

Theory Developments in VBS/VBF

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Thank you for the (in-person) invitation! 😊

Vector Boson Scattering Processes: Status and Prospects

Diogo Buarque Franzosi, Michele Gallinaro, Richard Ruiz, Thea K. Aarrestad, Flavia Cetorelli, Mauro Chiesa, Antonio Costantini, Ansgar Denner, Stefan Dittmaier, Robert Franken, Pietro Govoni, Tao Han, Ashutosh V. Kotwal, Jimmian Li, Lohwasser, Kenneth Long, Yang Ma, Luca Mantani, Matteo Marchegiani, Mathieu Pellen, Giovanni Pelliccioli, Karolos Potamianos, Jürgen Reuter, Timo Schmidt, Christopher Schwan, Michal Szleper, Rob Verheyen, Keping Xie, Rao Zhao

Insight into the electroweak (EW) and Higgs sectors can be achieved through measurements of vector boson scattering (VBS) processes. The scattering of EW bosons are rare processes that are precisely predicted in the Standard Model (SM) and are closely related to the Higgs mechanism. Modifications to VBS processes are also predicted in models of physics beyond the SM (BSM), for example through changes to the Higgs boson couplings to gauge bosons and the resonant production of new particles. In this review, experimental results and theoretical developments of VBS at the Large Hadron Collider, its high luminosity upgrade, and future colliders are presented.

Comments: Journal version with additional discussion and references. 58 pages (including toc and refs.), 71 image files, eight tables, and many references. VBSCan@Snowmass review

Subjects: High Energy Physics - Phenomenology (hep-ph), High Energy Physics - Experiment (hep-ex)

Report number: CP3-21-14, DESY-21-064, IFJAN-W-2021-8, PITT-PACC-2106, VBSCAN-PUB-04-21

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(or arXiv:2106.01393v2 [hep-ph] for this version)

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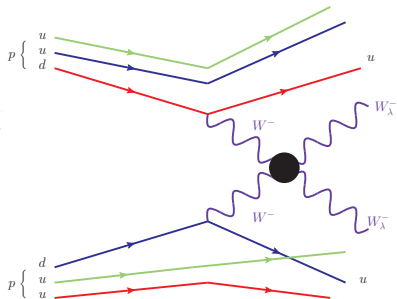
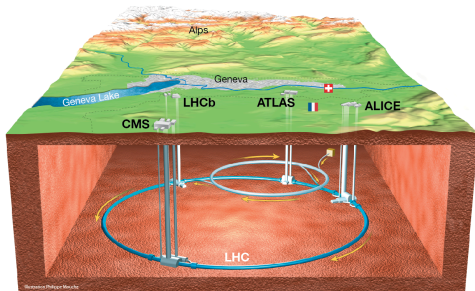
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- **VBSCan@Snowmass review:** “state-of-the-art” + (Snowmass) projections for HL-LHC + future colliders ($pp, e^+e^-, \mu^+\mu^-$) [2106.01393]
- **Updates:** follow-up work of Snowmass activities
- **Apologies:** skipping **lots** since only talk is only 15' 😊

Motivation: measuring rare processes, e.g., **vector boson scattering (VBS)**, is part of the **Large Hadron Collider's** long-term program

See review by Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, *Rev. Physics* ('22) [arXiv:2106.01393]



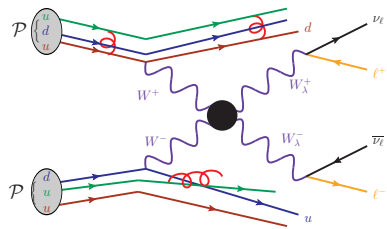
VBS probes spin & charge configurations inaccessible with **quarks/gluons**

⇒ **VBS is uniquely sensitive** to Standard Model and new physics!

polarization in vector boson fusion (VBF) / scattering (VBS)

The W_λ^\pm, Z_λ bosons are massive, spin-1 objects

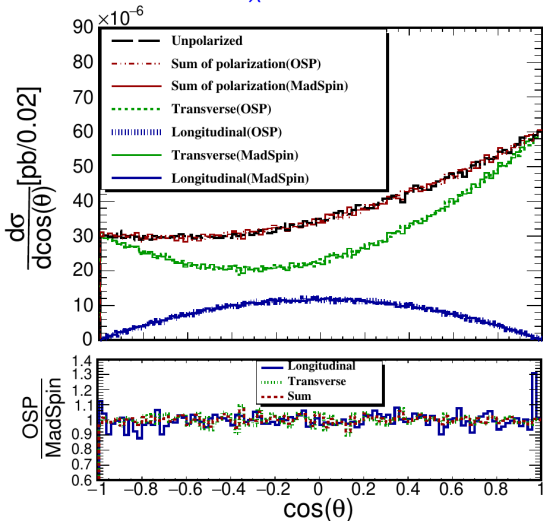
- 2 transverse polarizations (L,R)
- 1 longitudinal polarization (0)



polarizations also imprint on kinematics of decay products!

