CP-Violation in $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

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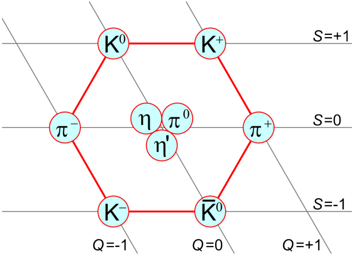
January 2023

CHIPP Winter School of Particle Physics 2023

based on
[M. Zillinger, B. Kubis, P. Sánchez-Puertas, JHEP 12 (2022)]

Properties of $\eta^{(\prime)}$ and Motivation for CP Violation

- $\eta^{(\prime)}$ are part of the pseudoscalar nonet of mesons
- $I^G J^{PC}(\eta^{(\prime)}) = 0^+0^{-+}$



[https://upload.wikimedia.org/wikipedia/commons/c/cd/Noneto_mes%C3%B4nico_de_spin_0.png]

- Most decays are flavor-conserving and mediated by the electromagnetic or strong interaction
- Any flavor-conserving decay channel violating CP is expected to be highly suppressed within the SM (CP violation from weak interaction is very small)
- Sakharov arguments for the explanation of matter-antimatter asymmetry
  - CP violation via the SM is not enough. Need additional sources

REDTOP Experiment

- High yield $\eta^{(\prime)}$ factory

- Its aim is to produce $5 \cdot 10^{12}\eta$ and $10^{10}\eta'$ in a few years of running
  - Compare: integrated η samples collected in earlier experiments amount to $10^9\eta$

- Study very rare $\eta^{(\prime)}$ decays and test conservations laws like CP

- Need to provide experimentalists with the most promising decay modes to search for CP violation

- Sanchez-Puertas et al. already investigated several decay modes:
  1. $\eta \to \mu^+\mu^-$
  2. $\eta \to \mu^+\mu^-\gamma$
  3. $\eta \to \mu^+\mu^-e^+e^-$
  4. $\eta^{(\prime)} \to \pi^0\mu^+\mu^-$
  5. $\eta' \to \eta\mu^+\mu^-$
- Now: study the decay mode $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

CP violation via SMEFT

- Use Standard Model effective field theory (SMEFT) to describe CP violation in a model-independent way
  - $\mathcal{L}_{\mathsf{SMEFT}} = \mathcal{L}^4_{\mathsf{SM}} + \frac{1}{v} \sum_k C^5_k \mathcal{O}^5_k + \frac{1}{v^2} \sum_k C^6_k \mathcal{O}^6_k + \mathcal{O}\left(\frac{1}{v^3}\right)$
- CP-violating operators receive very strong constraints from EDMs
- There is a loophole for operators of dimension 6 mixing strange quarks and muons with scalar/pseudoscalar structure

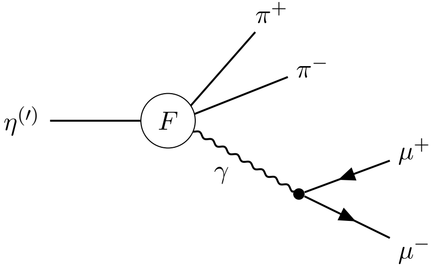
$$\mathcal{L}_{\mathsf{eff}} = \frac{\mathrm{Im}\, c^{2222}_{ledq}}{2v^2} \left[\left(\bar{\mu} i \gamma_5 \mu\right) \left(\bar{s} s\right) - \left(\bar{\mu}\mu\right) \left(\bar{s} i \gamma_5 s\right) \right]$$

- EDM contributions are suppressed since they begin at two loop order

SM and BSM contribution to $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

- SM contribution:

$$iM_{\mathsf{SM}} = -i\epsilon_{\mu\nu\rho\sigma}(p_1)^\nu(p_2)^\rho(p_{34})^\sigma F(s,t,u,s_\ell)\frac{e}{s_\ell}\left(\bar{u}_s(p_4)\gamma^\mu v_{s'}(p_3)\right)$$



- BSM contribution:

$$\left\langle \mu^-\mu^+\pi^-\pi^+ \middle| iT \middle| \eta^{(\prime)} \right\rangle = -\frac{i\,\mathrm{Im}\,c_{ledq}^{2222}}{2v^2}\left\langle \mu^-\mu^+\pi^-\pi^+ \middle| (\bar{\mu}\mu)\,(\bar{s}i\gamma_5 s) \middle| \eta^{(\prime)} \right\rangle$$
$$= -\frac{i\,\mathrm{Im}\,c_{ledq}^{2222}}{2v^2}\bar{u}_s(p_4)v_{s'}(p_3)\left\langle \pi^-\pi^+ \middle| \bar{s}i\gamma_5 s \middle| \eta^{(\prime)} \right\rangle$$

