

(A microscopic model of) quarkonia production (in heavy ion collisions)

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Based on arxiv 2206.01308 (accepted for publication in PRC) and on 2305.10750 (freshly submitted)

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and Pays de la Loire



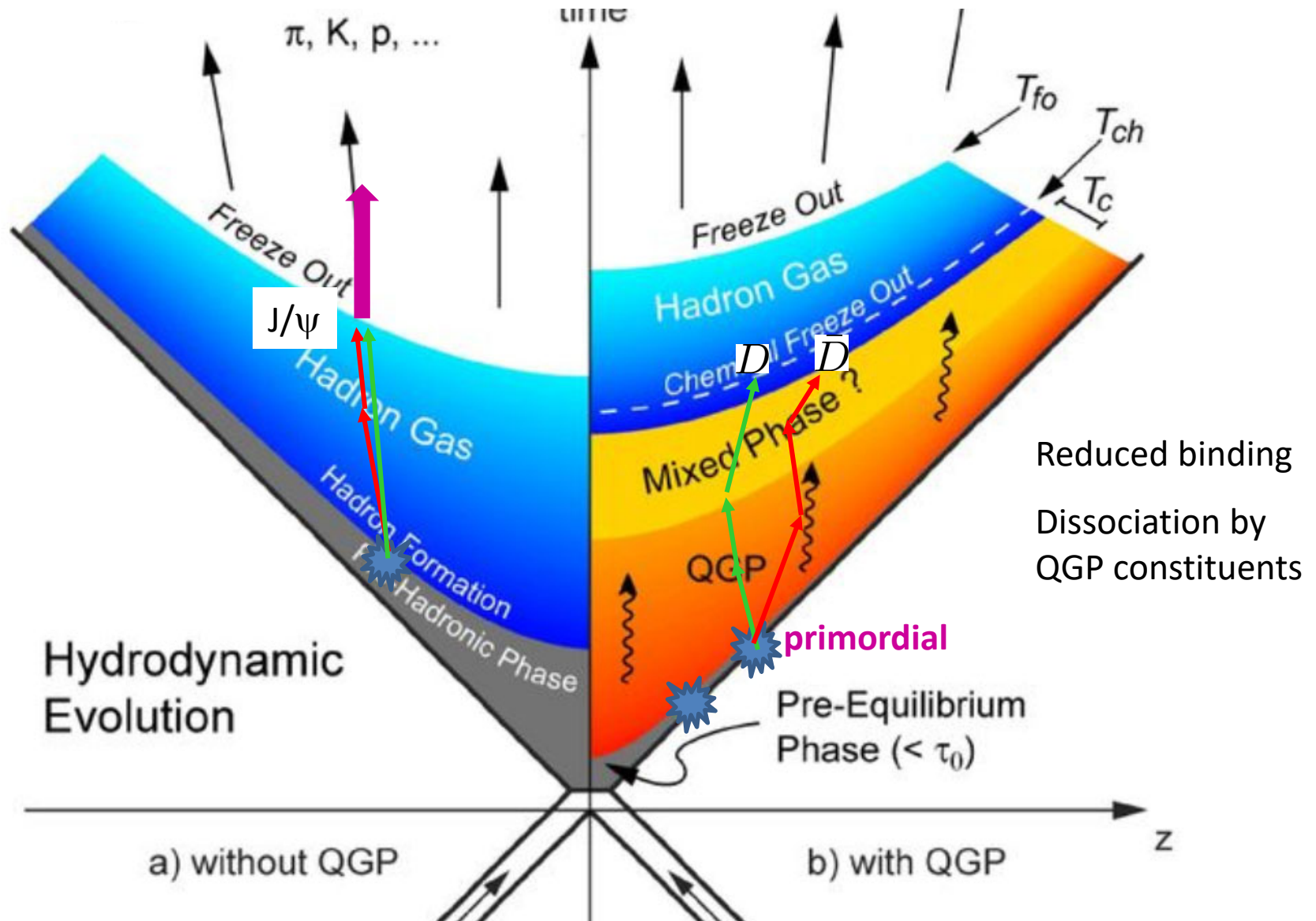
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Quarkonia production in Heavy Ion Collisions

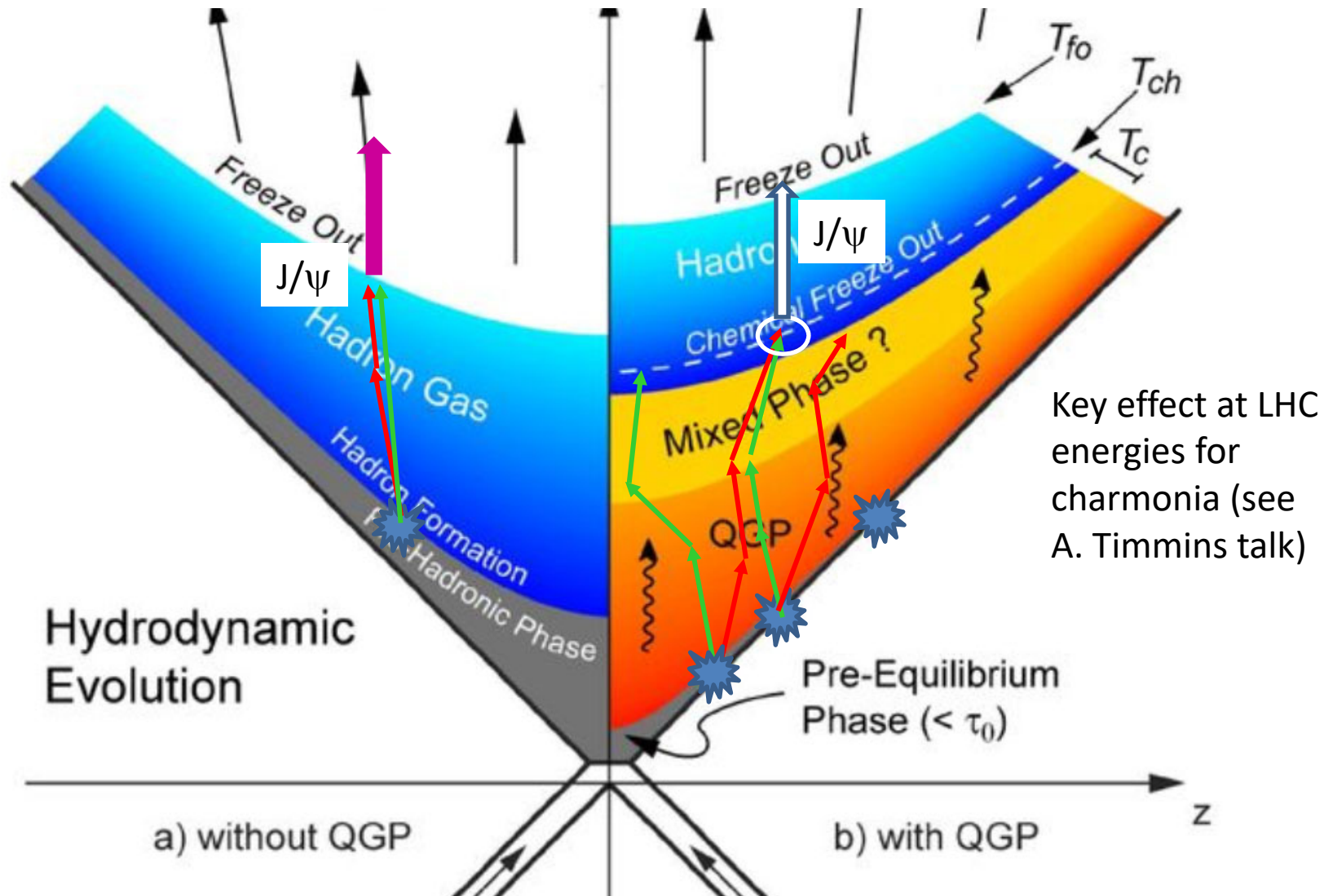
Two main effects if Quark Gluon Plasma: (sequential) **suppression**

Matsui & Satz 1986



Quarkonia production in Heavy Ion Collisions

Two main effects if Quark Gluon Plasma: **Regeneration (production from 2 distinct initial Q-Qbar pairs)** Braun-Munzinger, Stachel & Andronic + Thews (early 2000)

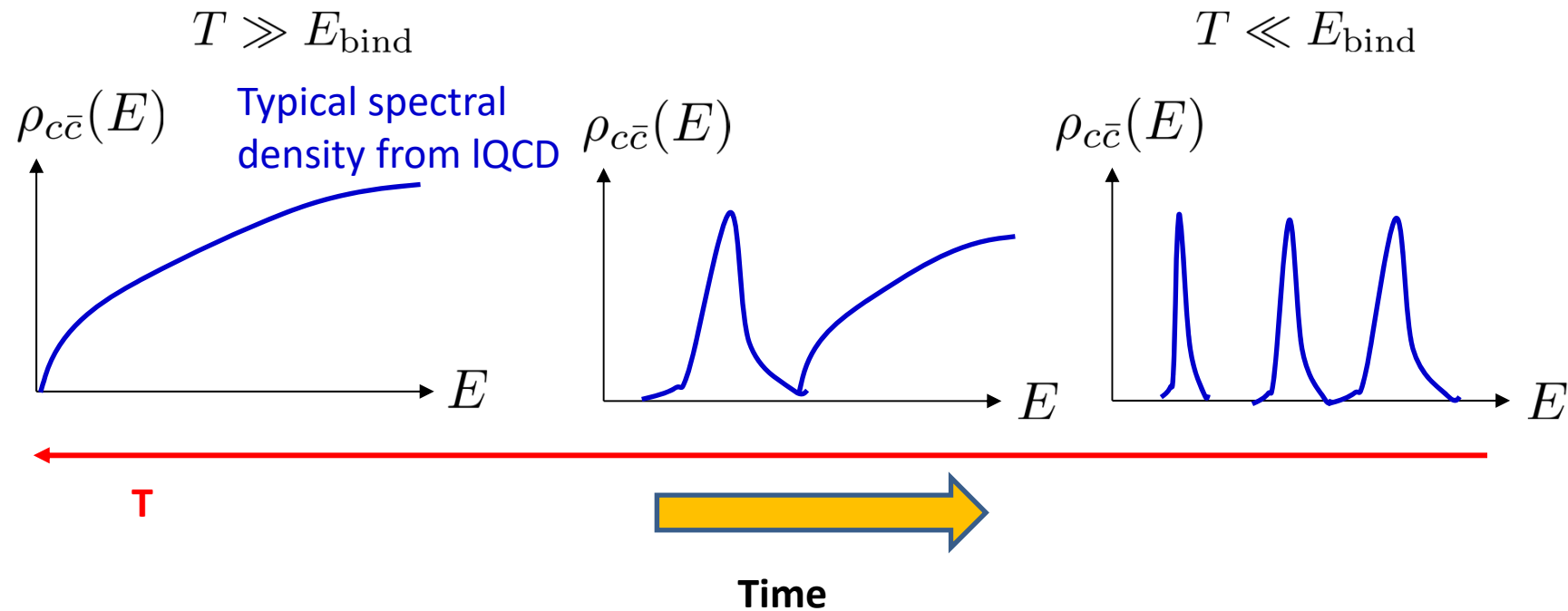


Quarkonia production in Heavy Ion Collisions

These 2 mechanisms and the abundance of experimental data have the potentialities to better understand the forces acting on HQ inside QGP and probe its (short lived) evolution.

BUT

One is facing a dual question : what is the nature of a quarkonia inside a QGP ?



Two types of dynamical modelling

(and a 3rd class of its own: statistical hadronization)

$$T \gg E_{\text{bind}}$$

Quantum Brownian Motion

$$T \sim E_{\text{bind}}$$

$$T \ll E_{\text{bind}}$$

Quantum Optical Regime

- Correlations growing with cooling QGP
- Best described in position-momentum space
- Time short wrt quantum decoherence time

Quantum Master Equations for **microscopic dof (QS and Qbars)**

?