MadGraph5_aMC@NLO now supports event simulation at LO of particles with fixed helicities

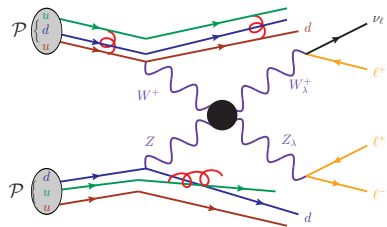
Plotted: angle of outgoing ℓ^- in $pp \rightarrow W^+ W_\lambda^- jj \rightarrow W^+ \ell^- \bar{\nu}_\ell jj$ via VBS



Buarque Franzosi, Mattelaer, RR, Shil [(JHEP'20)]

The $W_{\lambda}^{\pm}, Z_{\lambda}$ bosons are massive, spin-1 objects

- 2 transverse polarizations (L,R)
- 1 longitudinal polarization (0)

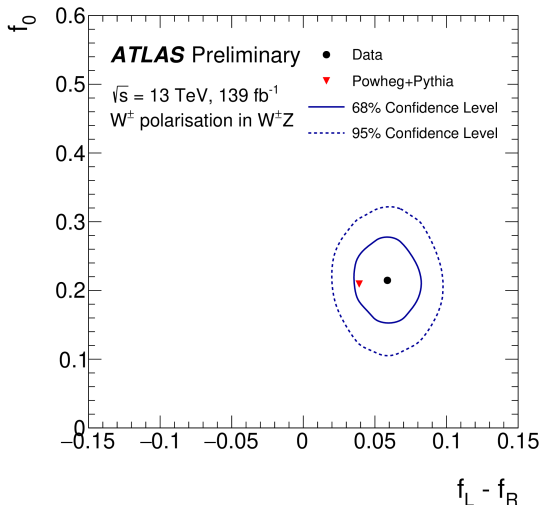


polarizations also imprint on kinematics of decay products!

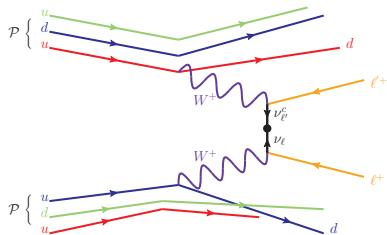
MadGraph5_aMC@NLO now supports event simulation at LO of particles with fixed helicities

First measurement of polarization fractions (f_{λ}) in $W^{\pm}Z$ scattering

ATLAS ('22) [2211.09435]



violation of lepton number symmetries (NEW!)



First high-energy constraints on $d = 5$

Weinberg operator

$$\Lambda / C_5^{\mu\mu} \gtrsim 5 \text{ TeV}$$

CMS [PRL'22]

NEW! w/ ATLAS ('23) [EXOT-2020-06]

Fun ideas using **VBS/VBF** to probe **lepton number violation** and **lepton flavor violation**:

– $W\gamma \rightarrow 3\ell + X$ or $\ell^\pm \ell'^\pm + X$
via heavy neutrinos w/ Alva, Han [1411.7305];

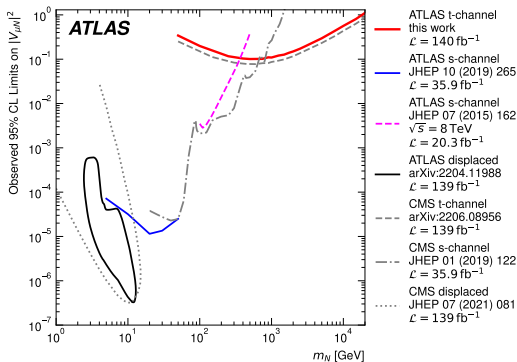
via heavy neutrinos w/ Pascoli, et al [1812.08750]

– $W^\pm W^\pm \rightarrow \ell^\pm \ell'^\pm$

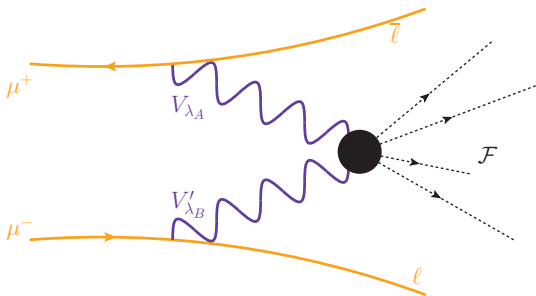
via heavy neutrinos w/ Fuks, et al [2011.02547]

