Status and Outlook for B-mixing parameters on the lattice

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Outline

Flavour Physics: The Cabibbo-Kobayashi-Maskawa matrix

experiment $\approx$ CKM-factors $\times$ non-perturbative inputs $\times$ known terms

CKM Matrix

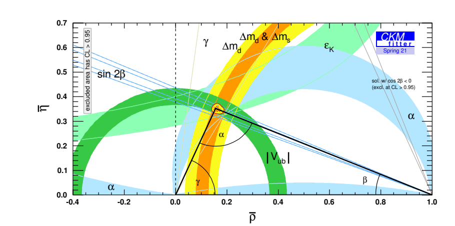
- parameterises transitions up-type $\leftrightarrow$ down-type

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- **fundamental SM parameters**
- **unitary matrix in the SM:**

$$VV^\dagger = \mathbb{1}_3$$

Unitarity Triangle



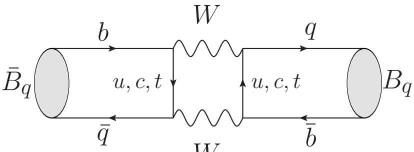
yellow & orange bands = exp. measurements of $\Delta m_{d(s)}$ + SM predictions for mixing parameters

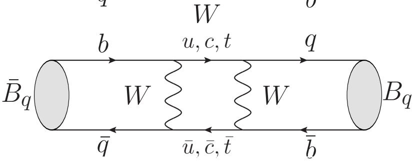
$\Rightarrow$ **Tests CKM unitarity!**

Non-unitarity of CKM $\Leftrightarrow$ New Physics Beyond the SM

Neutral $B_{(s)}$ meson mixing - background

Neutral mesons oscillate:





where $q = d, s$

mass eigenstate $\neq$ flavour eigenstate

$$|B_{L,H}\rangle = p\left|B_q^0\right\rangle \pm q\left|\bar{B}_q^0\right\rangle$$

$\Rightarrow$ **splittings** in mass eigenstates:

- mass splitting $\Delta m_q \equiv m_H - m_L$
- width splitting $\Delta\Gamma_q \equiv \Gamma_L - \Gamma_H$

Time dependence:

$$\left|B_q^0(t)\right\rangle = g_+(t)\left|B_q^0\right\rangle + \frac{q}{p}g_-(t)\left|\bar{B}_q^0\right\rangle$$

$$\left|\bar{B}_q^0(t)\right\rangle = \frac{p}{q}g_-(t)\left|B_q^0\right\rangle + g_+(t)\left|\bar{B}_q^0\right\rangle$$

Occurs at loop level in SM $\Rightarrow$ **sensitive probe of new physics**!

Neutral $B_{(s)}$ meson mixing - experiment

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos\left(\Delta m_q t\right)\right]$$

Δm experimentally accessible as a frequency!

B_d^0: Many results

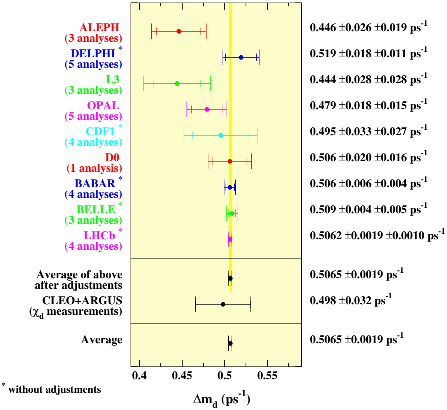
B_s^0: "Only" CDF, CMS and LHCb

$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

$$\Delta m_s = 17.765(06)\mathrm{ps}^{-1}$$

Well below per cent level!

[HFLAV 2206.07501]



Neutral $B_{(s)}$ meson mixing - theory

Short distance dominated $\Rightarrow$ described by $\mathcal{H}^{\Delta b=2}$ eff. weak Hamiltonian. OPE factorises this into

- **Perturbative model-dependent Wilson coefficients** $C_i(\mu)$
- **Non-perturbative model-independent matrix elements**

$$\left\langle B^0_{(s)} \right| \mathcal{H}^{\Delta b=2} \left| \bar{B}^0_{(s)} \right\rangle = \sum_i C_i(\mu) \left\langle B^0_{(s)} \right| \mathcal{O}_i^{\Delta b=2}(\mu) \left| \bar{B}^0_{(s)} \right\rangle$$

- 5 independent (parity even) operators $\mathcal{O}_i$, only $\mathcal{O}_1$ relevant for Δm:

$$\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$$

- Define bag parameters: $\hat{B}^{(i)}_{B_q} = \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle / \left\langle \bar{B}^0_q \right| \mathcal{O}_i \left| B^0_q \right\rangle_{VSA}$

$$\Delta m_q = \left| V^*_{tb} V_{tq} \right|^2 \times f^2_{B_q} \hat{B}^{(1)}_{B_q} \times m_{B_q} \mathcal{K}$$

