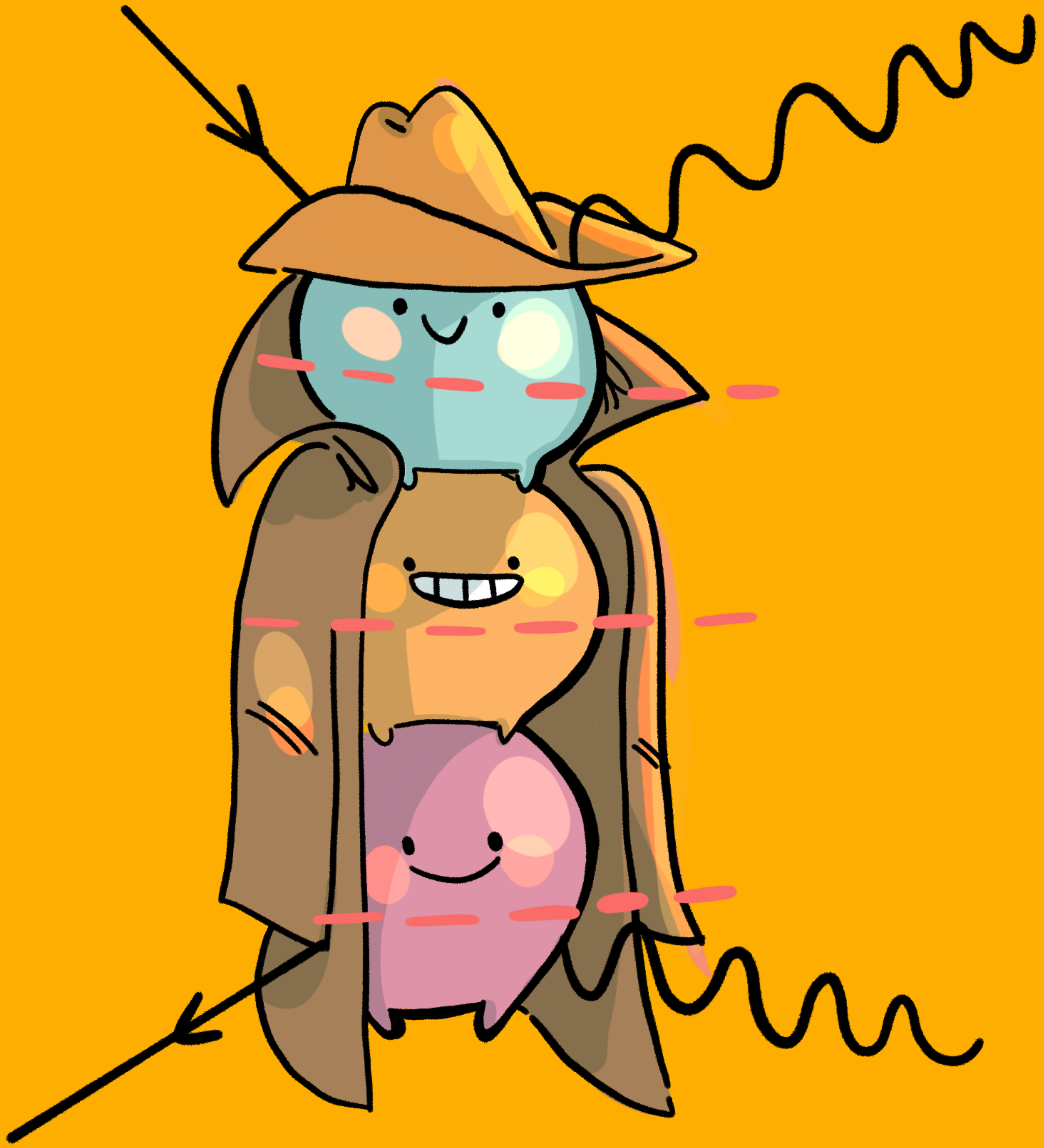
Phase Space Integrals for N^3LO QCD



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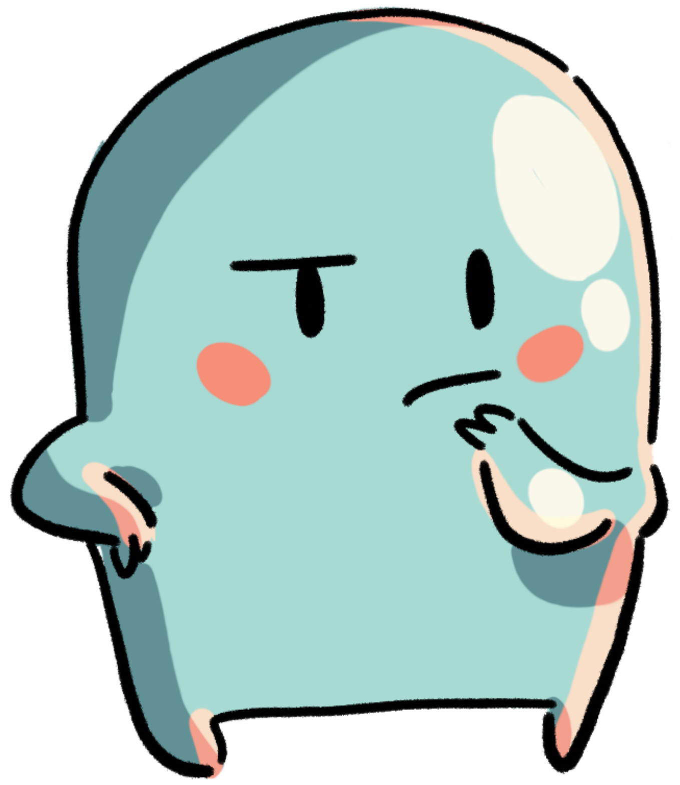
Higher order perturbation theory

Motivations



- High precision predictions for observables
- A lot of fun mathematics

State of the Art

- Solved at $1-$loop, $2-$loops are the current frontier, $3-$loops are being studied only recently
- Need loop integrals (virtual corrections) and phase space integrals (real corrections)
- Collider observables: use subtraction schemes

