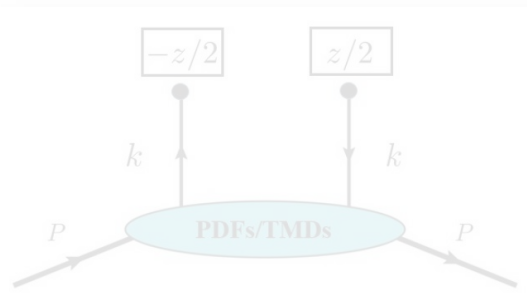
$\int d^2 \vec{k}_\perp$

$\Delta = 0$

$\int dx$

**PDFs**  $(x)$

**FFs**  $(\Delta)$



# Outline

## 1. Progress in the last 6 years

## 2. Accessing **O**rbital **A**ngular **M**omentum (OAM) through GTMDs

(Meissner, Metz, Schlegel, 2009)

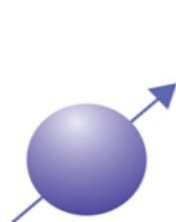
**GTMDs**  $(x, \vec{k}_\perp, \Delta)$

$\Delta = 0$

$\Delta = 0$

$\int d^2 \vec{k}_\perp$

An incomplete story:



$$\frac{1}{2}$$

$$= \frac{1}{2} \Delta \Sigma$$

Best known

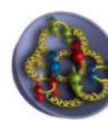
Quark helicity



$$+ \Delta G$$

How well do we know?

Gluon helicity



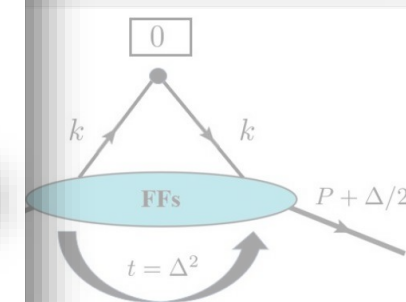
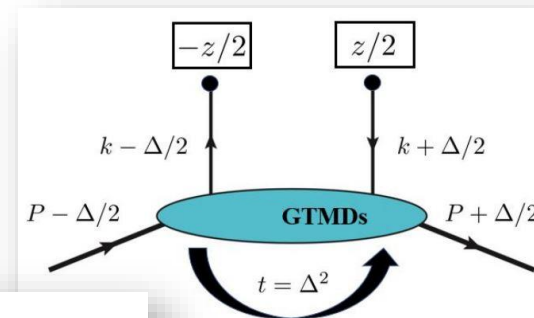
$$+ L^q$$

$$+ L^g$$

??

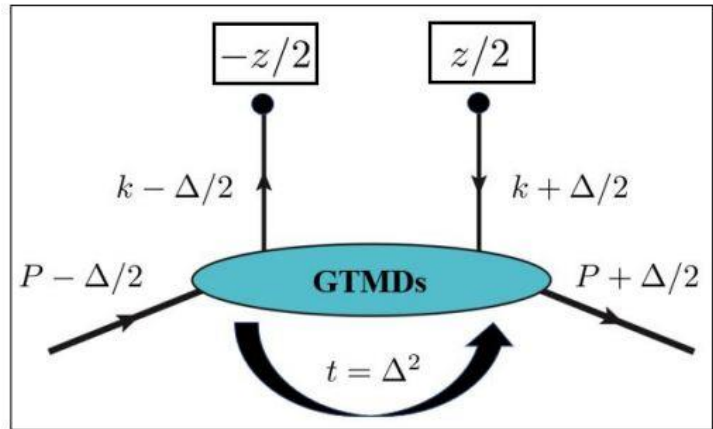
OAM of quarks & gluons

Another major scientific goal of the EIC





# Generalized Transverse Momentum dependent Distributions (GTMDs)



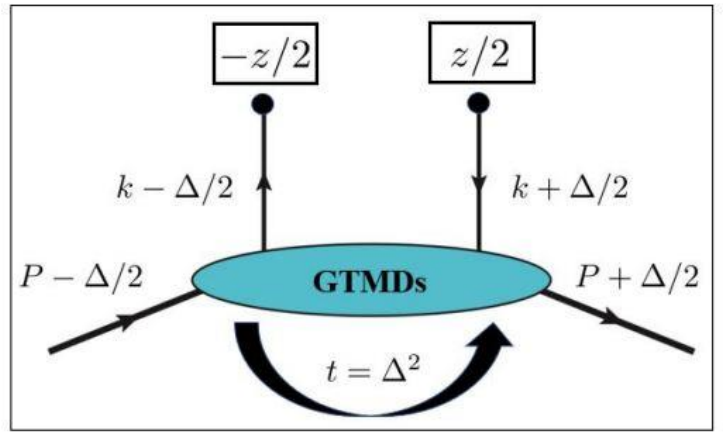
**Definition of a (quark) GTMD correlator:**

$$W_{\lambda, \lambda'}^{q[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 \vec{z}_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi^q(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0}$$





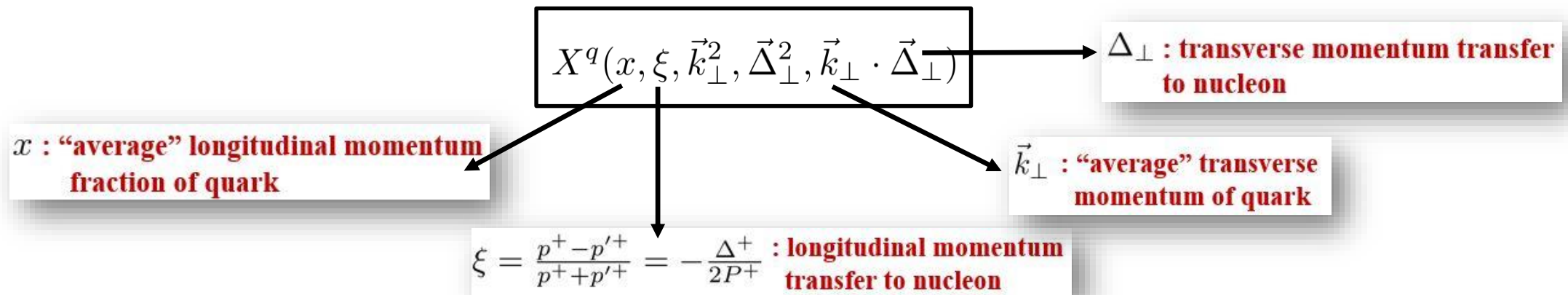
# Generalized Transverse Momentum dependent Distributions (GTMDs)



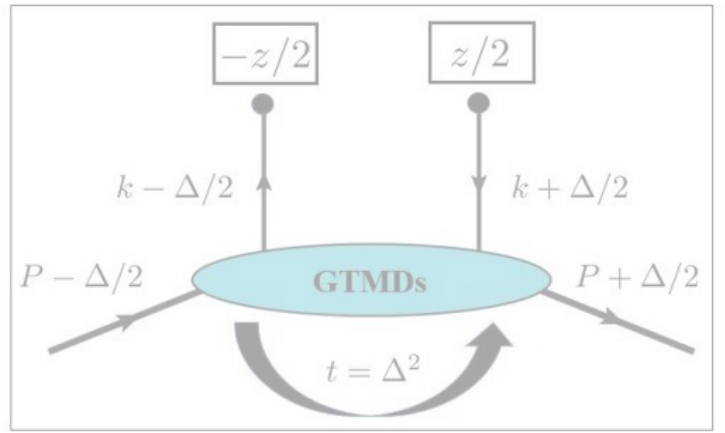
**Definition of a (quark) GTMD correlator:**

$$W_{\lambda, \lambda'}^{q[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 \vec{z}_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi^q(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0}$$

**Parameterization of correlator through GTMDs:**



# Generalized Transverse Momentum dependent Distributions (GTMDs)



Definition of a (quark) GTMD correlator:

$$W_{\lambda, \lambda'}^{q[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 \vec{z}_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{\psi}^q(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) \psi^q(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0}$$

## General results:

- i. 16 leading-twist GTMDs for quarks (Meissner, Metz, Schlegel, arXiv: 0906.5323)
- ii. 16 leading-twist GTMDs for gluons (Lorcé, Pasquini, arXiv: 1307.4497)
- iii. GTMDs are complex functions

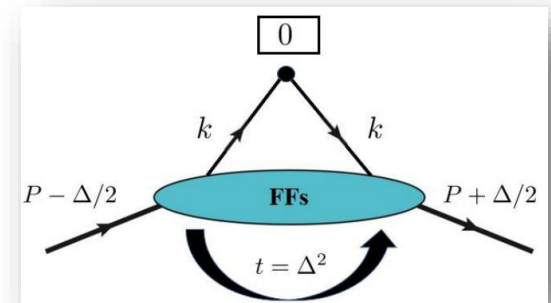
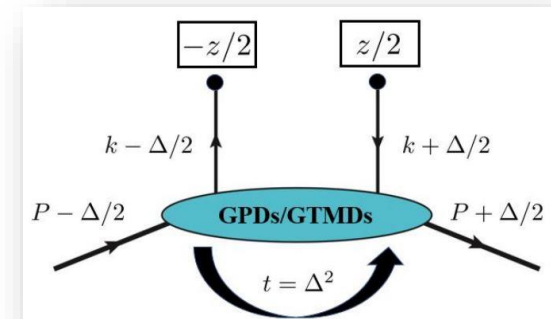
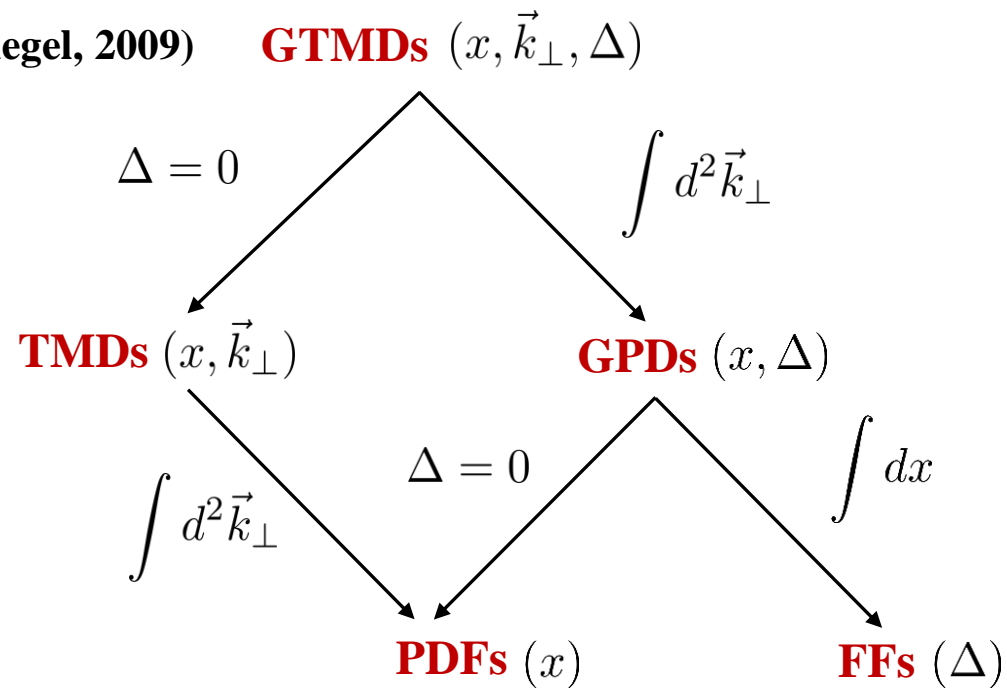
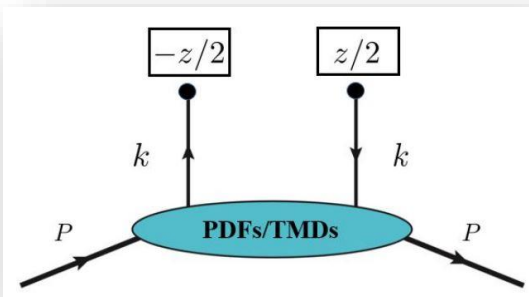


# **Why are GTMDs interesting?**

# Why are GTMDs interesting?

## 1) GTMDs are the “Mother Functions”

(Meissner, Metz, Schlegel, 2009) **GTMDs**  $(x, \vec{k}_\perp, \Delta)$

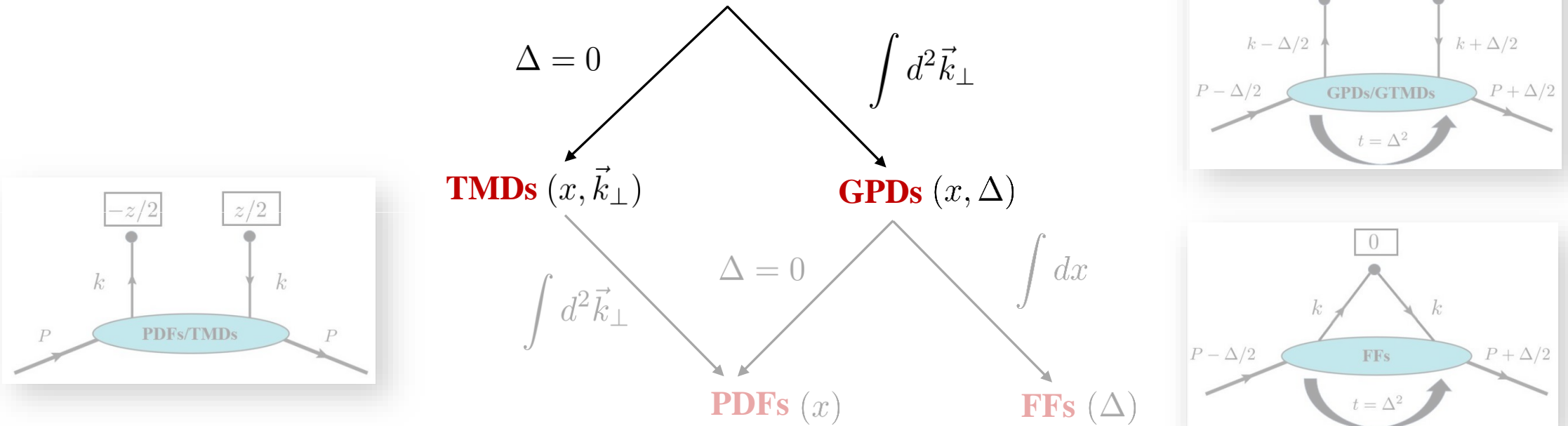


# Why are GTMDs interesting?

1) GTMDs are the “Mother Functions”

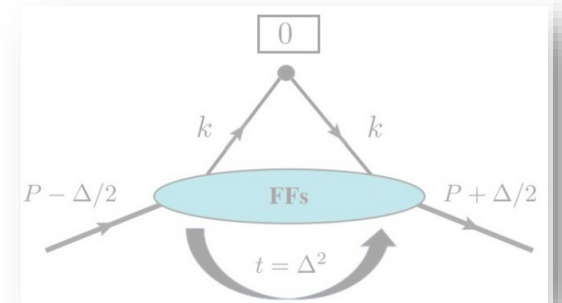
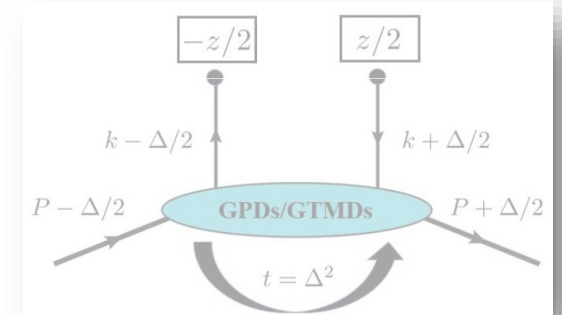
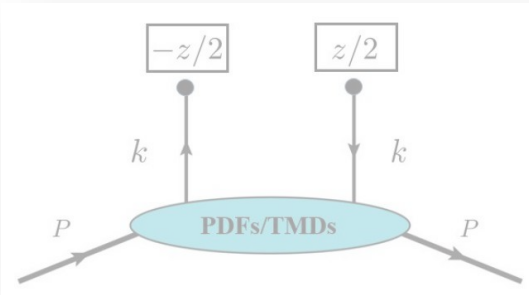
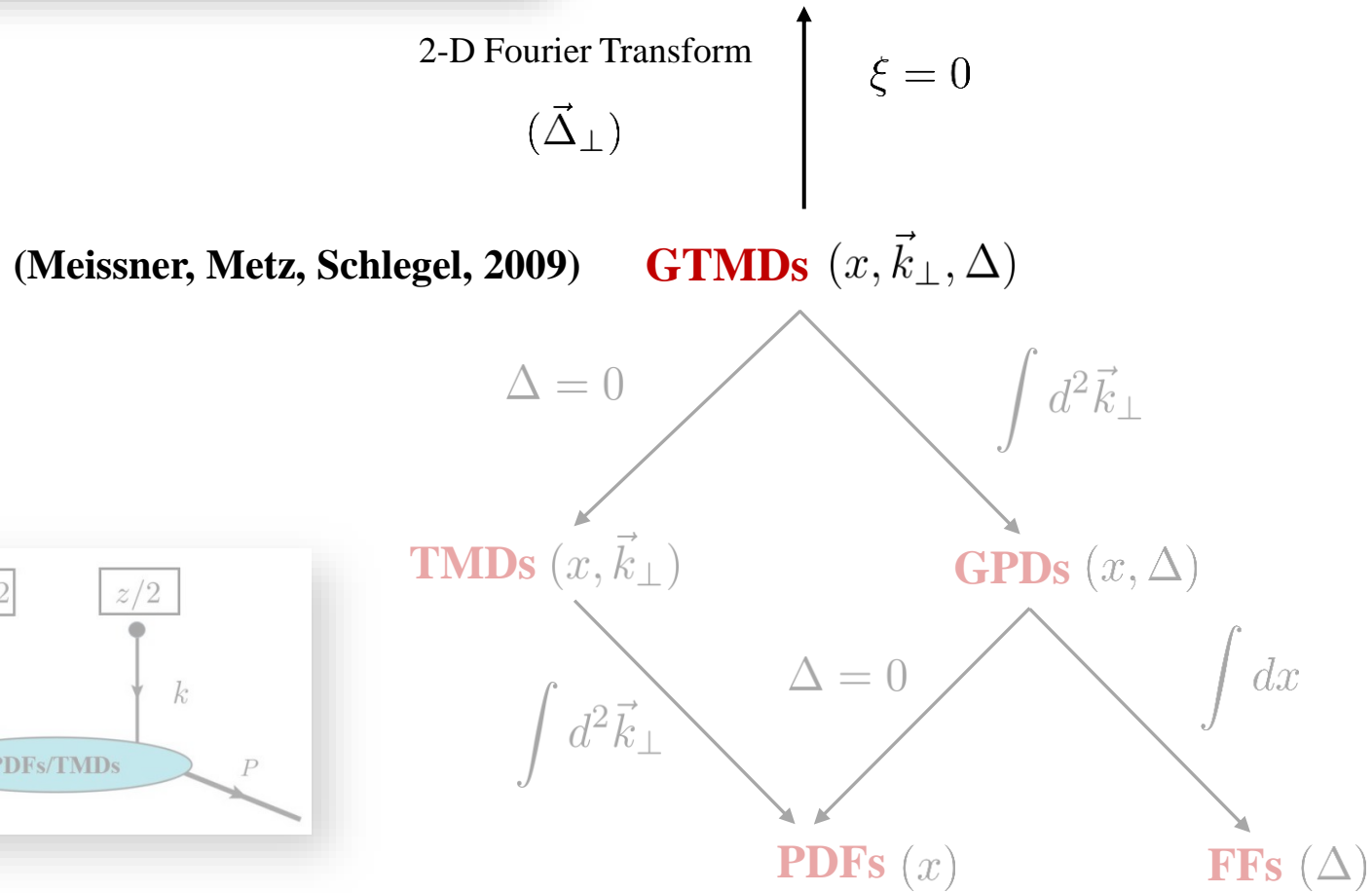
2) GTMDs contain physics beyond TMDs & GPDs