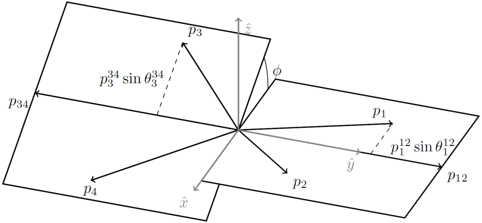
- Use $SU(3)$ χPT for the η decay and large-N_C χPT + Unitarization for the η' decay to calculate the hadronization of the operator

Angular Asymmetry and results for $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

- For a 4-body decay, we use an angular asymmetry for probing CP violation

$$A_\phi = \frac{N(\sin(\phi) > 0) - N(\sin(\phi) < 0)}{N(\text{all})}$$

- ϕ being the azimuthal angle between the pion and muon planes



$N_\eta = 5 \cdot 10^{12}$	$N_{\eta'} = 10^{10}$
$\Gamma(\eta \to \pi^+\pi^-\mu^+\mu^-) = 7.40 \cdot 10^{-15}$ GeV	$\Gamma(\eta' \to \pi^+\pi^-\mu^+\mu^-) = 3.72 \cdot 10^{-9}$ GeV
$N_{\eta\to\pi^+\pi^-\mu^+\mu^-} \sim 28244$	$N_{\eta'\to\pi^+\pi^-\mu^+\mu^-} \sim 197872$
$A_\phi = -0.4(2.2) \cdot 10^{-5}\, \mathrm{Im}\, c_{\ell edq}^{2222}$	$A_\phi = -1.4(5) \cdot 10^{-5}\, \mathrm{Im}\, c_{\ell edq}^{2222}$
$\left\lvert \mathrm{Im}\left\{c_{ledq}^{2222}\right\} \right\rvert \sim 1584$	$\left\lvert \mathrm{Im}\left\{c_{ledq}^{2222}\right\} \right\rvert \sim 77$

Comparison

Process	$\lvert \mathrm{Im}\, c_{ledq}^{2222} \rvert$
$\eta \to \mu^+\mu^-$	0.005
$\eta \to \mu^+\mu^-\gamma$	1
$\eta \to \mu^+\mu^-e^+e^-$	25
$\eta \to \pi^0\mu^+\mu^-$	0.7
$\eta \to \pi^+\pi^-\mu^+\mu^-$	1584
$\eta' \to \pi^0\mu^+\mu^-$	11
$\eta' \to \eta\mu^+\mu^-$	5
$\eta' \to \pi^+\pi^-\mu^+\mu^-$	77
nEDM	≤ 0.02

[M. Zillinger, B. Kubis, P. Sánchez-Puertas, JHEP 12 (2022)]

$\Rightarrow$ The leptonic decay $\eta \to \mu^+\mu^-$ remains the most promising channel (golden-channel) to be studied at REDTOP

Backup-Slides

CP violation via SMEFT

- Treat the Standard Model (SM) as a low energy approximation of some more fundamental theory
- Effective field theory framework: include higher dimensional operators in a model-independent way in the Lagrangian
  $\rightarrow$ "Standard Model effective field theory"(SMEFT)
- Higher-dimensional operators are suppressed by powers of the BSM scale Λ
  - $\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^4 + \frac{1}{v}\sum_k C_k^5 \mathcal{O}_k^5 + \frac{1}{v^2}\sum_k C_k^6 \mathcal{O}_k^6 + \mathcal{O}\left(\frac{1}{v^3}\right)$
- Dimensionless Wilson coefficients C_k^i

CP-Violating Operators

- CP-violating operators receive very strong constraints from EDMs
- There is a loophole for dimension 6 operators mixing light quarks and muons with scalar/pseudoscalar structure

