Classical and Statistical Physics version of Proton's pressure distribution

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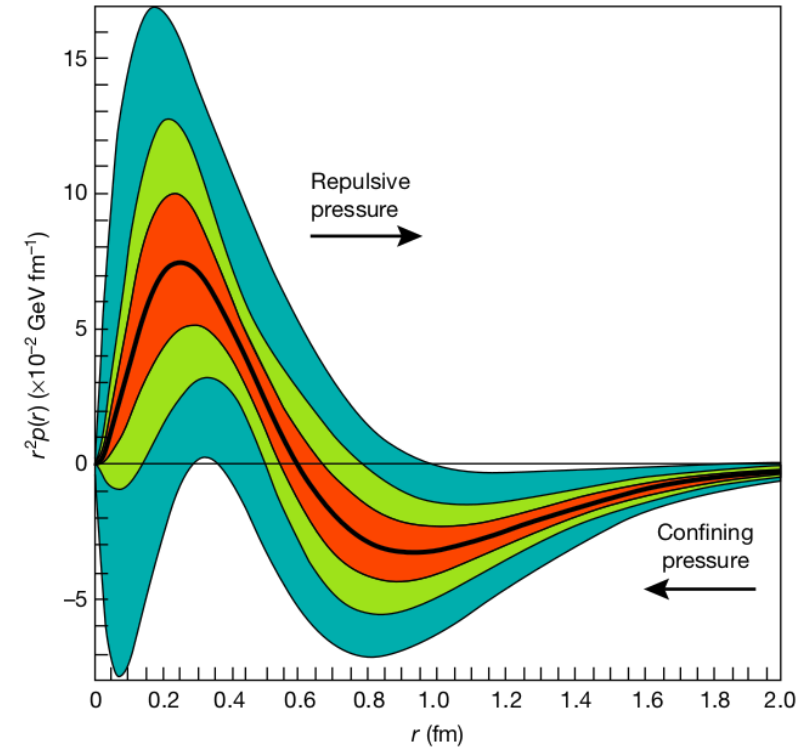Deependra Singh
(BS-MS, NISER)

Motivation came from .....

Thanks to....

QCD with Electron-Ion Collider (QEIC)

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Fig. 1 | Radial pressure distribution in the proton. The graph shows the pressure distribution $r^2 p(r)$ that results from the interactions of the quarks in the proton versus the radial distance r from the centre of the proton. The thick black line corresponds to the pressure extracted from the D-term parameters fitted to published data[22] measured at 6 GeV. The corresponding estimated uncertainties are displayed as the light-green shaded area shown. The blue area represents the uncertainties from all the data that were available before the 6-GeV experiment, and the red shaded area shows projected results from future experiments at 12 GeV that will be performed with the upgraded experimental apparatus[30]. Uncertainties represent one standard deviation.

Heavy Ion Collision (HIC) Direction..........