– $W^\pm W^\pm \rightarrow \ell^\pm \ell'^\pm$

via Weinberg operator w/ Fuks, et al [2012.09882]



$\mu^+ \mu^-$ collisions at many-TeV¹

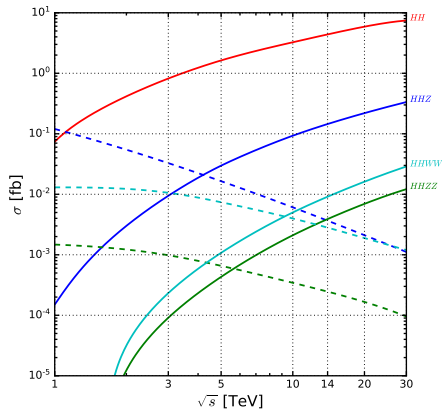
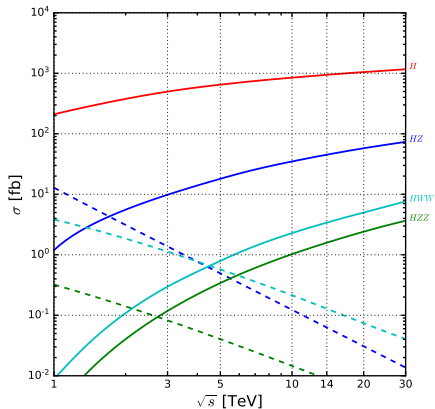


¹ Surge of motivation/interest, e.g., Al Ali, et al. [2103.14043]; R&D progress as reported in the European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass + US activities

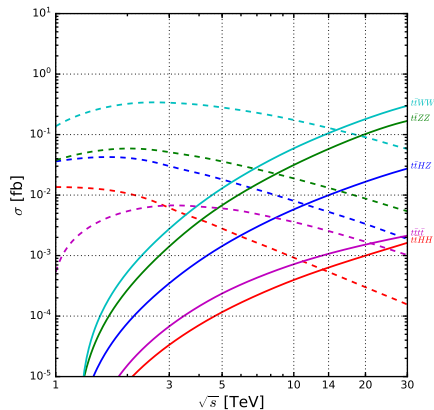
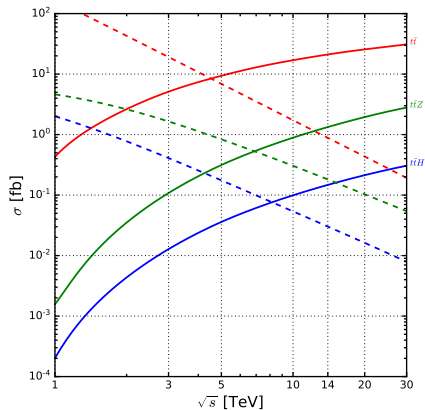
next several slides from w/ A. Costantini, et al [2005.10289]

Higgs production

cross sections (σ) vs \sqrt{s} for
s-channel annihilation (dash) vs VBF (solid)

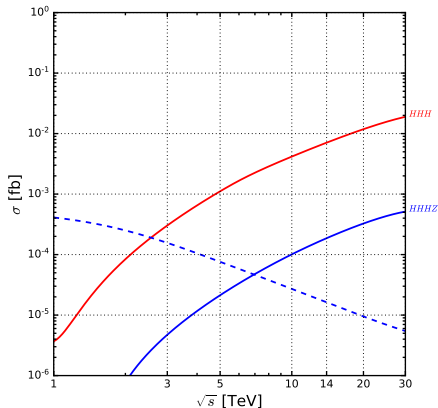
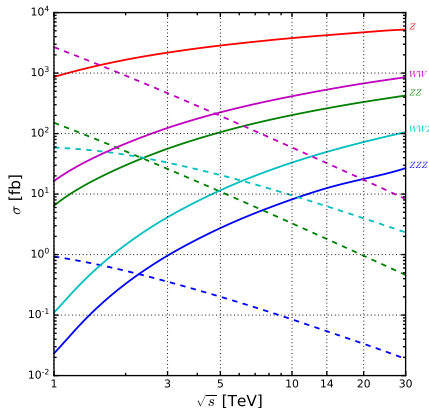


Top production



- Do you notice a pattern?