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2v^2}\Big\{ \operatorname{Im} c_{lequ}^{(1)2211} \left[(\bar{\mu} i\gamma_5 \mu)(\bar{u}u) + (\bar{\mu}\mu)(\bar{u}i\gamma_5 u) \right] \\ - \operatorname{Im} c_{ledq}^{2211} \left[(\bar{\mu} i\gamma_5 \mu)\left(\bar{d}d\right) - (\bar{\mu}\mu)\left(\bar{d}i\gamma_5 d\right) \right] \\ - \operatorname{Im} c_{ledq}^{2222} \left[(\bar{\mu} i\gamma_5 \mu)(\bar{s}s) - (\bar{\mu}\mu)(\bar{s}i\gamma_5 s) \right] \Big\}$$

- Recent bounds on Wilson coefficients from the neutron EDM are:

$$|\operatorname{Im} c_{lqeu}^{(1)2211}| < 0.001, \quad |\operatorname{Im} c_{ledq}^{2211}| < 0.002, \quad |\operatorname{Im} c_{ledq}^{2222}| < 0.02$$

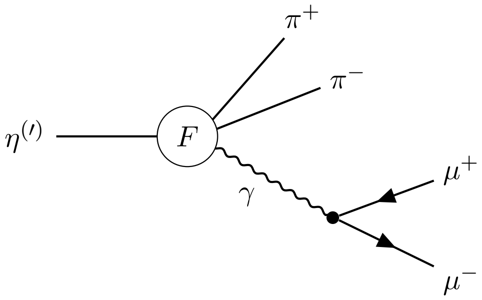
$\rightarrow$ Most promising operator is:

$$\mathcal{L}_{\text{eff}} = \frac{\operatorname{Im} c_{ledq}^{2222}}{2v^2} \left[(\bar{\mu} i\gamma_5 \mu)(\bar{s}s) - (\bar{\mu}\mu)(\bar{s}i\gamma_5 s) \right]$$

The Process $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ in the SM

- $\eta^{(\prime)}(P) \to \pi^+(p_1)\pi^-(p_2)\mu^+(p_3)\mu^-(p_4)$
  - $s = (p_1 + p_2)^2 = p_{12}^2, \qquad s_\ell = (p_3 + p_4)^2 = p_{34}^2$
  - $t = (P - p_1)^2, \qquad u = (P - p_2)^2$
- $s + t + u = M^2 + 2M_\pi^2 + s_\ell$

$$iM_{\mathsf{SM}} = -i\epsilon_{\mu\nu\rho\sigma}(p_1)^\nu (p_2)^\rho (p_{34})^\sigma F(s,t,u,s_\ell)\frac{e}{s_\ell}\left(\bar{u}_s(p_4)\gamma^\mu v_{s'}(p_3)\right)$$



The Process $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ in the SM

- Partial wave expansion:

$$F(s,t,u,s_\ell) = \sum_{\text{odd} l} P_l'(z) f_l(s,s_\ell)$$

$$z = \cos(\theta) = \frac{s(t-u)}{\lambda^{1/2}(s, M_\pi^2, M_\pi^2)\lambda^{1/2}(s, s_\ell, M^2)}$$

- Keep only the dominant P-wave: $F(s,t,u,s_\ell) = f_1(s,s_\ell) + \dots$
- Factorisation ansatz: $f_1(s,s_\ell) = f_1(s)\bar{F}(s_\ell) = P(s)\Omega_1^1(s)\bar{F}(s_\ell)$
  - $P(s)$ is a real polynomial free of cuts
  - $\Omega_1^1(s)$ is the P-wave Omnès function of isospin $I = 1$
  - $\bar{F}(s_\ell)$ models the coupling of the virtual photon to the hadronic system

Process dependence

- $P_\eta = A_\eta(1 + \alpha_\eta s)$
- $P_{\eta'} = A_{\eta'}(1 + \alpha_{\eta'} s + \beta_{\eta'} s^2)$
- Use VMD-model for $\bar{F}(s_\ell)$: Include ρ and ρ' meson

$$\bar{F}(s_\ell) = \frac{m_\rho^2 m_{\rho'}^2}{\left[m_\rho^2 - s_\ell - i\sqrt{s_\ell}\Gamma_\rho(s_\ell)\right]\left[m_{\rho'}^2 - s_\ell - i\sqrt{s_\ell}\Gamma_{\rho'}(s_\ell)\right]}$$

Source: S. Holz (2021)

BSM Contribution to $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

BSM calculation

- $\mathcal{L}_{\mathsf{SMEFT}} \supset -\frac{\operatorname{Im} c_{ledq}^{2222}}{2v^2} \left(\bar{\mu}\mu\right)\left(\bar{s}i\gamma_5 s\right)$