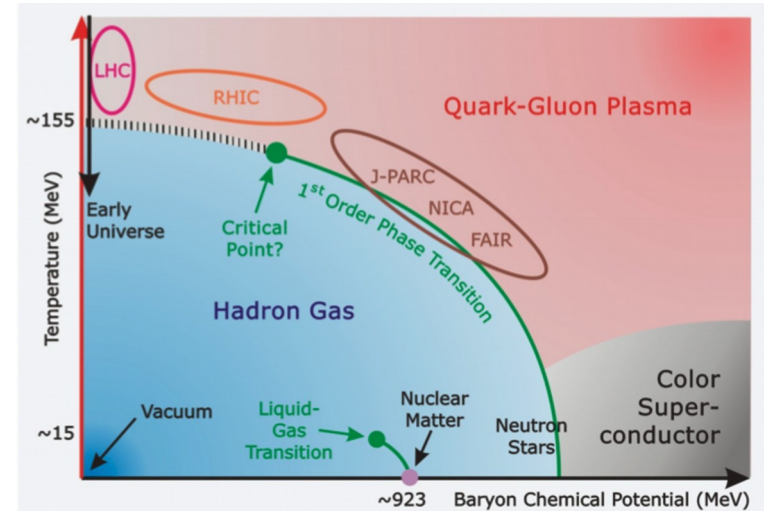Pressure of RHIC or LHC Matter
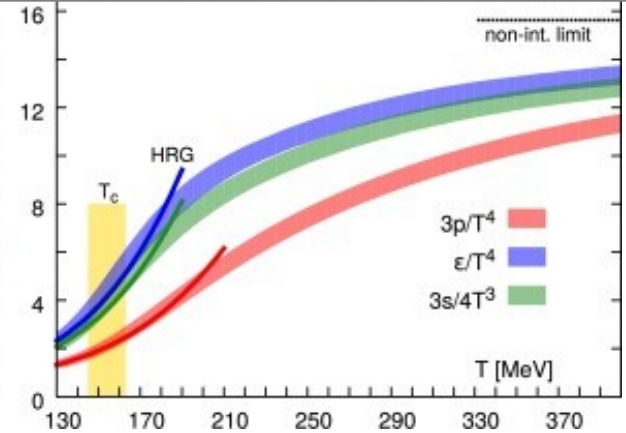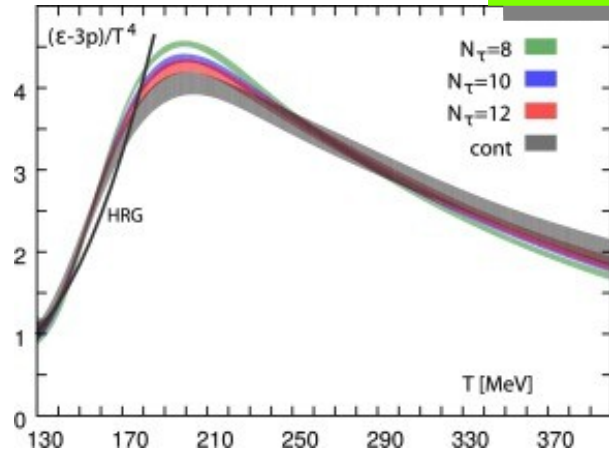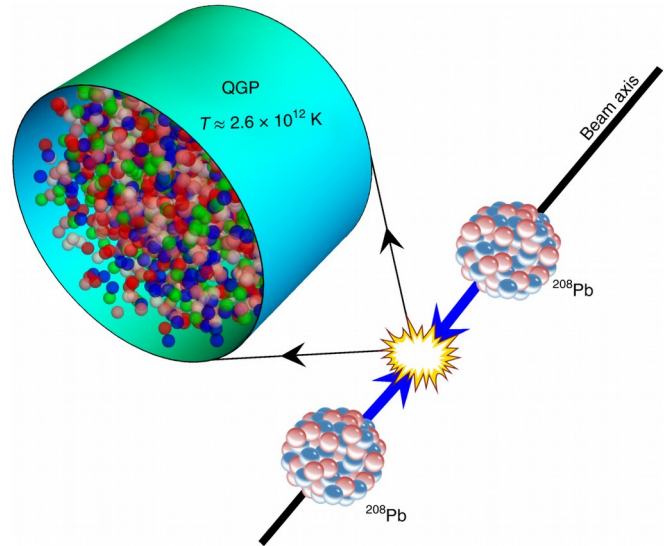
2.3.2 Pressure