(Meissner, Metz, Schlegel, 2009) **GTMDs**  $(x, \vec{k}_\perp, \Delta)$



# Why are GTMDs interesting?

## 3) Connection to Wigner functions **Wigner Distribution** $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)





# Why are GTMDs interesting?

## 3) Connection to Wigner functions **Wigner Distribution** $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)

### Wigner distributions in NRQM

(Wigner, 1932)

#### □ Calculate from wave-functions:

$$W(x, k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x + \frac{x'}{2}) \psi^*(x - \frac{x'}{2})$$

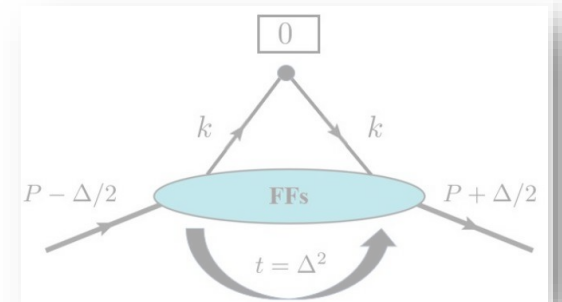
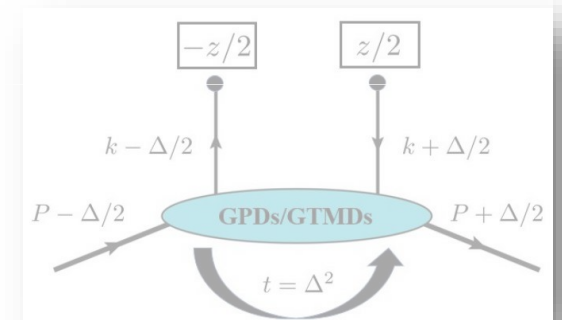
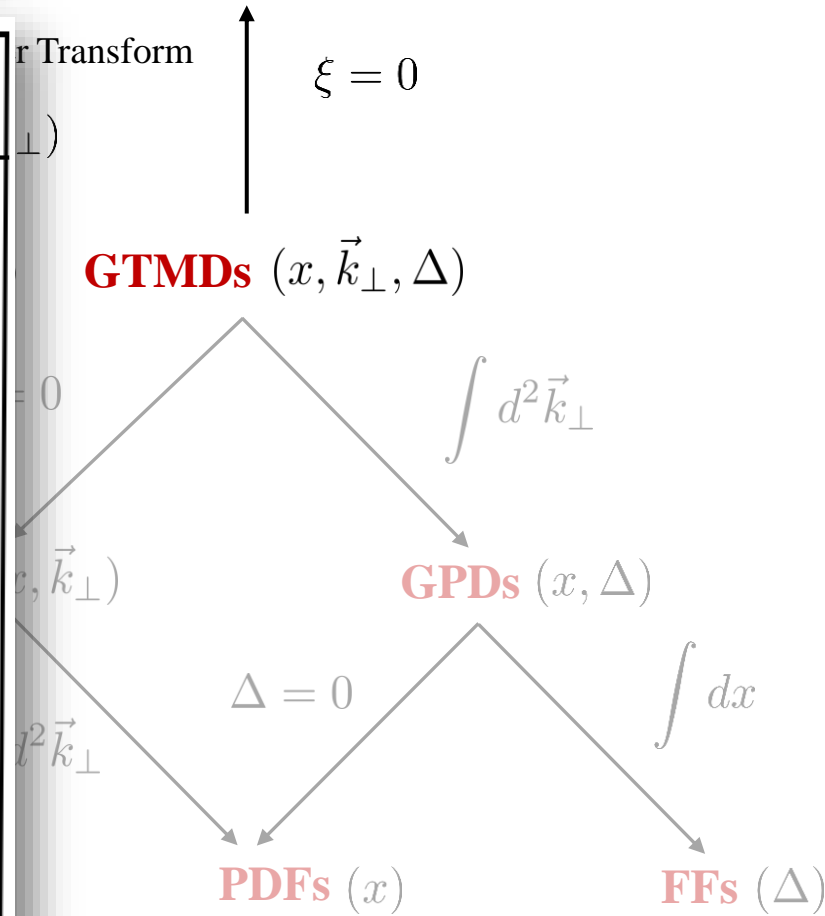
#### □ Connection with probability densities & observables:

#### • Position-space probability:

$$|\psi(x)|^2 = \int dk W(x, k)$$

#### • Expectation value of observables:

$$\langle O \rangle = \int dx \int dk O(x, k) W(x, k)$$





# Why are GTMDs interesting?

## 3) Connection to Wigner functions **Wigner Distribution** $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)

### Wigner distributions in NRQM

(Wigner, 1932)

#### □ Calculate from wave-functions:

$$W(x, k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x + \frac{x'}{2}) \psi^*(x - \frac{x'}{2})$$

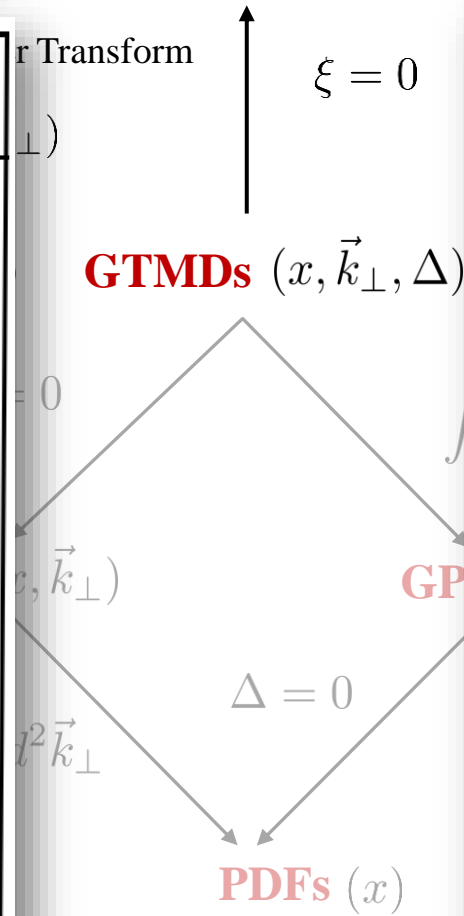
#### □ Connection with probability densities & observables:

#### • Position-space probability:

$$|\psi(x)|^2 = \int dk W(x, k)$$

#### • Expectation value of observables:

$$\langle O \rangle = \int dx \int dk O(x, k) W(x, k)$$



### Wigner distributions in parton physics

(Belitsky, Ji, Yuan, 2003)

#### □ Defined through F.T. of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

#### □ Application:

#### **Orbital Angular Momentum (OAM)**

#### **Intuitive definition of OAM**

$$\langle L_z^q \rangle = \int dx \int d^2 \vec{k}_\perp \int d^2 \vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \left. W_L^{q[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp) \right|_z$$

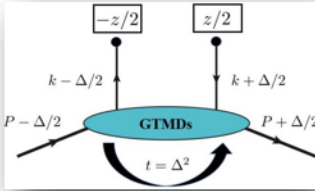
(Lorcé, Pasquini, 2011 / Hatta, 2011 / Ji, Xiong, Yuan, 2012)



# Why are GTMDs interesting?

## 3) Connection

Parameterization of a GTMD correlator (Meissner, Metz, Schlegel, arXiv: 0906.5323):



$$= \frac{1}{2M} \bar{u}(p', \lambda') \left[ \mathbf{F}_{1,1} + \frac{i\sigma^{i+} k_{\perp}^i}{P^+} \mathbf{F}_{1,2} + \frac{i\sigma^{i+} \Delta_{\perp}^i}{P^+} \mathbf{F}_{1,3} + \frac{i\sigma^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} \mathbf{F}_{1,4} \right] u(p, \lambda)$$

Wigner distribution  
(Wigner, 1932)

□ Calculate from wave-functions:

$$W(x, k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x + \frac{x'}{2}) \psi^*(x - \frac{x'}{2})$$

□ Connection with probability densities & observables:

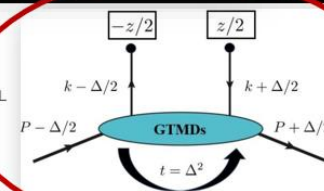
• **Position-space probability:**

$$|\psi(x)|^2 = \int dk W(x, k)$$

• **Expectation value of observables:**

$$\langle O \rangle = \int dx \int dk O(x, k) W(x, k)$$

$$L_z^q = \int dx \int d^2 k_{\perp} d^2 b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z \left\{ \int e^{i\vec{b}_{\perp} \cdot \vec{\Delta}_{\perp}} \right\}$$



(Yuan, 2003)

distributions in parton physics  
(Melitsky, Yuan, 2003)

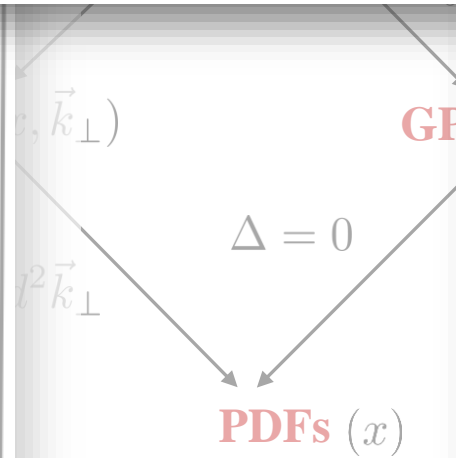
lator:

**Orbital Angular Momentum (OAM)**

**Intuitive definition of OAM**

$$\langle L_z^q \rangle = \int dx \int d^2 \vec{k}_{\perp} \int d^2 \vec{b}_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z \left| W_L^{q[\gamma^+]}(x, \vec{k}_{\perp}, \vec{b}_{\perp}) \right|_z$$

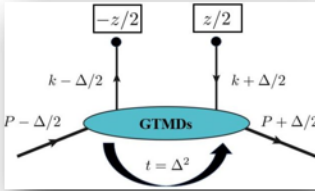
(Lorcé, Pasquini, 2011 / Hatta, 2011 /  
Ji, Xiong, Yuan, 2012)



# Why are GTMDs interesting?

## 3) Connection

Parameterization of a GTMD correlator (Meissner, Metz, Schlegel, arXiv: 0906.5323):



$$= \frac{1}{2M} \bar{u}(p', \lambda') \left[ \mathbf{F}_{1,1} + \frac{i\sigma^{i+} k_{\perp}^i}{P^+} \mathbf{F}_{1,2} + \frac{i\sigma^{i+} \Delta_{\perp}^i}{P^+} \mathbf{F}_{1,3} + \frac{i\sigma^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} \mathbf{F}_{1,4} \right] u(p, \lambda)$$

(Yuan, 2003)

Wigner distribution

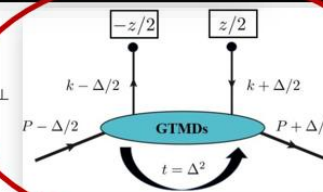
distributions in parton physics  
(Melitsky, Yuan, 2003)

□ Calculate from wave-functions:

$$W(x, k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x + \frac{x'}{2}) \psi^*(x - \frac{x'}{2})$$

□ Connection with probability densities & observables:

$$L_z^q = \int dx \int d^2 k_{\perp} d^2 b_{\perp} (\vec{b}_{\perp} \times \vec{k}_{\perp})_z \left\{ \int e^{i\vec{b}_{\perp} \cdot \vec{\Delta}_{\perp}} \right\}$$



lator:

Orbital Angular Momentum (OAM)

Intuitive definition of OAM

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{M^2} \mathbf{F}_{1,4}^{q,g}(x, \vec{k}_{\perp}^2)$$

Relation between GTMD  $\mathbf{F}_{1,4}^{q,g}$  & OAM

$$(\vec{b}_{\perp} \times \vec{k}_{\perp})_z \left| W_L^{q[\gamma^+]}(x, \vec{k}_{\perp}, \vec{b}_{\perp}) \right|_z$$

(Latta, 2011 /

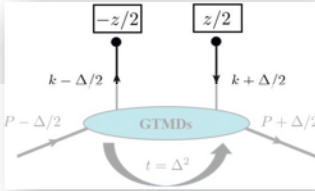
Ji, Xiong, Yuan, 2012)



# Why are GTMDs interesting?

## 3) Connection

Parameterization of a GTMD correlator (Meissner, Metz, Schlegel, arXiv: 0906.5323):



$$= \frac{1}{2M} \bar{u}(p', \lambda') \left[ \mathbf{F}_{1,1} + \frac{i\sigma^{i+} k_{\perp}^i}{P^+} \mathbf{F}_{1,2} + \frac{i\sigma^{i+} \Delta_{\perp}^i}{P^+} \mathbf{F}_{1,3} + \frac{i\sigma^{ij} k_{\perp}^i \Delta_{\perp}^j}{M^2} \mathbf{F}_{1,4} \right] u(p, \lambda)$$

(Yuan, 2003)

distributions in parton physics  
(Melitsky, Yuan, 2003)

**Big question: Is this measurable?**

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{M^2} \mathbf{F}_{1,4}^{q,g}(x, \vec{k}_{\perp}^2)$$

Relation between GTMD  $\mathbf{F}_{1,4}^{q,g}$  & OAM

Intuitive definition of OAM

$$(\vec{b}_{\perp} \times \vec{k}_{\perp}) \Big|_z W_L^{q[\gamma^+]}(x, \vec{k}_{\perp}, \vec{b}_{\perp})$$

(Latta, 2011 /

Ji, Xiong, Yuan, 2012)

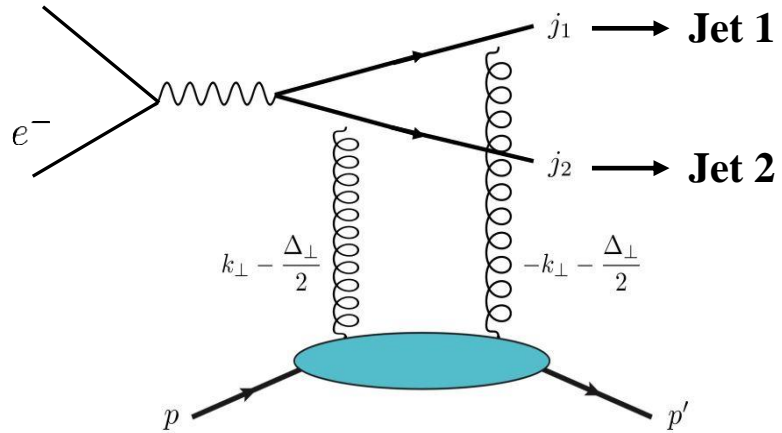


# **Observables for GTMDs: State of the art**



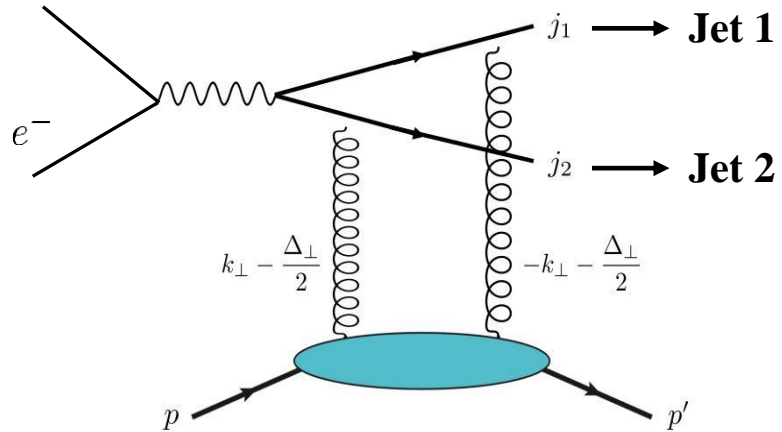
# Observables for GTMDs

**Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)**



# Observables for GTMDs

**Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)**



“Elliptic” Wigner distribution:

$$W(x, \vec{k}_\perp, \vec{b}_\perp) \approx W_0(x, |\vec{k}_\perp|, |\vec{b}_\perp|)$$

$$+ 2 \cos 2(\phi_k - \phi_b) W_1(x, |\vec{k}_\perp|, |\vec{b}_\perp|)$$

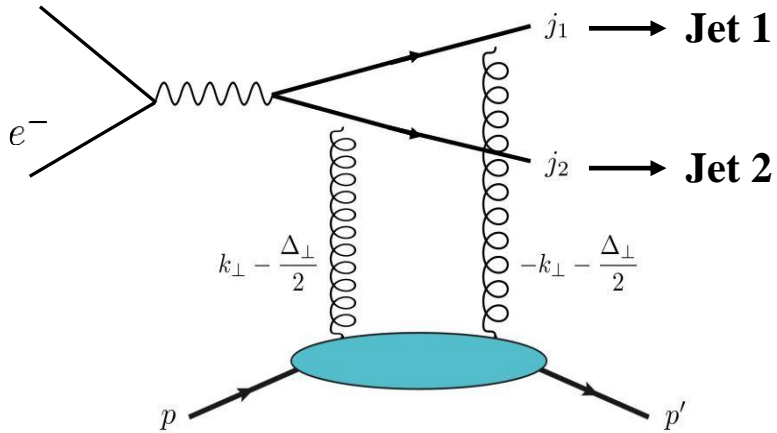
**Symmetric part**

**Elliptic part**



# Observables for GTMDs

**Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)**



**“Elliptic” Wigner distribution:**

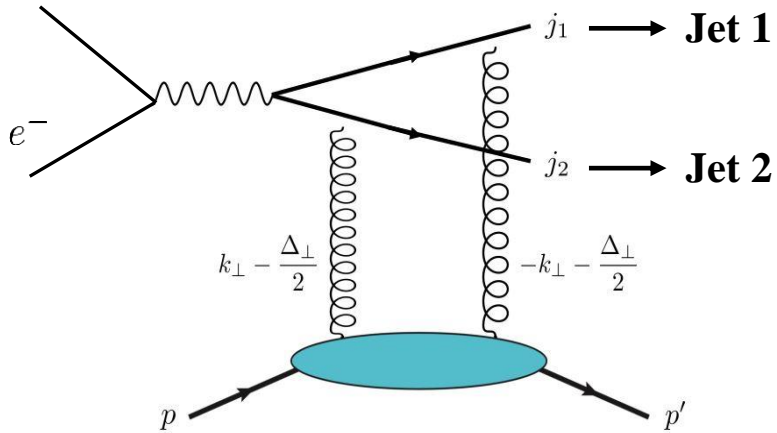
$$W(x, \vec{k}_\perp, \vec{b}_\perp) \approx W_0(x, |\vec{k}_\perp|, |\vec{b}_\perp|) + 2 \cos 2(\phi_k - \phi_b) W_1(x, |\vec{k}_\perp|, |\vec{b}_\perp|)$$