- Leading order contribution:

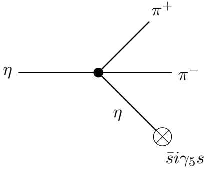
$$\left\langle \mu^- \mu^+ \pi^- \pi^+ \middle| iT \middle| \eta^{(\prime)} \right\rangle = -\frac{i \operatorname{Im} c_{ledq}^{2222}}{2v^2} \left\langle \mu^- \mu^+ \pi^- \pi^+ \middle| \left(\bar{\mu}\mu\right)\left(\bar{s}i\gamma_5 s\right) \middle| \eta^{(\prime)} \right\rangle$$
$$= -\frac{i \operatorname{Im} c_{ledq}^{2222}}{2v^2} \bar{u}_s(p_4) v_{s'}(p_3) \left\langle \pi^- \pi^+ \middle| \bar{s}i\gamma_5 s \middle| \eta^{(\prime)} \right\rangle$$

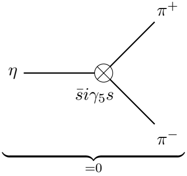
- Use $SU(3)$ ChPT for $\langle \pi^- \pi^+ | \bar{s} i \gamma_5 s | \eta \rangle$
- Use large-N_C ChPT for $\langle \pi^- \pi^+ | \bar{s} i \gamma_5 s | \eta' \rangle$
- Incorporate pseudoscalar sources via $\chi = 2B_0(s + ip)$
- Find the coupling to $\bar{s} i \gamma_5 s$ by setting $p = \text{diag}(0, 0, 1)$
- We restrict to the isospin limit $m_u = m_d \equiv m$

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

BSM calculation in $\eta \to \pi^+\pi^-\mu^+\mu^-$

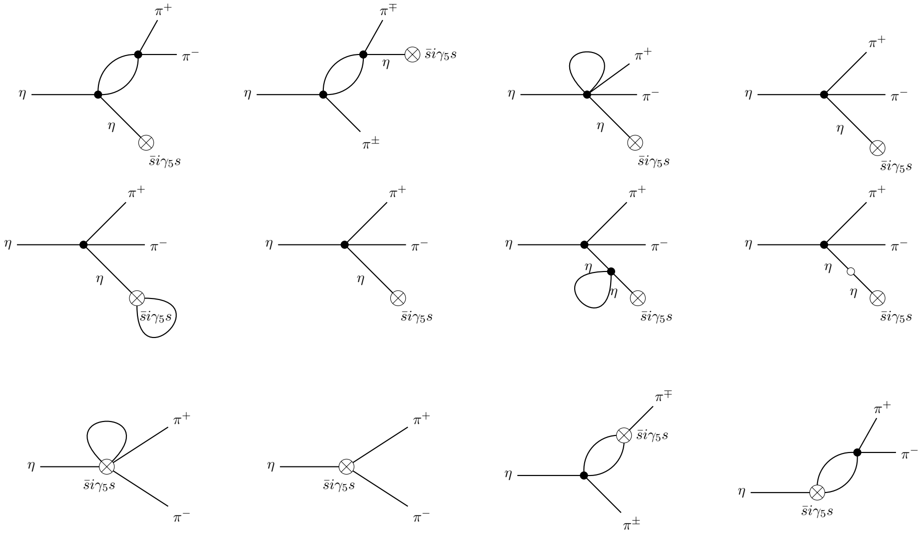
- At LO:





BSM calculation in $\eta \to \pi^+\pi^-\mu^+\mu^-$

- At NLO:



BSM calculation in $\eta \to \pi^+\pi^-\mu^+\mu^-$

- Organize matrix element into two parts:

$$\left\langle \pi^-\pi^+ \middle| \bar{s}i\gamma_5 s \middle| \eta \right\rangle_{\text{NLO}} = \underbrace{M_{\eta\eta\to\pi^+\pi^-} \frac{1}{M_\eta^2 - s_\ell} \langle 0|\bar{s}i\gamma_5 s|\eta\rangle}_{\eta-\text{pole}} + \underbrace{M_{\text{non-pole}}}_{\text{non-pole}}$$