- Well identified resonances
- Time long enough wrt quantum decoherence time

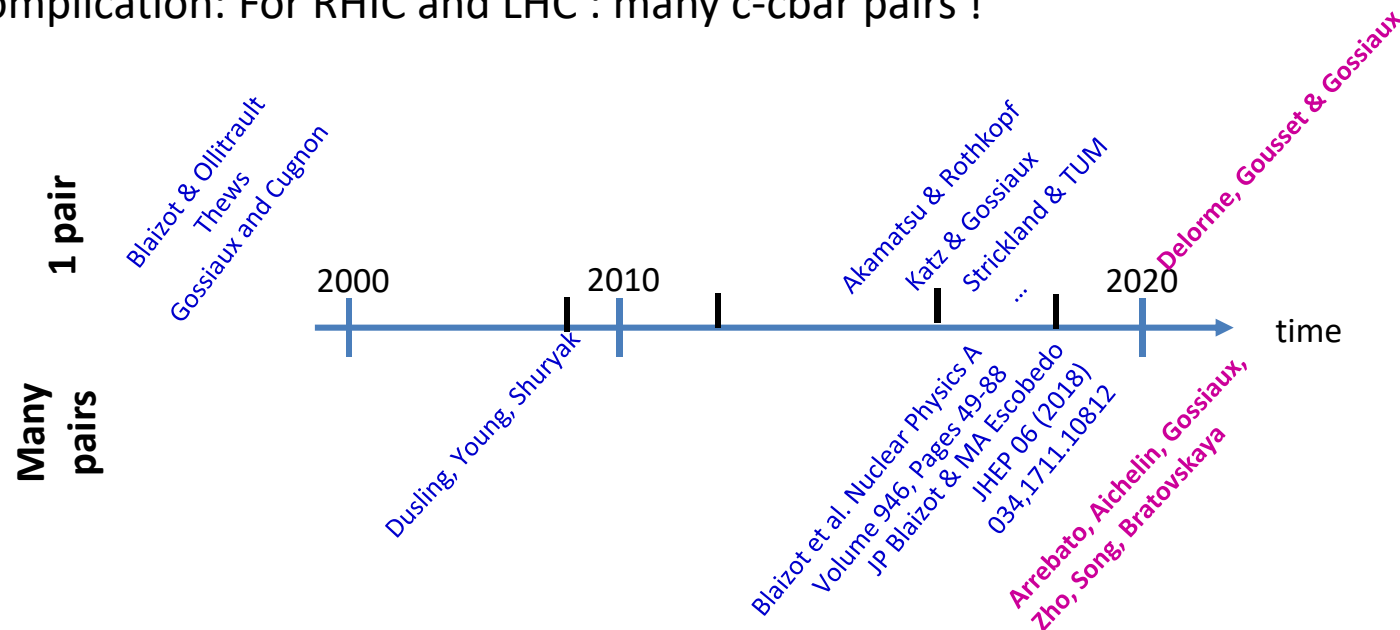
Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections,
decay rates

Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these model is an important prerequisite !!!

Several motivations to go microscopic & quantum

- The in-medium quarkonia are not born as such. One needs to develop an **initial compact state** to fully bloomed quarkonia
- The dissociation-recombination reactions affecting quarkonia are **not instantaneous**... In dense medium, the notion of cross section should be replaced by the more rigorous open-quantum system approach (**continuous transitions**)
- Better suited for « **from small to large** »
- Extra complication: For RHIC and LHC : many c-cbar pairs !



Pioneering work of **Blaizot and Escobedo** for many c-cbar pairs => Semi-Classical Fokker-Planck + gain/loss rates for color transitions; awaits for implementation in realistic conditions

The spirit of the method...

$$P^\Psi(t) = \text{Tr} \left[\hat{\rho}_{Q\bar{Q}}^\Psi \hat{\rho}_N(t) \right] \quad \text{Simply taken at the end of the evolution (ideal world)}$$

$$\hat{\rho}_{Q\bar{Q}}^\Psi = \sum_i |\Psi_{Q\bar{Q}}^i\rangle \langle \Psi_{Q\bar{Q}}^i|$$

$$\frac{d\hat{\rho}_N(t)}{dt} = -i \left[\hat{H}_N, \hat{\rho}_N(t) \right]$$

Various Quarkonia bound states (in vacuum)

Unfortunately... all N-body practionners know that **modelling the full system up to the last stage is quite challenging !** Issues of stability, energy conservation,...

Clear lesson from the « old » cascade and QMD codes for fragment formation



Replace « final » => « initial » + Sum of time steps and chop off at the appropriate time scale

$$P^\Psi(t) = P^\Psi(t_0) + \int_{t_0}^t \Gamma(t') dt'$$

Caution : Not the usual decay rate

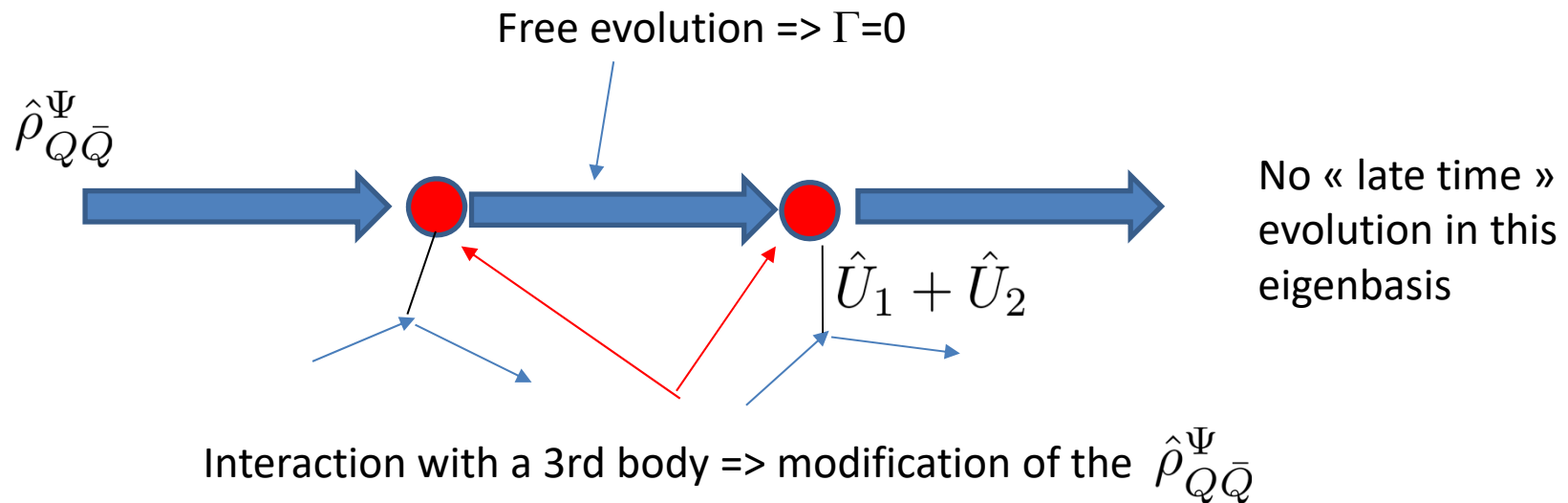
Convergence towards statistical equilibrium in a fixed temperature QGP recently demonstrated in arxiv 2302.14001

The spirit of the method...

Dealing with the dynamics ?

Von Neumann equation

If eigenstates of the « internal » 2-body (QQbar) interaction



...



$$\Gamma^\Psi(t) = -iTr[\hat{\rho}^\Psi[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Total interaction of Q and Qbar with all light partons

Source of « destruction » \Leftrightarrow imaginary potential

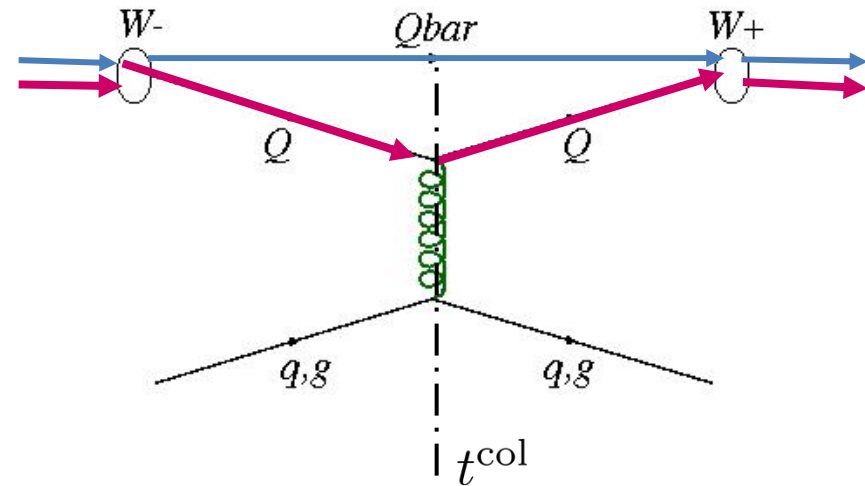
Remler Formalism at work

Level of the modelling : **semi classical** for the Q-Qbar evolution => Wigner distributions instead of density operators

Combining the expression of the Wigner's distribution and substituting in the **effective rate equation** :

$$\Gamma^\Psi(t) \approx \sum_{i=1,2} \sum_{j \geq 3} \delta(t-t_{ij}^{\text{col}}) \int \frac{d^3 p_i d^3 x_i}{h^3} \left[W_{Q\bar{Q}}^\Psi(p_1, x_1; p_2, x_2) \Big|_{t+\epsilon} - W_{Q\bar{Q}}^\Psi(p_1, x_1; p_2, x_2) \Big|_{t-\epsilon} \right]$$

- The quarkonia production in this model is a three body process; the HQs interact only by collisions with the QGP !!!
- The “details” of H_{int} between HQ and bulk partons are incorporated into the evolution of W_N after each collision / time step (nice feature for the MC simulations)
- Dissociation and recombination treated in the same scheme



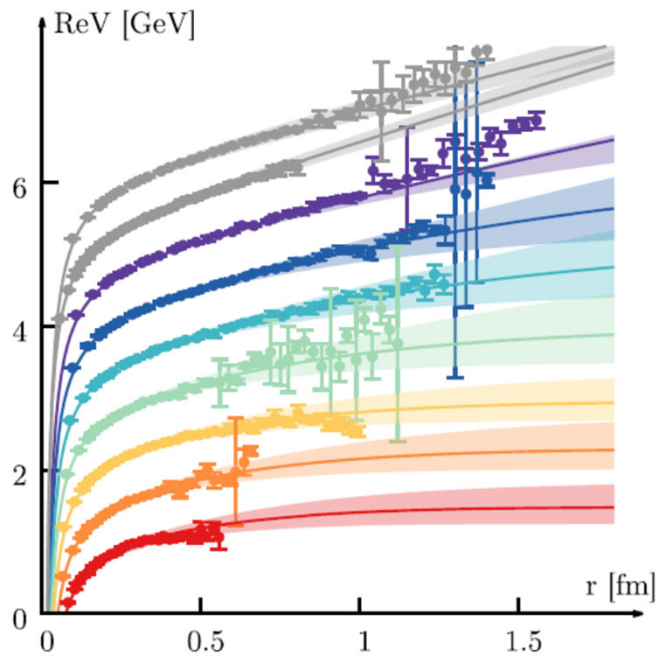
Interaction of HQ with the QGP are carried out by EPOS2+MC@HQ (good results for D and B mesons production) Eur. Phys. J. C (2016) 76:107

Then: $P^\Psi(t) = P^\Psi(t_0) + \int_{t_0}^t \Gamma(t') dt'$

NB: Also possible to generate similar relations for differential rates

Extension of the Remler formalism

- **Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs... **and correlated trajectories.****



(No internal potential in early applications dedicated to deuteron production in low energy AA collisions; advocated to be negligible... as only the « hot zone » was contributing

But for quarkonia, it turns out not to be the case => need for in-medium potential