Observables expressed as series expansions

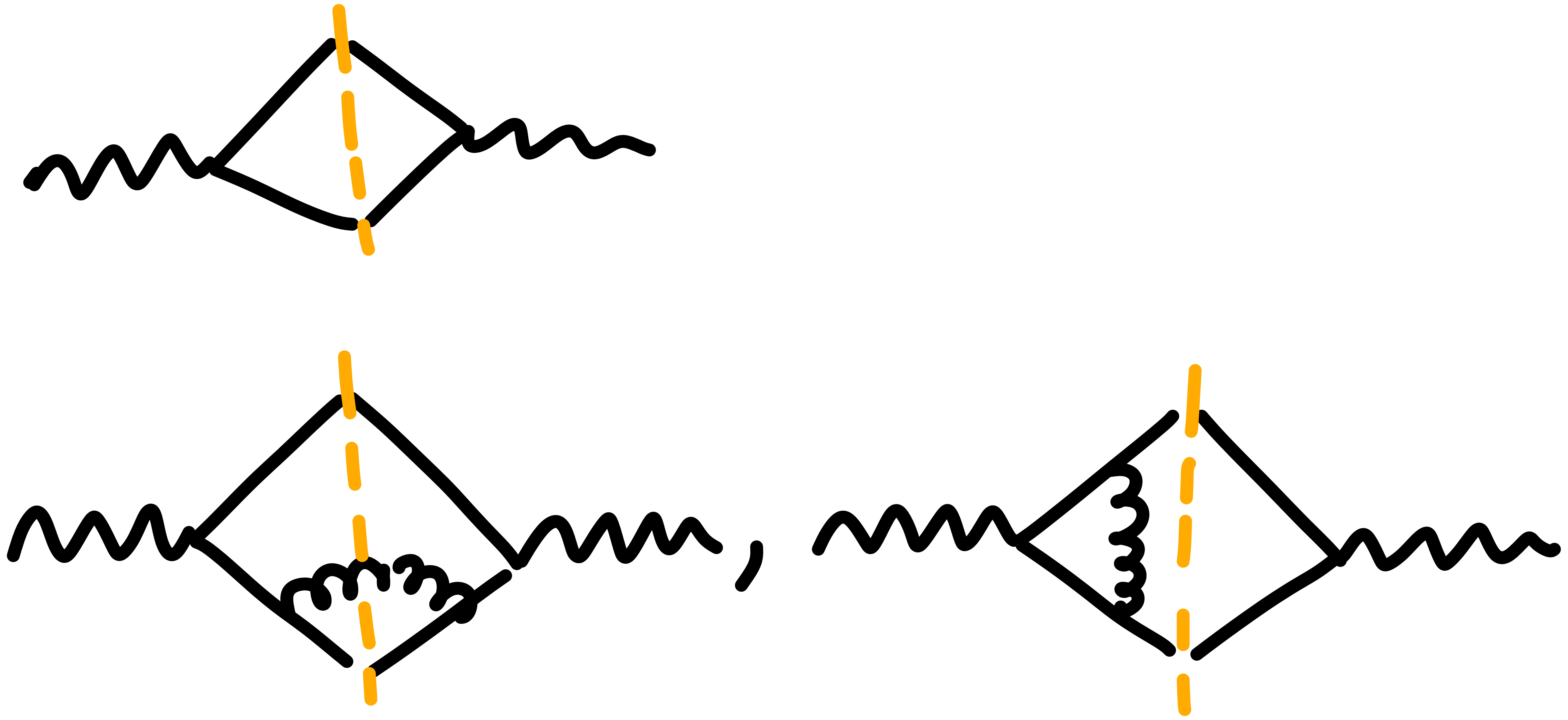
$$\mathrm{d}\sigma = \mathrm{d}\sigma_0 + \alpha_S \mathrm{d}\sigma_{NLO} + \alpha_S^2 \mathrm{d}\sigma_{NNLO} + \alpha_S^3 \mathrm{d}\sigma_{N^3LO} + \ldots$$

Phase space integral

$$\int \mathrm{d}\Phi_n M_n^l$$

LO

NLO



...

$$\mathrm{d}\Phi_n = \frac{\mathrm{d}^{d-1}p_1}{2E_1(2\pi)^{d-1}} \cdots \frac{\mathrm{d}^{d-1}p_n}{2E_n(2\pi)^{d-1}} (2\pi)^d \delta^d(q - p_1 - \cdots - p_n)$$

phase space measure

Reverse Unitarity

Formalism to treat phase space integrals

$$\int \mathrm{d}\Phi_n M_n^l$$

$$\mathrm{d}\Phi_n = \frac{\mathrm{d}^{d-1}p_1}{2E_1(2\pi)^{d-1}} \cdots \frac{\mathrm{d}^{d-1}p_n}{2E_n(2\pi)^{d-1}} (2\pi)^d \delta^d(q - p_1 - \cdots - p_n)$$

$$\frac{\mathrm{d}^{d-1}p}{2E} = \mathrm{d}^d p \, \underbrace{\delta(p^2)\,\Theta(p^0)}_{\delta^+(p^2)}$$