The general expression

$$P_{UR} = \frac{2}{\pi^2} T^4 f_4 (A) \qquad (37)$$

- **Case:1** T=0K

$$P_{UR} = \frac{1}{12\pi^2} \mu^4 \qquad (38)$$

- **Case:2** $\mu = 0$

$$P_{UR} = \frac{1.89}{\pi^2} T^4 \qquad (39)$$

Pressure (Microscopic expression) from Statistical Mechanics

$$\Phi(T, V, \mu) = -PV = -k_B T \ln Z_{GCE}$$

$$
\begin{aligned}
\Phi &= PV \\
&= U - TS - \mu N \\
\Rightarrow d\Phi &= dU - TdS - SdT - \mu dN - Nd\mu \\
dU &= TdS - PdV + \mu dN \\
\Rightarrow d\Phi &= -SdT - PdV - Nd\mu
\end{aligned}
$$

$$P = k_B T \frac{\partial}{\partial V} \ln Z_{GCE}$$

$$P = g \int \frac{d^3 p}{(2\pi)^3} \frac{pv}{3} f_0(\epsilon)$$

Massless case

2.3.2 Pressure

The general expression

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Pressure (Microscopic expression) in the direction of Form Factor
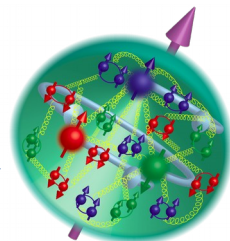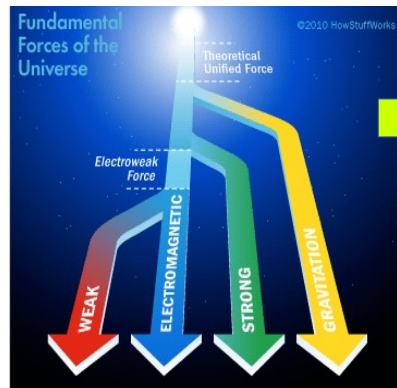
$$\textbf{em:}\quad \partial_\mu J^\mu_{\textbf{em}} = 0 \quad \langle N'|J^\mu_{\textbf{em}}|N\rangle \longrightarrow Q = 1.602176487(40) \times 10^{-19}\,\textrm{C}$$
$$\mu = 2.792847356(23)\mu_N$$

$$\textbf{weak:}\quad \textrm{PCAC} \qquad \langle N'|J^\mu_{\textbf{weak}}|N\rangle \longrightarrow g_A = 1.2694(28)$$
$$g_p = 8.06(55)$$

$$\textbf{gravity:}\quad \partial_\mu T^{\mu\nu}_{\textbf{grav}} = 0 \; \langle N'|T^{\mu\nu}_{\textbf{grav}}|N\rangle \longrightarrow m = 938.272013(23)\,\textrm{MeV}/c^2$$
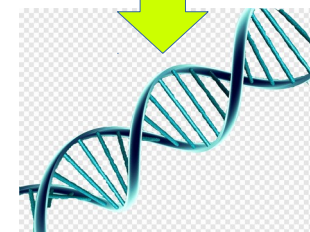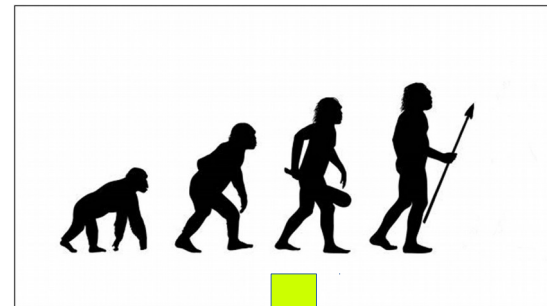$$J = \tfrac{1}{2}$$
$$D = \; ?$$

Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.
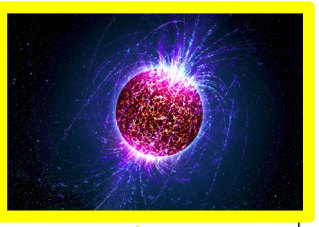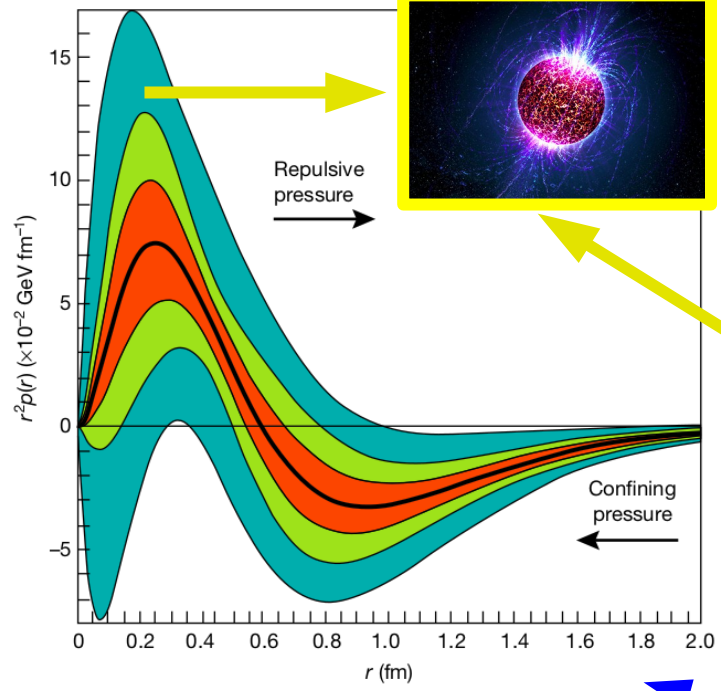
Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov and Peter Schweitzer ✉

Pressure (Microscopic expression) in the direction of Form Factor

Energy Momentum Tensor

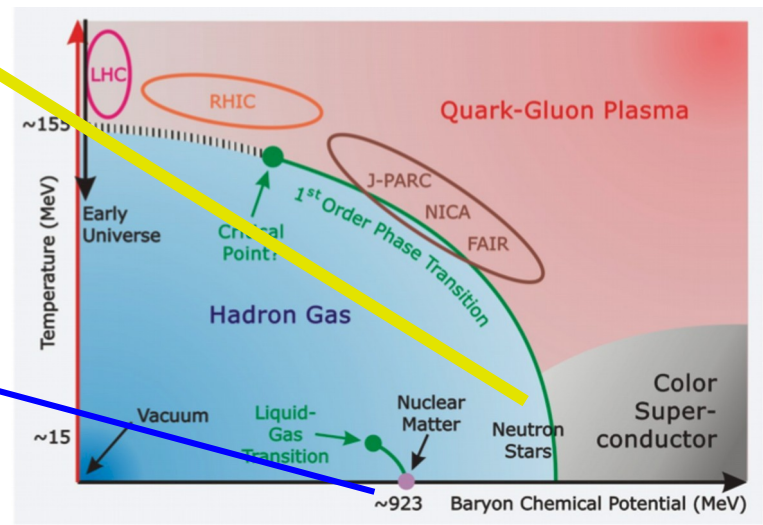$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij} \right) s(r) + \delta^{ij}\, p(r)\,.$$