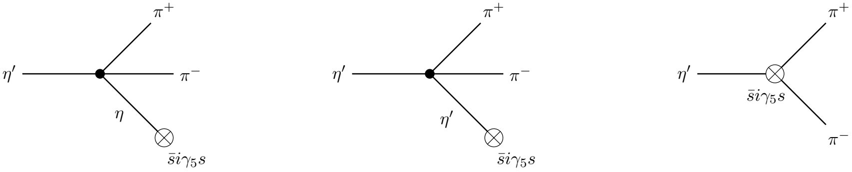
- Scattering amplitude and non-pole part can be decomposed into single-variable functions

$$M_{\eta\eta\to\pi^+\pi^-}(s,t,u) = \frac{M_\pi^2}{3F_\pi^2} + W_{\eta\eta}(s) + U_{\eta\eta}(t) + U_{\eta\eta}(u)$$

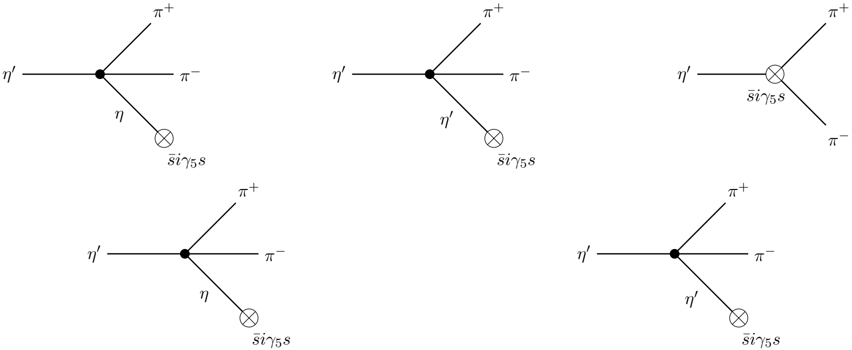
$$M_{\text{non-pole}} = W_{\text{non-pole}}(s) + U_{\text{non-pole}}(t) + U_{\text{non-pole}}(u)$$

BSM calculation in $\eta' \to \pi^+\pi^-\mu^+\mu^-$

- At LO:



- At NLO:



- Additional η'-pole part emerges

$$\left\langle \pi^-\pi^+ \middle| \bar{s}i\gamma_5 s \middle| \eta' \right\rangle_{\text{NLO}} = M_{\eta'\eta\to\pi^+\pi^-} \frac{1}{M_\eta^2 - s_\ell} \langle 0|\bar{s}i\gamma_5 s|\eta\rangle + M_{\eta'\eta'\to\pi^+\pi^-} \frac{1}{M_{\eta'}^2 - s_\ell} \langle 0|\bar{s}i\gamma_5 s|\eta'\rangle + M_{\text{non-pole}}$$

BSM calculation in $\eta' \to \pi^+\pi^-\mu^+\mu^-$

- Can assign $\bar{s}i\gamma_5 s$ to the same quantum numbers as η-meson
  $\to$ Rescattering effects will play a similar role as in the decay $\eta' \to \eta\pi^+\pi^-$

- Include $\pi\pi$ rescattering in the s-channel

$$\left\langle \pi^-\pi^+ \middle| \bar{s}i\gamma_5 s \middle| \eta' \right\rangle \Big|_{\text{Omnes}} = M^{\text{Omnes}}_{\eta'\eta\to\pi^+\pi^-} \frac{1}{M_\eta^2 - s_\ell} \langle 0|\bar{s}i\gamma_5 s|\eta\rangle + M^{\text{Omnes}}_{\eta'\eta'\to\pi^+\pi^-} \frac{1}{M_{\eta'}^2 - s_\ell} \langle 0|\bar{s}i\gamma_5 s|\eta'\rangle + M^{\text{Omnes}}_{\text{non-pole}}$$

- $M^{\text{Omnes}}_{\eta'\eta\to\pi^+\pi^-} = 32\pi M^{J=0}_{\eta'\eta\to\pi^+\pi^-}\Omega_0^0(s) + 80\pi(3\cos^2\theta_\pi - 1)M^{J=2}_{\eta'\eta\to\pi^+\pi^-}$

- $M^{\text{Omnes}}_{\eta'\eta'\to\pi^+\pi^-} = 32\pi M^{J=0}_{\eta'\eta'\to\pi^+\pi^-}\Omega_0^0(s) + 80\pi(3\cos^2\theta_\pi - 1)M^{J=2}_{\eta'\eta'\to\pi^+\pi^-}$