**Main result:**

$$\begin{aligned} \frac{d\sigma}{dy_1 dy_2 d^2 \vec{\Delta}_\perp d^2 \vec{P}_\perp} &\propto z(1-z)[z^2 + (1-z)^2] \int d^2 k_\perp d^2 k'_\perp S(k_\perp, \Delta_\perp) S(k'_\perp, \Delta_\perp) \\ &\quad \times \left[ \frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{k}_\perp}{(P_\perp - k_\perp)^2 + \epsilon^2} \right] \cdot \left[ \frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{k}'_\perp}{(P_\perp - k'_\perp)^2 + \epsilon^2} \right] \\ &\approx d\sigma_0 + 2 \cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp}) d\tilde{\sigma} \end{aligned}$$

# Observables for GTMDs

**Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)**



**“Elliptic” Wigner distribution:**

$$W(x, \vec{k}_\perp, \vec{b}_\perp) \approx W_0(x, |\vec{k}_\perp|, |\vec{b}_\perp|) + 2 \cos 2(\phi_k - \phi_b) W_1(x, |\vec{k}_\perp|, |\vec{b}_\perp|)$$

**Main result:**

$$\frac{d\sigma}{dy_1 dy_2 d^2 \vec{\Delta}_\perp d^2 \vec{P}_\perp} \propto z(1-z)[z^2 + (1-z)^2] \int d^2 k_\perp d^2 k'_\perp S(k_\perp, \Delta_\perp) S(k'_\perp, \Delta_\perp)$$

**Cosine angular modulation**

$$\approx d\sigma_0 + 2 \cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp}) d\tilde{\sigma}$$

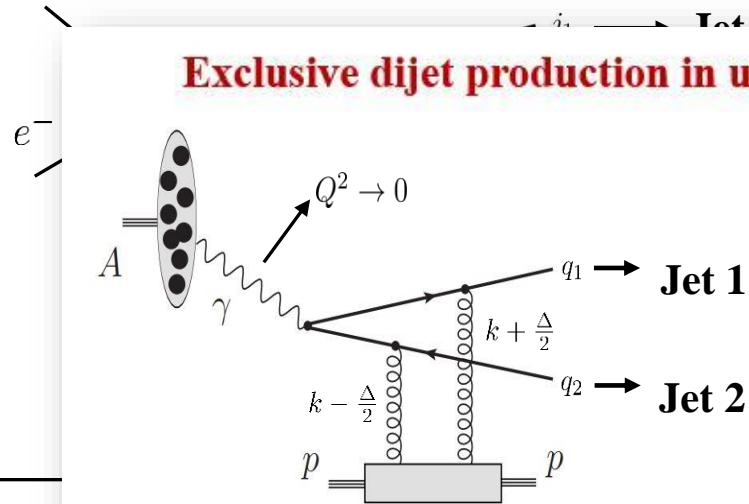
$$\vec{P}_\perp = \frac{1}{2}(\vec{j}_{2\perp} - \vec{j}_{1\perp})$$

$$\left[ \frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{k}_\perp}{(P_\perp - k_\perp)^2 + \epsilon^2} \right] \cdot \left[ \frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{k}'_\perp}{(P_\perp - k'_\perp)^2 + \epsilon^2} \right]$$



# Observables for GTMDs

Exclusive dijet production in lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)



Same cosine angular correlation observed in UPC

Main result.

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{\Delta}_\perp d^2\vec{P}_\perp} \propto z(1-z)[z^2 + (1-z)^2] \int d^2k_\perp d^2k'_\perp S(k_\perp, \Delta_\perp) S(k'_\perp, \Delta_\perp)$$

Cosine angular modulation

$$\approx d\sigma_0 + 2 \cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp}) d\tilde{\sigma}$$

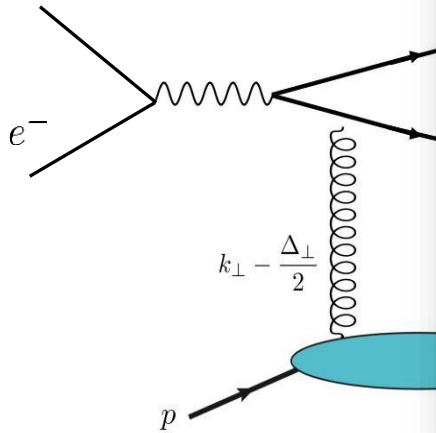
$$\vec{P}_\perp = \frac{1}{2}(\vec{j}_{2\perp} - \vec{j}_{1\perp})$$



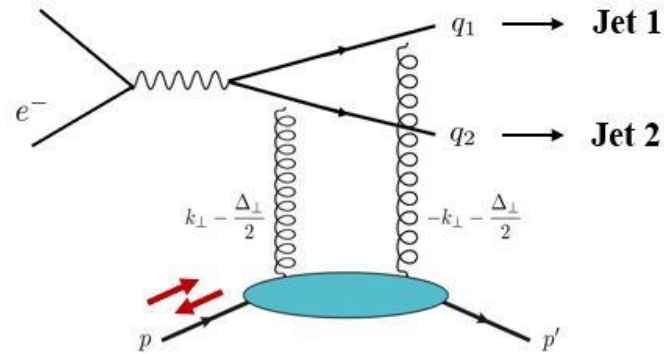
# Observables for GTMDs

Exclusive **Coming up:**

lepton-ion collisions at small-x (Hatta, Xiao, Yuan, arXiv: 1601.01585)



What happens if target is polarized?



arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the  
Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>

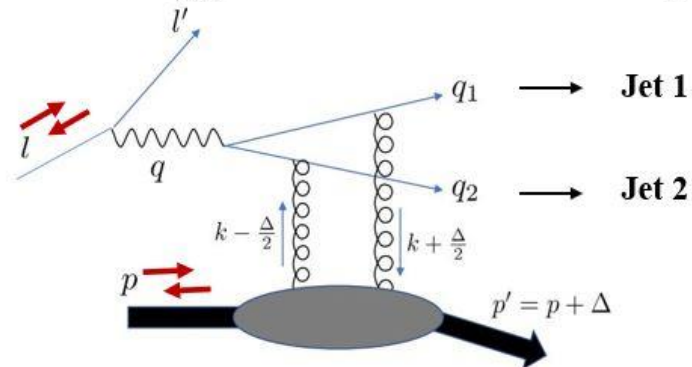
Main result:

$$\frac{d\sigma}{dy_1 dy_2 d^2 \vec{\Delta}_\perp d^2 \vec{P}_\perp} \propto z(1-z)$$

**Cosine angular**

$$\approx d\sigma_0$$

What happens if in addition lepton is polarized?



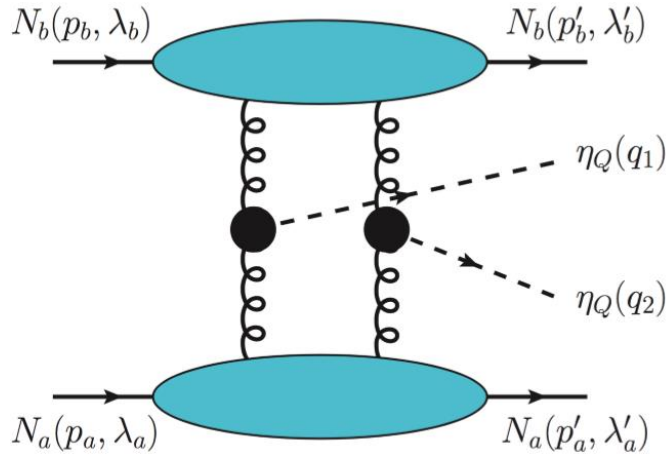
arXiv: 2201.08709 (2022)

Signature of the gluon orbital angular momentum

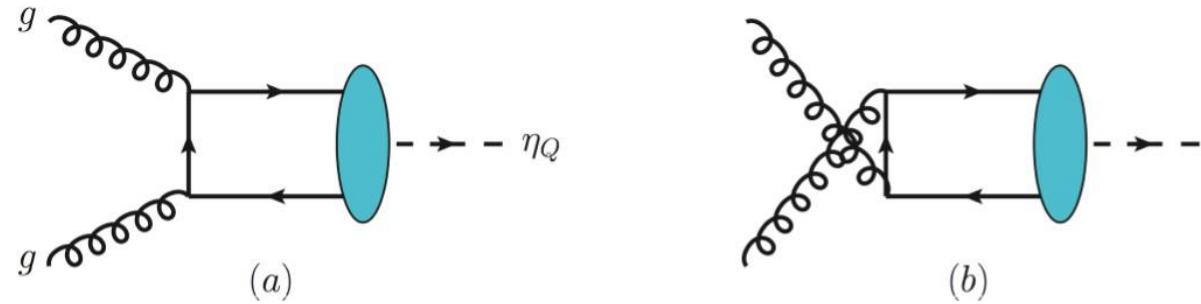
Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>

# Observables for GTMDs

## Exclusive double quarkonium production (SB, Metz, Ojha, Tsai, Zhou, arXiv: 1802.10550)



### Color Singlet Model: (Kuhn et. al., 1979, ...)

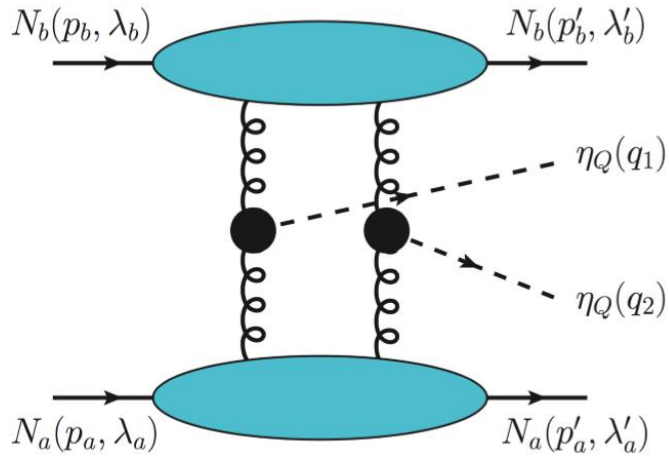


### Main result:

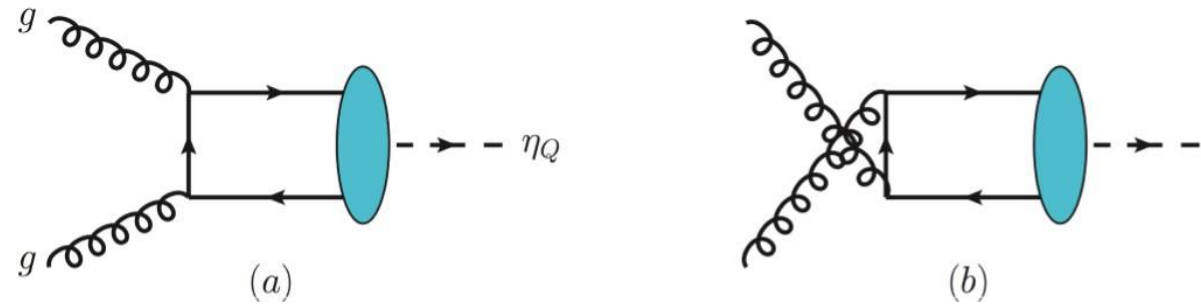
$$\frac{1}{2}(\tau_{XY} - \tau_{YX}) \approx 2 \text{Re.} \left\{ -\frac{\varepsilon_{\perp}^{ij} \Delta_{a\perp}^j}{M} C \left[ \frac{k_{a\perp}^i}{M} F_{1,4}(x_a, \vec{k}_{a\perp}) F_{1,1}(x_b, \vec{k}_{b\perp}) \right] C \left[ F_{1,1}^*(x_a, \vec{p}_{a\perp}) F_{1,1}^*(x_b, \vec{p}_{b\perp}) \right] \right\}$$

# Observables for GTMDs

## Exclusive double quarkonium production (SB, Metz, Ojha, Tsai, Zhou, arXiv: 1802.10550)



### Color Singlet Model: (Kuhn et. al., 1979, ...)



### Main result:

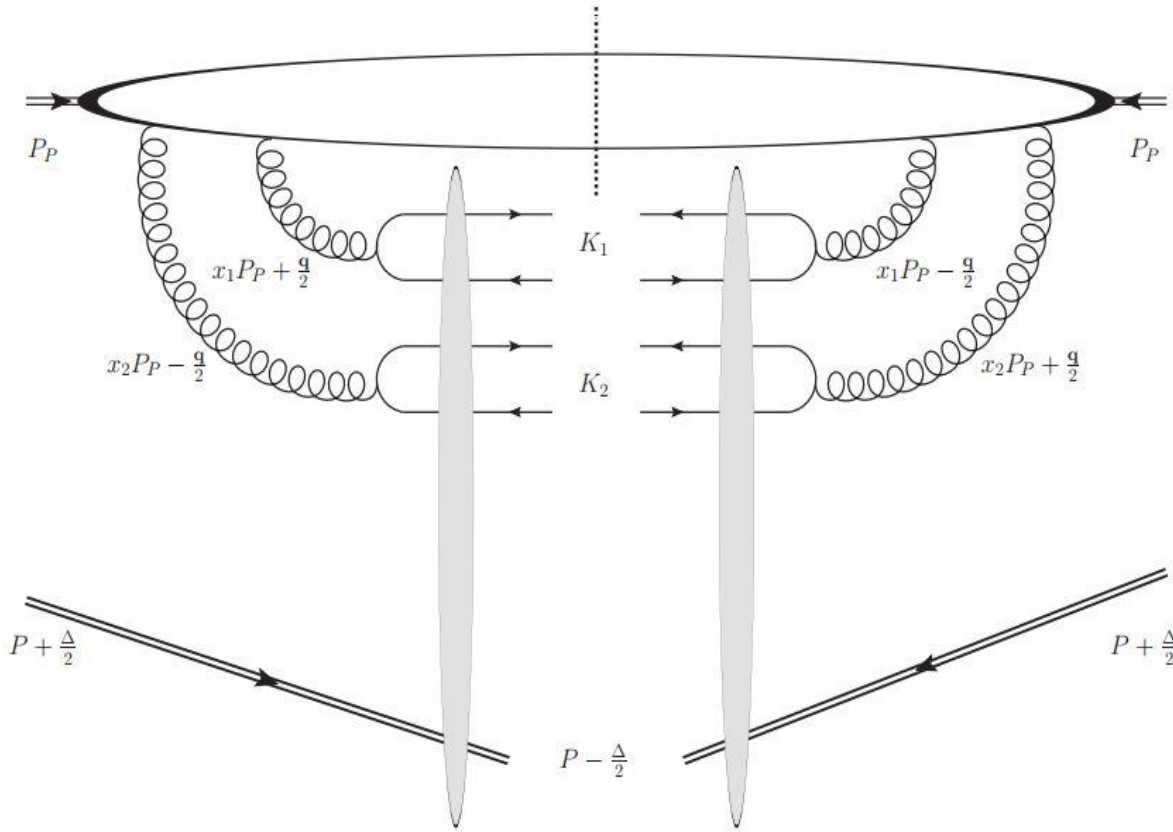
$$\frac{1}{2}(\tau_{XY} - \tau_{YX}) \approx 2 \text{Re.} \left\{ -\frac{\varepsilon_{\perp}^{ij} \Delta_{a\perp}^j}{M} C \left[ \frac{k_{a\perp}^i}{M} F_{1,4}(x_a, \vec{k}_{a\perp}) F_{1,1}(x_b, \vec{k}_{b\perp}) \right] C \left[ F_{1,1}^*(x_a, \vec{p}_{a\perp}) F_{1,1}^*(x_b, \vec{p}_{b\perp}) \right] \right\}$$



**This linear combination of polarization observables is sensitive to gluon OAM**

# Observables for GTMDs

**Single-exclusive pp collisions (Boussarie, Hatta, Xiao, Yuan, arXiv: 1807.08697)**



**Main result:**

**Access Weizacker-Williams gluon GTMD**

**Example: Result for  $\chi_1 \chi_1$  production**

**Double PDF**

$$d\sigma \approx F(x_1, x_2)$$

$$\times \left( G_1(\vec{K}_\perp, \vec{\Delta}_\perp) + \frac{\vec{K}_\perp^2}{2M^2} G_2(\vec{K}_\perp, \vec{\Delta}_\perp) \right)^2$$

**Unpolarized & Linearly-polarized GTMDs**



## More developments ...

$$\text{Im. } \mathbf{F}_{1,2} \Big|_{\Delta=0} = -f_{1T}^\perp$$

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target:  
GTMD distributions and the Odderons

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>1</sup> Lech Szymanowski,<sup>2</sup> and Samuel Wallon<sup>3,4</sup>