$$s(r) = -\frac{1}{4m}\, r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r), \quad \widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta r}}\, D(-\boldsymbol{\Delta}^2).$$
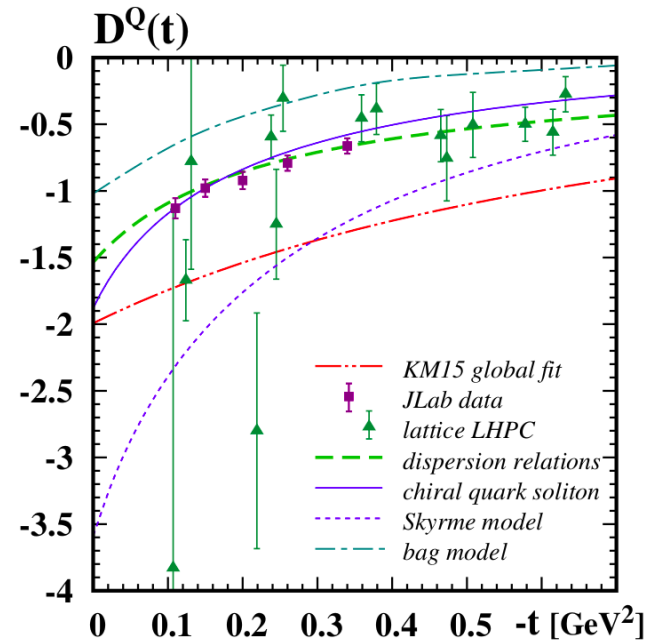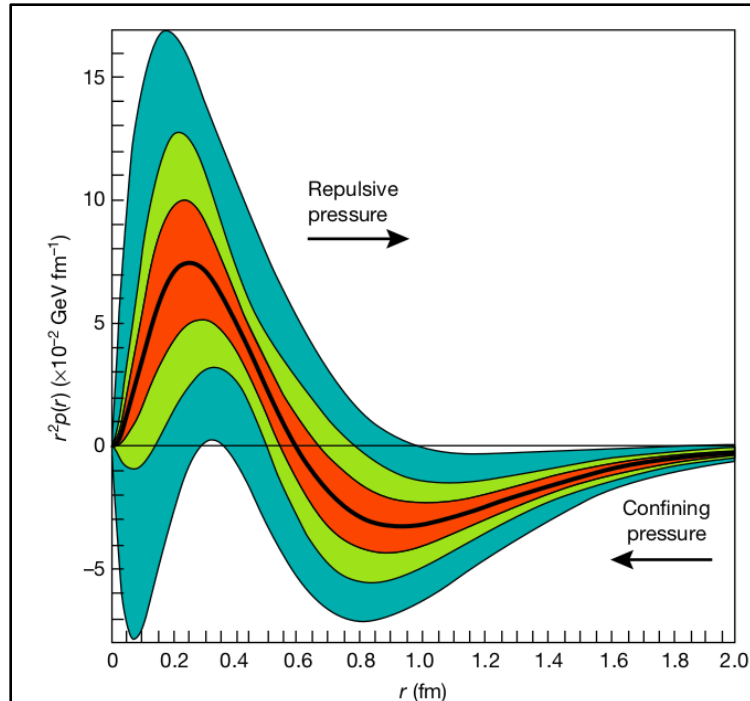