Many-boson production



- **VBF is the dominant production vehicle for many processes**

Evidence for trend that VBF/S rates will always exceed s -ch. rates

Is this obvious? (not to me at first!) **Is there intuition for this?** (yes!)

w/ A. Costantini, et al [[2005.10289](#)]

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Idea: crudely compare the production of X by writing generically

$$\sigma^{s\text{-ch.}} \sim \frac{(s-M_X^2)}{(s-M_V^2)^2} \sim \frac{(s-M_X^2)}{s^2} \quad \leftarrow \text{assumes } s \gg M_V^2$$

$$\frac{d\sigma^{VBF}}{dz_1 dz_2} \sim \underbrace{f_V(z_1) f_{V'}(z_2)}_{\text{"}\mu\text{PDFs"}}$$
$$\underbrace{\frac{(M_{VV'}^2 - M_X^2)}{(M_{VV'}^2 - M_V^2)^2}}_{M_{VV'}^2 = z_1 z_2 s \gg M_V^2} \sim f_V(z_1) f_{V'}(z_2) \frac{(z_1 z_2 s - M_X^2)}{(z_1 z_2)^2} \frac{\sigma^{s\text{-ch.}}}{(s - M_X^2)}$$

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PDFs are largest when $z = E_V/E_\mu \ll 1$ but $E_V \sim \sqrt{s} \gg M_V$

$$\implies f_V(z_i) \sim \frac{g_W^2}{4\pi} \frac{1}{z_i} \log\left(\frac{s}{M_V^2}\right) \quad \leftarrow \text{crude approximation}$$

Evidence for trend that VBF/S rates will always exceed s-ch. rates

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Observation: $\sigma^{VBF} = \sigma^{s\text{-ch.}} \times \int dz_1 dz_2 \dots$ is solvable for $M_{VV'} \gg M_X!$

Universal behavior: when production of X by **VBF** and **annihilation** are driven by same physics, **VBF dominates** when \sqrt{s} satisfies

$$\frac{\sigma^{\text{VBF}}}{\sigma^{s\text{-ch.}}} \sim \mathcal{S} \left(\frac{g_W^2}{4\pi} \right)^2 \left(\frac{s}{M_X^2} \right) \log^2 \frac{s}{M_V^2} \log \frac{s}{M_X^2} > 1$$

Scaling estimate not so bad if $M_X \gg M_V$. Difference is about $\mathcal{O}(10\%)$

mass (M_X) [TeV]	SZ (Singlet)	H_2Z (2HDM)	$t'\bar{t}'$ (VLQ)	$\tilde{t}\bar{\tilde{t}}$ (MSSM)	$\tilde{\chi}^0\tilde{\chi}^0$ (MSSM)	$\tilde{\chi}^+\tilde{\chi}^-$ (MSSM)	Scaling (Eq. 7.7)
400 GeV	2.1 TeV	2.1 TeV	11 TeV	2.9 TeV	3.2 TeV	7.5 TeV	1.0 (1.7) TeV
600 GeV	2.5 TeV	2.5 TeV	16 TeV	3.8 TeV	3.8 TeV	8.1 TeV	1.3 (2.4) TeV
800 GeV	2.8 TeV	2.8 TeV	22 TeV	4.3 TeV	4.3 TeV	8.5 TeV	1.7 (3.1) TeV
2.0 TeV	4.0 TeV	4.0 TeV	>30 TeV	7.8 TeV	6.9 TeV	11 TeV	3.7 (6.8) TeV
3.0 TeV	4.8 TeV	4.8 TeV	>30 TeV	10 TeV	9.0 TeV	13 TeV	5.3 (9.8) TeV
4.0 TeV	5.5 TeV	5.5 TeV	>30 TeV	13 TeV	11 TeV	15 TeV	6.8 (13) TeV

Table 9. For representative processes and inputs, the required muon collider energy \sqrt{s} [TeV] at which the VBF production cross section surpasses the s -channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale M_X ($2M_X$).

Evidence that PDF prescription works quantitatively

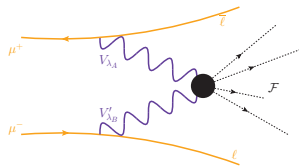
The Effective W/Z Approximation (EWA)²

a.k.a. weak boson parton distribution functions

²Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

Idea: one can write the following scattering formula

$$\sigma(\mu^+ \mu^- \rightarrow \mathcal{F} + \text{anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$



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$$\sigma(\mu^+ \mu^- \rightarrow \mathcal{F} + \text{anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$

$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^1 d\xi_1 \int_{\tau_0/\xi_1}^1 d\xi_2 \int dPS_{\mathcal{F}}}_{\text{sum over all configurations / phase space integral}}$$

sum over all configurations / phase space integral

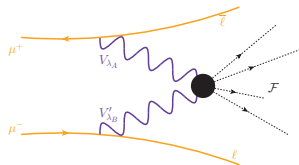
$$\times \left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f) f_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f)}_{W_{\lambda}^+/W_{\lambda}^-/Z_{\lambda}/\gamma_{\lambda} \text{ PDFs}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \rightarrow \mathcal{F})}{dPS_n}}_{\text{"hard scattering" at LO}}$$

$$+ \underbrace{\mathcal{O}\left(\frac{M_{V_k}^2}{M_{VV'}^2}\right) + \mathcal{O}\left(\frac{p_{T,V_k}^2}{M_{VV'}^2}\right)}_{\text{perturbative power-law corrections}}$$

