Simulating the non-linear QED on the lattice

Gabriele Pierini, *work with Prof. Dr. Marina K. Marinkovic*

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Outline

1. Motivation
2. Lattice
3. Results

Motivation

Electrodynamics

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

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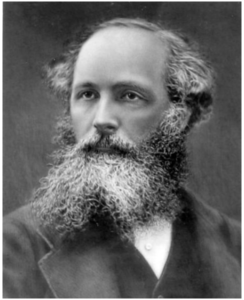

[James Maxwell, 1865]

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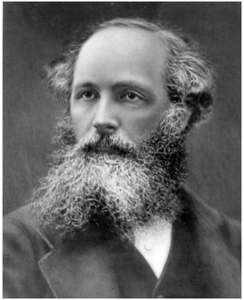

[James Maxwell, 1865]

Richard Feynman (in picture), Sin-Itiro Tomonaga and Julian Schwinger were awarded the Nobel Prize in 1965

Non-linear extensions of QED

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The most well known example: Born-Infeld [Born & Infeld, 1934]

$$\mathcal{L} = b^2\left[1 - \sqrt{1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16b^4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2}\right]$$

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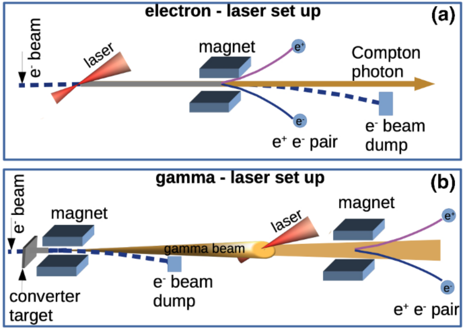
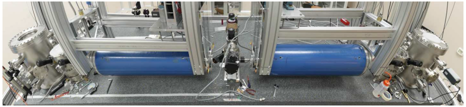
$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \tilde{F}_{\mu\nu} = \tfrac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

Coming soon



LUXE experiment @DESY [Abramowicz et al. 2102.02032 [hep-ex]]



PVLAS experiment @Università degli Studi di Ferrara [Ejlli et al. 2005.12913 [phys.optics]]

Lattice

Lattice?

Lattice?

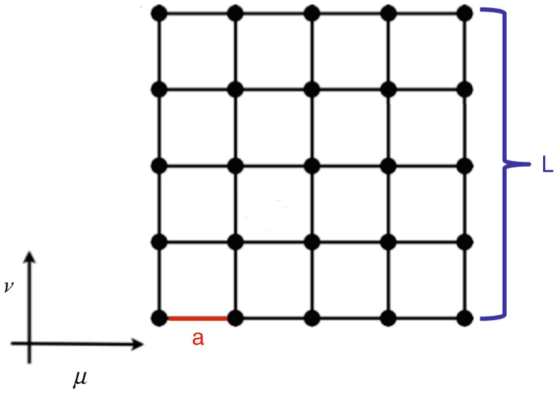
For the Italian speakers: **NOT** *the theory of mattresses*

Lattice approach

What's a lattice?

"Discretized" space-time

[Grattringer & Lang, Springer Berlin, 2010]



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"Discretized" space-time
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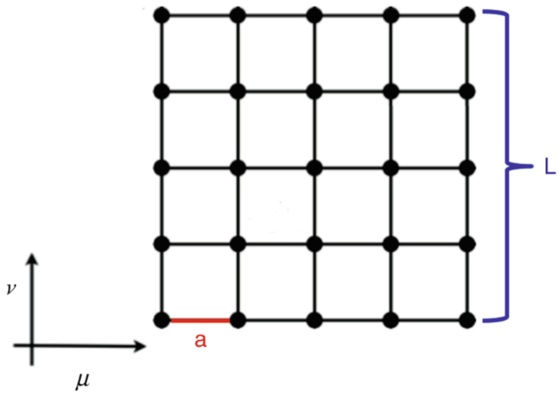
- L points per dimension, a is the space between two points;



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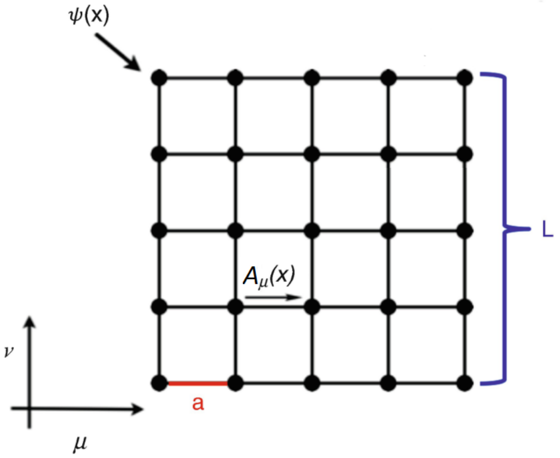