- $M^{\text{Omnes}}_{\text{non-pole}} = 32\pi M^{J=0}_{\text{non-pole}}\Omega_0^0(s) + 80\pi(3\cos^2\theta_\pi - 1)M^{J=2}_{\text{non-pole}}$

CP-Violating Angular Asymmetry

Angular Asymmetry

- $M = M_{\mathsf{SM}} + M_{\mathsf{BSM}}$

$$\begin{aligned} |M|^2 =& |M_{\mathsf{SM}}|^2 + |M_{\mathsf{BSM}}|^2 + 2\,\mathrm{Re}\{M_{\mathsf{SM}} M_{\mathsf{BSM}}^*\} \\ \xrightarrow{CP}& |M_{\mathsf{SM}}|^2 + |M_{\mathsf{BSM}}|^2 - 2\,\mathrm{Re}\{M_{\mathsf{SM}} M_{\mathsf{BSM}}^*\} \end{aligned}$$

$\rightarrow$ Probing CP violation will require the interference term

$$2\sum_{s,s'} \mathrm{Re}\{M_{\mathsf{SM}} M_{\mathsf{BSM}}^*\} = \frac{8 m_\mu e\, \mathrm{Im}\left\{c_{ledq}^{2222}\right\}}{v^2 s_\ell}\, \mathrm{Re}\left\{F(s,t,u,s_\ell)\left\langle \pi^+\pi^- \middle| \bar{s} i \gamma^5 s \middle| \eta^{(\prime)} \right\rangle^*\right\} \cdot \epsilon_{\mu\nu\rho\sigma}(p_1)^\mu (p_2)^\nu (p_3)^\rho (p_4)^\sigma$$

- $\epsilon_{\mu\nu\rho\sigma}(p_1)^\mu (p_2)^\nu (p_3)^\rho (p_4)^\sigma \sim \sin\phi$ with ϕ being the azimuthal angle between the pion and muon planes.

Angular Asymmetry

- Most suitable observable is the angular asymmetry defined as

$$A_\phi = \frac{N(\sin(\phi) > 0) - N(\sin(\phi) < 0)}{N(\text{all})}$$

- Or when using $d\Gamma$

$$A_\phi = \frac{\int_0^\pi d\phi \frac{d\Gamma}{d\phi} - \int_\pi^{2\pi} d\phi \frac{d\Gamma}{d\phi}}{\int_0^{2\pi} d\phi \frac{d\Gamma}{d\phi}}$$

- Numerator is given by

$$\int_0^\pi d\phi \frac{d\Gamma}{d\phi} - \int_\pi^{2\pi} d\phi \frac{d\Gamma}{d\phi} = -\frac{m_\mu \sqrt{\pi\alpha}\,\mathrm{Im}\left\{c_{ledq}^{2222}\right\}}{2^{13}\pi^5 v^2 M^3} \int ds_\ell \int ds \int dy_{12} \lambda(M^2, s, s_\ell) \sqrt{\frac{s}{s_\ell}}$$
$$\cdot \lambda_{34}^2 \sqrt{\lambda_{12}^2 - y_{12}^2} \Big(\mathrm{Re}\, F(s,t,u,s_\ell)\, \mathrm{Re}\, \left\langle \pi^+\pi^- \middle| \bar{s} i \gamma^5 s \middle| \eta^{(\prime)} \right\rangle$$
$$+ \mathrm{Im}\, F(s,t,u,s_\ell)\, \mathrm{Im}\, \left\langle \pi^+\pi^- \middle| \bar{s} i \gamma^5 s \middle| \eta^{(\prime)} \right\rangle \Big)$$

Results

Interlude: Sensitivity

- Estimate the size of the Wilson coefficient that can be accessed at REDTOP (Sensitivity)

- Example:
  1. Assume N_η η-mesons will be produced
  2. Use $\mathrm{Br}(\eta \to \pi^+\pi^-\mu^+\mu^-)$ to calculate the number of $\eta \to \pi^+\pi^-\mu^+\mu^-$ events: $N_{\eta\to\pi^+\pi^-\mu^+\mu^-} = \mathrm{Br}(\eta \to \pi^+\pi^-\mu^+\mu^-) \cdot N_\eta$
  3. Angular asymmetry A_ϕ is non-vanishing in the SM due to statistical fluctuations/noise: $\Delta A_\phi^{\mathrm{SM}} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$
  4. If BSM signal is below $\Delta A_\phi^{\mathrm{SM}}$, new physics would be buried by the noise: $A_\phi^{\mathrm{BSM}} \overset{!}{>} \Delta A_\phi^{\mathrm{SM}}$
  5. $A_\phi^{\mathrm{BSM}} = c \,\mathrm{Im}\, c_{ledq}^{2222} \Rightarrow \mathrm{Im}\, c_{ledq}^{2222} \sim \Delta A_\phi^{\mathrm{SM}}/c$