$\Rightarrow$ **Non-perturbative matrix elements calculable on the lattice**

Extracting CKM matrix elements

We write Δm_q in terms of the Renormalisation Group Independent (RGI) bag parameter $\hat{B}_{B_q}$:

$$\Delta m_q = |V_{tb}^* V_{tq}|^2 \frac{G_F^2 m_W^2 m_{B_q}^2}{6\pi^2} S_0(x_t)\eta_{2B} f_{B_q}^2 \hat{B}_{B_q}^{(1)}$$

- Δm_d and Δm_s are known experimentally to $\ll 1\%$ **accuracy**
- Combined other inputs are known to **0.4%**
- Typical precision for $f_{B_q}\sqrt{\hat{B}_{B_q}^{(1)}}$ is currently **a few percent**.
- Part of statistic and systematic errors cancel in $SU(3)$ breaking ratios:

$$\xi^2 \equiv \frac{f_{B_s}^2 \hat{B}_{B_s}^{(1)}}{f_{B_d}^2 \hat{B}_{B_d}^{(1)}} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}} \qquad \Rightarrow \text{access to } |V_{td}/V_{ts}|$$

Lattice QCD in a nutshell

Based on the **Path Integral** formulation.

$$\langle \mathcal{O} \rangle_M = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{iS[\psi,\overline{\psi},U]}$$

Minkowski: Highly oscillatory, infinite dimensional integral. ✗

⇒ Wick rotate to Euclidean (i.e. imaginary) time ($t \to i\tau$).

$$\langle \mathcal{O} \rangle_E = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \overline{\psi}, U]\, \mathcal{O}_E[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

Euclidean: Exponentially decaying, infinite dimensional integral. ✗

⇒ Discretise space-time and **interpret as a probability distribution**.

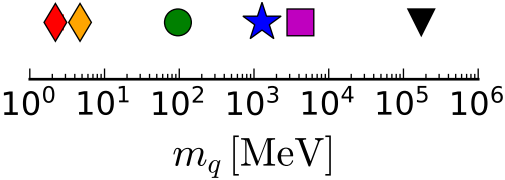
- Lattice spacing a (UV regulator)
- Box of length L (IR regulator)
- $\int \to \sum$, $\partial \to$ finite differences
- Evaluate **stochastically**

Lattice: Exponentially decaying and finite dimensional ✓

Multiple scale problem on the lattice: back of the envelope

Control effects of IR (finite volume) and UV (discretisation) regulators:

$m_\pi L \gtrsim 4$ $\qquad\qquad$ $a^{-1} \gg$ Mass scale of interest



For $m_\pi = m_\pi^{\mathrm{phys}} \sim 140\,\mathrm{MeV}$ and $\overline{m}_b(m_b) \approx 4.2\,\mathrm{GeV}$:

$L \gtrsim 5.6\,\mathrm{fm}$ $\qquad\qquad$ $a^{-1} \gg 4.2\,\mathrm{GeV} \approx (0.05\,\mathrm{fm})^{-1}$

Requires $N \equiv L/a \gg 120 \Rightarrow N^3 \times (2N) \gg 4 \times 10^8$ lattice sites.

VERY EXPENSIVE to satisfy both constraints simultaneously...

... needs to be repeated for different values of a.

How to simulate the b-quark?

For now choose between:

Effective action for b

- Can tune to m_b
- comes with systematic errors which are hard to estimate/reduce

Relativistic action for b

- Theoretically cleaner and systematically improvable
- Need to control extrapolation in heavy quark mass

Different properties:

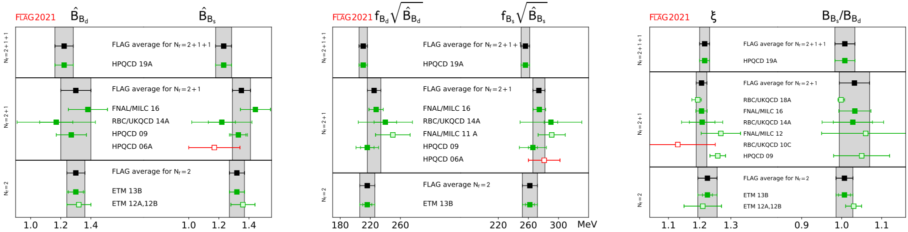
- computational cost
- chirality
- tuning errors
- systematic errors
- cut off effects
- renormalisation

BUT SOON:

Huge efforts in the community to produce **very fine lattice spacings**:

$\Rightarrow$ **Direct simulation of $\approx m_b^{\mathrm{phys}}$ will become possible!**