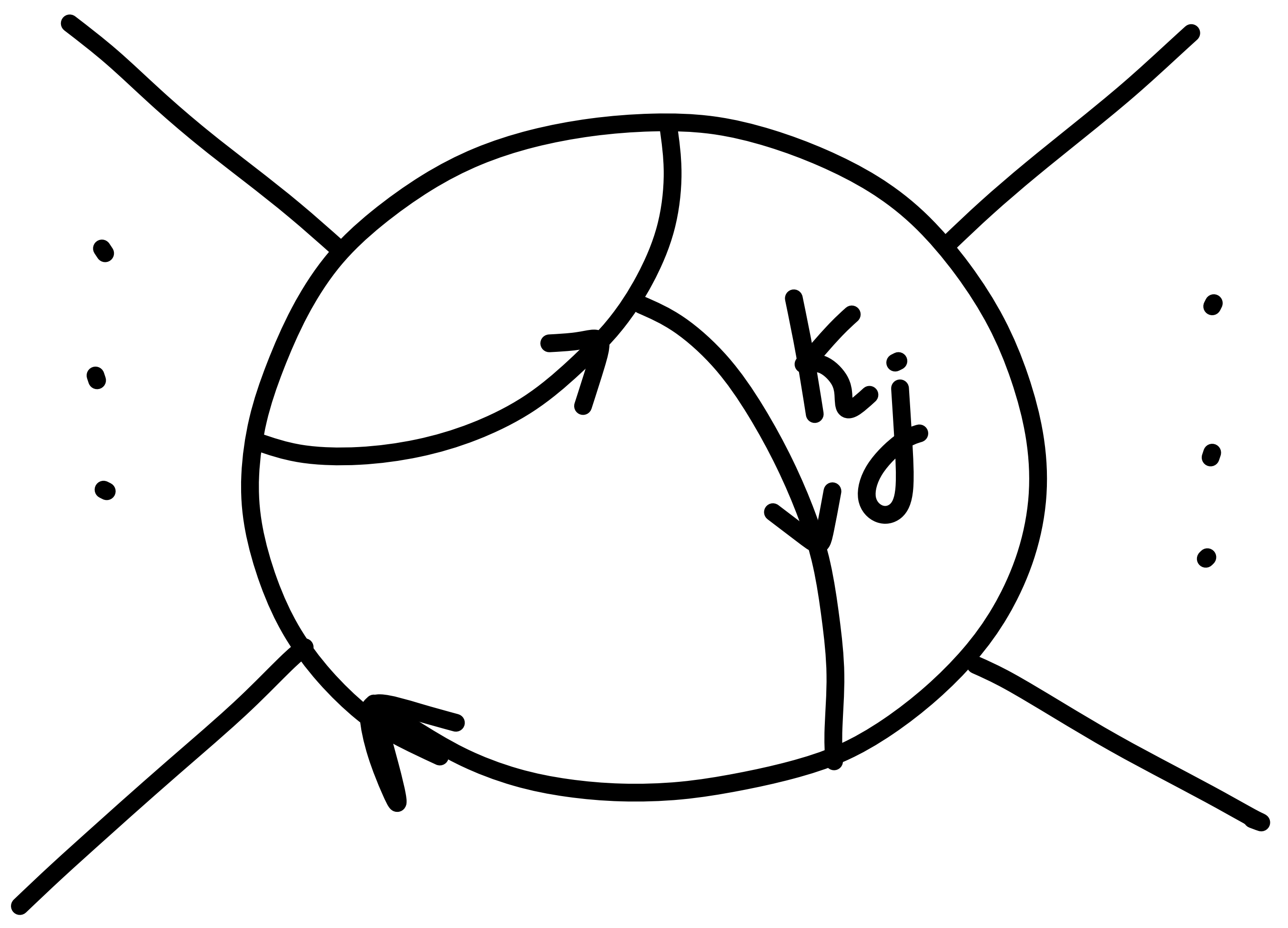
Replace mass-shell condition with cut propagators

$$\frac{1}{D_i} = 2\pi i \delta^+(p_i^2) = \frac{1}{p_i^2 + i0} - \frac{1}{p_i^2 - i0}$$

phase space integrals $\longrightarrow$ multiloop integrals

How do we do multiloop calculations?

We work in dimensional regularization for integrals like…



$$\int \frac{1}{D_1^{\alpha_1} \dots D_n^{\alpha_n}} \qquad \alpha_i \lesseqgtr 0$$

$$D_i = \begin{cases} k_j^2 - m_j^2 \\ k_j \cdot v_j - m_j^2 \end{cases}$$

Not all of them are linearly independent!

$$\int \mathrm{d}^d k \frac{\partial}{\partial k^\mu} \left(\frac{v^\mu}{\prod_i D_i^{\alpha_i}} \right) = 0$$

⇒we don't need to calculate ALL of them

- Take the list of integrals you need to calculate
- Reduce it to a linearly independent set (Laporta algorithm, intersection theory….)
- Calculate only the **master integrals**