Results for $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$

$N_\eta = 5 \cdot 10^{12}$	$N_{\eta'} = 10^{10}$
$\Gamma(\eta \to \pi^+\pi^-\mu^+\mu^-) = 7.40 \cdot 10^{-15}$ GeV	$\Gamma(\eta' \to \pi^+\pi^-\mu^+\mu^-) = 3.72 \cdot 10^{-9}$ GeV
$N_{\eta\to\pi^+\pi^-\mu^+\mu^-} \sim 28244$	$N_{\eta'\to\pi^+\pi^-\mu^+\mu^-} \sim 197872$
$A_\phi = -5.41 \cdot 10^{-6}\,\mathrm{Im}\left\{c_{ledq}^{2222}\right\}$	$A_\phi = -3.78 \cdot 10^{-5}\,\mathrm{Im}\left\{c_{ledq}^{2222}\right\}$
$\left\|\mathrm{Im}\left\{c_{ledq}^{2222}\right\}\right\| \sim 1100.15$	$\left\|\mathrm{Im}\left\{c_{ledq}^{2222}\right\}\right\| \sim 26.58$

Comparison

Process	$\lvert \mathrm{Im}\, c_{ledq}^{2222} \rvert$
$\eta \to \mu^+\mu^-$	0.005
$\eta \to \mu^+\mu^-\gamma$	1
$\eta \to \mu^+\mu^-e^+e^-$	25
$\eta \to \pi^0\mu^+\mu^-$	0.69
$\eta \to \pi^+\pi^-\mu^+\mu^-$	1100.15
$\eta' \to \pi^0\mu^+\mu^-$	10.96
$\eta' \to \eta\mu^+\mu^-$	4.46
$\eta' \to \pi^+\pi^-\mu^+\mu^-$	26.58
nEDM	≤ 0.02

Source: Sanzches-Puertas (2019, 2022)

$\Rightarrow$ The leptonic decay $\eta \to \mu^+\mu^-$ remains the most promising channel (golden-channel) to be studied at REDTOP

Summary

Summary

- We employed the SMEFT framework to include higher-dimensional CP-violating operators
- Studied their contribution to the processes $\eta^{(\prime)} \to \pi^+\pi^-\mu^+\mu^-$ via the angular asymmetry
- The η' decay gives rise to a better sensitivity than the corresponding η decay
- Both processes can not compete with the golden channel $\eta \to \mu^+\mu^-$ and do not exceed the bound from the nEDM with the projected statistics at REDTOP

Strong CP-Problem

- Possible P & CP-violating operator appearing in the QCD Lagrangian
  - θ-term: $\mathcal{L}_\theta = \frac{g_s^2\theta}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$
- Bounds from the neutron EDM yield vanishing small parameter $\theta \leq 10^{-10}$

  $\Rightarrow$ Strong CP-problem
- Usually excluded from the QCD Lagrangian

  $\Rightarrow$ QCD is CP-conserving

SMEFT

- Treat the Standard Model (SM) as a low energy approximation of some more fundamental theory
- Effective field theory framework: include higher dimensional operators in the Lagrangian
  $\rightarrow$ "Standard Model effective field theory"(SMEFT)
- Higher-dimensional operators are suppressed by powers of the BSM scale Λ
  - $\mathcal{L}_{\mathsf{SMEFT}} = \mathcal{L}^4_{\mathsf{SM}} + \frac{1}{v}\sum_k C^5_k \mathcal{O}^5_k + \frac{1}{v^2}\sum_k C^6_k \mathcal{O}^6_k + \mathcal{O}\left(\frac{1}{v^3}\right)$
- Dimensionless Wilson coefficients C^i_k
- At Dimension 6: 59 independent operators