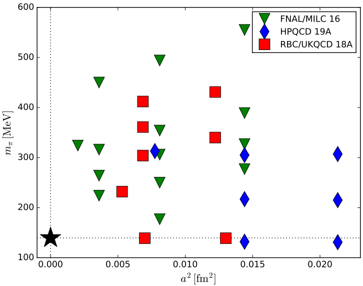
$B - \bar{B}$ mixing results: FLAG 2021 [2111.09849]



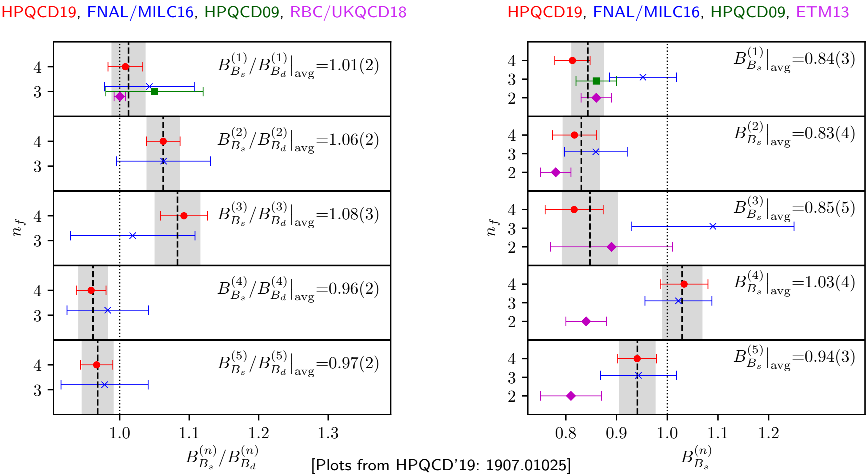
Fewer results than in the light sector, but very complementary results from

- different gauge field configurations
- different valence light actions
- different valence heavy actions
- different methodologies

Includes computations with $m_\pi^{\rm phys}$!



Going beyond $\hat{B}^{(1)}_{B_q}$

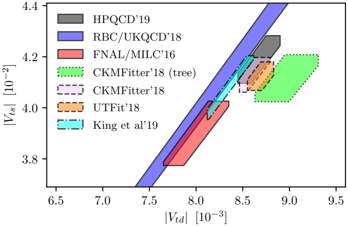


[Plots from HPQCD'19: 1907.01025]

Mostly in agreement, but some spread in $B^{(i)}_{B_s}$ parameters.

CKM matrix elements

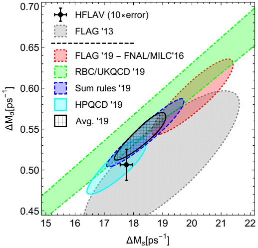
	$f_{B_d}(\hat{B}_{B_d})^{1/2}$ [GeV]	$f_{B_s}(\hat{B}_{B_s})^{1/2}$ [GeV]	ξ	$\lvert V_{td}\rvert$	$\lvert V_{ts}\rvert$	$\lvert V_{ts}/V_{td}\rvert$
HPQCD19	0.2106(55)	0.2561(57)	1.216(16)	0.00867(23)	0.04189(93)	0.2071(27)
FNAL/MILC16	0.2277(98)	0.2746(88)	1.206(18)	0.00800(35)	0.0390(13)	0.2052(33)
RBC/UKQCD18			$1.194\left(^{+12}_{-19}\right)$			$0.2033\left(^{+16}_{-30}\right)$



- Reasonable agreement between lattice results, but some spread
- Tree-only fit somewhat differs
- Uncertainty dominated by theory

$\Rightarrow$ **Further theory work required!**

[Plots from ↑ 1907.01025, ↓ 1909.11087]



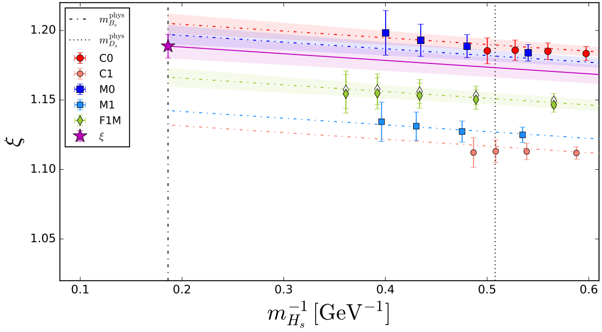
Current lattice uncertainties dominated by

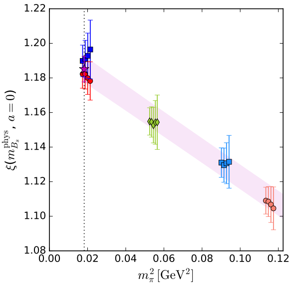
- Renormalisation/matching
- Heavy quarks (discretisation/extrapolation)

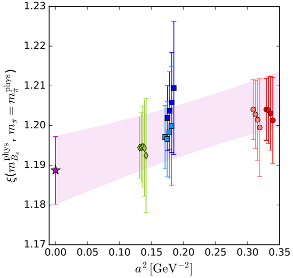
$\Rightarrow$ we are working on improving both!