A lot of multiloop integrals → smaller set of master integrals

Once we have the master integrals…

We integrate them by differentiating:
method of differential equations

$$\partial_i \vec{I} = M_i(x,\epsilon)\vec{I}$$

ε FACTORIZES :)

integrating is easier if we put them in a canonical form

$$\partial_i \vec{I} = \epsilon A_i(x)\vec{I}$$

$$\vec{I} = \epsilon^k \sum_{n=0}^{\infty} \vec{I}^{(n)}\epsilon^n$$

INTEGRALS EXPRESSED AS POWER SERIES IN ε

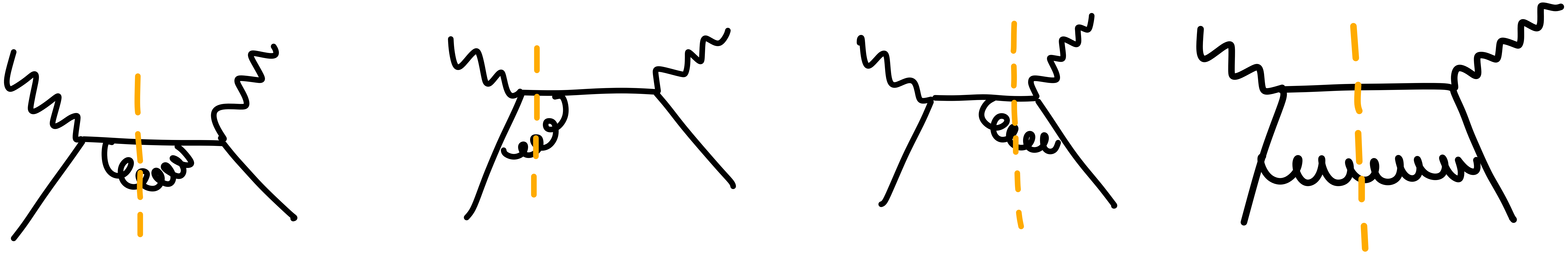
ITERATIVE INTEGRALS

$$\begin{cases} \partial_i \vec{I}^{(0)} = 0 \\ \partial_i \vec{I}^{(1)} = A_i(x)\vec{I}^{(0)} \\ \partial_i \vec{I}^{(2)} = A_i(x)\vec{I}^{(1)} \\ \dots \end{cases}$$

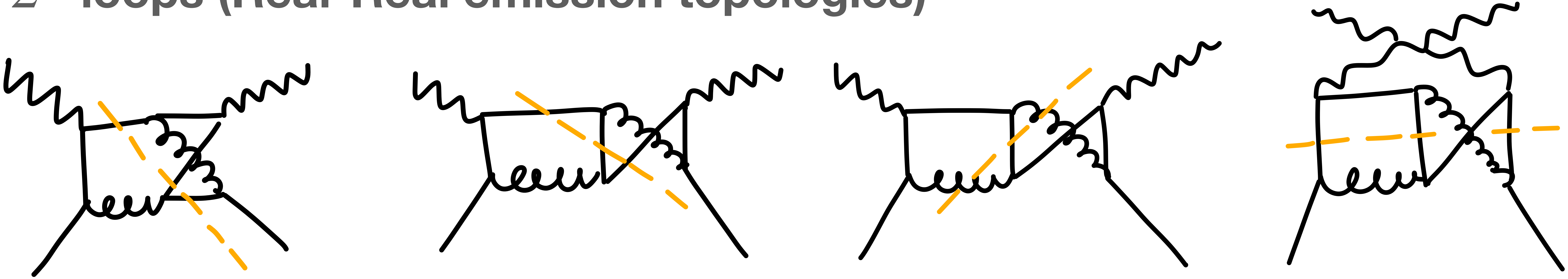
My integrals

Currently working on Deep Inelastic Scattering at $3-$loops

$1-$loop (Real emission topology)



$2-$loops (Real-Real emission topologies)



$3-$loops (Real-Real-Real emission topologies)

coming soon :)