← (appear from expanding $\mu_{\lambda} \rightarrow V_{\lambda}$ matrix elements)

$$+ \underbrace{\mathcal{O}\left(\log \frac{\mu_f^2}{M_V^2}\right)}_{\text{log corrections}}$$

← (μ_f is only an UV regulator here at LO)



We studied the **red** terms

w/ Antonio Costantini, Fabio Maltoni, Olivier Mattelaer [2111.02442]

some results on $V_\lambda V'_{\lambda'} \rightarrow X$ in $\mu^+ \mu^-$ collisions³

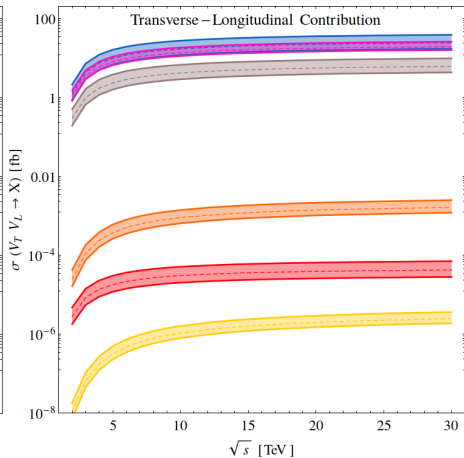
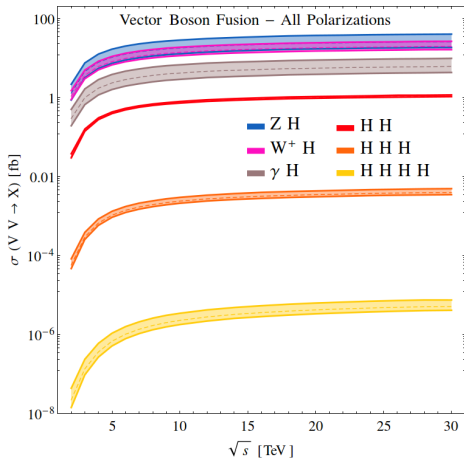
³ w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2111.02442]

Higgs production in EVA

We then had fun looking into *many* processes

$$(L) \sum_{\lambda_A, \lambda_B} V_{\lambda_A} V_{\lambda_B} \rightarrow HX$$

$$(R) V_T V_0 \rightarrow HX$$

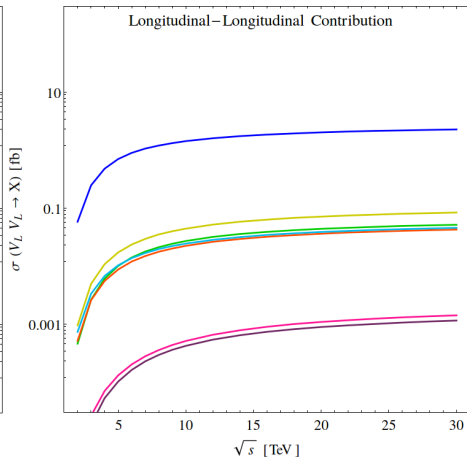
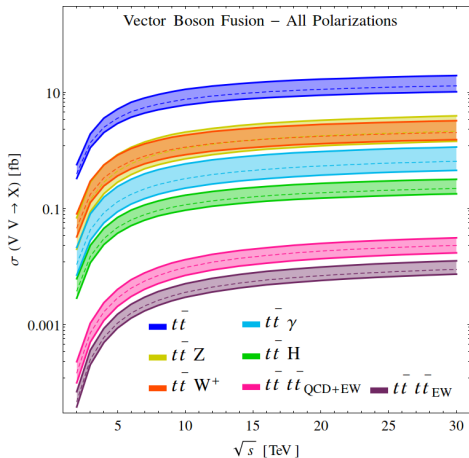