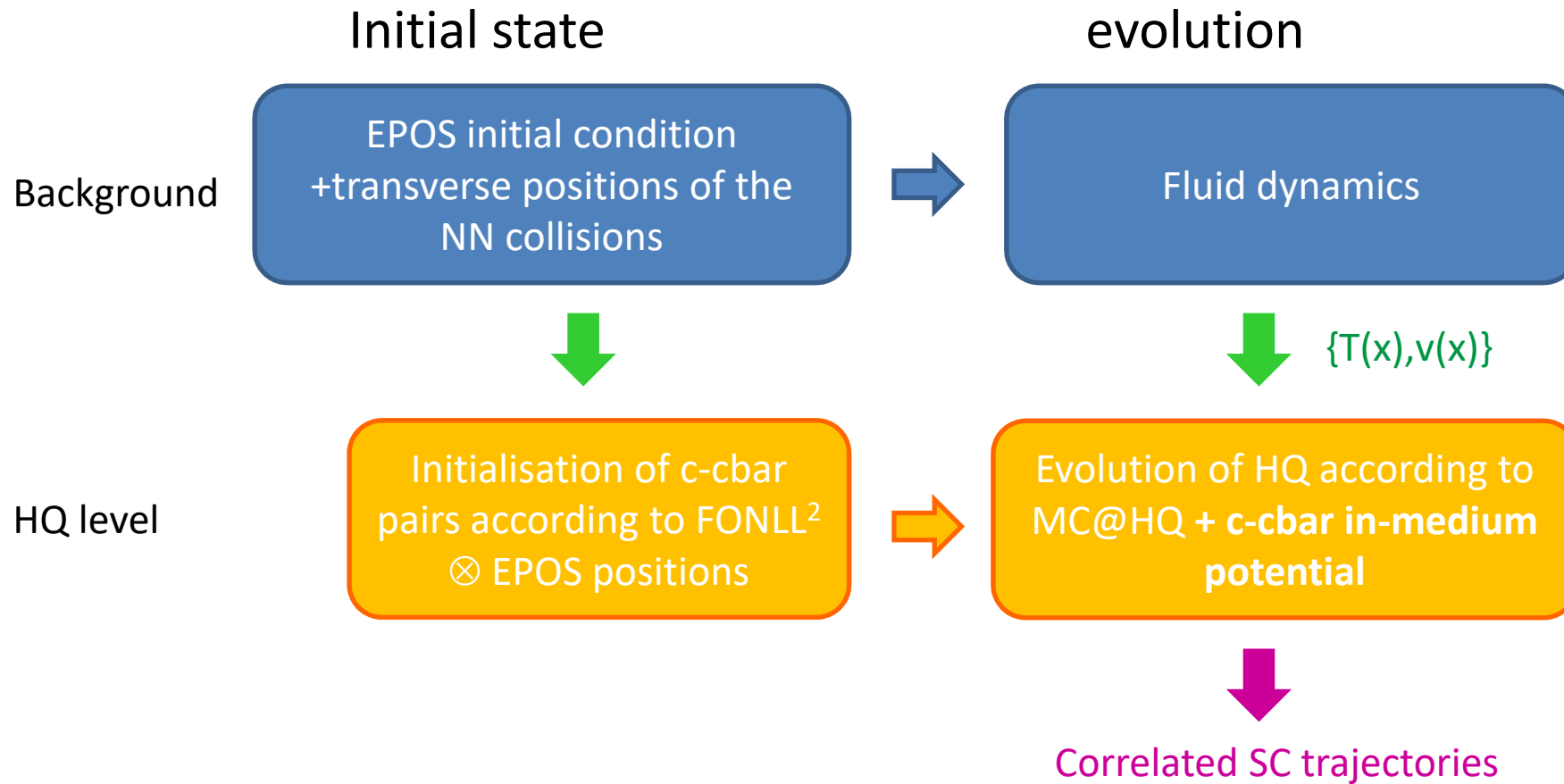
D. Lafferty and A. Rothkopf,
PHYS. REV. D 101, 056010 (2020)



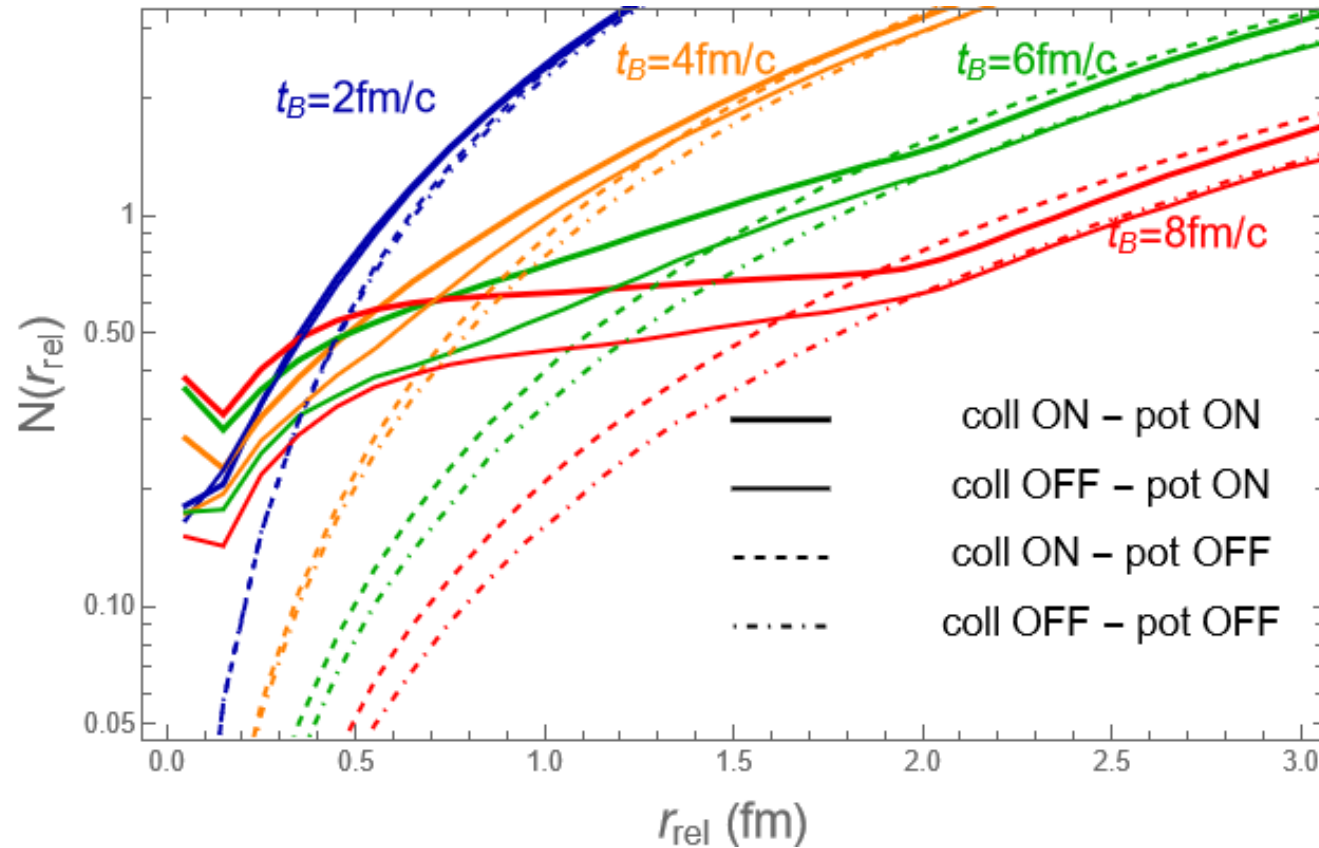
Correlated SC trajectories

Complicated relativistic N-body problem... Only stable at « not too high » p_T

The 3 layers of the numerical modelling

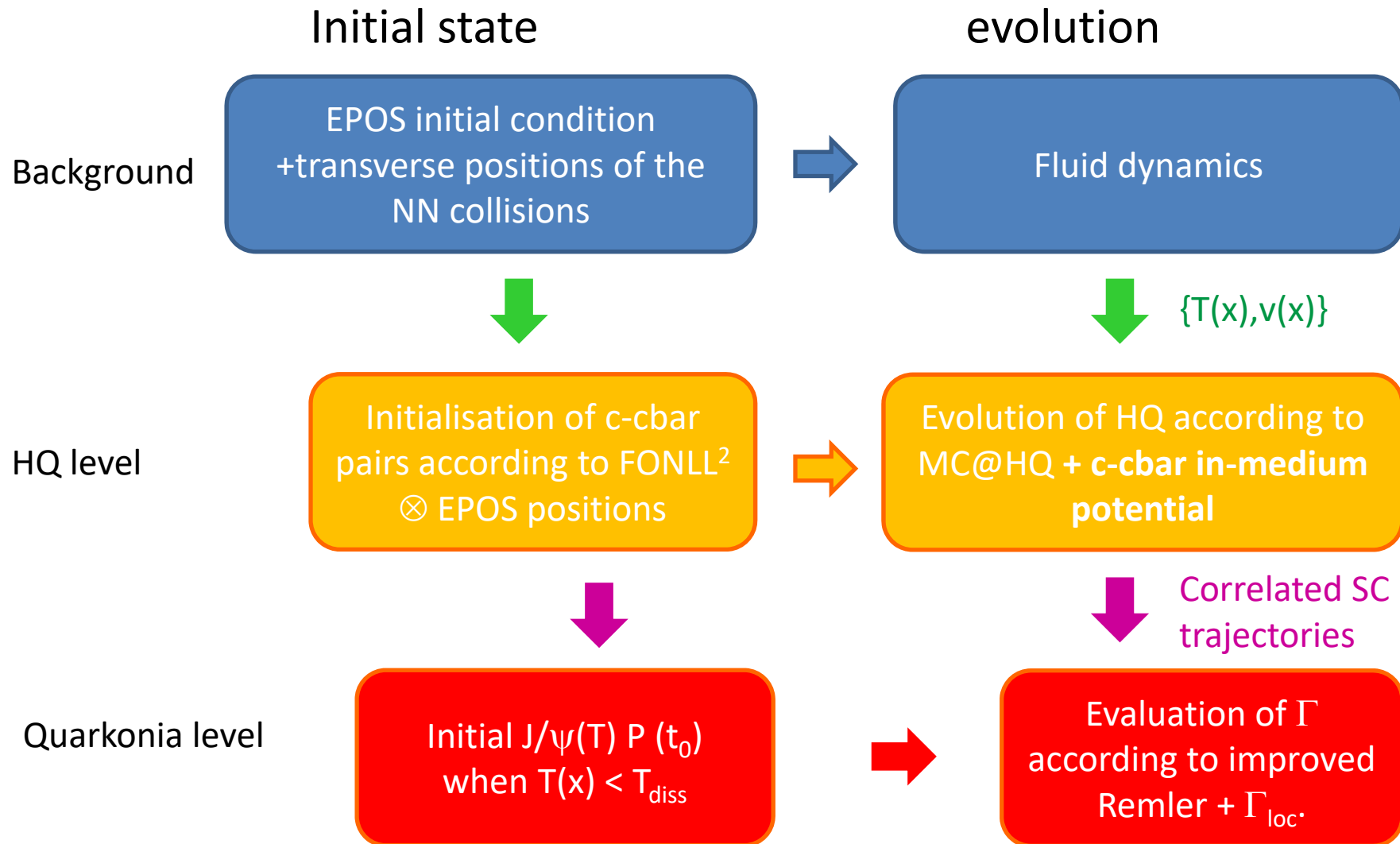


The dynamics of c-cbar correlation



- The c-cbar potential (« pot ON ») leads to a **huge increase of the c-cbar probability at close distance** at large times (not a random Poisson distribution !)...
- ... Especially when the collisions with the QGP (« coll ») are switched ON as well

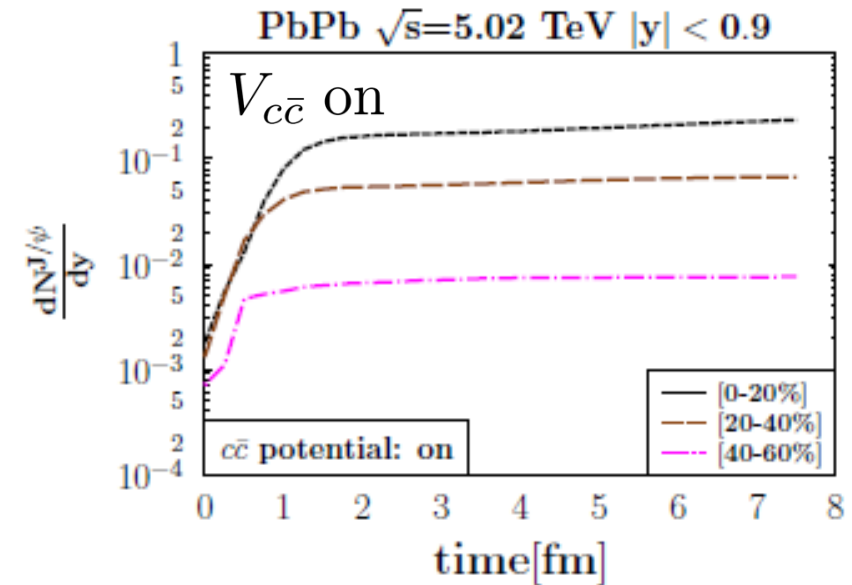
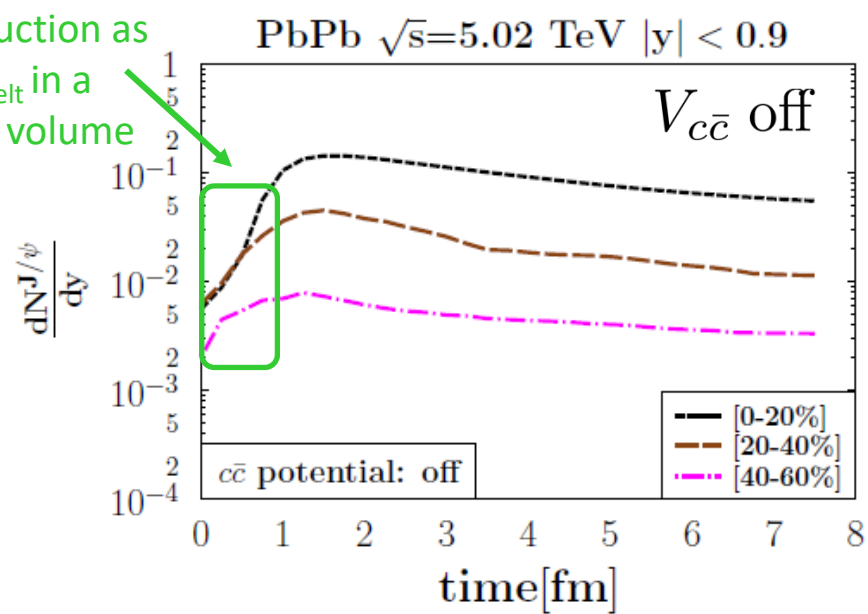
The 3 layers of the numerical modelling