Classification of dim=6 Operators

- CP-violating operators start with dimension 6
- Purely hadronic operators CP_{H}: $\mathcal{O}_{uG} = \left(\bar{q}_p \sigma^{\mu\nu} T^A d_r\right) \varphi G^A_{\mu\nu}$
- Operators mixing lepton and quark bilinears CP_{HL}:
  $\mathcal{O}_{ledq} = \left(\bar{l}^j_p e_r\right) \left(\bar{d}_s q^j_t\right) + \text{h.c.}$
- Purely leptonic operators CP_{L}: $\mathcal{O}_{le} = \left(\bar{l}_p \gamma_\mu l_r\right) \left(\bar{e}_s \gamma^\mu e_t\right) + \text{h.c.}$
  - Irrelevant for the decay mode $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$
- Category affecting lepton-photon interactions: $\mathcal{O}_{eB} = \left(\bar{l}_p \sigma_{\mu\nu} e_r\right) \varphi B_{\mu\nu}$
  - Will be discarded as there are too strong constraints arising from e/μEDMs

CP_H category

- Main contribution from purely hadronic operators results in a CP-violating shift of the $\eta^{(\prime)}\pi^+\pi^-\gamma^\star$ coupling
- Corresponding amplitude can be parameterized as

$$\mathcal{A}(\eta \to \pi^+\pi^-\gamma^\star) = M\epsilon_{\mu\nu\rho\sigma}(p_1)^\mu(p_2)^\nu k^\rho \epsilon^\sigma + E\left((\epsilon \cdot p_1)(k \cdot p_2) - (\epsilon \cdot p_2)(k \cdot p_1)\right)$$

- M: magnetic transitions driven by the chiral anomaly in the SM
- E: electric transitions that are P and CP odd

CP_{H} category

- Hadronic operators also generate CP-violating shifts in the $\eta\rho\gamma^\star$ coupling
  - The $\pi^+\pi^-$ system has to be in an odd partial wave due to C-symmetry: effective resonance is a ρ

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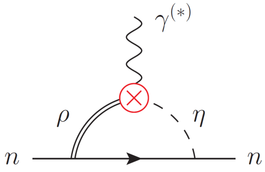
- Hadronic operators also generate CP-violating shifts in the $\eta\rho\gamma^\star$ coupling
  - The $\pi^+\pi^-$ system has to be in an odd partial wave due to C-symmetry: effective resonance is a ρ



- $\eta\rho\gamma^\star$ coupling contributes to the neutron EDM at one loop
  - Generates constraints on E: $E/M \leq 10^{-11}$

$\rightarrow$ No chance to observe CP-violation: neglect CP_{H} category

CP_{HL} category

- Operators made up of a lepton bilinear plus a quark bilinear

- Most operators of this category are of VV, AA or VA structure:

$$\left(\bar{l}_p \gamma_\mu l_r\right)\left(\bar{u}_s \gamma^\mu u_t\right) + \text{h.c.}$$

$\Rightarrow$ CP-conserving

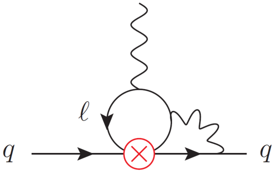
- Left with 3 operators:

$$\begin{aligned}
\mathcal{O}_{ledq} &= \left(\bar{l}_p^j e_r\right)\left(\bar{d}_s q_t^j\right) + \text{h.c.} \\
\mathcal{O}_{lequ}^{(1)} &= \left(\bar{l}_p^j e_r\right)\epsilon_{jk}\left(\bar{q}_s^k u_t\right) + \text{h.c.} \\
\mathcal{O}_{lequ}^{(3)} &= \left(\bar{l}_p^j \sigma_{\mu\nu} e_r\right)\epsilon_{jk}\left(\bar{q}_s^k \sigma^{\mu\nu} u_t\right) + \text{h.c.}
\end{aligned}$$

- Only $\mathcal{O} - \mathcal{O}^\dagger$ is CP-odd, requiring a non-vanishing imaginary part of the corresponding Wilson coefficient

CP_{HL} category

- Operators with a scalar/pseudoscalar lepton structure only contribute at two-loop to the neutron EDM
  - Green's functions $\langle 0|T\{V^\mu(x)S(P)(0)\}|0\rangle$ vanish in QED+QCD due to C symmetry



- Operators made up of vector, axial or tensor structure already contribute at one-loop
  - Stronger bounds from neutron EDM: Neglect operator $\mathcal{O}_{lequ}^{(3)}$

- Only keep light quarks u, d and s as well as muonic operators
  - Operators mixing electrons with quarks receive very stright bounds from EDMs of heavy atoms and molecules