Top production in EVA

... ***many*** processes

$$(L) \sum_{\lambda_A, \lambda_B} V_{\lambda_A} V_{\lambda_B} \rightarrow t\bar{t}X$$

$$(R) V_0 V_0 \rightarrow t\bar{t}X$$



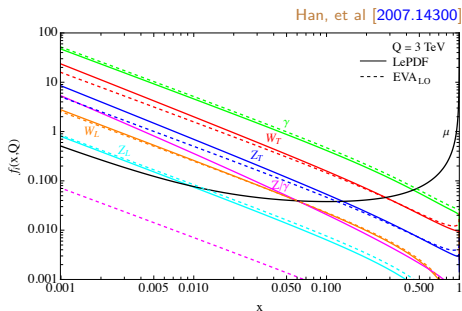
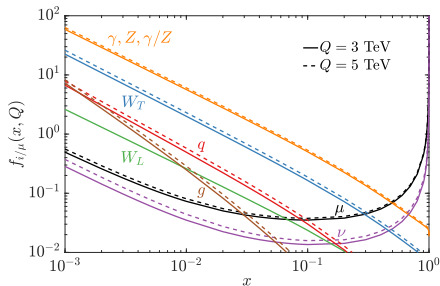
Diboson production in EVA

(4 polarization plots + 1 table) \times each class of processes

	mg5amc syntax	$\sqrt{s} = 3$ TeV	$\sqrt{s} = 14$ TeV	$\sqrt{s} = 30$ TeV
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow W+W^-$	vp vm > w+ w-	$2.2 \cdot 10^2$ ^{+98%} _{-35%}	$7.0 \cdot 10^2$ ^{+91%} _{-33%}	$8.6 \cdot 10^2$ ^{+88%} _{-32%}
$V_T V'_T \rightarrow W+W^-$	vp{T} vm{T} > w+ w-	$2.0 \cdot 10^2$ ^{+99%} _{-35%}	$6.6 \cdot 10^2$ ^{+93%} _{-34%}	$8.0 \cdot 10^2$ ^{+92%} _{-33%}
$V_0 V'_0 \rightarrow W+W^-$	vp{0} vm{T} > w+ w-	$1.2 \cdot 10^1$ ^{+54%} _{-27%}	$4.4 \cdot 10^1$ ^{+50%} _{-25%}	$5.2 \cdot 10^1$ ^{+49%} _{-24%}
$V_0 V'_0 \rightarrow W+W^-$	vp{0} vm{0} > w+ w-	$4.2 \cdot 10^{-1}$	$1.7 \cdot 10^0$	$2.0 \cdot 10^0$
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow W+Z$	vp vm > w+ z	$5.3 \cdot 10^1$ ^{+105%} _{-40%}	$1.8 \cdot 10^2$ ^{+97%} _{-37%}	$2.2 \cdot 10^2$ ^{+95%} _{-37%}
$V_T V'_T \rightarrow W+Z$	vp{T} vm{T} > w+ z	$5.0 \cdot 10^1$ ^{+111%} _{-42%}	$1.6 \cdot 10^2$ ^{+103%} _{-39%}	$2.0 \cdot 10^2$ ^{+100%} _{-38%}
$V_0 V'_0 \rightarrow W+Z$	vp{0} vm{T} > w+ z	$3.4 \cdot 10^0$ ^{+36%} _{-18%}	$1.4 \cdot 10^1$ ^{+34%} _{-17%}	$1.7 \cdot 10^1$ ^{+34%} _{-17%}
$V_0 V'_0 \rightarrow W+Z$	vp{0} vm{0} > w+ z	$3.9 \cdot 10^{-2}$	$2.1 \cdot 10^{-1}$	$2.6 \cdot 10^{-1}$
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow ZZ$	vp vm > z z	$4.4 \cdot 10^1$ ^{+164%} _{-52%}	$1.6 \cdot 10^2$ ^{+144%} _{-48%}	$1.9 \cdot 10^2$ ^{+143%} _{-48%}
$V_T V'_T \rightarrow ZZ$	vp{T} vm{T} > z z	$4.0 \cdot 10^1$ ^{+171%} _{-54%}	$1.4 \cdot 10^2$ ^{+153%} _{-50%}	$1.7 \cdot 10^2$ ^{+150%} _{-49%}
$V_0 V'_0 \rightarrow ZZ$	vp{0} vm{T} > z z	$4.2 \cdot 10^0$ ^{+66%} _{-33%}	$1.8 \cdot 10^1$ ^{+61%} _{-30%}	$2.2 \cdot 10^1$ ^{+60%} _{-30%}