We do not have J/ψ quasi particles in our approach, just correlated c-cbar trajectories

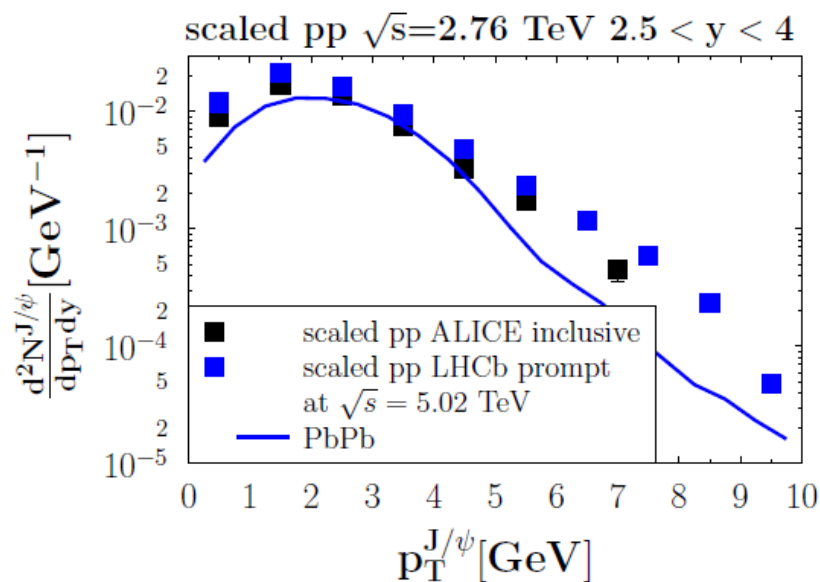
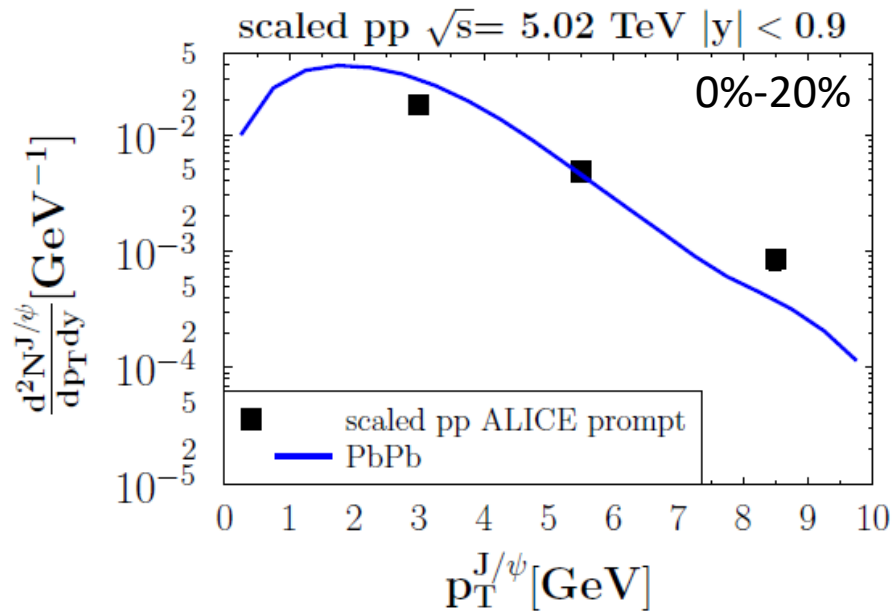
Results : J/ψ production vs time

Delayed initial production as $T > T_{\text{melt}}$ in a large volume



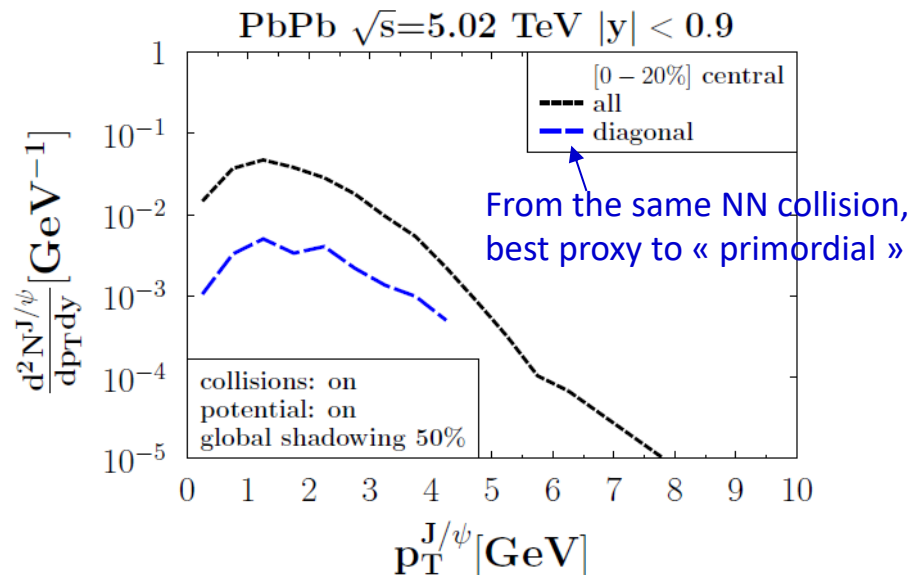
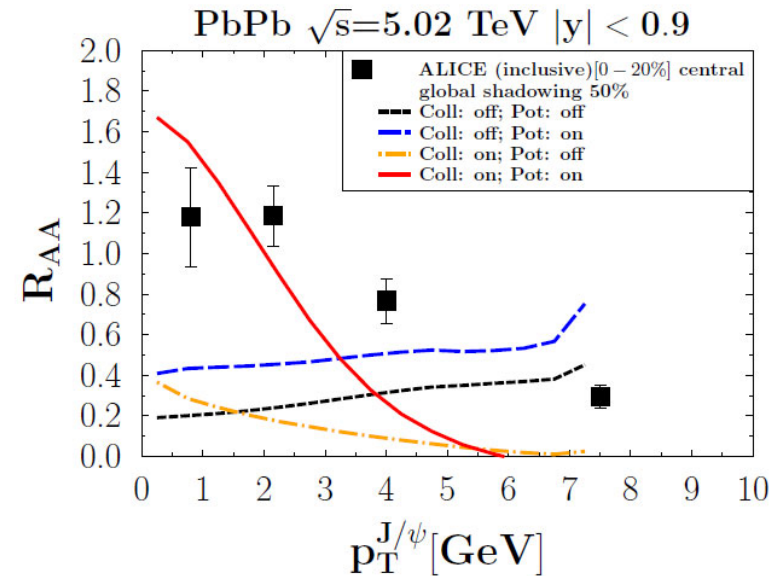
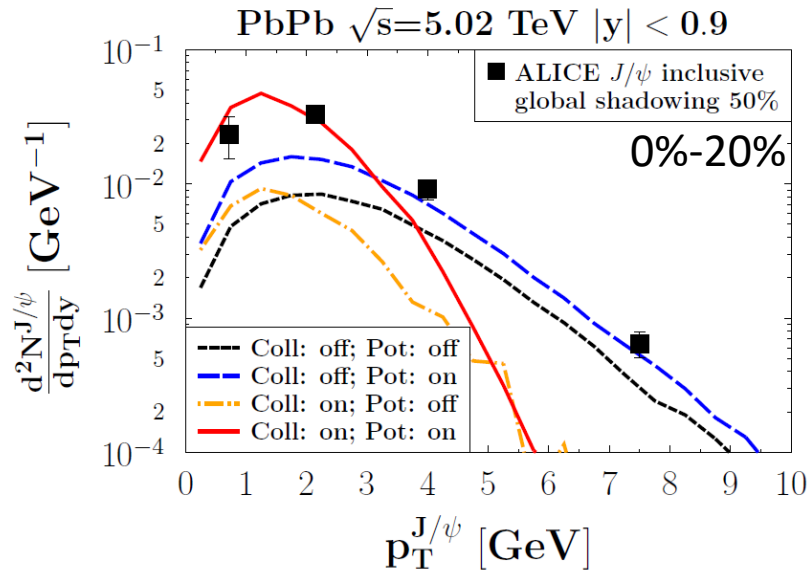
- Without interaction potential between c and $c\bar{c}$, the collisions with the medium manage to destroy the native J/ψ (left)
- With the interaction potential between c and $c\bar{c}$ « on », one observes a steady rate of J/ψ creation (increase of Γ^{col} , increase of Γ^{local})... No adiabaticity, but **no instantaneous formation** either.

Results : J/ψ production vs p_T



- **Equivalent pp production** (the denominator of the R_{AA}) : c-cbar according to FONLL² without any correlation, then coalescence with the Wigner distribution.
- No feed-down from higher states (to be implemented)
- Acceptable for $p_T < 5$ GeV/c, but deviations for higher p_T .
- To investigate : more appropriate scheme for c-cbar production, including c-cbar correlation : **EPOS4**

Results : J/ψ production vs p_T

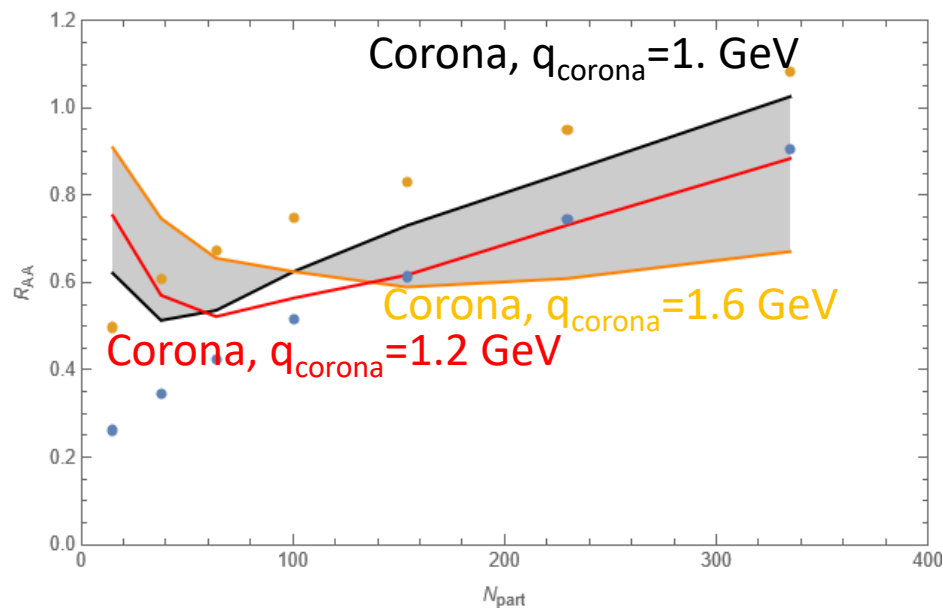
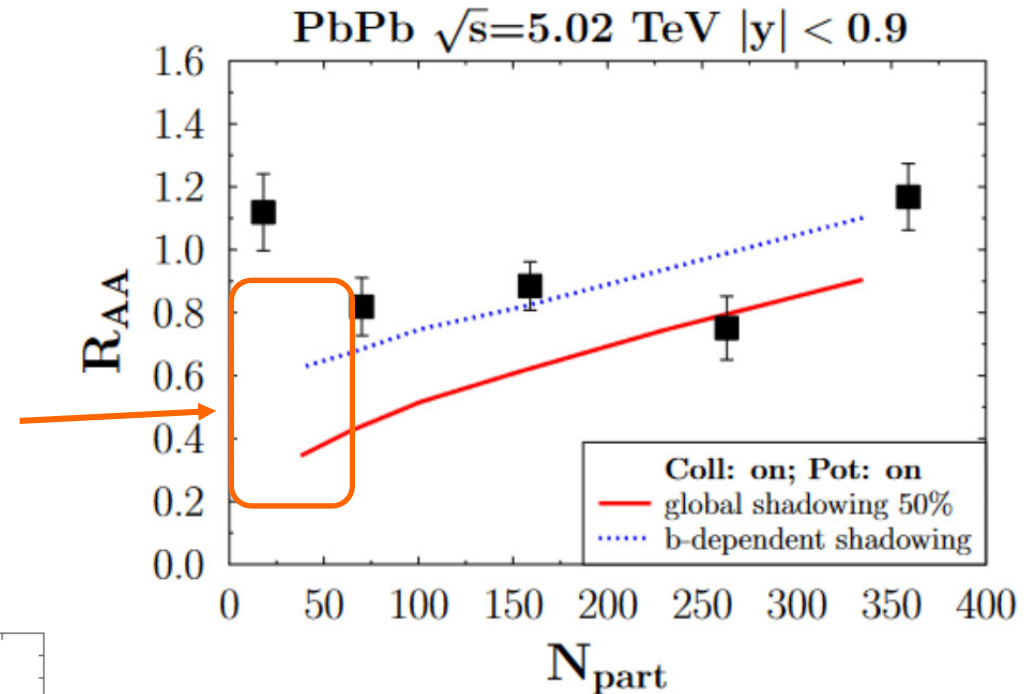


- Dynamical recombination is quite effective at low p_T
- At higher p_T , we are missing J/ψ as compared to the experimental value.
- Several possible reasons, under investigation:
 - in terms of transport model : « primordial too much suppressed »
 - lack of c - \bar{c} correlation in the IS
 - ...