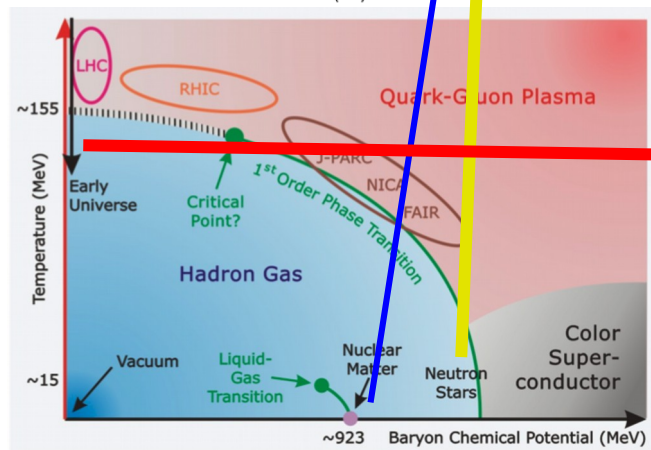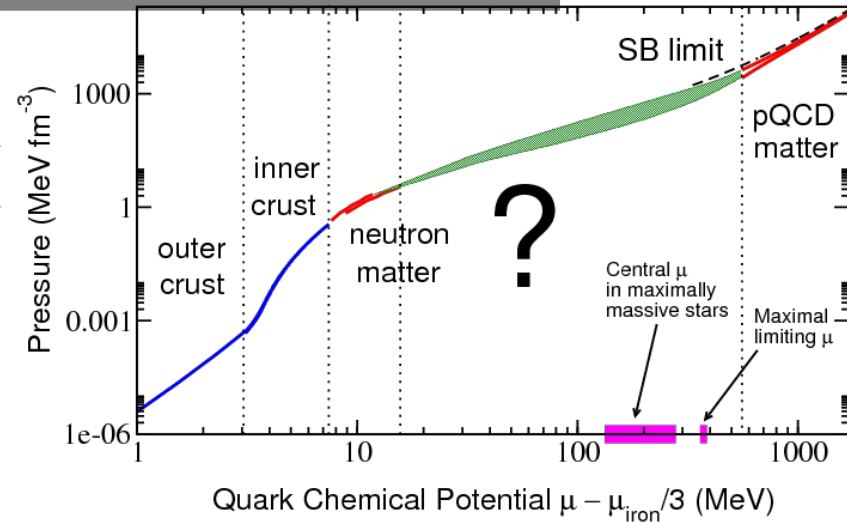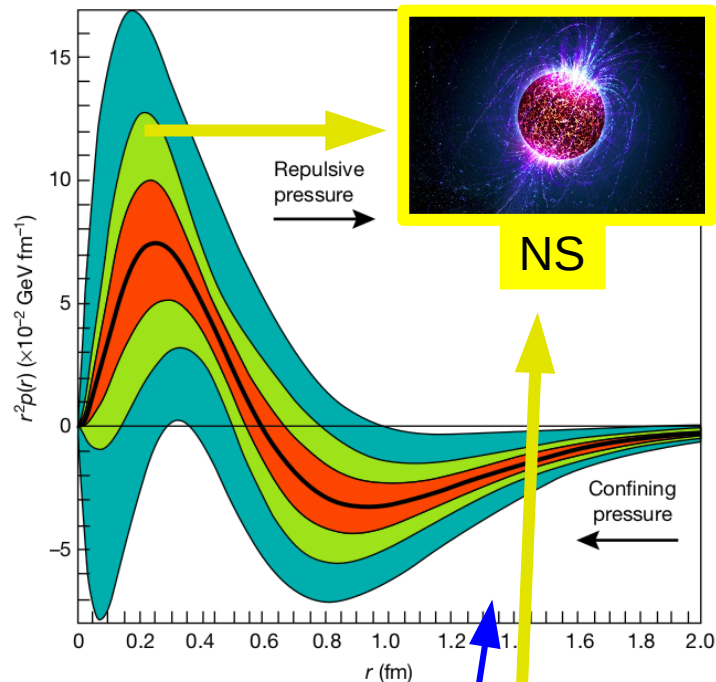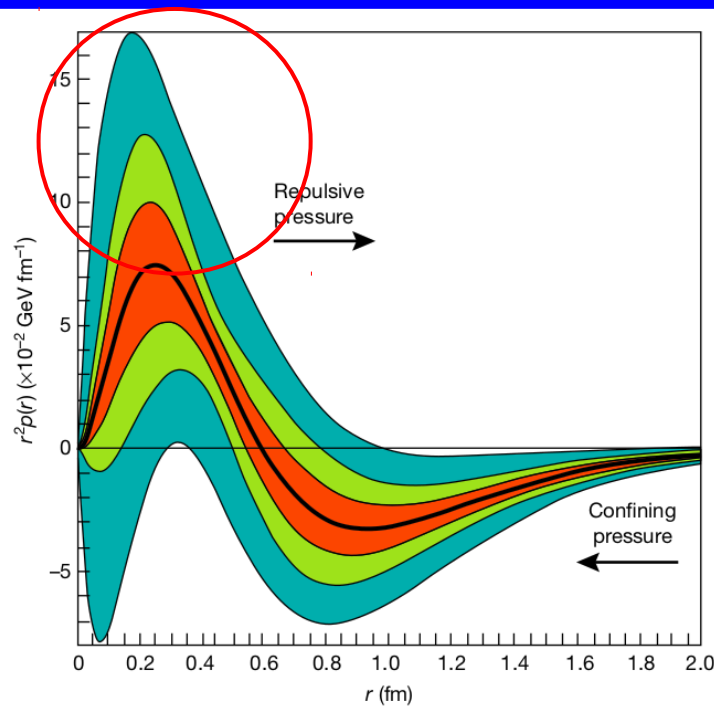
The average peak pressure near the centre is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars.