$V_0 V'_0 \rightarrow ZZ$	vp{0} vm{0} > z z	$1.1 \cdot 10^{-1}$	$6.0 \cdot 10^{-1}$	$7.2 \cdot 10^{-1}$
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow \gamma Z$	vp vm > a z	$1.9 \cdot 10^1$ ^{+169%} _{-53%}	$7.1 \cdot 10^1$ ^{+149%} _{-49%}	$8.8 \cdot 10^1$ ^{+145%} _{-48%}
$V_T V'_T \rightarrow \gamma Z$	vp{T} vm{T} > a z	$1.8 \cdot 10^1$ ^{+172%} _{-54%}	$6.8 \cdot 10^1$ ^{+153%} _{-50%}	$8.4 \cdot 10^1$ ^{+149%} _{-49%}
$V_0 V'_0 \rightarrow \gamma Z$	vp{0} vm{T} > a z	$9.5 \cdot 10^{-1}$ ^{+67%} _{-33%}	$4.4 \cdot 10^0$ ^{+61%} _{-30%}	$5.5 \cdot 10^0$ ^{+60%} _{-30%}
$V_0 V'_0 \rightarrow \gamma Z$	vp{0} vm{0} > a z	$5.6 \cdot 10^{-4}$	$4.5 \cdot 10^{-3}$	$6.5 \cdot 10^{-3}$
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow \gamma W^+$	vp vm > a w+	$1.1 \cdot 10^1$ ^{+111%} _{-42%}	$4.0 \cdot 10^1$ ^{+101%} _{-39%}	$4.9 \cdot 10^1$ ^{+99%} _{-38%}
$V_T V'_T \rightarrow \gamma W^+$	vp{T} vm{T} > a w+	$1.1 \cdot 10^1$ ^{+111%} _{-42%}	$3.9 \cdot 10^1$ ^{+102%} _{-39%}	$4.8 \cdot 10^1$ ^{+100%} _{-38%}
$V_0 V'_0 \rightarrow \gamma W^+$	vp{0} vm{T} > a w+	$1.6 \cdot 10^{-2}$ ^{+62%} _{-31%}	$7.3 \cdot 10^{-1}$ ^{+56%} _{-28%}	$9.2 \cdot 10^{-1}$ ^{+54%} _{-27%}
$V_0 V'_0 \rightarrow \gamma W^+$	vp{0} vm{0} > a w+	$1.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$
$\sum V_{\lambda A} V'_{\lambda B} \rightarrow \gamma \gamma$	vp vm > a a	$2.1 \cdot 10^0$ ^{+172%} _{-54%}	$8.5 \cdot 10^0$ ^{+152%} _{-50%}	$1.1 \cdot 10^1$ ^{+147%} _{-48%}
$V_T V'_T \rightarrow \gamma \gamma$	vp{T} vm{T} > a a	$2.1 \cdot 10^0$ ^{+172%} _{-54%}	$8.5 \cdot 10^0$ ^{+152%} _{-50%}	$1.1 \cdot 10^1$ ^{+147%} _{-48%}
$V_0 V'_0 \rightarrow \gamma \gamma$	vp{0} vm{T} > a a	$7.8 \cdot 10^{-4}$ ^{+70%} _{-35%}	$3.4 \cdot 10^{-3}$ ^{+67%} _{-34%}	$4.2 \cdot 10^{-3}$ ^{+67%} _{-33%}
$V_0 V'_0 \rightarrow \gamma \gamma$	vp{0} vm{0} > a a	$5.8 \cdot 10^{-4}$	$4.7 \cdot 10^{-3}$	$6.8 \cdot 10^{-3}$