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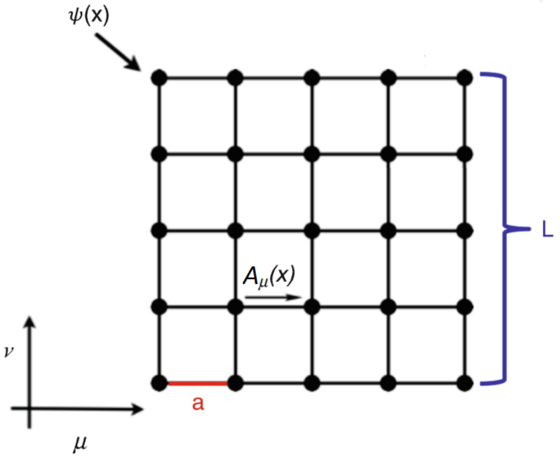

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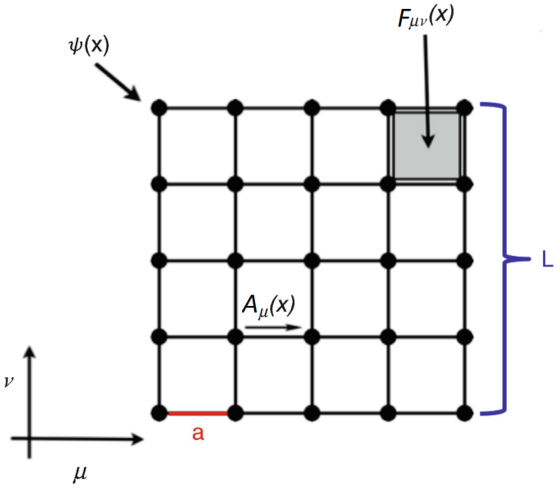
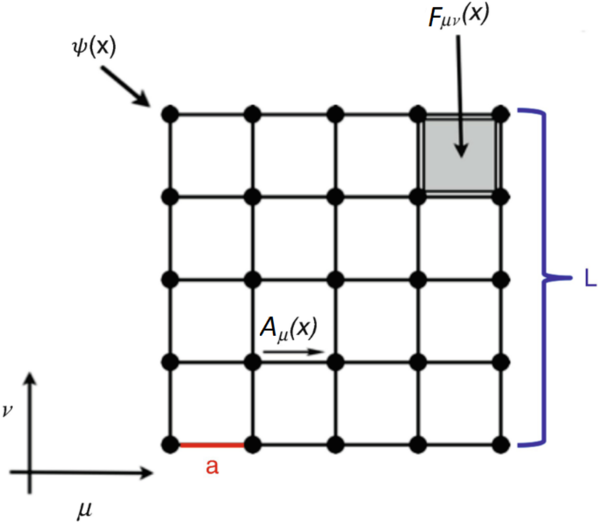
- $F_{\mu\nu}$ is called "plaquette";



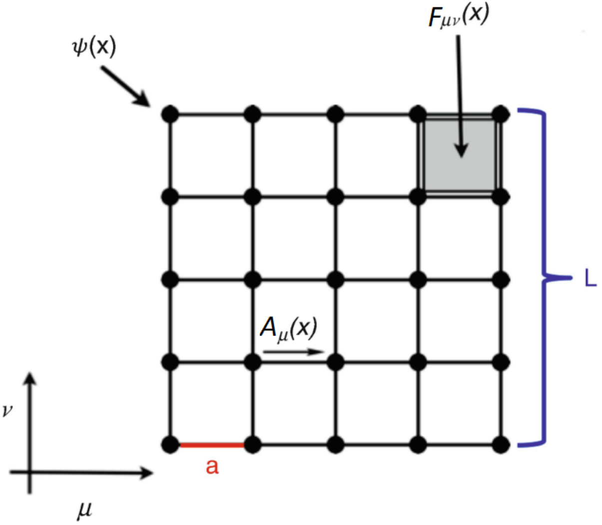
Lattice approach

- Non-perturbative approach;