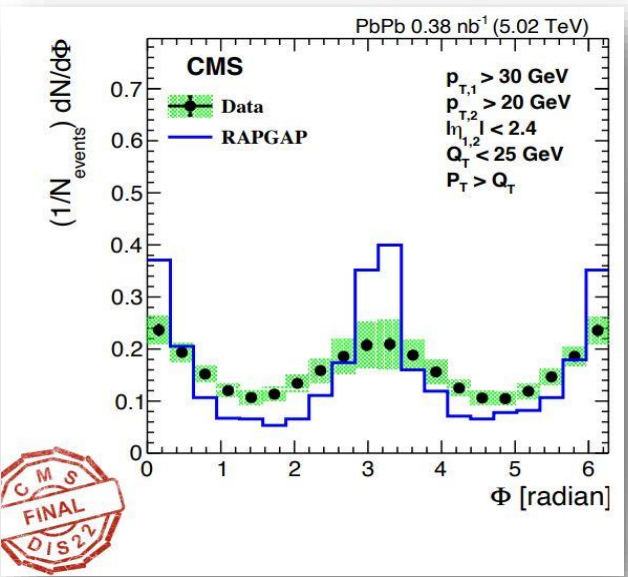
# More developments ...

$$\text{Im. } \mathbf{F}_{1,2} \Big|_{\Delta=0} = -f_{1T}^\perp$$

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target:  
GTMD distributions and the Odderons

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>1</sup> Lech Szymanowski,<sup>2</sup> and Samuel Wallon<sup>3,4</sup>



The CMS Collaboration **Michael Murray's talk, DIS 2022**

Angular correlations in exclusive dijet photoproduction in  
ultra-peripheral PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV





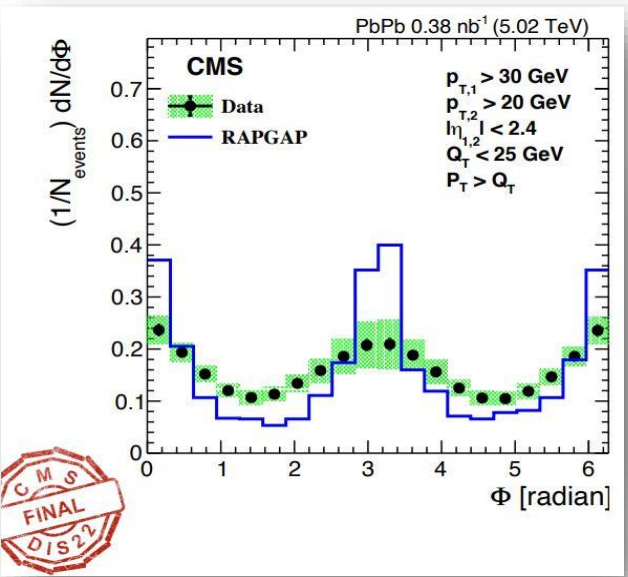
# More developments ...

$$\text{Im. } \mathbf{F}_{1,2} \Big|_{\Delta=0} = -f_{1T}^\perp$$

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target:  
GTMD distributions and the Odderons

Renaud Boussarie,<sup>1</sup> Yoshitaka Hatta,<sup>1</sup> Lech Szymanowski,<sup>2</sup> and Samuel Wallon<sup>3,4</sup>



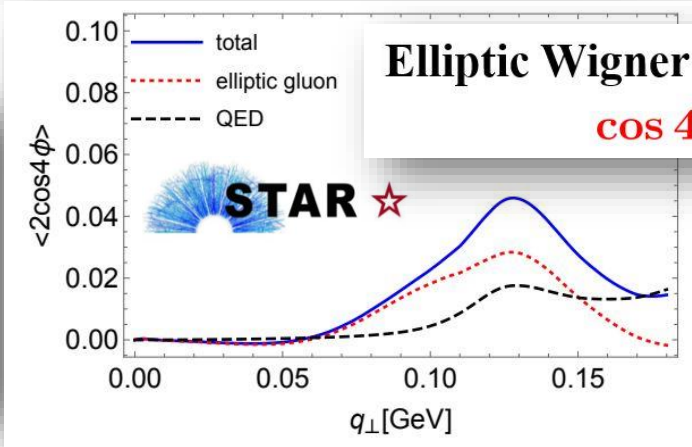
The CMS Collaboration **Michael Murray's talk, DIS 2022**

Angular correlations in exclusive dijet photoproduction in  
ultra-peripheral PbPb collisions at  $\sqrt{s_{NN}} = 5.02 \text{ TeV}$

arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou



Elliptic Wigner distribution contributes to  
**cos 4φ** asymmetry

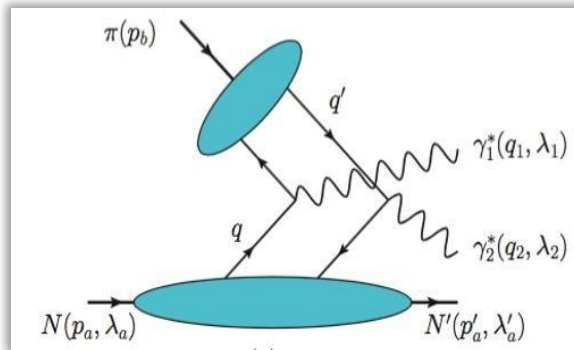


# More developments ...

arXiv: 1702.04387 (2017)

## Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,<sup>1</sup> Andreas Metz,<sup>1</sup> and Jian Zhou<sup>2</sup>



**First & only process sensitive to quark GTMDs**



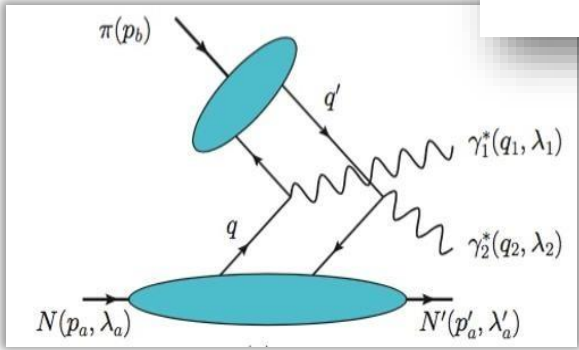
arXiv: 1702.04387 (2017)

Example of an observable

Generalized TMDs and the exclusive double spin asymmetry  
Shohini Bhattacharya,<sup>1</sup> Andreas Metz,<sup>1</sup> and ...

$$\frac{1}{2}(\tau_{XY} - \tau_{YX}) = \frac{4}{M_a^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta_{a\perp}^j) \text{Re} \left\{ C^{(-)} [F_{1,1} \phi_{\pi}] C^{(+)} [\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} \mathbf{F}_{1,4}^* \phi_{\pi}^*] - C^{(+)} [G_{1,4} \phi_{\pi}] C^{(-)} [\vec{\beta}_{\perp} \cdot \vec{p}_{a\perp} \mathbf{G}_{1,1}^* \phi_{\pi}^*] \right\}$$

This observable is sensitive to **OAM** & **spin-orbit correlation**



First & only process sensitive to quark GTMDs

GTMDs related to strength of spin-orbit correlation  
(Lorcé, Pasquini, 2011 / Lorcé, 2014)

Recall Spin-Orbit coupling in H atom!

$\text{Re. } \mathbf{F}_{1,4}^{q/g}$   $\vec{L}_{q/g} \cdot \vec{S}_N$  Nucleon

$\text{Re. } \mathbf{G}_{1,1}^{q/g}$   $\vec{L}_{q/g} \cdot \vec{S}_{q/g}$  Quark & Gluon



arXiv: 1702.04387 (2017)

Example of an observable

Generalized TMDs and the exclusive double Drell-Yan

Shohini Bhattacharya,<sup>1</sup> Andreas Metz,<sup>1</sup> and ...

$$\frac{1}{2}(\tau_{XY} - \tau_{YX}) = \frac{4}{M_a^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta_{a\perp}^j) \text{Re.} \left\{ C^{(-)} [F_{1,1} \phi_{\pi}] C^{(+)} [\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} \mathbf{F}_{1,4}^* \phi_{\pi}^*] - C^{(+)} [G_{1,4} \phi_{\pi}] C^{(-)} [\vec{\beta}_{\perp} \cdot \vec{p}_{a\perp} \mathbf{G}_{1,1}^* \phi_{\pi}^*] \right\}$$

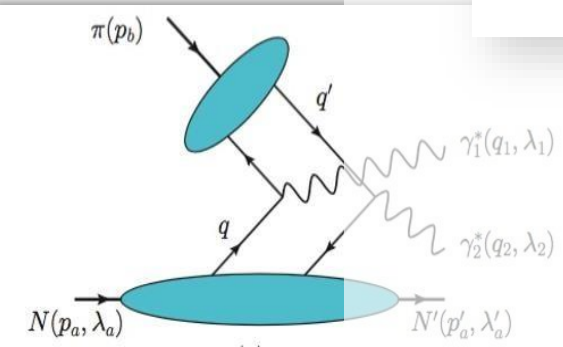
arXiv: 2208.00021 (2022)

First proof of factorization

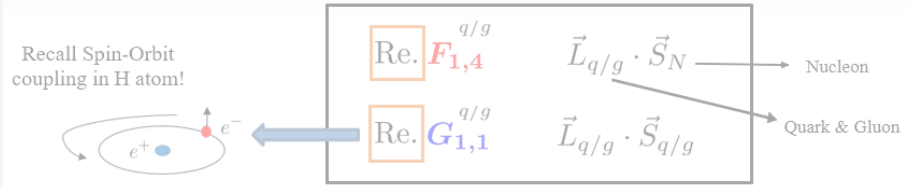
This ...

GTMDs and the factorization of exclusive double Drell-Yan

Miguel G. Echevarria<sup>a,b</sup>, Patricia A. Gutierrez Garcia<sup>c</sup>, Ignazio Scimemi<sup>c</sup>



First & only process sensitive to quark GTMDs





# Our recent work

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>

In Collaboration with:

**Renaud Boussarie** (CPHT, CNRS)

**Yoshitaka Hatta** (BNL)

Based on:

**PRL 128, 182002 (arXiv: 2201.08709)**

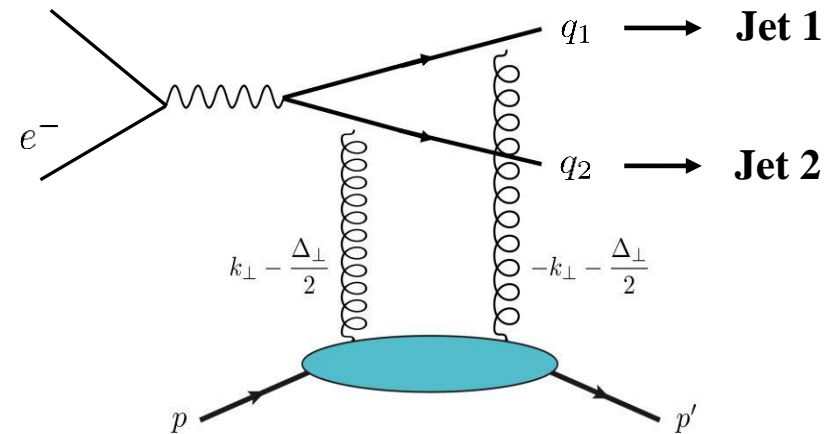
# Probing gluon OAM through exclusive dijet production

## Inspiration

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the  
Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



**We took a fresh look at this 2016 paper**



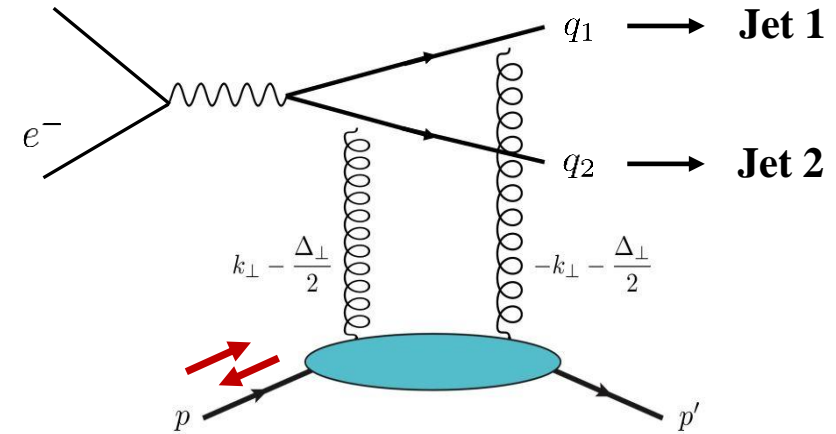
# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

### Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



## Longitudinal single spin asymmetry (SSA):

$$\frac{d\Delta\sigma}{dydQ^2d\Omega} = \sigma_0 h_p \frac{2(\bar{z} - z)(q_\perp \times \Delta_\perp)}{q_\perp^2 + \mu^2} \left[ 16\beta(1 - y) \Im[F_g^* + 4\xi^2 \bar{\beta} F_g'^*] [\mathcal{L}_g + 8\xi^2 \bar{\beta} \mathcal{L}_g'] \right. \\ \left. + (1 + (1 - y)^2) \Im[F_g^* + 2\xi^2(1 - 2\beta) F_g'^*] [\mathcal{L}_g + 2\bar{\beta}(1/z\bar{z} - 2)(\mathcal{L}_g + 4\xi^2(1 - 2\beta) \mathcal{L}_g')] \right]$$

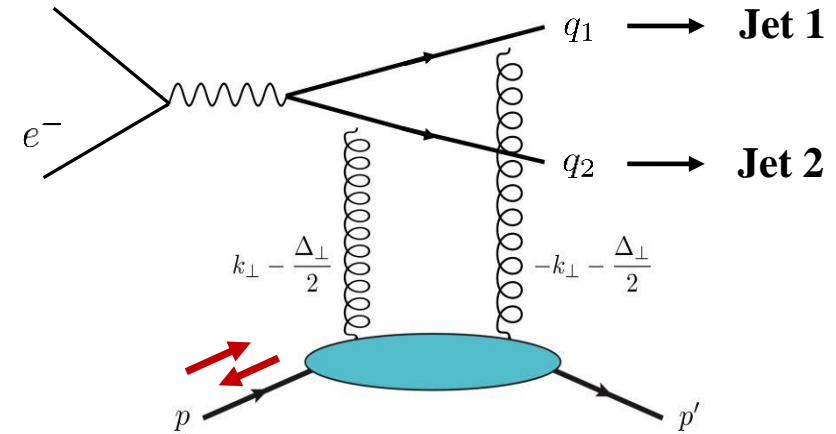
# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

### Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



## Schematic structure of SSA (oversimplified):

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q\perp} - \phi_{\Delta\perp}) (\bar{z} - z) \left[ \Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

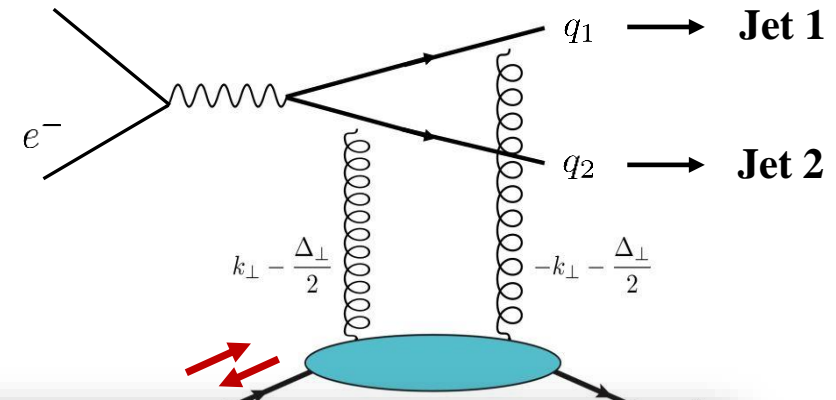
# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



**Signature of OAM is sinusoidal angular modulation**

**Longitudinal single spin asymmetry (SSA):**

$$q_{\perp} = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p$$

$$\sin(\phi_{q_{\perp}} - \phi_{\Delta_{\perp}})(\bar{z} - z) \left[ \Im \left( F_g^*(\xi) \mathcal{L}_g(\xi) \right) \right]$$

Moment of GPD

Moment of OAM



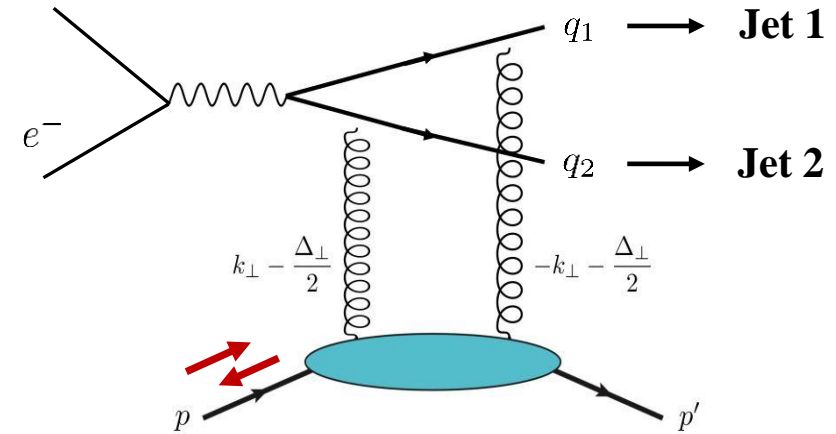
# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

### Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



## Issues with SSA:

$$q_{\perp} = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_{\perp}} - \phi_{\Delta_{\perp}}) (\bar{z} - z) \left[ \Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

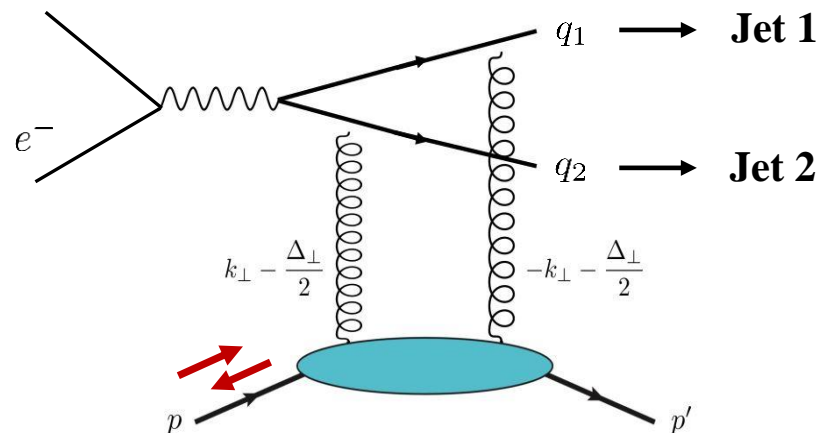
# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



## Issues with SSA:

$$q_{\perp} = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_{\perp}} - \phi_{\Delta_{\perp}}) (\bar{z} - z) \left[ \text{Im}(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