R_{AA} vs N_{part}

Caveat : too crude modelling of the thermalization in the bulk... assumed to happen after 0.35 fm/c independent of the centrality

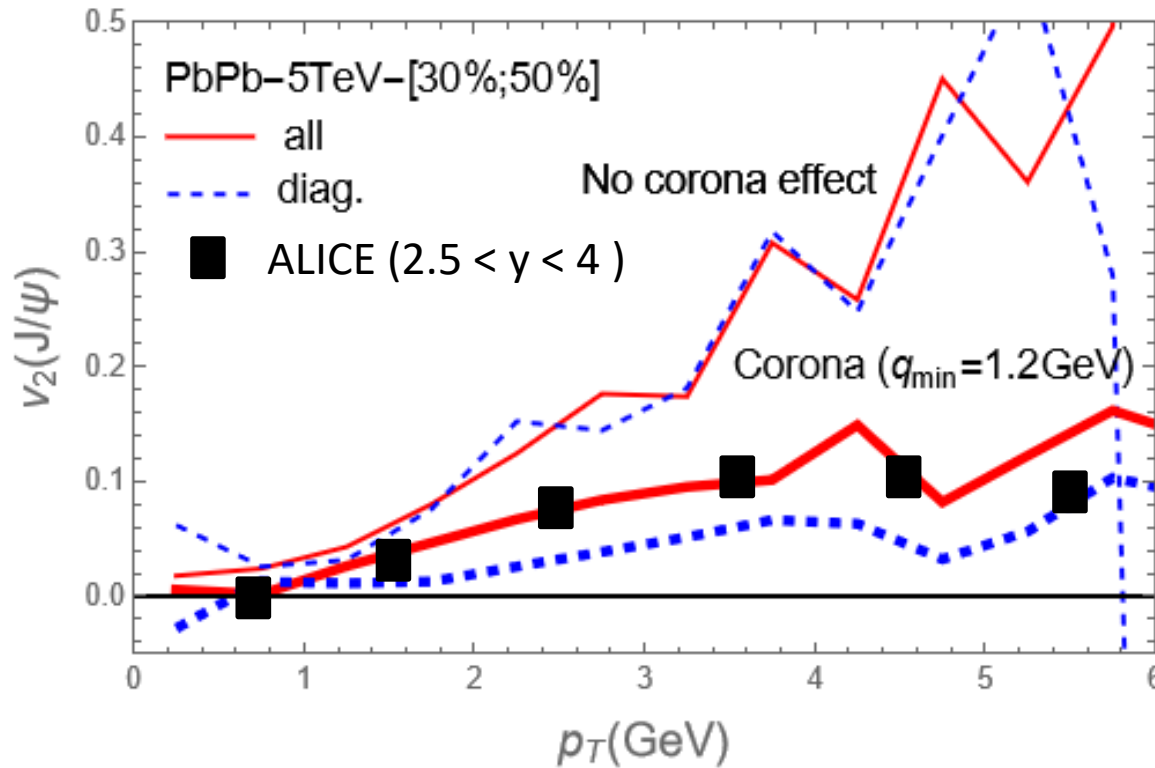
=> c and cbar created at $t \approx 0$ have the time to diffuse away => reduction of the production



One possible solution : core – corona model for c-quarks : c-quarks with momentum transfer $< q_{corona}$ are considered to combine -> quarkonia as in vacuum...

Optimal value : Corona, $q_{corona}=1.2 \text{ GeV}$

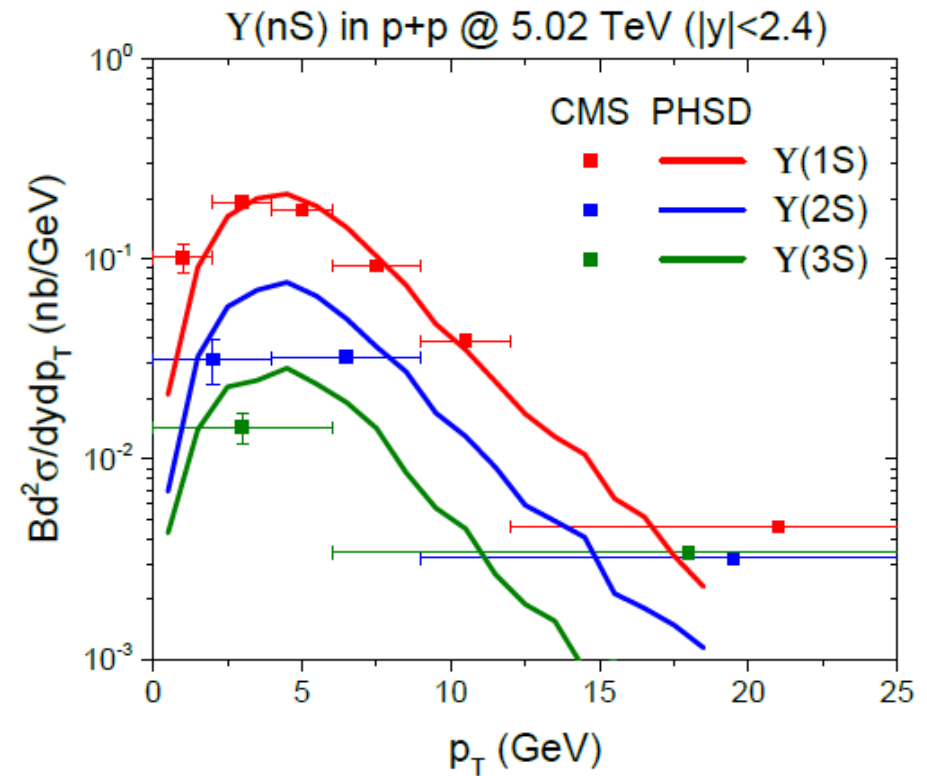
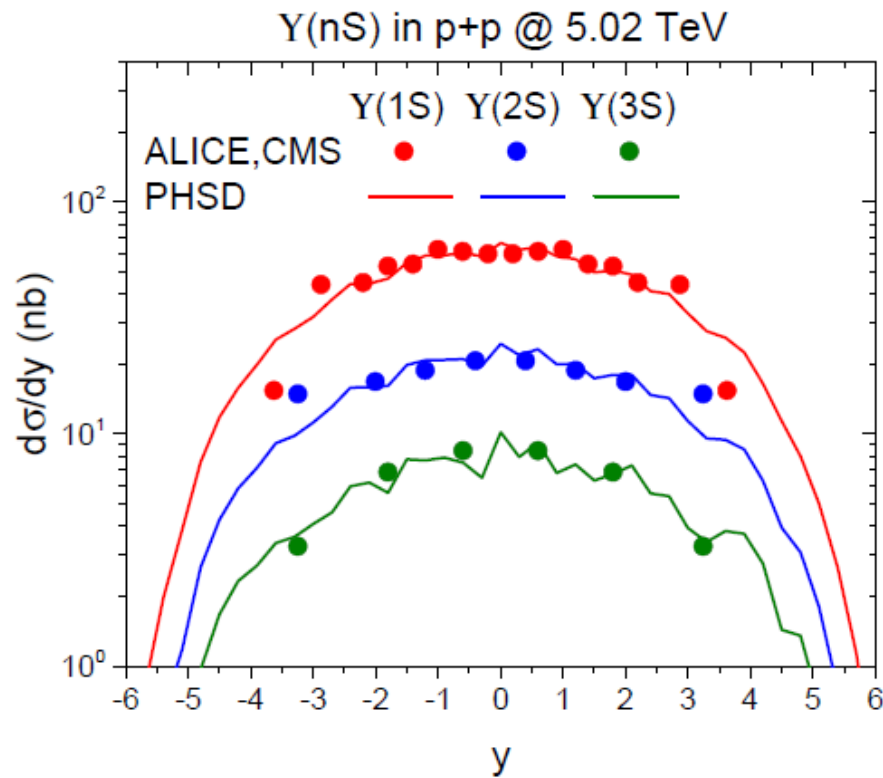
Results : J/ ψ v_2



- v_2 excess as compared to experimental data (**late formation** of the J/ ψ due to binding potential under restoration)
- Without corona, the « diagonal » contribution shows no difference wrt the full production, what is a bit counter-intuitive
- Corona has large effect on v_2 , even with moderate q_{thresh} . As the corona mostly affects the diagonal part, one recovers $v_2^{\text{diag}} < v_2^{\text{all}}$

Recent news

Applying the Wigner coalescence approach for bottomonia in pp (and in PbPb), starting from b-bbar pQCD spectra: 2305.10750



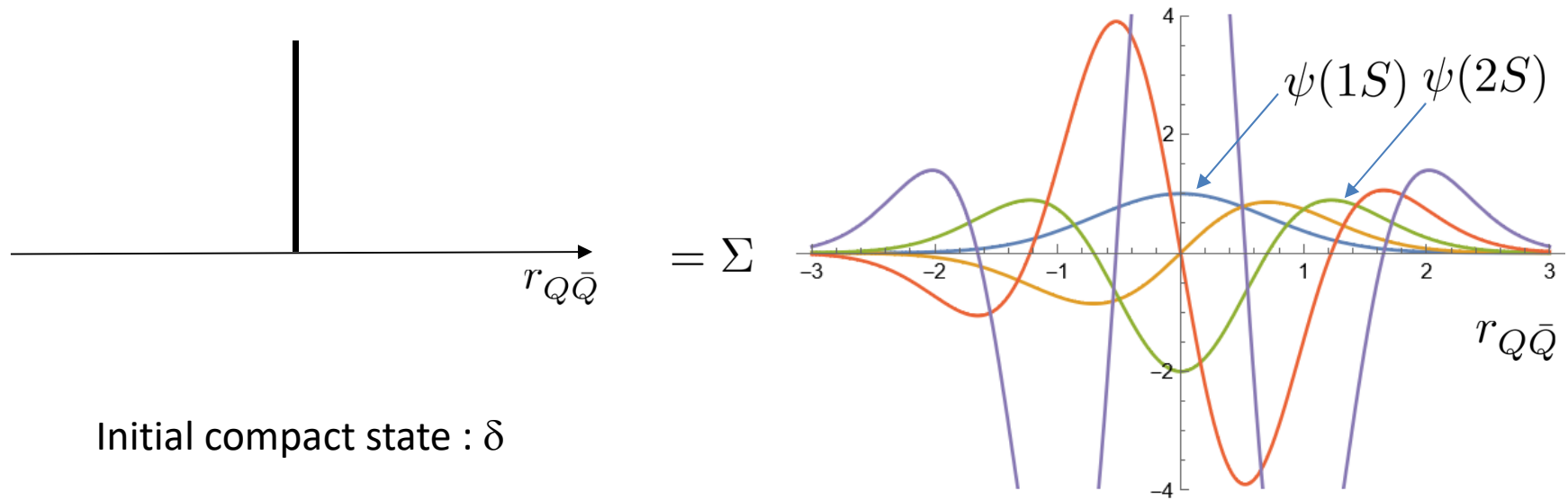
Good overall description with a pretty simple model

Conclusions and Perspectives

- First move towards a microscopic approach based on individual c and \bar{c} dynamics implementing some of the open quantum systems features for charmonia production in realistic HI conditions :
dynamical coalescence
- Encouraging results, but still many features to be refined
- Rooms for improvement :
 - Include excited states decay
 - Including color transparency and more generally color dynamics
 - More reliable treatment of the fully relativistic evolution of a N-body coupled system (under construction)
 - Upgrading to EPOS4 => More realistic « initial state » for the c - \bar{c} pairs, including correlations at intermediate p_T .
 - Late interactions with hadronic phase
 - ...