$$1 \text{ GeV}^4 \rightarrow 2.0852 \times 10^{37} \text{ J m}^{-3}$$

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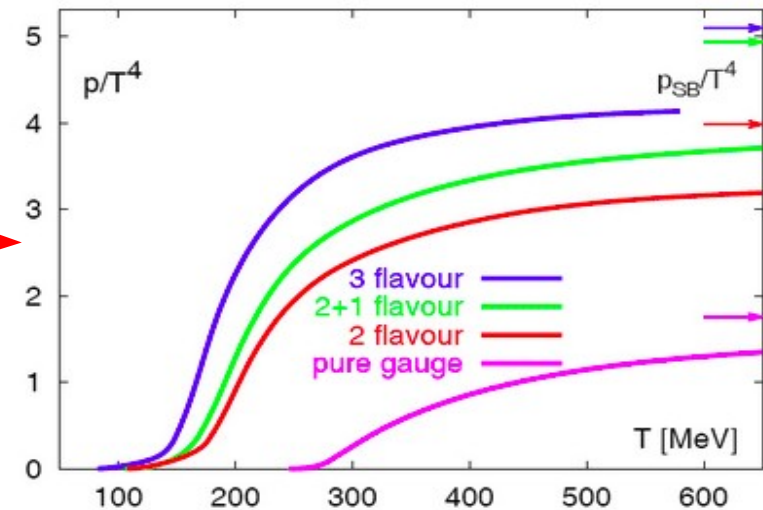
$$P(r)V = \frac{g}{h^3} \int_V \int_0^\infty \left(\frac{pv}{3}\right) f(\epsilon_i) \, d\tau \, d^3p$$

$$P(r) = \frac{4\pi(2s+1)}{h^3} \int_0^{p_f(r)} \left(\frac{pv}{3}\right) p^2 \, dp$$

where $g = 2s + 1$ is the degenracy, $p_f(r)$ is the fermi momentum obtained from the relation $\epsilon_f(\mu) = \sqrt{p_f^2 c^2 + m^2 c^4}$ and $d\tau$ is an infinitsimal volume element.

Velocity v is given by:
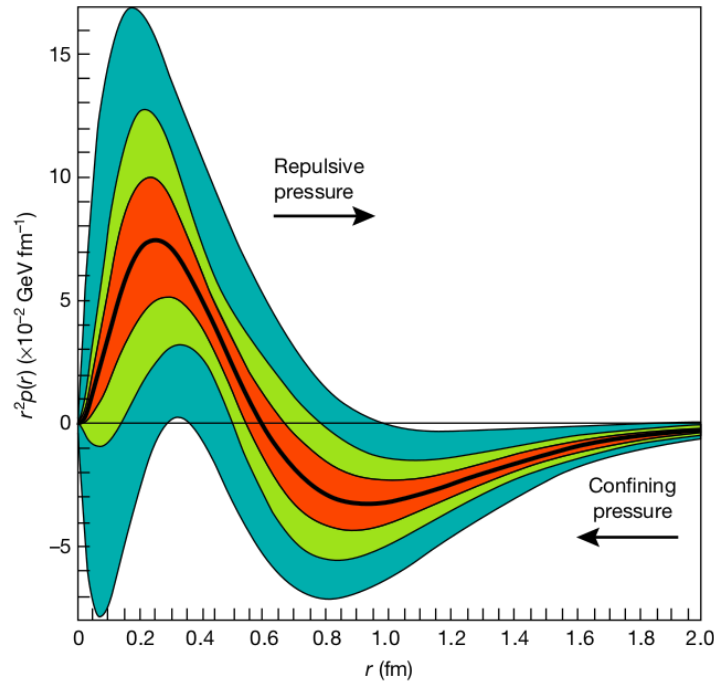
$$v = \frac{pc^2}{\sqrt{p^2 c^2 + m^2 c^4}}$$