SSA vanishes for symmetric jet configurations  $z = \bar{z} = \frac{1}{2}$



# Probing gluon OAM through exclusive dijet production

## Summary of the 2016 paper

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum  
Electron-Ion Collider

Xiangdong Ji,<sup>1,2</sup> Feng Yuan,<sup>3</sup> and Yong Zhao<sup>1,3</sup>



“Compton Form Factor”:

$$\mathcal{L}_g(\xi) = \int dx \frac{x^2 \xi L_g(x, \xi)}{(x^2 - \xi^2 + i\xi\epsilon)^3}$$

Third pole at  $x = \pm\xi \longrightarrow$  potentially dangerous for collinear factorization  
(See Cui, Hu, Ma, 1804.05293)

Issues with SSA:

$$q_\perp = q_{1\perp} - q_{2\perp}$$

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) (\bar{z} - z) \left[ \Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

SSA vanishes for symmetric jet configurations  $z = \bar{z} = \frac{1}{2}$

## Probing gluon OAM through exclusive dijet production

## Our work

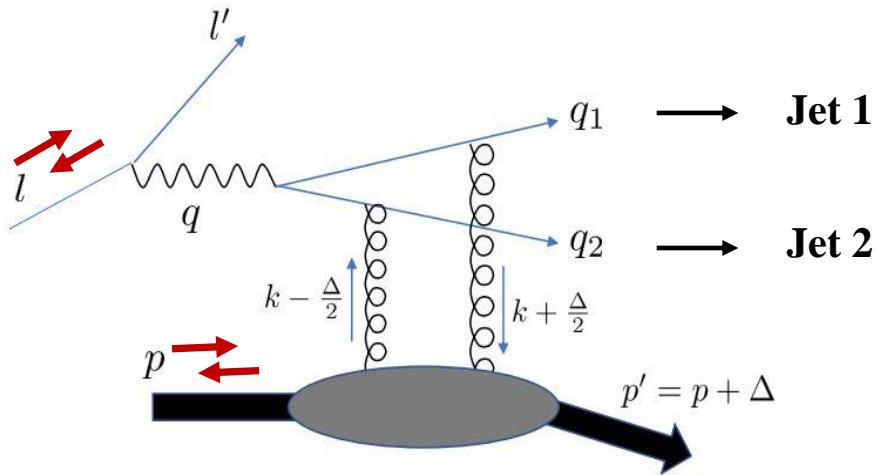
## Signature of the gluon orbital angular momentum

Shohini Bhattacharya,<sup>1,\*</sup> Renaud Boussarie,<sup>2,†</sup> and Yoshitaka Hatta<sup>1,3,‡</sup>

## Distinct feature in our work

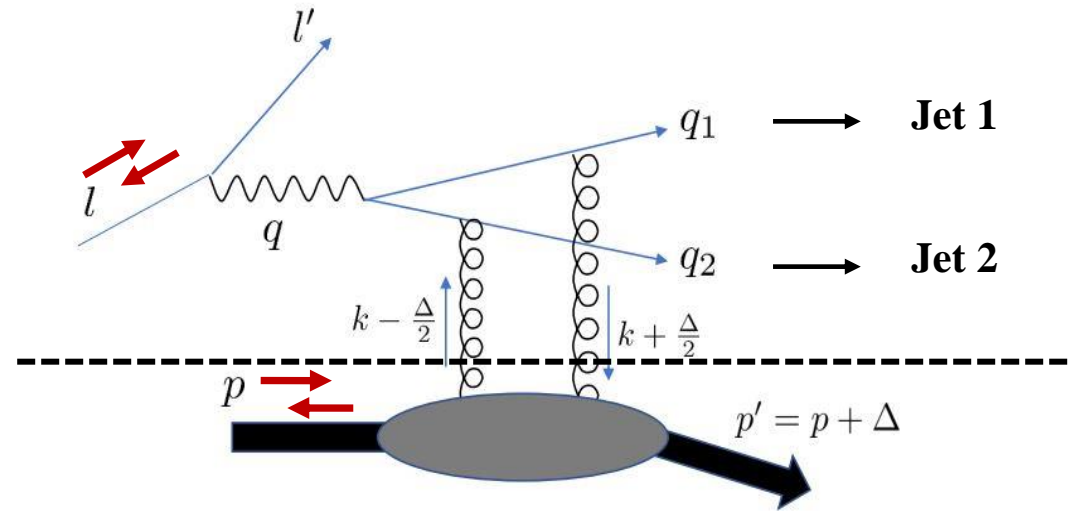
### Double spin asymmetry (DSA):-

Both electron & incoming proton are longitudinally polarized



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude



- 6 leading-order Feynman diagrams
- Scattering amplitude:

$$A \propto \int dx \int d^2 k_{\perp} \mathcal{H}(x, \xi, q_{\perp}, k_{\perp}, \Delta_{\perp}) x f_g(x, \xi, k_{\perp}, \Delta_{\perp})$$

Hard part

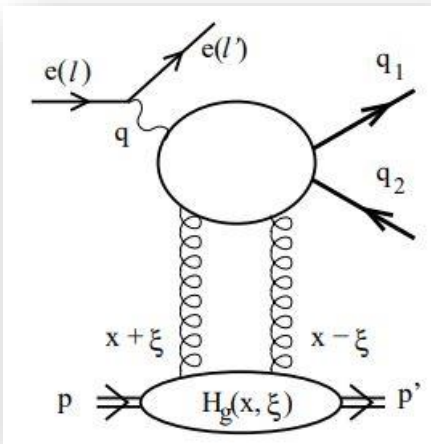
Soft part

# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Twist expansion:

- **Twist-2 amplitude:** Proportional to gluon GPD



Braun, Ivanov, 0505263

$$A_T^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{q_\perp^2 + \mu^2} (\bar{u}(q_1) \not{\epsilon}_\perp v(q_2)) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \times \left( 1 + \frac{2\xi^2(1 - 2\beta)}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$A_L^2 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{1}{(q_\perp^2 + \mu^2)^2} 4\xi z \bar{z} QW (\bar{u}(q_1) \gamma^- v(q_2)) \int dx \frac{1}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \times \left( 1 + \frac{4\xi^2 \bar{\beta}}{(x + \xi - i\varepsilon)(x - \xi + i\varepsilon)} \right) \int d^2 k_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



# High exclusive dijet production

g amplitude

$$L_z^{q,g} = - \int dx \int d^2 \vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} \mathbf{F}_{1,4}^{q,g}(x, \vec{k}_\perp^2)$$

Relation between GTMD  $\mathbf{F}_{1,4}^{q,g}$  & OAM

Twist expansion:

- **Twist-3 amplitude:** Proportional to gluon OAM

$$A_T^3 = - \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$- \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2 k_\perp \epsilon_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Twist expansion:

- **Twist-3 amplitude:** Proportional to gluon OAM

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$- \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

**Factorization-breaking third poles at  $x = \pm\xi$**

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$





# Probing gluon OAM through exclusive dijet production

Twist expansion:

- **Twist-3 amplitude:** Proportion

**Note: Gluon GPDs may contain  $\sim \theta(\xi - |x|)(x^2 - \xi^2)^2$**   
**(See Radyushkin, 9805342)**

**Hence, integrals containing third poles are divergent**

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$- \frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

**Factorization-breaking third poles at  $x = \pm\xi$**

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting  $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Factorization-breaking third poles at  $x = \pm\xi$

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 1 + \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting  $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_1^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( 2\xi + \frac{(2\xi)^3(1 - 2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

Recall: Not possible in SSA

Factorization-breaking third poles at  $x = \pm\xi$

Issues with SSA:

$$A_L^3 \Big|_{q_\perp = q_{1\perp} - q_{2\perp}} \frac{d\sigma}{dy dQ^2 d\Omega} \sim \sigma_0 h_p \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) (\bar{z} - z) \left[ \sin(F_g^*(\xi) \mathcal{L}_g(\xi)) \right] \frac{8\xi^2(1 - \beta)}{(x^2 - \xi^2 + i\xi\varepsilon)} \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

SSA vanishes for symmetric jet configurations  $z = \bar{z} = \frac{1}{2}$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Twist expansion:

Switch off the factorization-breaking third poles by setting  $z = \bar{z} = \frac{1}{2}$

$$A_T^3 = -\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(\bar{z} - z)}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \epsilon_\perp \cdot \gamma_\perp v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( \cancel{\frac{(2\xi)^3(1-2\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)}} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

$$-\frac{ig_s^2 e_{em} e_q}{N_c} \frac{2(2\xi)^2 z \bar{z} W}{(q_\perp^2 + \mu^2)^2} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \int d^2 k_\perp \epsilon_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$

DSA is sensitive to OAM through an interference between twist-2 amplitude  $A^2$  & twist-3 amplitude  $A_T^3$  (No third pole)

$$A_L^3 = \frac{ig_s^2 e_{em} e_q}{N_c} \frac{16\xi^2(\bar{z} - z)z\bar{z}QW}{(q_\perp^2 + \mu^2)^3} \bar{u}(q_1) \gamma^- v(q_2) \int dx \frac{x}{(x^2 - \xi^2 + i\xi\varepsilon)^2} \left( \cancel{\frac{8\xi^2(1-\beta)}{(x^2 - \xi^2 + i\xi\varepsilon)}} \right) \int d^2 k_\perp q_\perp \cdot \mathbf{k}_\perp x f_g(x, \xi, k_\perp, \Delta_\perp)$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

**Main result** ( $z = 1/2$ ):

**DSA's OAM part:**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

**Main result** ( $z = 1/2$ ): **DSA does not vanish for symmetric jet configurations**  $z = \bar{z} = \frac{1}{2}$

**DSA's OAM part:**

**Consequence:**

**Elimination of factorization-breaking third poles at  $x = \pm\xi$**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

**“Compton Form Factors”:**

$$\mathcal{H}_g^{(1)}(\xi) = \int_{-1}^1 dx \frac{H_g(x, \xi)}{(x - \xi + i\epsilon)(x + \xi - i\epsilon)}$$

$$\mathcal{H}_g^{(2)}(\xi) = \int_{-1}^1 dx \frac{\xi^2 H_g(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$

$$\mathcal{L}_g(\xi) = \int_{-1}^1 dx \frac{x^2 L_g(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

**Main result** ( $z = 1/2$ ):

**DSA's OAM part:**

Scattered lepton angle

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

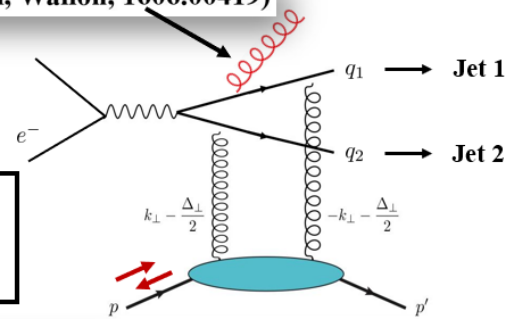
**Signature of gluon OAM is cosine angular modulation**

(Boussarie, Grabovsky, Szymanowski, Wallon, 1606.00419)

Schematic structure of SSA (oversimplified):

$$\frac{d\sigma}{dydQ^2d\Omega} \sim \sigma_0 h_p \sin(\phi_{q\perp} - \phi_{\Delta\perp}) (\bar{z} - z) \left[ \Im(F_g^*(\xi) \mathcal{L}_g(\xi)) \right]$$

Jet angle affected by gluon emissions



Scattered lepton angle

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

Signature of gluon OAM is cosine angular modulation





# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

**Main result** ( $z = 1/2$ ):

**DSA's OAM part:**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

**“Compton Form Factors”:**

$$O(x, \xi) \equiv \int d^2 \tilde{k}_\perp \frac{\tilde{k}_\perp^2}{M^2} F_{1,2}(x, \xi, \tilde{\Delta}_\perp = 0)$$

$$\mathcal{O}(\xi) = \int_{-1}^1 dx \frac{x O(x, \xi)}{(x - \xi + i\epsilon)^2 (x + \xi - i\epsilon)^2}$$

# Probing gluon OAM through exclusive dijet production



**Scattering amplitude**

**Not the end of the story:**



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** ( $z = 1/2$ ):

Helicity GPD



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$



# Probing gluon OAM through exclusive dijet production

## Scattering amplitude

Not the end of the story:

- **Interference between unpolarized & helicity GPD** ( $z = 1/2$ ):

Helicity GPD

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \Re \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$

**Helicity contributes to the same angular modulation as that of OAM**

**DSA is a simultaneous probe of gluon OAM & it's helicity**



# Probing gluon OAM through exclusive dijet production

**Numerical estimate of cross section**

**See backup slides for details on how we modelled  
GPDs and OAM**



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Realistic EIC kinematics

$\sqrt{s}$ [GeV]	$Q^2$ [GeV <sup>2</sup> ]	$y$	$\xi$
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

**Focus on:**  
 $z = \bar{z} = \frac{1}{2}$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Realistic EIC kinematics

$\sqrt{s}$ [GeV]	$Q^2$ [GeV <sup>2</sup> ]	$y$	$\xi$
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:  
 $z = \bar{z} = \frac{1}{2}$

### Cross section:

$$\frac{d\sigma}{dydQ^2d\phi_{l\perp}dzdq_{\perp}^2d^2\Delta_{\perp}} = \frac{\alpha_{em}y}{2^{11}\pi^7Q^4} \frac{\int d\phi_{q\perp} L^{\mu\nu} A_{\mu}^* A_{\nu}}{(W^2 + Q^2)(W^2 - M_J^2)z\bar{z}}$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Realistic EIC kinematics

$\sqrt{s}$ [GeV]	$Q^2$ [GeV <sup>2</sup> ]	$y$	$\xi$
120	2.7	0.7	$\lesssim 10^{-3}$
	4.8		
	10.0		

Focus on:  
 $z = \bar{z} = \frac{1}{2}$

Study cross section as differential in the skewness variable

Cross section:

$$\frac{d\sigma}{dy dQ^2 d\phi_{l\perp} dz dq_{\perp}^2 d\Delta_{\perp}} = \frac{\alpha_{em} y}{2^{11} \pi^7 Q^4} \left( \frac{d\sigma}{dz d\xi d\delta\phi} \right)$$

Relation between skewness & jet momenta:

$$\xi = \frac{q_{\perp}^2 + z\bar{z}Q^2}{-q_{\perp}^2 + z\bar{z}(Q^2 + 2W^2)}$$

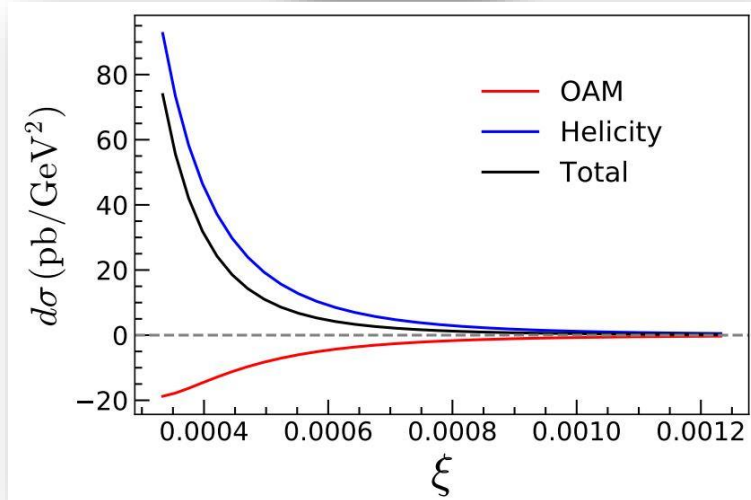


# Probing gluon OAM through exclusive dijet production



## Numerical estimate of cross section

$$Q^2 = 2.7$$

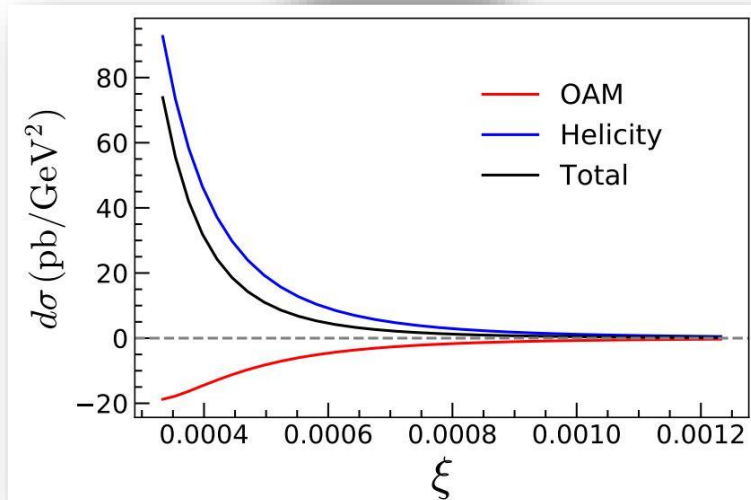




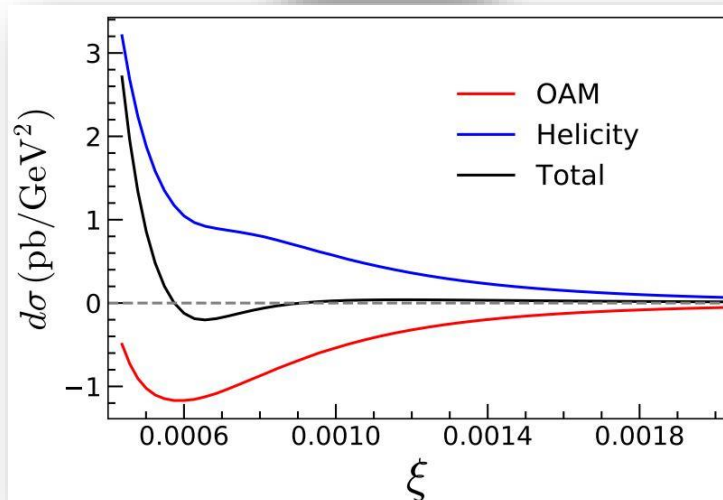
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