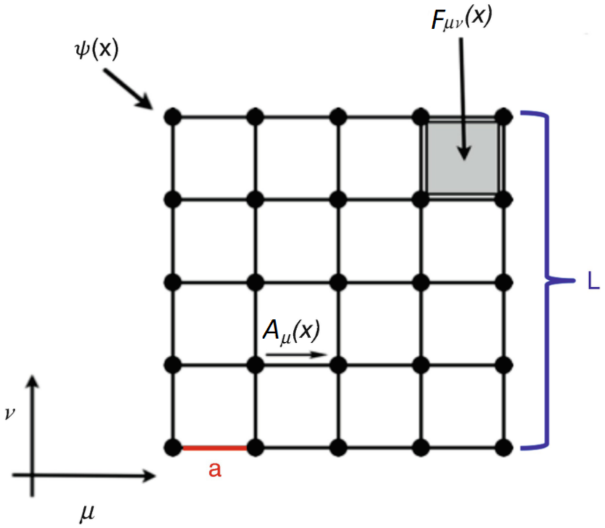
Lattice approach

- Non-perturbative approach;
- Wick rotation: $t \to i\tau$, $g_{\mu\nu} \to 1$, "classical" partition function;



Lattice approach

- Non-perturbative approach;
- Wick rotation: $t \rightarrow i\tau$, $g_{\mu\nu} \rightarrow 1$, "classical" partition function;
- Periodic boundary conditions;



Born-Infeld on a lattice

Only one simulation done so far, in 2005 [Kogut-Sinclaire, 0509097 [hep-lat]]

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Action:

$$S = b^2 \int d^4x \left[\sqrt{1 + \frac{1}{2b^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{16b^4} F_{\mu\nu} \tilde{F}^{\mu\nu}} - 1 \right]$$

How to discretize?

Continuum	Discrete
$\int d^4x$	$\sum_x$
$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu$	$F_{\mu\nu}(x) = A_\nu(x+\hat{\mu}) - A_\nu(x) - A_\mu(x+\hat{\nu}) + A_\mu(x)$
$A_\mu(x) \to A_\mu(x) - \partial_\mu \chi$	$A_\mu(x) \to A_\mu(x) - \chi(x+\hat{\mu}) + \chi(x)$

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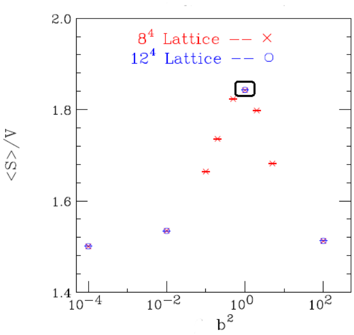
$F_{\mu\nu}$ invariant under $U(1)$ gauge transformation

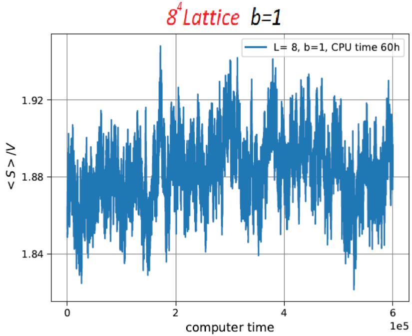
Results

Energy

Monte-Carlo algorithm used to run the simulation



Energy found by Kogut-Sinclair



Energy in my simulation

Wilson line

In continuum

$$W[\gamma] = \exp\left\{ i \oint_\gamma A_\mu(x) dx^\mu \right\}$$

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On the lattice (Kogut-Sinclair)

$$W[x] = \exp\left\{ ie \sum_t \left[A_4(\mathbf{x}, t) - \boxed{\frac{1}{L^3} \sum_{\mathbf{y}} A_4(\mathbf{y}, t)} \right] \right\}$$

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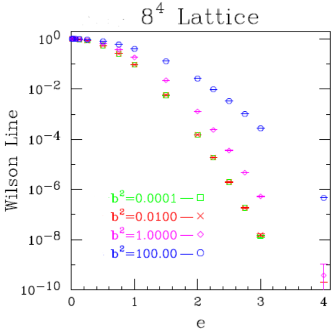
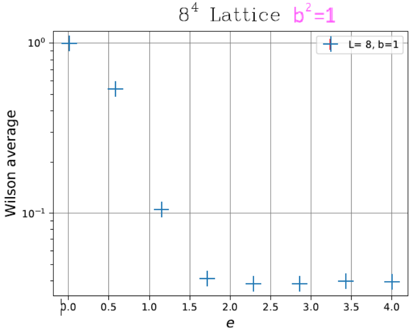
Net charge $= 0$