What about impact of DGLAP/RG evolution?

- shift in normalization
- perturbative generation of g, q, t
- PDF mixing
- Some PDF tables now public!
- **look out for updates here!**



Summary and Conclusion

Exploring VBS/VBF is part of LHC's long-term program

See review by Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, *Rev. Physics* ('22) [arXiv:2106.01393]

- **Helicity-polarized simulations available with MadGraph5aMC@NLO**

up to LO+LL(PS) [1912.01725]; NLO is under dev.; see also Poncelet, et al [2102.13583], + others

- **At high-energies, VBS/VBF eventually becomes leading production mechanism in $\ell^+\ell^-$ collisions** [2005.10289]

+ dedicated updates to support VBS/VBF at higher energies in mg5amc, e.g., [2102.00773]

- **Clarity on quantitative success of W/Z PDFs in $\ell^+\ell^-$** [2111.02442]

EWA@LO in mg5amc is now available and plans underway to merge parallel Snowmass efforts

- **EW PDFs at higher orders actively being studied**

Han, et al [2007.14300], Garosi, et al [2303.16964], +others

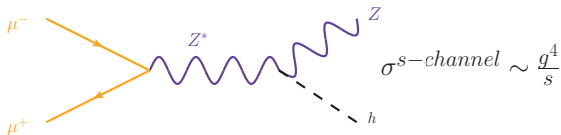


backup

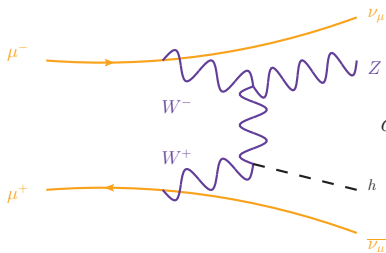
Implementing EW boson PDFs in MadGraph5

- **NEW: (Polarized) Effective Vector Boson Approx. (EVA)**
 - ▶ Bare (LO) PDFs for helicity-polarized $W_\lambda, Z_\lambda, \gamma_\lambda$ from ℓ_λ^\pm
 - ▶ Automatically support PDFs for unpolarized W/Z (**EWA**) from ℓ_λ^\pm
- **KEPT: Improved Weizsäcker-Williams approximation (iWWA)**
 - ▶ Unpolarized γ PDF + power corrections from ℓ^\pm (Frixione, et al [[hep-ph/9310350](https://arxiv.org/abs/hep-ph/9310350)])
- **Technicalities:**
 - ▶ M_W, M_Z always nonzero in PDFs and matrix elements!
 - ▶ static and dynamic μ_f
 - ▶ n -point μ_f variation
 - ▶ Choice of p_T and q as evolution variable (this gives extra $\log(1 - \xi)$ terms in PDFs!)
 - ▶ Also enabled **EVA+DIS** collider configuration
- **Technical appendix** rederiving W_λ, Z_λ PDFs to provide standard reference and mapping between different approaches in the literature
 - ▶ Released in v3.3.0 (Major milestone for lepton colliders; see Frixione, et al [[2108.10261](https://arxiv.org/abs/2108.10261)])

s-channel annihilation vs VBF/S



$$\sigma^{s\text{-channel}} \sim \frac{g^4}{s}$$



$$\sigma^{VBF} \sim \frac{g^8}{M_{WW}^2} \log^2 \left(\frac{M_{WW}^2}{M_W^2} \right)$$

More legs \implies more propagators $\implies \int dk^2 / (k^2 - M_W^2) \sim \log(\Lambda^2 / M_W^2)$
 Larger $s \implies$ larger $(M_{WW}^2 / M_W^2) \implies$ collinear V compensate for g

PDFs for $e^\pm, \mu^\pm \rightarrow W_\lambda/Z_\lambda/\gamma_\lambda + \ell$ depend on helicities (λ)

- Subtle but important differences if evolving by q^2 of V vs p_T^2 of ℓ

(this can account for some differences in literature!)

$$f_{V_+/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)^2}{2z} \log \left[\frac{\mu_f^2}{M_V^2} \right],$$

$$f_{V_-/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2}{2z} \log \left[\frac{\mu_f^2}{M_V^2} \right],$$

$$f_{V_0/f_L}(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z, \mu_f^2) = \left(\frac{g_R}{g_L} \right)^2 \times f_{V_-/f_L}(z, \mu_f^2)$$

$$f_{V_-/f_R}(z, \mu_f^2) = \left(\frac{g_R}{g_L} \right)^2 \times f_{V_+/f_L}(z, \mu_f^2)$$

$$f_{V_0/f_R}(z, \mu_f^2) = \left(\frac{g_R}{g_L} \right)^2 \times f_{V_0/f_L}(z, \mu_f^2)$$

```

59 c /* *****
60 c EVA (1/6) for f L > v +
61 double precision function eva_fl_to_vp(gg2,gL2,mv2,x,mu2,ievo)
62 implicit none
63 integer ievo ! evolution by q2 or pT2
64 double precision gg2,gL2,mv2,x,mu2
65 double precision coup2,split,xxlog,fourPi5q
66 data fourPi5q/39.47841760435743d0/ ! = 4pi**2
67
68 c print*, 'gg2,gL2,mv2,x,mu2,ievo', gg2, !3,gL2,mv2,x,mu2,ievo
69 coup2 = gg2*gL2/fourPi5q
70 split = (1.d0-x)**2 / 2.d0 / x
71 if(ievo.eq.0) then
72 | xxlog = dlog(mu2/mv2)
73 else
74 | xxlog = dlog(mu2/mv2/(1.d0-x))
75 endif
76
77 eva_fl_to_vp = coup2*split*xxlog
78 return
79 end
80 c /* *****
81 c EVA (2/6) for f L > v -
82 double precision function eva_fl_to_vm(gg2,gL2,mv2,x,mu2,ievo)
83 implicit none
84 integer ievo ! evolution by q2 or pT2
85 double precision gg2,gL2,mv2,x,mu2
86 double precision coup2,split,xxlog,fourPi5q
87 data fourPi5q/39.47841760435743d0/ ! = 4pi**2
88
89 coup2 = gg2*gL2/fourPi5q
90 split = 1.d0 / 2.d0 / x
91 if(ievo.eq.0) then
92 | xxlog = dlog(mu2/mv2)
93 else
94 | xxlog = dlog(mu2/mv2/(1.d0-x))
95 endif
96
97 eva_fl_to_vm = coup2*split*xxlog
98 return
99 end

```