$Q^2 = 2.7$



$Q^2 = 4.8$

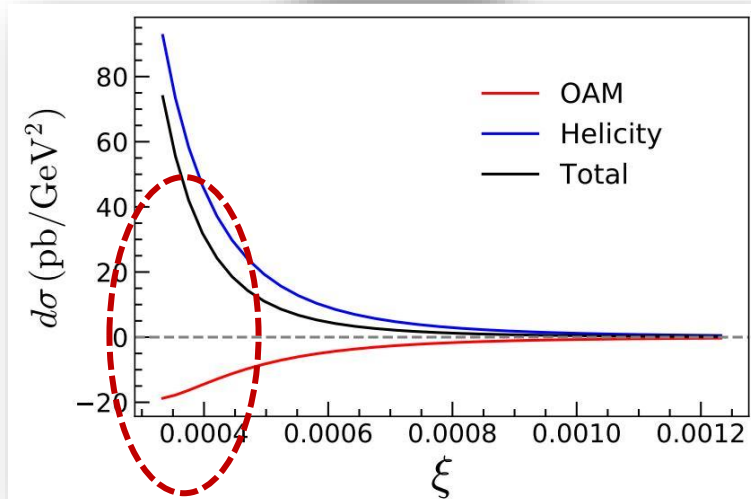




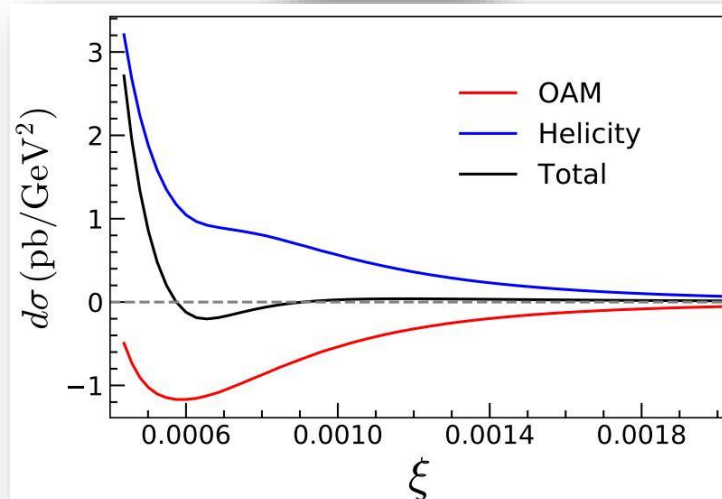
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

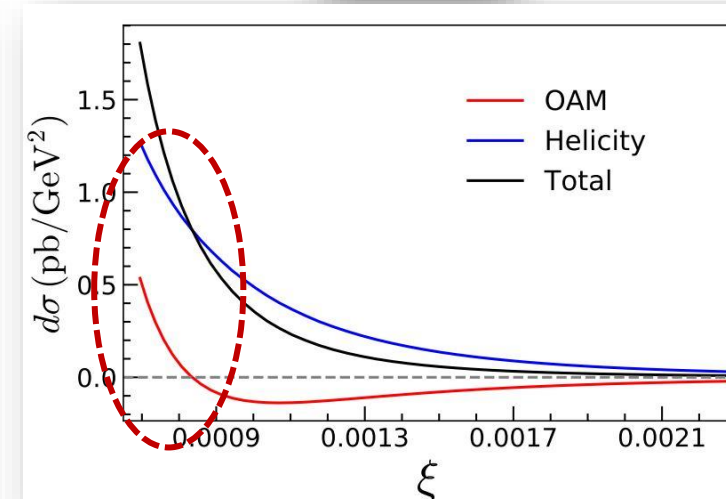
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$

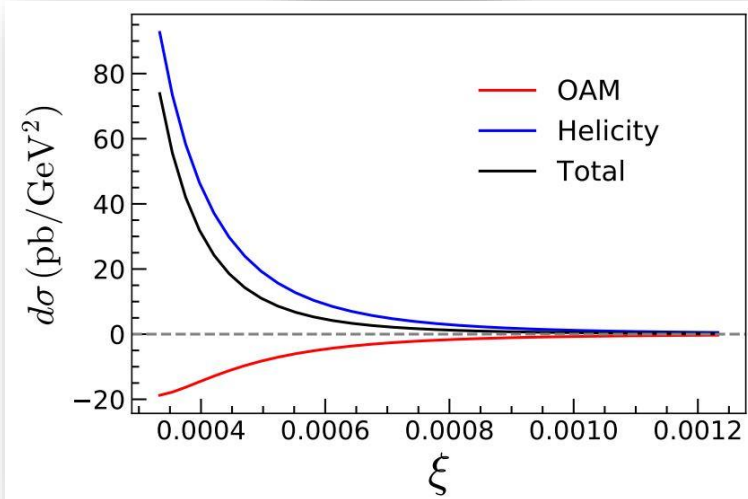




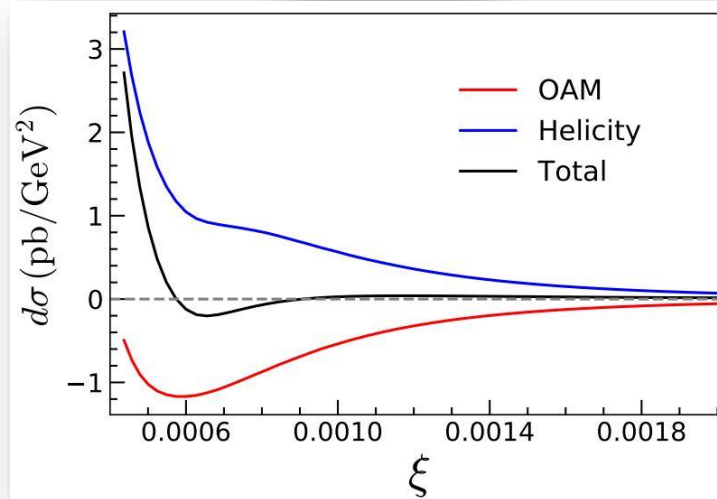
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

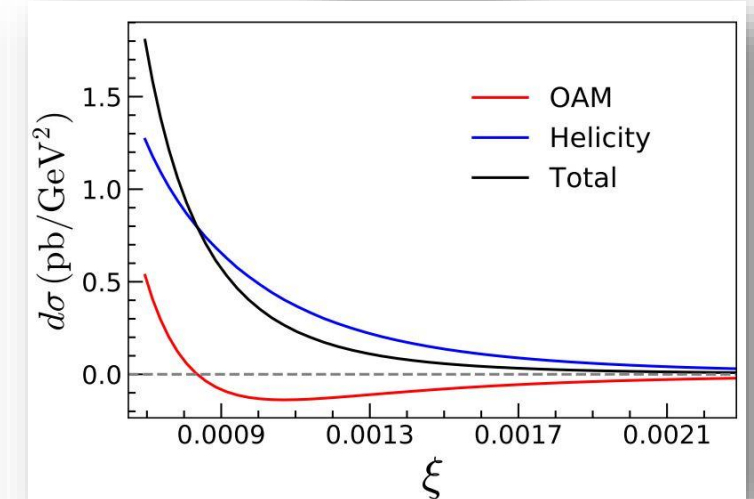
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$

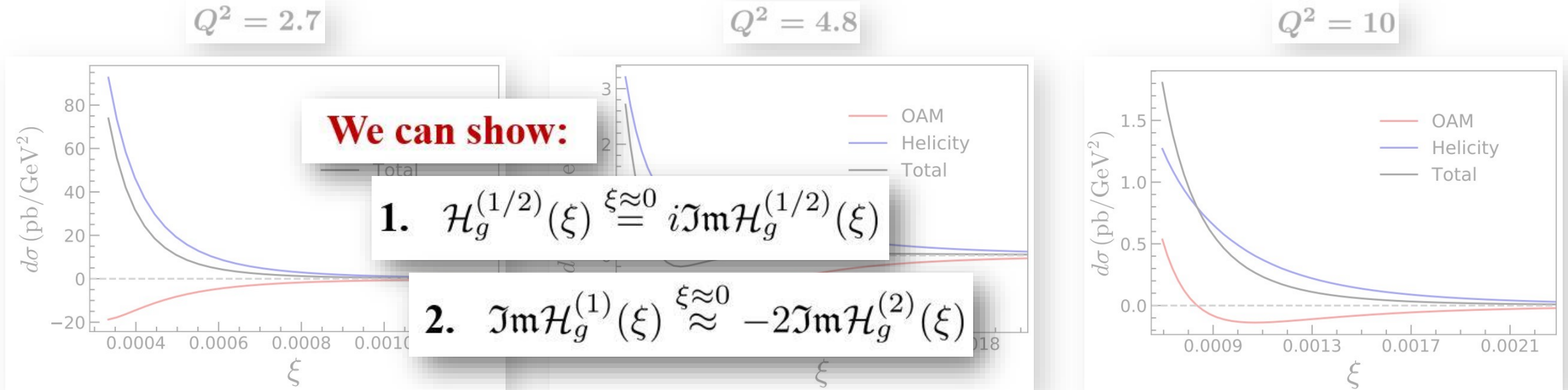


**DSA:** 
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \big|_{\delta\phi=0} \sim \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] - \Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right]$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

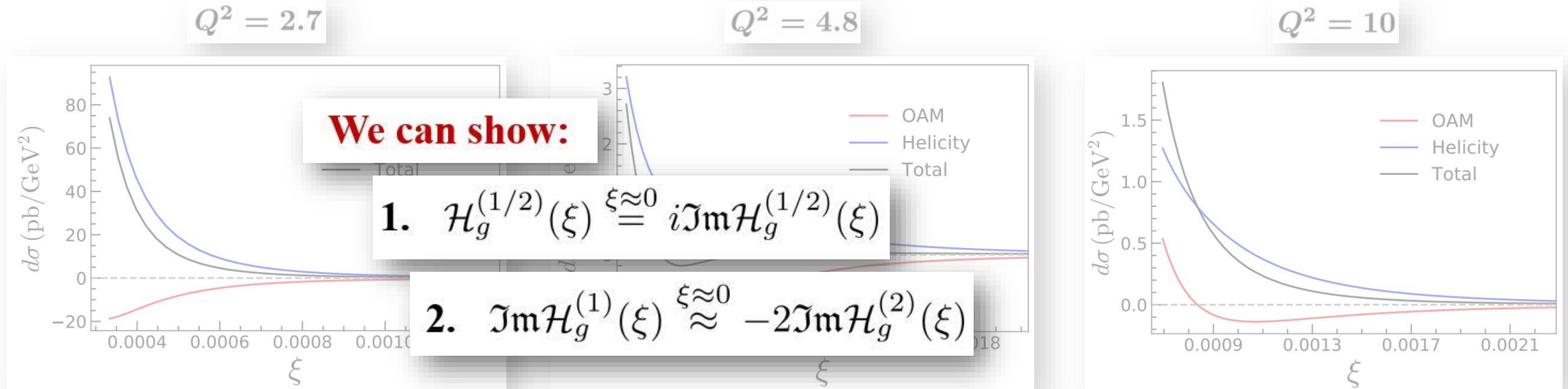


**DSA:** 
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \big|_{\delta\phi=0} \sim \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] - \Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right]$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section



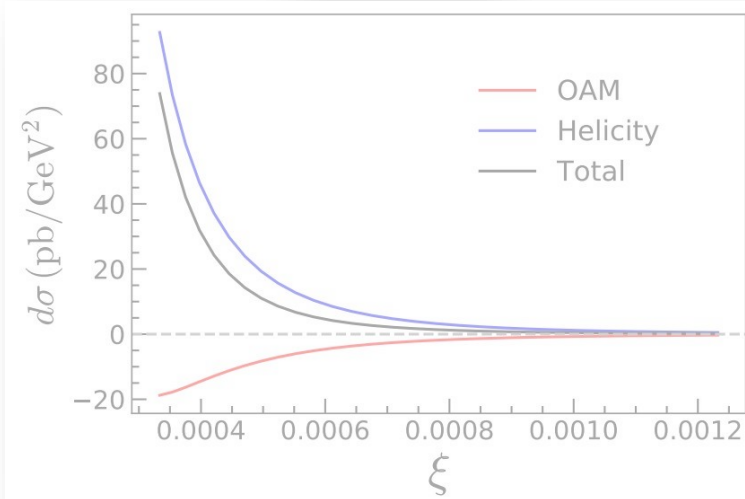
**DSA:** 
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left( \tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$



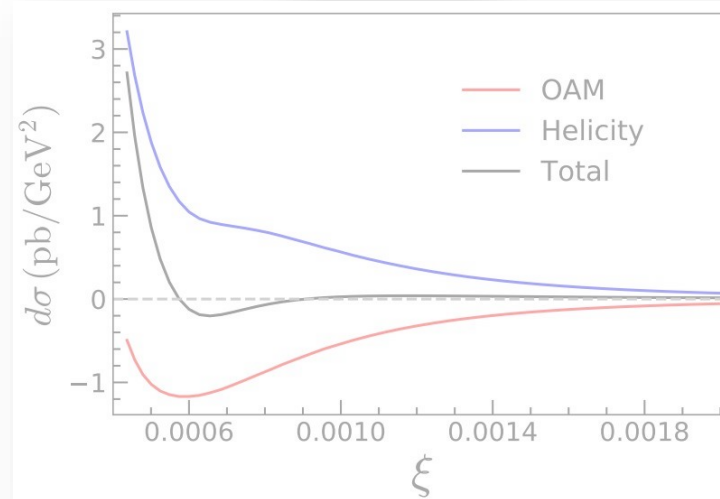
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

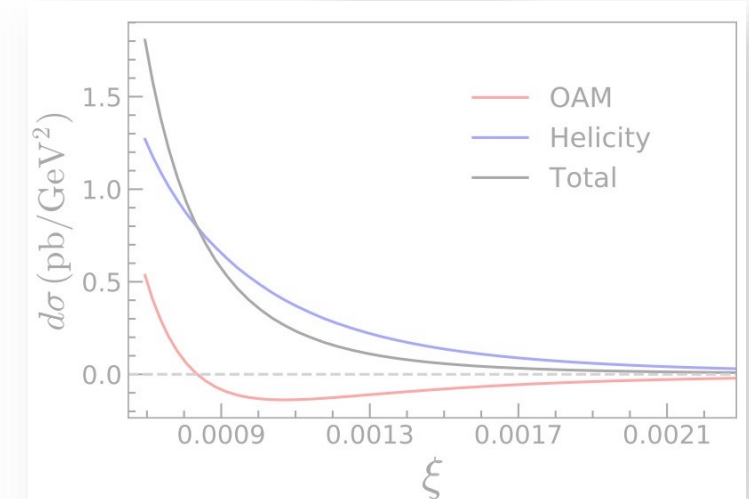
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



**DSA:** 
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left( \tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

$\tilde{\mathcal{H}}_g^{(2)}$  &  $\mathcal{L}_g$  interfere positively/negatively depending upon sign of  $q_\perp^2 - \frac{Q^2}{4}$

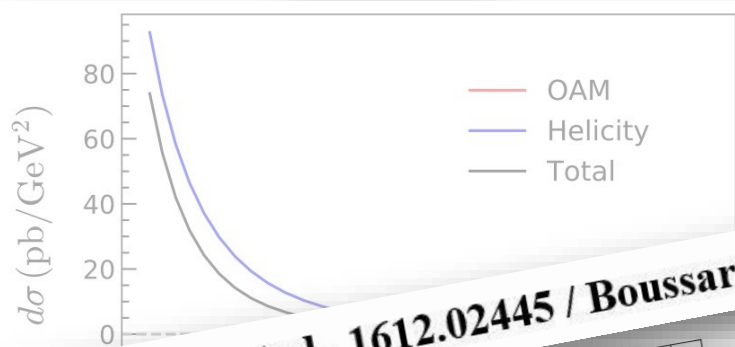




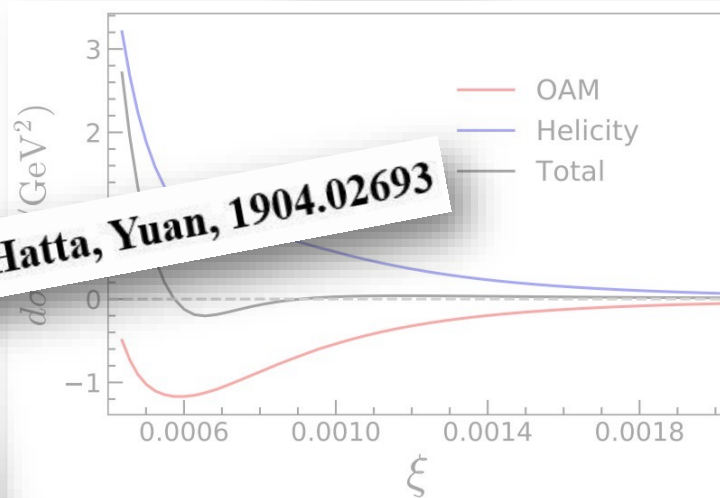
# Cancellation expected between Helicity & OAM at small $x$