Wilson line

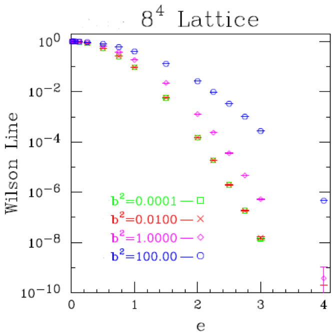


Wilson lines found by Kogut-Sinclair



Wilson lines in my simulation

Wilson line



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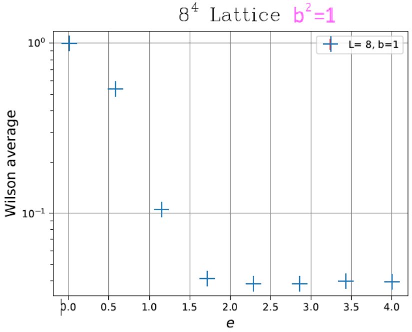

Wilson lines in my simulation

More statistics needed for $e > 1.5$

Final recap

- Non-linear QED to explain soon within experimental reach non-perturbative phenomena;
- Lattice approach is non perturbative, ideal to simulate non-linear QED;
- Some thermalization issues still
- Once the code is improved:
    1. Improved statistics;
    2. Phenomenological comparison between different theories.

Me in Zuerich

Professor Dr. Marina K. Marinkovic, my supervisor

Final recap

And many many thanks also to my co-supervisor Veronica Errasti Diez (LMU Munich - Excellence Cluster Origin)

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Supplementary slides

Gauge fixing

The Landau gauge is imposed:

$$\sum_{\mu} (A_\mu(x+\hat{\mu}) - A_\mu(x)) = 0 \tag{1}$$

Two different gauge fixing method used:

Gauge fixing

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Two different gauge fixing method used:

1. Relaxation
    - Gauge fixing is equivalent to maximize the function $F^g[U] = \frac{1}{V}\operatorname{Re}\sum_x f^g(x)$
    - Value locally optimized, iteratively maximizing $f^g(x)$
    - unitary transformations, $O(N^2)$

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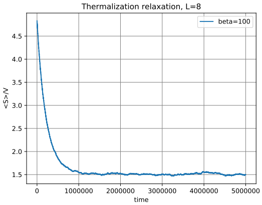
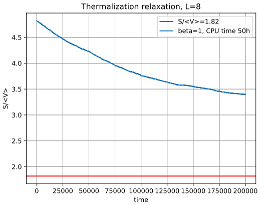
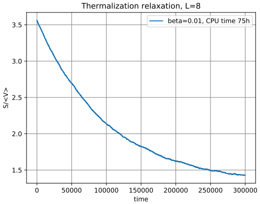
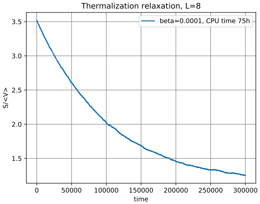
2. Since $\sum_\mu \left(A_\mu(x+\hat{\mu}) - A_\mu(x)\right) = 0$ is a system of N linear equations it can be solved with an algorithm, which is $O(N)$. The transformation is not unitary.

Relaxation method









Thermalization

Metropolis algorithm and heat bath

Steps of metropolis algorithm:

1. Start with a configuration X with energy $E[X]$;
2. propose a new configuration X' with energy $E[X']$;
3. if $E[X'] < E[X]$ accept the new configuration;
4. if $E[X'] > E[X]$ accept the new configuration with a probability of $e^{-\Delta E}$;
5. repeat.

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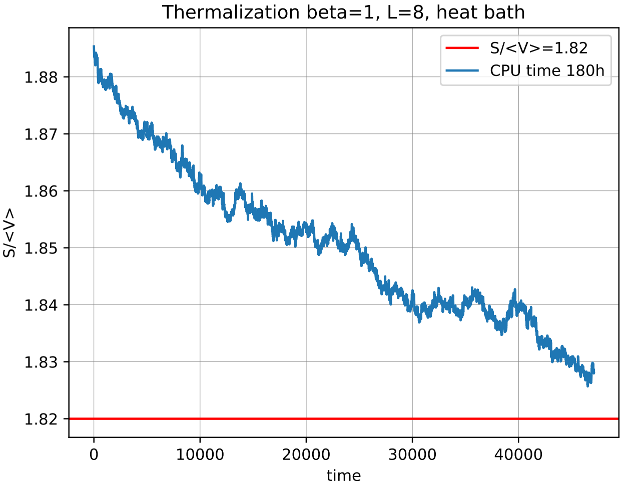
Heat bath algorithm combines the steps of Metropolis: sample X directly according to the probability distribution

$$dP(X) = dX exp(-E[X])$$

Heat bath method



Thermalization