$$p(r)r^2 = 25.93 \times 10^{-2}(4 - 6.325r)r \, exp(-6.325r/2).$$

Pressure inside Proton from the angle of Statistical Mechanics

$$P(r) = 1.047 \left((2p_f^3 - 3p_f m^2)\sqrt{p_f^2 + m^2} - 3m^4 \left(\frac{p_f}{m}\right) \right)$$

WORK IN PROGRESS

Pressure inside Proton from the angle of Classical Mechanics

$$\dot{V}(r, \theta, \phi) = -(A - B(br^2 + d)\exp(-fr^2))$$

$$\frac{\partial V(r)}{\partial r} = 2Br \cdot (bfr^2 + df - b)e^{-fr^2}$$

$$p(r)r^2 = 25.93 \times 10^{-2}(4 - 6.325r)r\,exp(-6.325r/2).$$

WORK IN PROGRESS

potential inside the proton

- - - -(A+1.5(10r^2-1.4)exp(-4.54r^2/2));A=0.0
- - - -(A+1.5(10r^2-1.4)exp(-4.54r^2/2));A=0.2
- - - -(A+1.5(10r^2-1.4)exp(-4.54r^2/2));A=0.3
- - - -(A+1.5(10r^2-1.4)exp(-4.54r^2/2));A=0.4
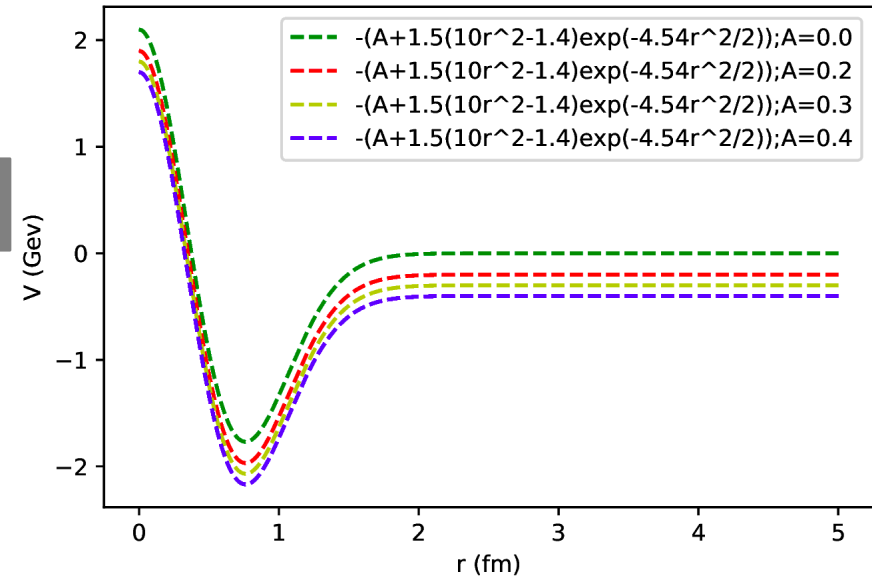
Summary

- We have attempted to estimate degenerate quark distribution inside Proton (motivated from the fact – peak pressure can denser that neutron star environment).

- Starting with this simple deenerate quark fluid motion, our future aim to project the formalism towards the standard Wigner function approach and also towards Kubo formalism approch with an expectation of non-zero dissipation!

- We have sketch the mechanical potential profile for the pressure distribution inside proton, which might be interesting to compare with our known form of potential like Yukawa, Culomb, string type potentials.

- Our statistical and Classical mechanical version attempt for the pressure profile may be considered as a beging steps from the understanding of heavy ion collision (HIC) direction, which may be eventually matured via mapping between EIC and HIC tools.