$$\Delta G(x) \approx -L_g(x)$$

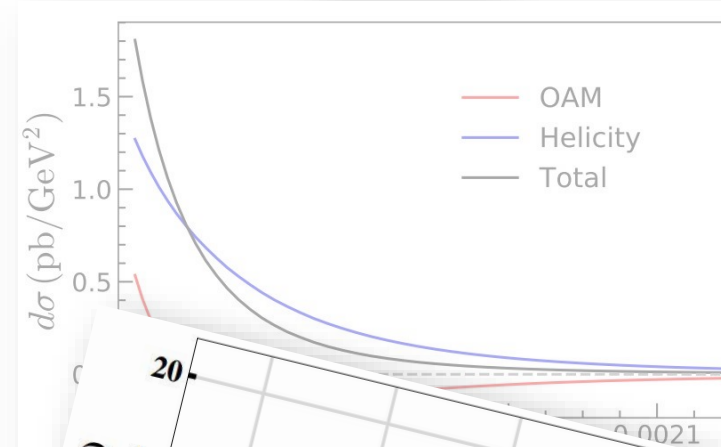
$Q^2 = 2.7$



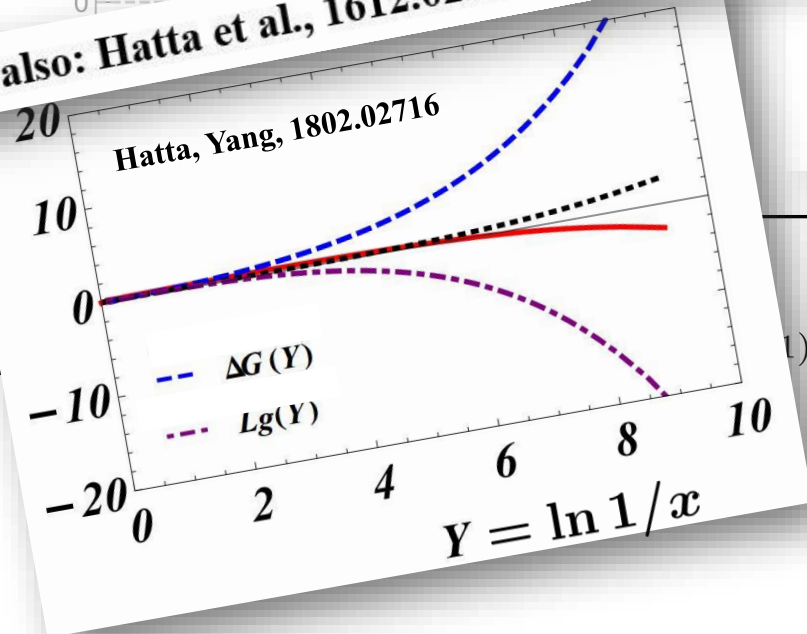
$Q^2 = 4.8$



$Q^2 = 10$

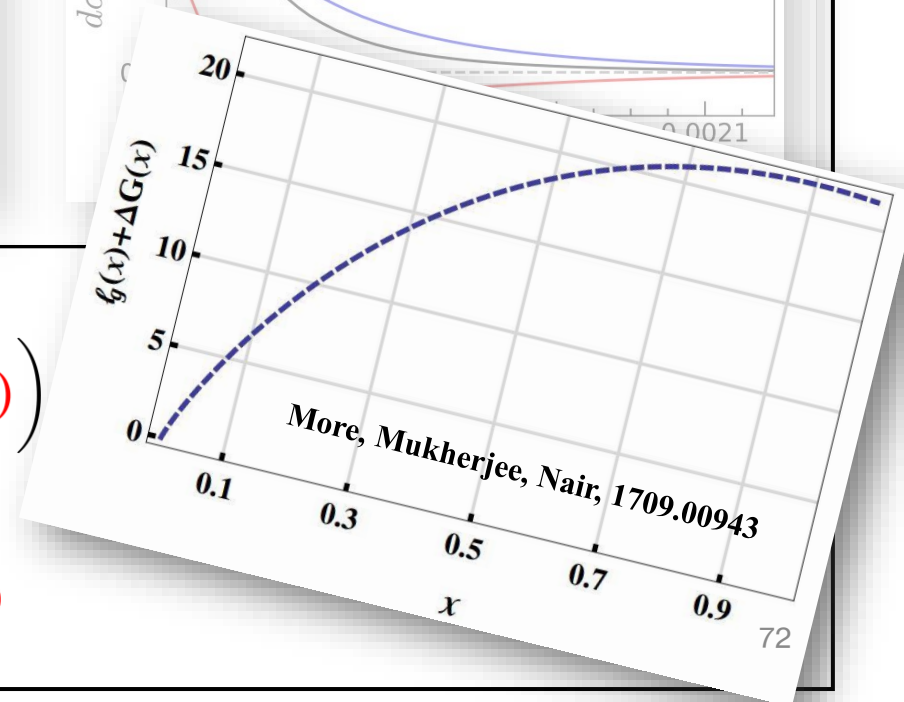


See also: Hatta et al., 1612.02445 / Boussarie, Hatta, Yuan, 1904.02693



$$l^*(\xi) \left( \tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \mathcal{L}_g(\xi) \right)$$

$\downarrow$   $\downarrow$   
 $\Delta G(x)$   $L_g(x)$



More, Mukherjee, Nair, 1709.00943

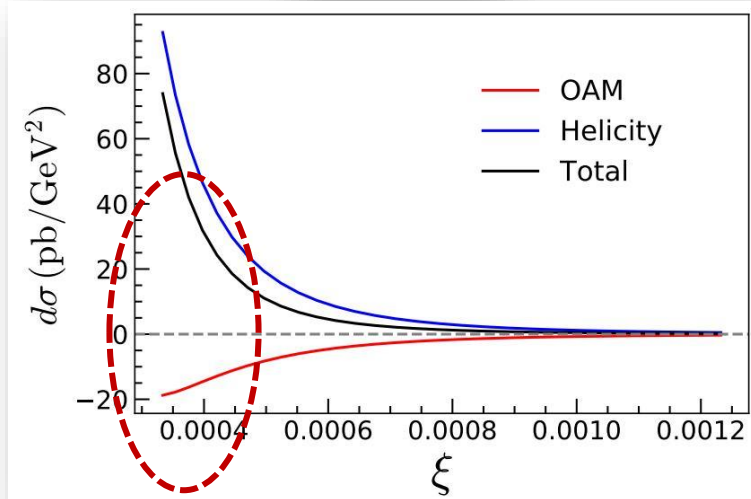




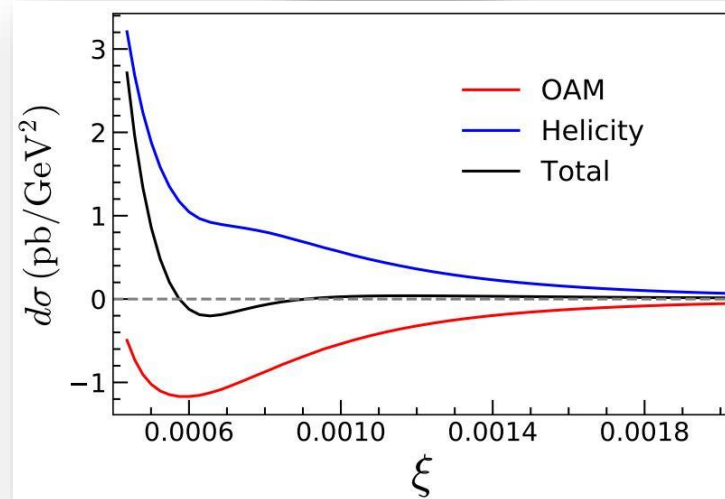
# Cancellation expected between Helicity & OAM at small $x$

$$\Delta G(x) \approx -L_g(x)$$

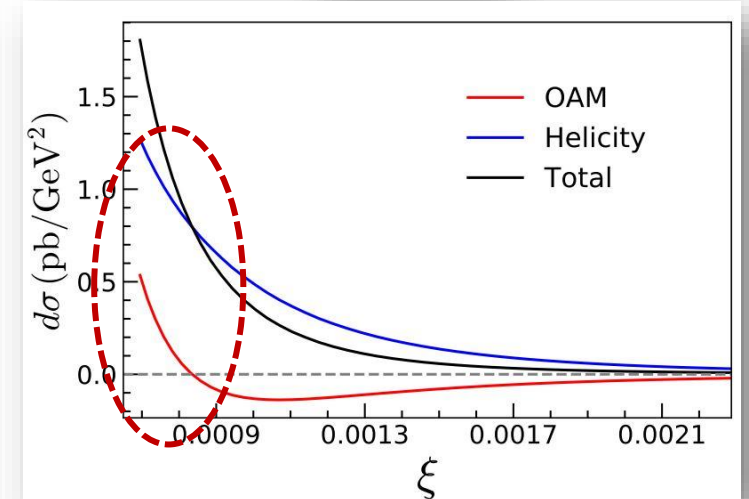
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



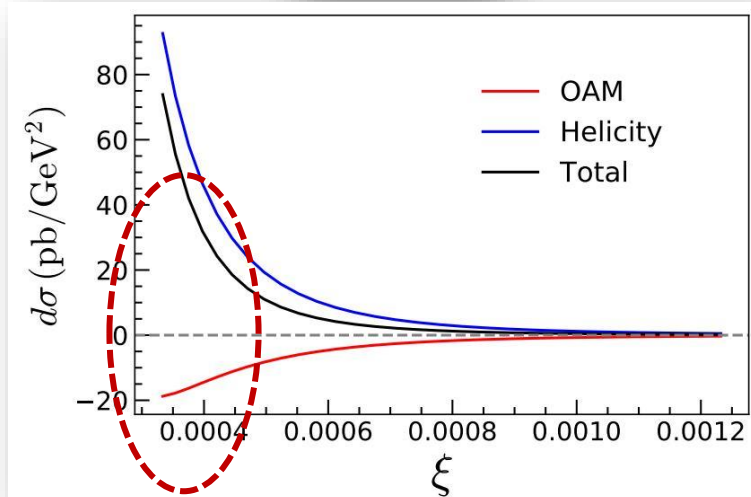
**DSA:** 
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \Big|_{\delta\phi=0} \sim \mathcal{H}_g^{(1)*}(\xi) \left( \underbrace{\tilde{\mathcal{H}}_g^{(2)}(\xi)}_{\Delta G(x)} + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4} \underbrace{\mathcal{L}_g(\xi)}_{L_g(x)} \right)$$



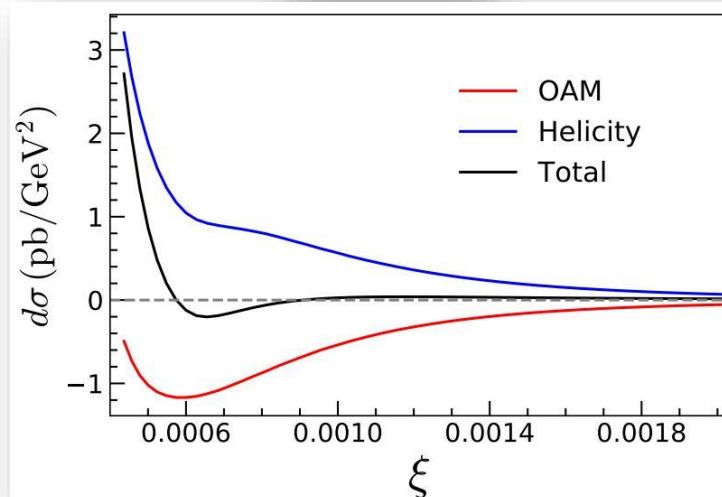
# Cancellation expected between Helicity & OAM at small $x$

$$\Delta G(x) \approx -L_g(x)$$

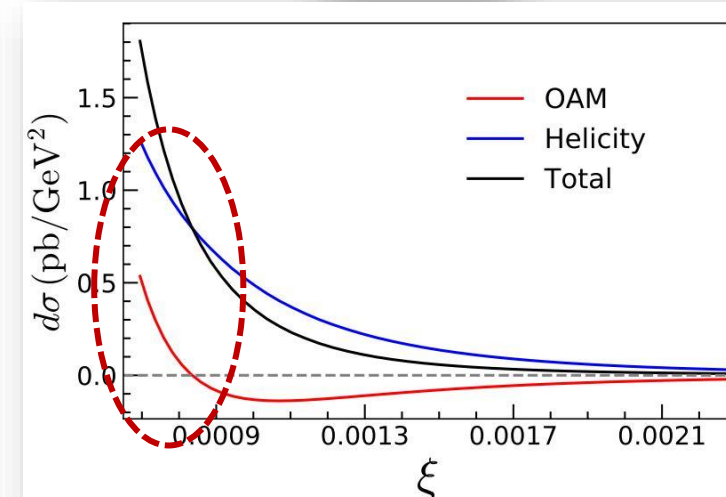
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



DSA:  $\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu$

Unique opportunity to study interplay between

$$\Delta G(x) \text{ \& } L_g(x)$$

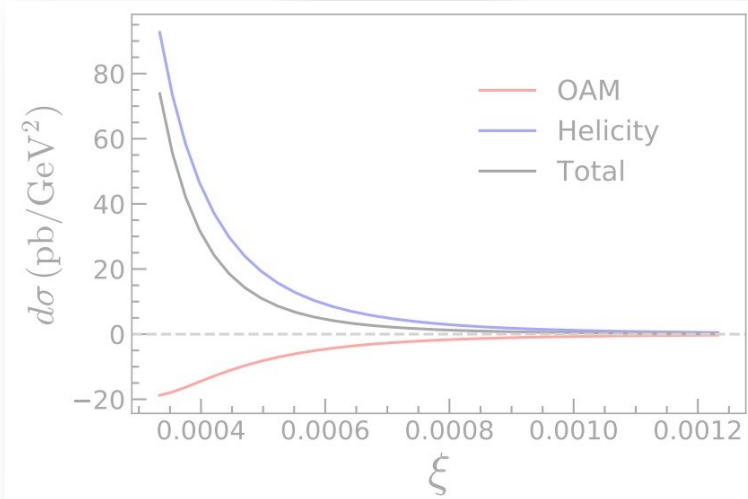
which has been so far only studied theoretically!



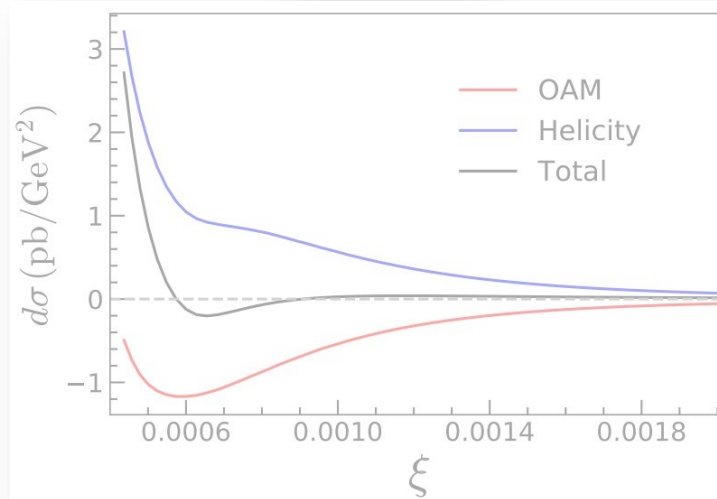
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

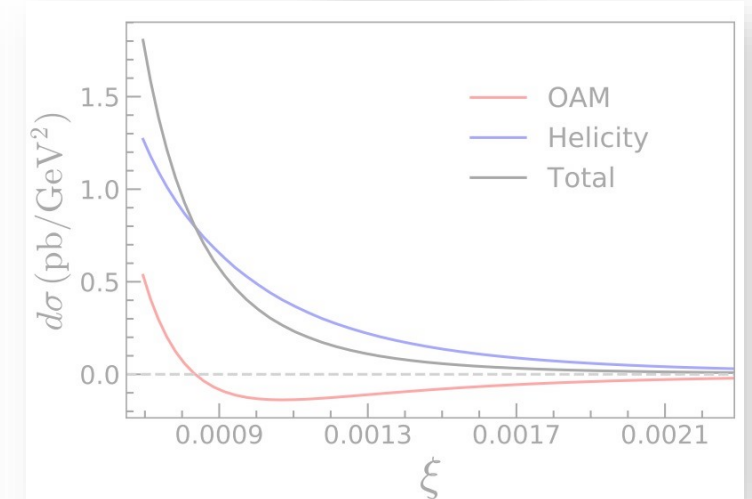
$Q^2 = 2.7$



$Q^2 = 4.8$



$Q^2 = 10$



### Caveat:

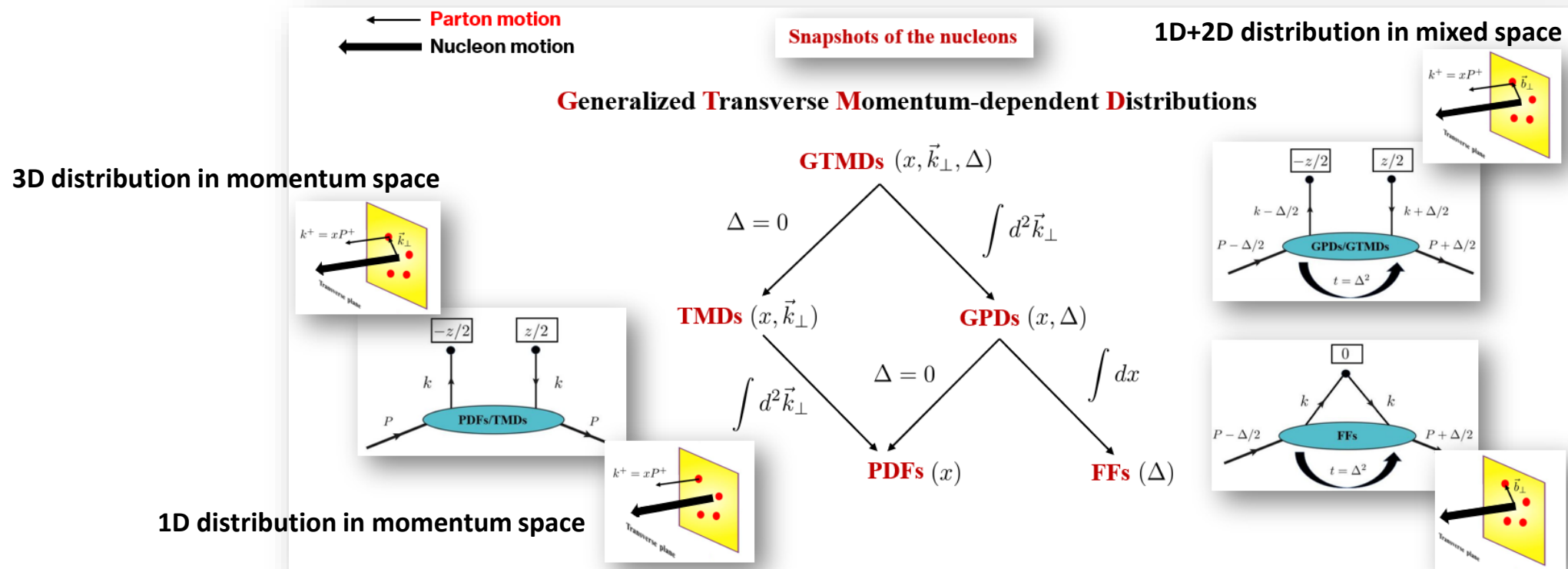
- In practice, measurements are done in a window in  $z$  around  $z = 1/2$

Corrections of order  $\sim (z - 1/2)^2$  should be calculable in  $k_t$ -factorization approach