Back up

Quantum coherence at early time



Dissociation rate: $\Gamma(r_{Q\bar{Q}}) \propto \alpha_S T \times \Phi(m_D r_{Q\bar{Q}}) \sim \alpha_S^2 T^3 \times r_{Q\bar{Q}}^2$

Coherence

Neglect of coherence

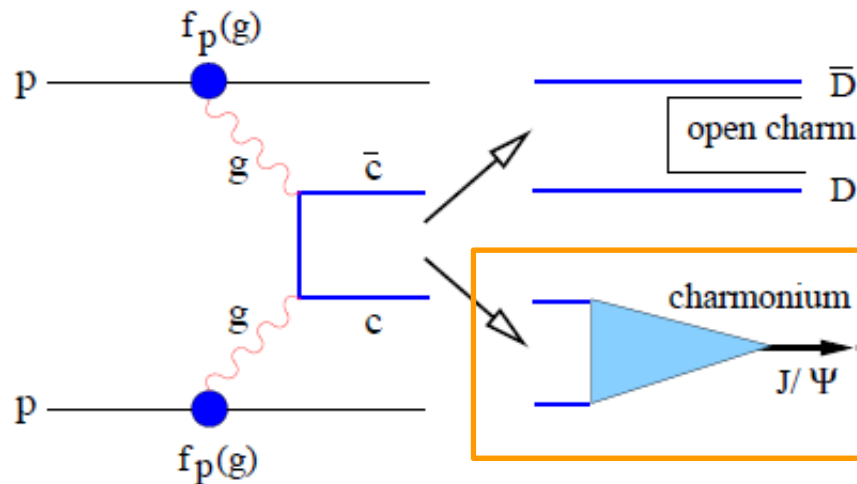
$$\Gamma(r_{Q\bar{Q}}) \approx 0 \propto \sum c_j^* c_i \langle \psi_j | r^2 | \psi_i \rangle \longrightarrow \Gamma \propto \sum_i |c_i|^2 \langle \psi_i | r^2 | \psi_i \rangle \approx \sum_i |c_i|^2 \Gamma_i \neq 0$$

Crucial to include coherence !

N.B. : one can model this effect by phenomenological formation time, but lack of control

Quantum coherence

Picture behind transport theory :



Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some “formation time” τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Common belief in the transport community:

Quarkonia initially « formed » in QGP and survive with an *individual* survival probability

$$S(t) = e^{-\int_{\tau_f}^t \Gamma(T(t')) dt'}$$

Remler's formalism for dynamical coalescence

Generic idea : describe charmonia (Ψ) production using density matrix

$$P^\Psi(t) = \text{Tr} \left[\hat{\rho}_{Q\bar{Q}}^\Psi \hat{\rho}_N(t) \right]$$

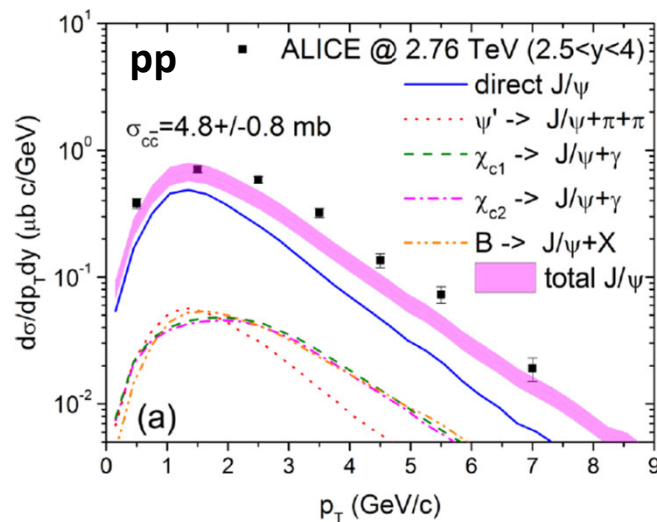
$$\hat{\rho}_{Q\bar{Q}}^{\Psi_i} = \sum_i |\Psi_{Q\bar{Q}}^i\rangle \langle \Psi_{Q\bar{Q}}^i|$$

Single quarkonia density operator

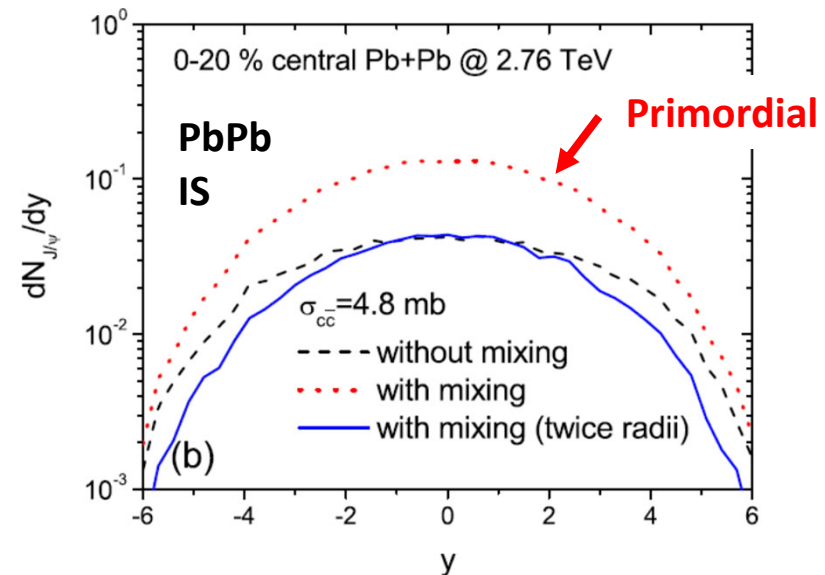
N-body density matrix (bulk partons + many c and many cbar)

“Just” looking at the **initial stage** brings interesting features:

T. Song, J. Aichelin and E. Bratkovskaya,
PRC 96. 014907 (2017)



Good reproduction of pp \rightarrow J/ ψ + x !!!

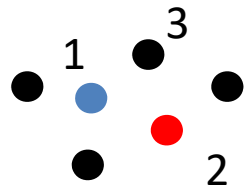


considerable enhancement of primordial J/ ψ (in the **initial state**): **large off-diagonal contributions**

Remler formalism at work

Combining the rate definition + VN equation: $\Gamma^\Psi(t) = -iTr[\rho^\Psi[H_N, \rho_N(t)]]$

Generic case where $H_N = \sum_i K_i + \sum_{i>j} V_{ij}$



1 & 2: c & \bar{c}
3, 4, : light quarks

Strictly speaking, not QCD. Important process partly missing : gluo-dissociation



$$H_N = H_{1,2} + H_{N-2} + U_1 + U_2$$

$c\bar{c}$ Internal Hamiltonian

Light quarks

$$\sum_{i>2} V_{i1} \quad \sum_{i>2} V_{i2}$$

Heavy – light interaction

$$\Gamma^\Psi(t) = -iTr[\rho^\Psi[H_N, \rho_N(t)]] = -iTr[\rho_N(t)[\rho^\Psi, H_N]]$$



Only U_1 and $U_2 \Rightarrow \neq 0$ (as $[\rho^\Psi, H_{1,2}] = 0$)

$$\Gamma^\Psi(t) = -iTr[\hat{\rho}^\Psi[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Sub-part of the VN equation, still impossible to deal with exactly at the quantum n-body level

Remler formalism at work

Passing to the Wigner representation:

$$W_N(\{r\}, \{p\}) = \int \Pi d^3 y e^{ipy} \langle r - \frac{y}{2} | \hat{\rho}_N | r + \frac{y}{2} \rangle$$

Direct space

$$\partial \rho_N(t) / \partial t = -i \sum_j [K_j, \rho_N(t)] - i \sum_{j>k} [V_{jk}, \rho_N(t)]$$

Wigner space....

$$\partial W_N(t) / \partial t = \langle \sum_i v_i \cdot \partial_r W_N(\mathbf{r}, \mathbf{p}, t) \rangle + \langle \sum_{i \geq j} \sum_n \delta(t - t_{ij}(n)) \times (W_N(\mathbf{r}, \mathbf{p}, t + \epsilon) - W_N(\mathbf{r}, \mathbf{p}, t - \epsilon)) \rangle$$

One to one correspondance

... treated at the semi-classical level :

Wigner distribution \Leftrightarrow {trajectories in phase space}

➡ $[\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]$ can be modelized from the trajectories evolution in Wigner space

Remler formalism at work

The effective rate for quarkonia state creation (dissociation) in the medium is

$$\Gamma^\Psi(t) = -i \text{Tr}[\hat{\rho}^\Psi [\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

Working in the phase space through Wigner distribution

$$W_{Q\bar{Q}}^{\Psi_i} = \int d^3y e^{ipy} \langle r - \frac{y}{2} | \Psi^i \rangle \langle \Psi^i | r + \frac{y}{2} \rangle$$

Quarkonia: Double Gaussian approximation

$$W_{Q\bar{Q}}^\Psi(r_{\text{rel}}, p_{\text{rel}}) = C e^{r_{\text{rel}}^2 \sigma^2} \times e^{-\frac{p_{\text{rel}}^2}{\sigma^2}}$$

Parameter: The Gaussian width $\sigma \approx 0.35$ fm

$$[\frac{\hbar^2}{2\mu} \nabla^2 + V(r)] \Psi_{Q\bar{Q}}(r) = E_{Q\bar{Q}} \Psi_{Q\bar{Q}} \rightarrow \langle r^2 \rangle \rightarrow W^\Psi$$

W_N : Semi-classical approach

$$W_N = \prod_i \hbar^3 \delta(x_i - x_{i0}(t)) \delta(p_i - p_{i0}(t))$$

... but no explicit description of W_N required (as it appears in the trace)

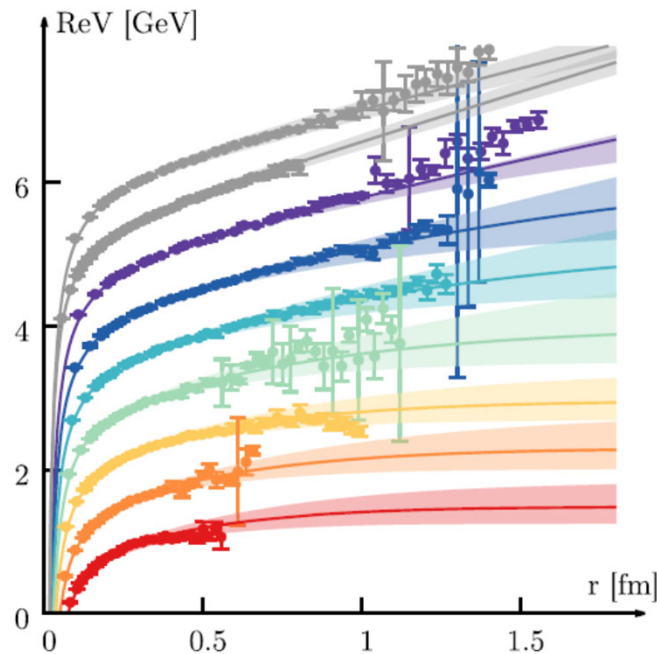
and (less trivial) : generalisation at finite 4-velocity u ; fully relativistic... to warrant orthogonality of states

$$\text{Tr}[W_u^{J/\psi} W_u^{\psi'}] = 0$$

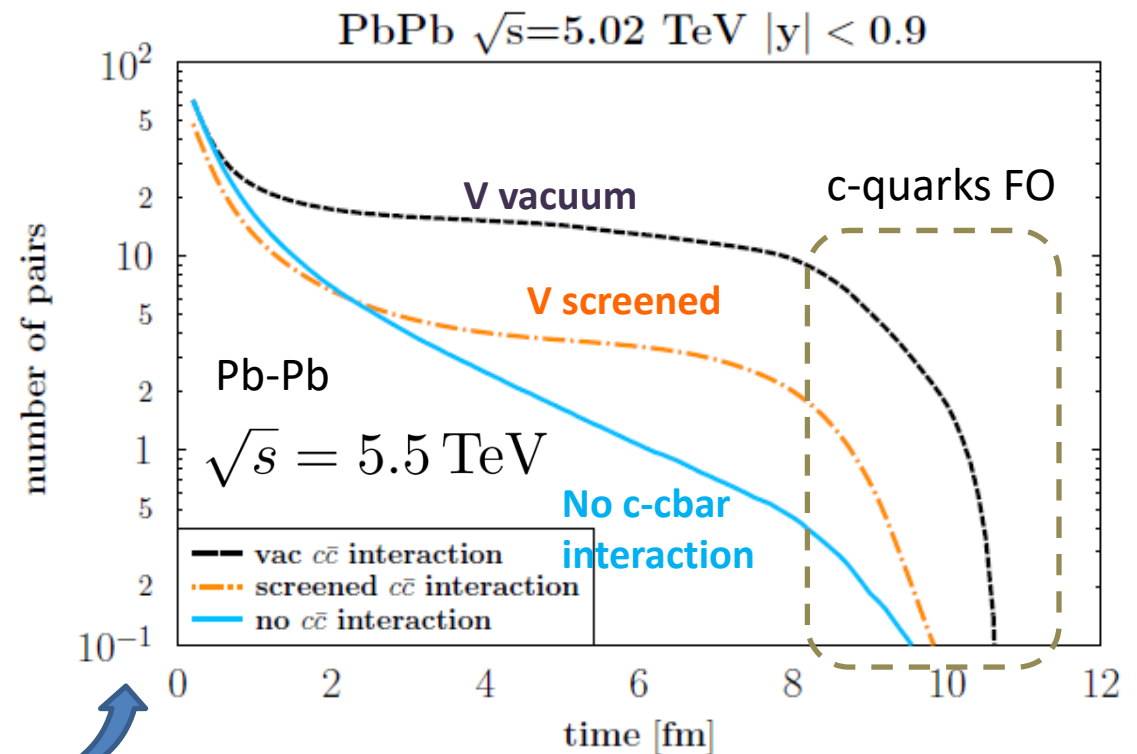
Extension of the Remler formalism

- Confining $Q\bar{Q}$ forces inside the MC evolution ; large impact on the # of close pairs... **and correlated trajectories.**