# Summary: GTMDs

## GTMDs: Big picture

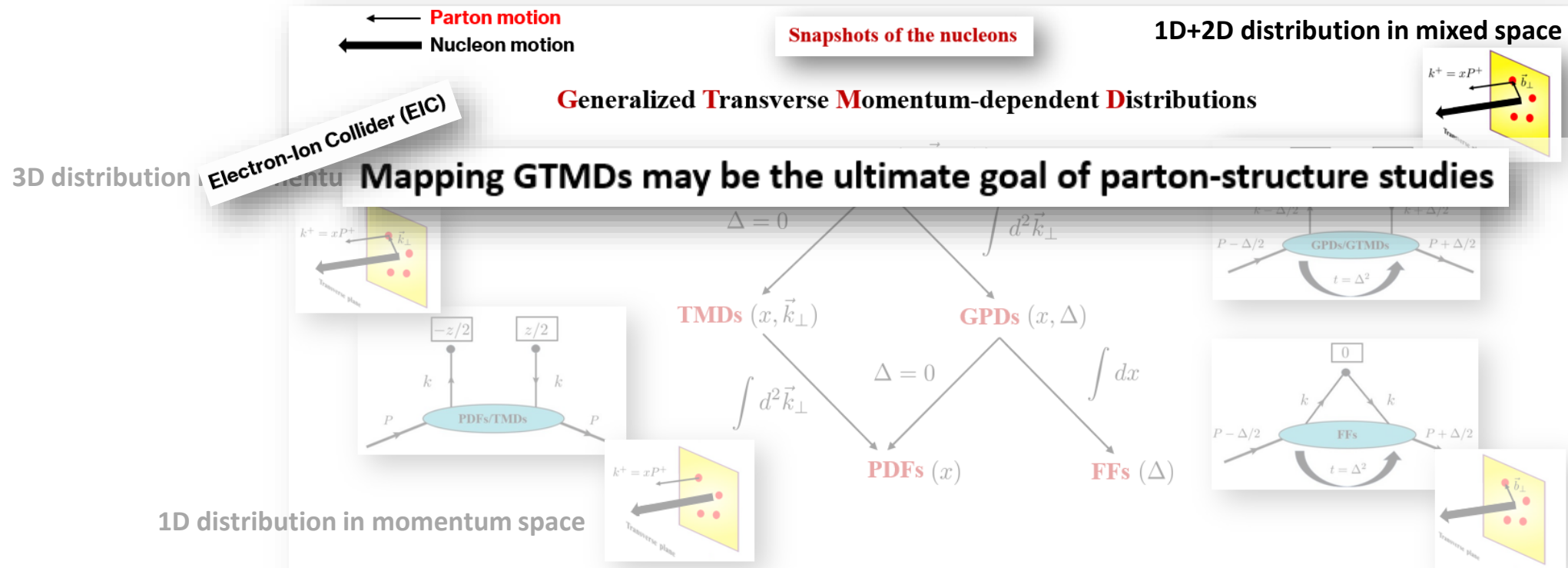
- GTMDs are the holy grail of spin physics



## Summary: GTMDs

# GTMDs: Big picture

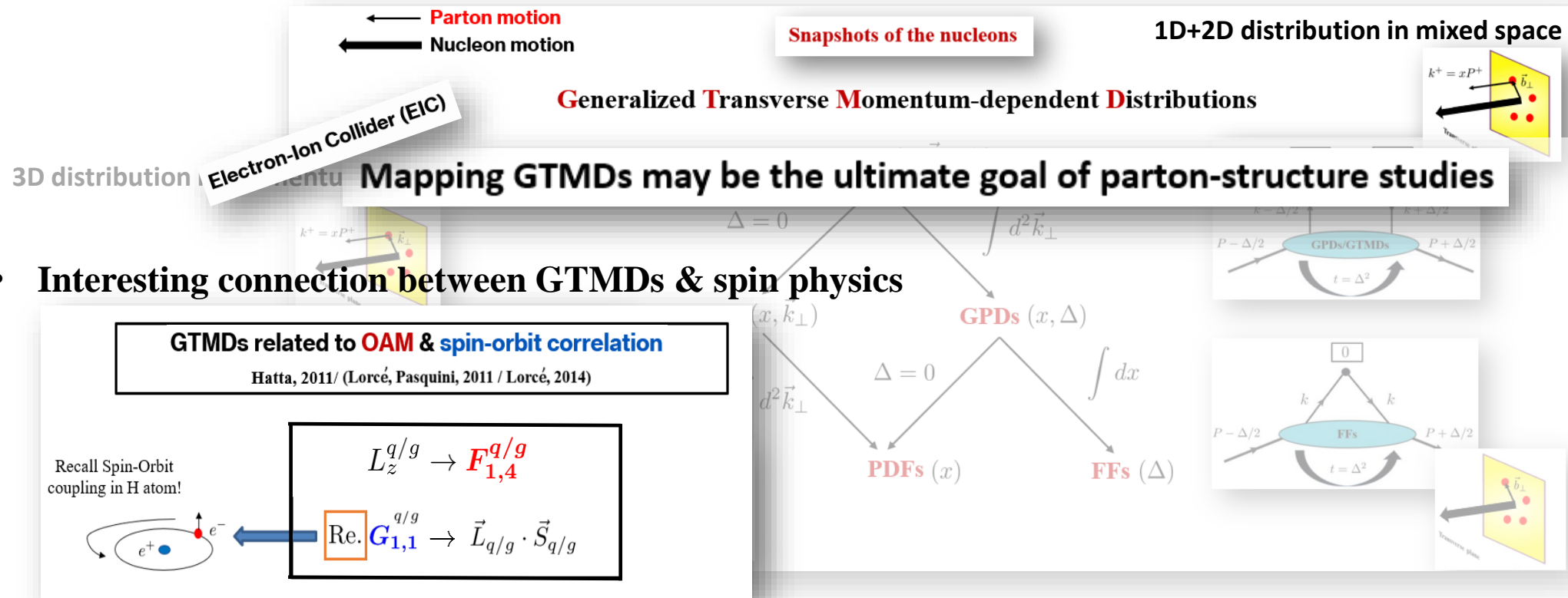
- **GTMDs are the holy grail of spin physics**



## Summary: GTMDs

# GTMDs: Big picture

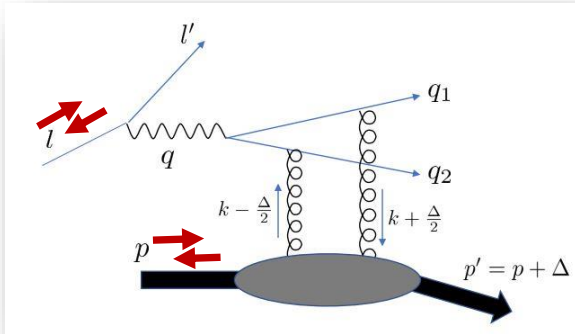
- **GTMDs are the holy grail of spin physics**



# Summary: GTMDs & OAM

## Summary of our work

- GTMDs are the holy grail of spin physics
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:



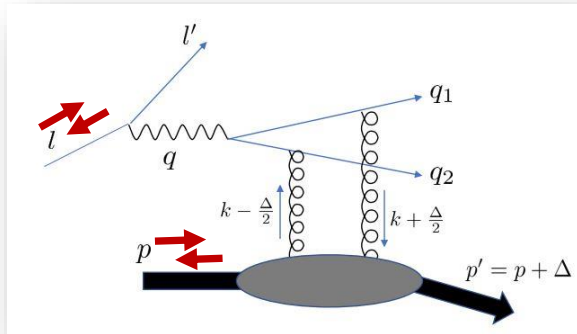
$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

# Summary: GTMDs & OAM

## Summary of our work

- GTMDs are the holy grail of spin physics
- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

**Signature of gluon OAM is cosine angular modulation**

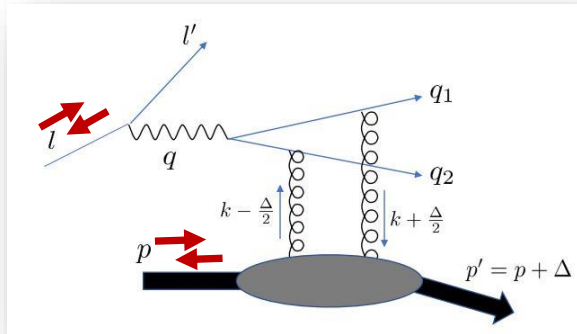


# Summary: GTMDs & OAM

## Summary of our work

**DSA does not vanish for symmetric jet configurations**  $z = \bar{z} = \frac{1}{2}$

- DSA in exclusive dijet production is a unique observable to access the gluon OAM @ EIC:



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

**Signature of gluon OAM is cosine angular modulation**

# Summary: GTMDs & OAM

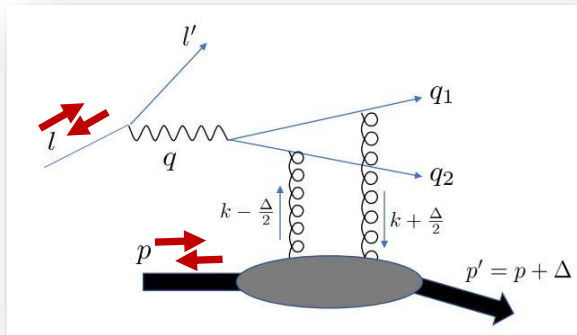
## Summary of our work

**DSA does not vanish for symmetric jet configurations**  $z = \bar{z} = \frac{1}{2}$

### Consequence:

**Elimination of factorization-breaking third poles at  $x = \pm\xi$**

- DSA in exclusive dijet production is



$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu \sim -\Re \left[ \left\{ \mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{H}_g^{(2)*}(\xi) \right\} \mathcal{L}_g(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l\perp} - \phi_{\Delta\perp})$$

**Signature of gluon OAM is cosine angular modulation**



# Summary: GTMDs & OAM

## Summary of our work

**DSA does not vanish for symmetric jet configurations**  $z = \bar{z} = \frac{1}{2}$

## Consequence:

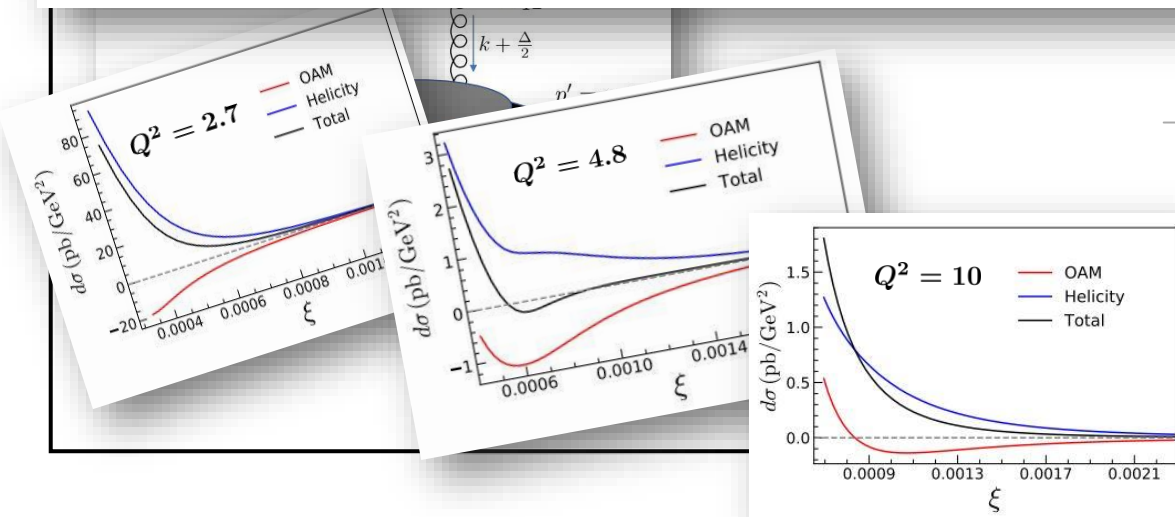
**Elimination of factorization-breaking third poles at  $x = \pm\xi$**

- DSA in exclusive dijet production is

**DSA is a unique observable to study interplay between gluon OAM & helicity**

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

**Signature of gluon OAM is cosine angular modulation**





# Summary: GTMDs & OAM

## Summary of our work

**DSA does not vanish for symmetric jet configurations  $z = \bar{z} = \frac{1}{2}$**

## Consequence:

**Elimination of factorization-breaking third poles at  $x = \pm\xi$**

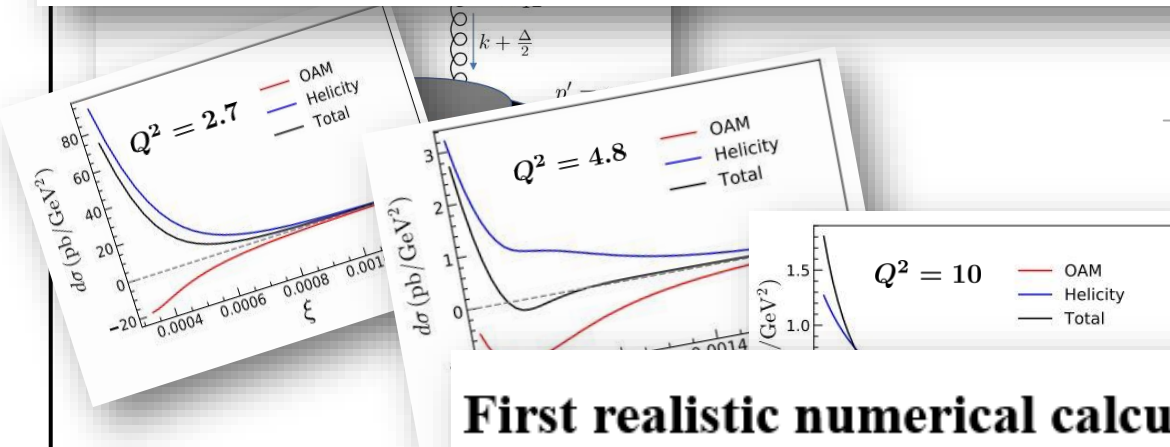
- DSA in exclusive dijet production is

**DSA is a unique observable to study interplay between gluon OAM & helicity**

$$+ \Re \left[ \mathcal{H}_g^{(1)*}(\xi) \tilde{\mathcal{H}}_g^{(2)}(\xi) \right] \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

**Signature of gluon OAM is cosine angular modulation**

**First realistic numerical calculation of observable sensitive to OAM @ EIC**



# Backup slides



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

**OAM**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

**Helicity**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$
$$\times \Re \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula

**OAM**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu^* A_\nu = -\frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1+\xi)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[ \left\{ \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \left( \mathcal{H}_g^{(2)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(2)*} \right) \right\} \mathcal{L}_g + \left( \mathcal{E}_g^{(1)*} + \frac{4q_\perp^2}{q_\perp^2 + \mu^2} \mathcal{E}_g^{(2)*} \right) \frac{\mathcal{O}}{2} \right]$$

**Helicity**

$$\int d\phi_{q\perp} L^{\mu\nu} A_\mu A_\nu = \frac{2^{10}\pi^4}{N_c} h_l h_p \alpha_s^2 \alpha_{em} e_q^2 \frac{(1-\xi^2)\xi Q^2}{(q_\perp^2 + \mu^2)^2} |l_\perp| |\Delta_\perp| \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$\times \Re \left[ \left( \mathcal{H}_g^{(1)*} - \frac{\xi^2}{1-\xi^2} \mathcal{E}_g^{(1)*} \right) \left( \tilde{\mathcal{H}}_g^{(2)} - \frac{\xi^2}{1-\xi^2} \tilde{\mathcal{E}}_g^{(2)} \right) \right]$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)

$$\begin{pmatrix} H_g(x, \xi) \\ \tilde{H}_g(x, \xi) \end{pmatrix} = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) \times \frac{15}{16} \frac{[(1 - |\beta|)^2 - \alpha^2]^2}{(1 - |\beta|)^5} \times \begin{cases} \beta G(\beta) \\ \beta \Delta G(\beta) \end{cases}$$



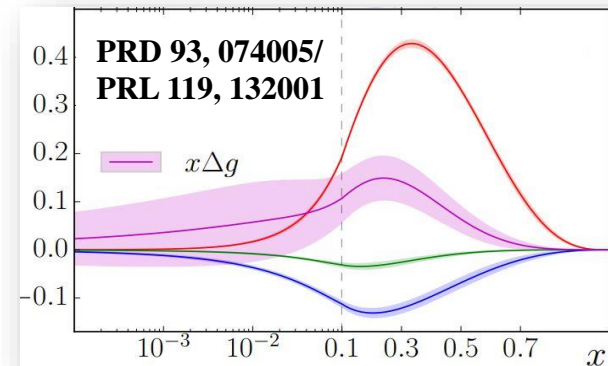
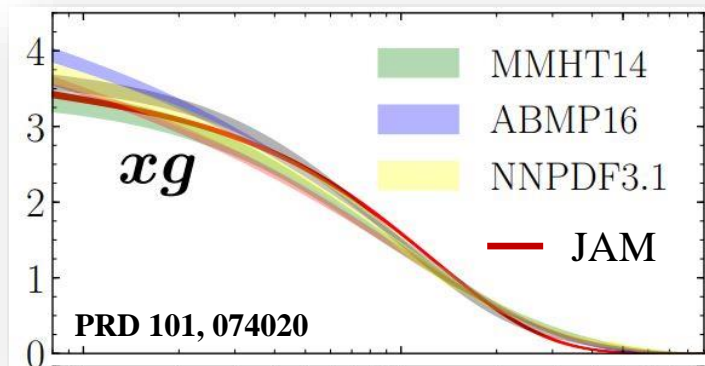
# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)

$$\begin{pmatrix} H_g(x, \xi) \\ \tilde{H}_g(x, \xi) \end{pmatrix} = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) \times \frac{15}{16} \frac{[(1 - |\beta|)^2 - \alpha^2]^2}{(1 - |\beta|)^5} \times \begin{cases} \beta G(\beta) \\ \beta \Delta G(\beta) \end{cases} \longleftrightarrow \text{JAM PDFs}$$





# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:

1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\boldsymbol{x}) = x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:

1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\textcolor{red}{x}) \overset{\text{WW approx}}{\approx} x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$

$$H_g(x') = x' G(x')$$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H, \tilde{H})$  according to the Double distribution approach (see for instance Radyushkin, 9805342) arXiv: 2207.03378 (2022)

### Small- $x$ evolution of the gluon GPD $E_g$

Yoshitaka Hatta<sup>1,2</sup> and Jian Zhou<sup>3</sup>

$$E_g(x) \propto H_g(x)$$

$$L_{can}^g(\textcolor{red}{x}) \overset{\text{WW approx}}{\approx} x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$

$$H_g(x') = x' G(x')$$

Neglect  $E_g$



# Probing gluon OAM through exclusive dijet production

## Numerical estimate of cross section

### Ingredients for non-perturbative functions

- Neglect contributions from  $(E_g, \tilde{E}_g), F_{1,2} \longrightarrow$  Very simple formula
- Model  $(H_g, \tilde{H}_g)$  according to the Double distribution approach (see for instance Radyushkin, 9805342)
- Model for OAM:

1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L_{can}^g(\textcolor{red}{x}) \overset{\text{WW approx}}{\approx} x \int_x^1 \frac{dx'}{x'^2} (H_g(x') + E_g(x')) - 2x \int_x^1 \frac{dx'}{x'^2} \Delta G(x') + \text{genuine twist-three}$$

2. Use the Double distribution approach to construct  $xL_g(x, \textcolor{red}{\xi})$  from  $xL_g(x)$  (GPD-like approach)