D. Lafferty and A. Rothkopf,
PHYS. REV. D 101, 056010 (2020)



Instantaneous # of Q-Qbar at (invariant) distance < 1fm

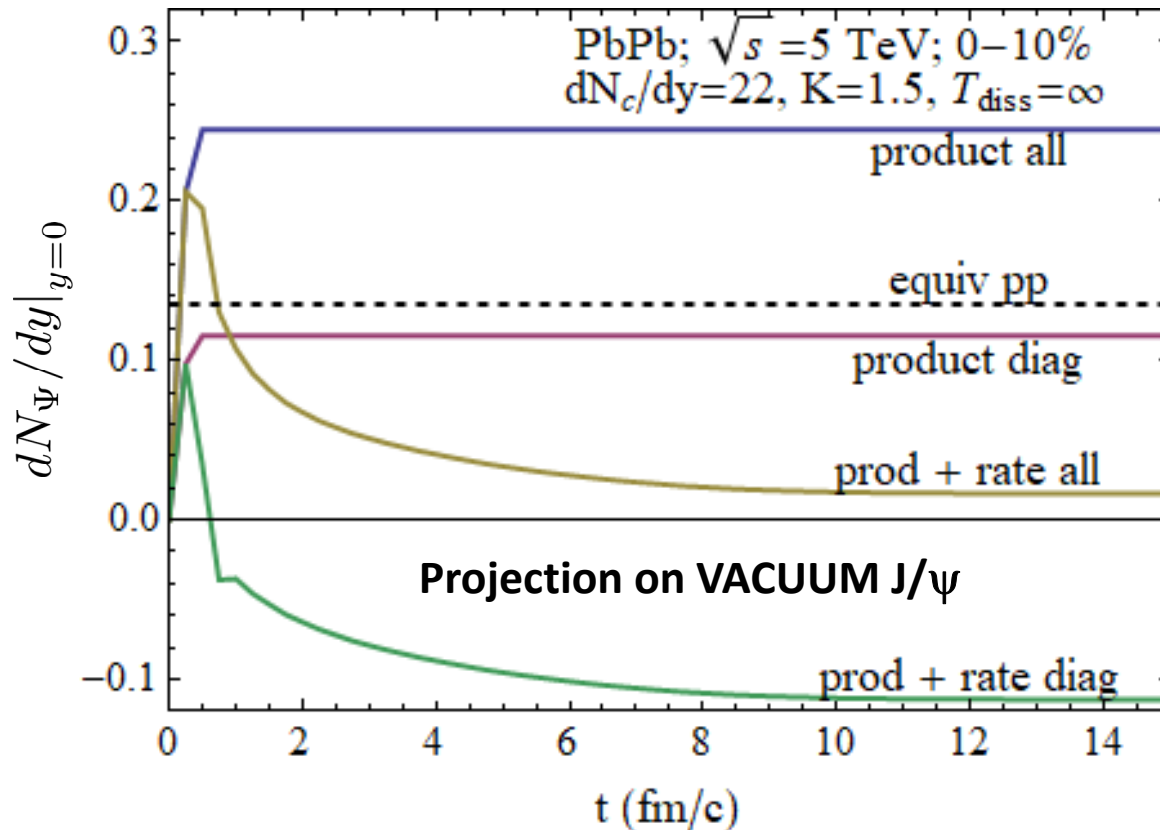


- Extra source of Γ due to “local-T” basis evolution with time : Γ^{loc}
- Generalization to relativistic Wigner density (boosted quarkonium states)

Preliminary results for J/ψ production in Pb-Pb

Word of caution: Exploratory phase => not meant to have an exact comparison with exp. data

$$P^\Psi(t) = P^\Psi(t_0) + \int_{t_0}^t \Gamma(t') dt'$$



Cumulated « production » (if no rate equation), indeed overshoots pp due to off-diagonal contributions

The denominator in the R_{AA}

The full production (i.e. the numerator in the R_{AA})

First answer to puzzle found in Song et al: the primordial production is killed rather fast by the « loss » rate.

Remler formalism for the QGP : last ingredient

Combining the rate definition + VN equation: $\Gamma^\Psi(t) = -i\text{Tr}[\rho^\Psi [H_N, \rho_N(t)]]$

$$\begin{array}{ccccccc} \rightarrow & & H_N = & H_{1,2} & + & H_{N-2} & + & U_1 & + & U_2 \\ & & & \uparrow & & \uparrow & & \uparrow & & \swarrow \\ & & c\bar{c} & \text{Internal Hamiltonian} & & \dots & & \dots & & \dots \end{array}$$

In QGP, 2 body T-dependent effective potential =>

$$\Gamma^\Psi(t) = -i\text{Tr}[\rho^\Psi [H_N, \rho_N(t)]] = -i\text{Tr}[\rho_N(t) [\rho^\Psi, H_N]]$$

$$\downarrow [\rho^\Psi, H_{1,2}(T)] = 0$$

$$= -i\text{Tr}[\hat{\rho}^\Psi(T) [\hat{U}_1 + \hat{U}_2, \hat{\rho}_N(t)]]$$

One only preserves the structure of the Remler « collisional rate » if one works in the « local » basis $\rho^\Psi(T)$!!!

Accessible for $T > T_{\text{dissoc}}^\Psi (=0.4 \text{ GeV for } J/\psi)$

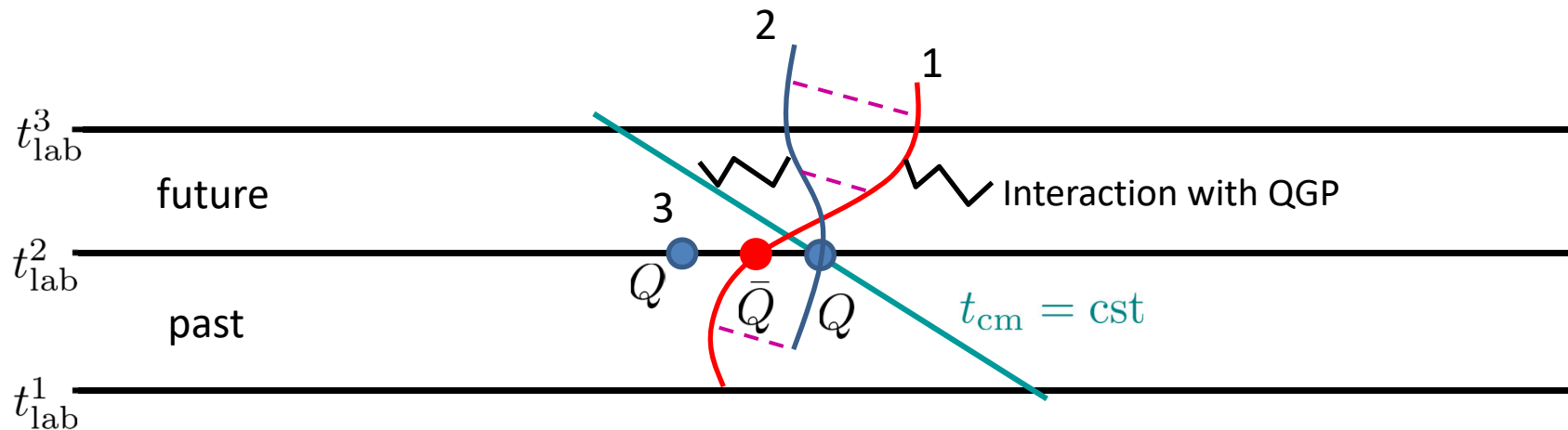
Back to the rate : $\Gamma^\Psi(t) = \frac{dP^\Psi(t)}{dt} = \text{Tr} \left[\hat{\rho}_{Q\bar{Q}}^\Psi \frac{d\hat{\rho}_N(t)}{dt} \right]$

$$\rightarrow \Gamma^\Psi(t) = \text{Tr} \left[\hat{\rho}_{Q\bar{Q}}^\Psi(T(t)) \frac{d\hat{\rho}_N(t)}{dt} \right] + \underbrace{\frac{dT}{dt} \text{Tr} \left[\frac{\partial \hat{\rho}_{Q\bar{Q}}^\Psi(T)}{\partial T} \hat{\rho}_N(t) \right]}_{\text{New contribution to the rate (so-called « local rate »)}}$$

New contribution to the rate (so-called « local rate »)

The Q-Qbar dynamics... the CM strategy

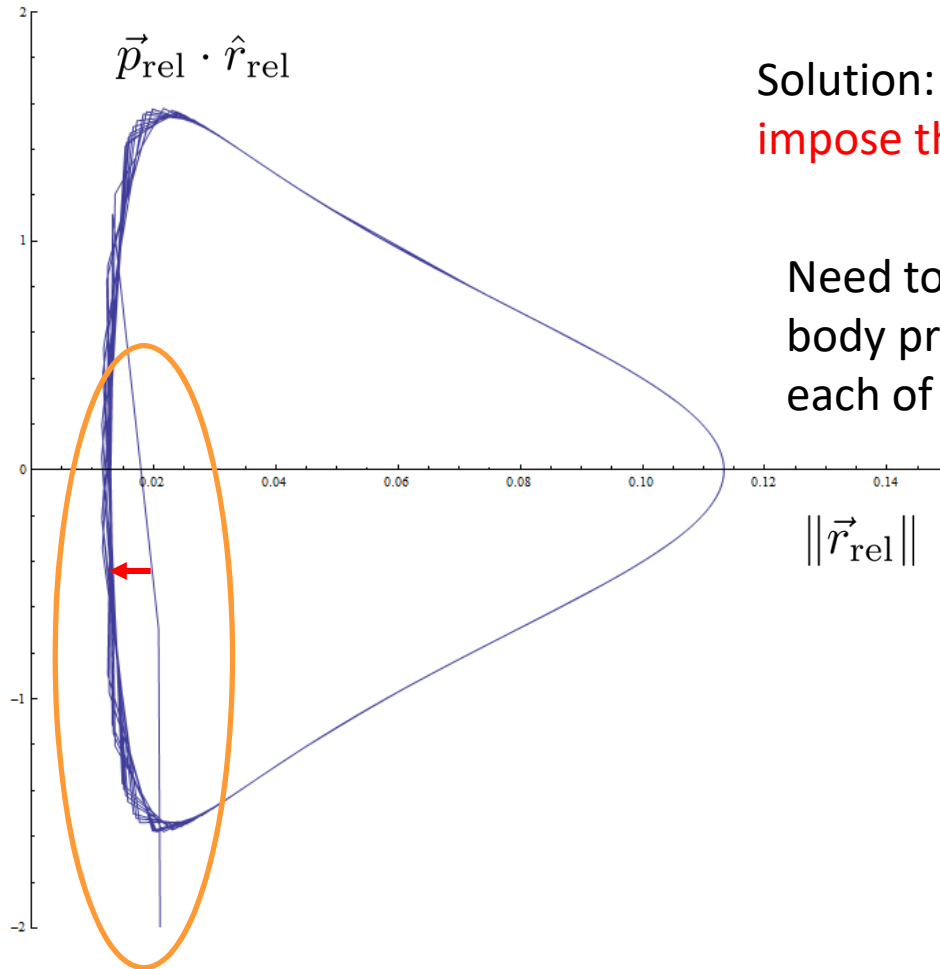
- Main objective : evaluate the propagation towards future of N Q-Qbar **interacting** pairs
- Strategy (adopted presently) : For each time step in the laboratory frame, pass to the cm frame and perform the evolution in the cm frame (where the potential is well defined)



- “Issue” : slicing the global time evolution (usual strategy in MC) is not 100% compatible with passing to c.m. frame as 2 particles are usually not at the same relative time in both frames (residual glitches \Leftrightarrow numerical noise)

The Q-Qbar dynamics... the CM strategy

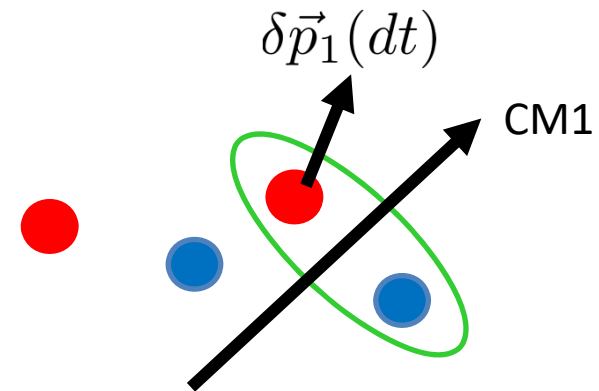
- “Minor problem” #1: Classical equations of motion are **unstable** (in the CM):



Solution: **Work in Hamilton – Jacobi coordinates or impose the conserved quantities (L and Etot)**

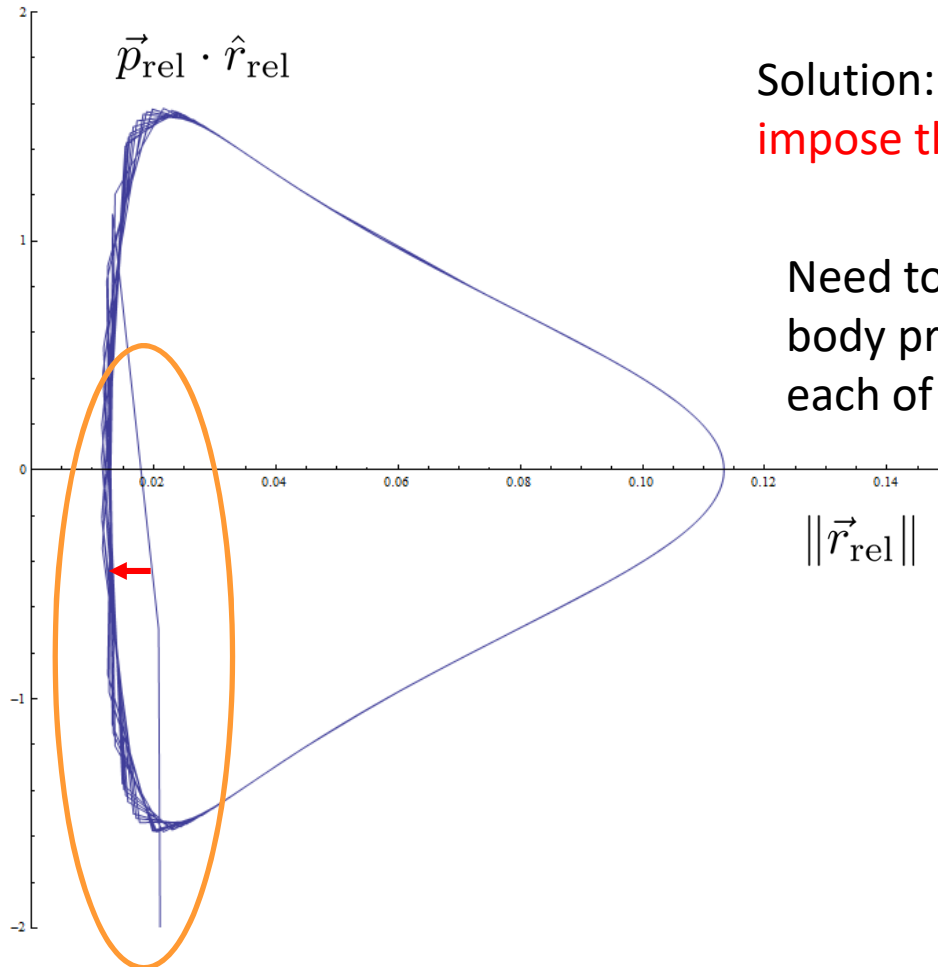


Need to factorize the N-body problem as an {} of 2-body problems for some evolution over time step dt, each of them to be solved in the CM



The Q-Qbar dynamics... the CM strategy

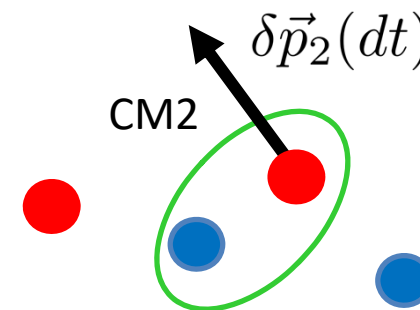
- “Minor problem” #1: Classical equations of motion are **unstable** (in the CM):



Solution: **Work in Hamilton – Jacobi coordinates or impose the conserved quantities (L and Etot)**

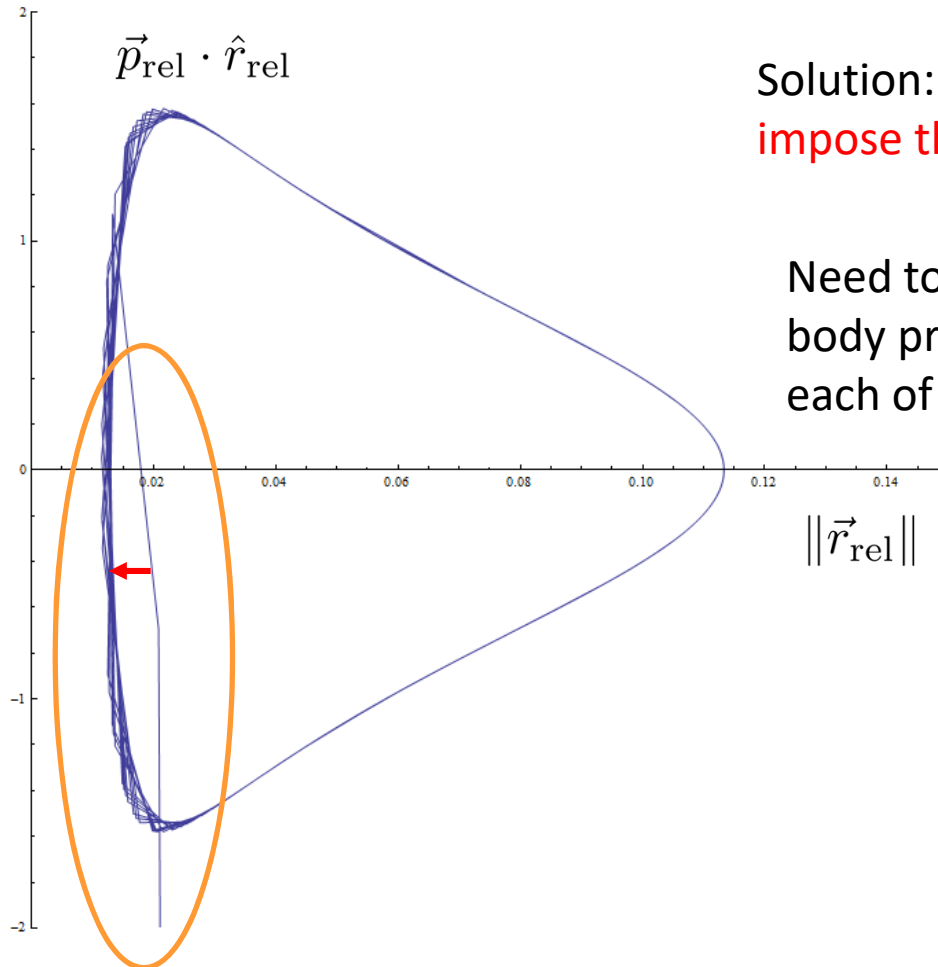


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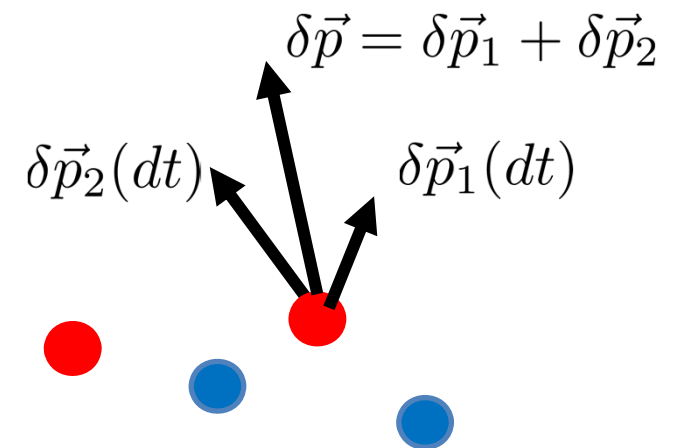
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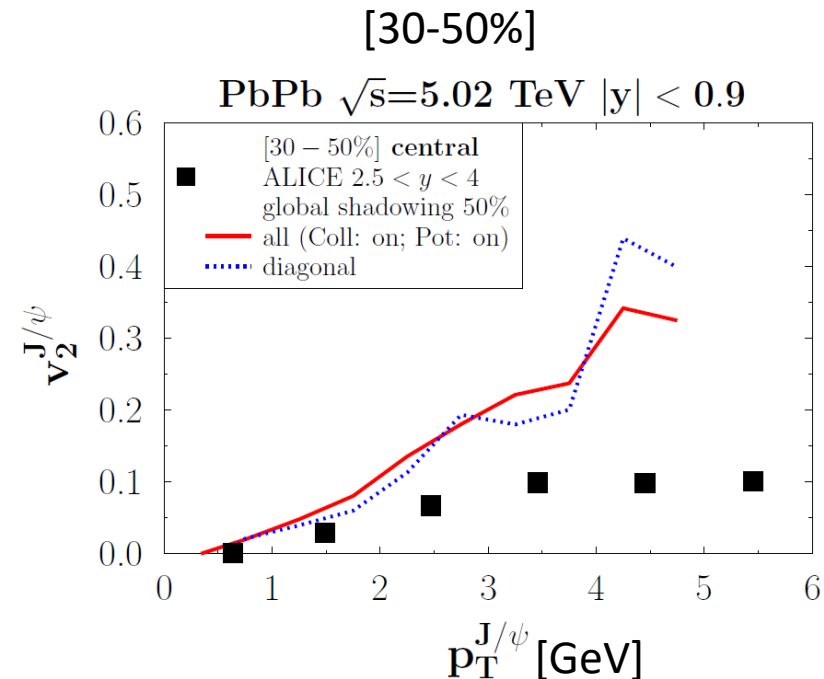
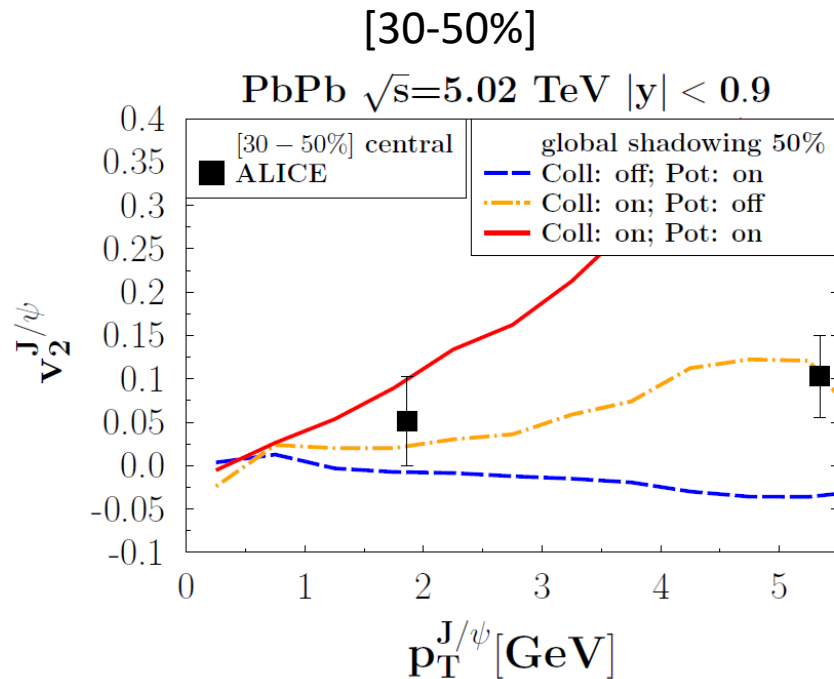
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Results : J/ψ v_2



- v_2 excess as compared to experimental data (**late formation** of the J/ψ due to binding potential under restoration)
- The « diagonal » contribution shows no difference wrt the full production, what is a bit counter-intuitive