RBC/UKQCD'18: strategy [1812.08791]

- Computation of $SU(3)$ breaking ratios f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_{B_d} and ξ
- Renormalisation constants cancel
- Extrapolation from $m_h < m_b$ to m_b
- Simultaneous fit to m_π^2, a^2, $\frac{1}{m_H}$







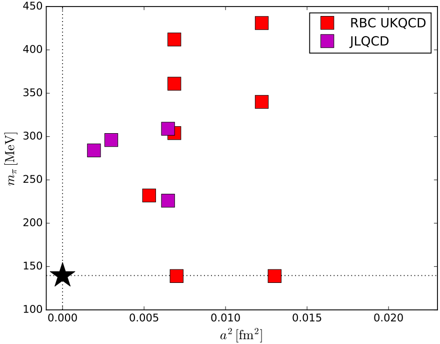
RBC/UKQCD/JLQCD: ongoing work [F. Erben @Lattice21: 2111.11287]

RBC/UKQCD'18 dominated by

- chiral-continuum fit

⇒ heavy quark extrapolation

⇒ estimates of higher order $1/m_H$ terms



- Supplement existing dataset with finer JLQCD ensembles
  ⇒ **reduce extrapolation** in m_H significantly
- all domain wall fermion set-up
  ⇒ only **physical block-diagonal renormalisation pattern**
- all 5 operators $\hat{B}^{(i)}_{B_d}$ and $\hat{B}^{(i)}_{B_s}$
- (3+3) lattice spacings, 2 ensembles with physical pion mass
  ⇒ good control over all required limits
- new correlator fitting strategy

RBC/UKQCD/JLQCD: analysis strategy

Huge data-set

- 15 ensembles with varying $a, L, m_l(\to M_\pi), m_s$.
- wide range of heavy-quark masses (4-6) per ensemble $m_{H_s}^{\max} \sim 75\% m_{B_s}^{\mathrm{phys}}$
- 6 lattice spacings (2 different discretisations)
- 2 m_π^{phys} ensembles.
- 2 pairs of ensembles which only differ in m_s.
- 1 pair which differs in V.

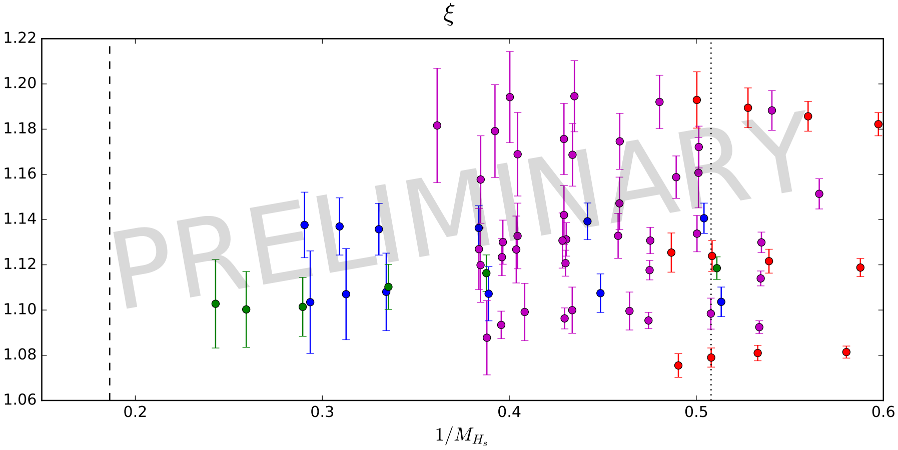
Fitting strategy & extrapolations

- Two independent correlation function analyses. (✓)
- RI/sMOM renormalisation analysis. (✓, preliminary)
- Fit to perform (ongoing)
  - $a \to 0$, $L \to \infty$
  - $M_\pi \to M_\pi^{\mathrm{phys}}$, $m_s \to m_s^{\mathrm{phys}}$
  - $M_{Hs} \to M_{B_s}$

  for $\hat{B}_{B_q}^{(i)}$, f_{B_q}, $f_{B_q}\sqrt{\hat{B}_{B_q}^{(i)}}$, ξ, ratios.
- Assess robustness of fit and all systematic uncertainties.

GOAL: Controlled prediction of all observables and their correlations

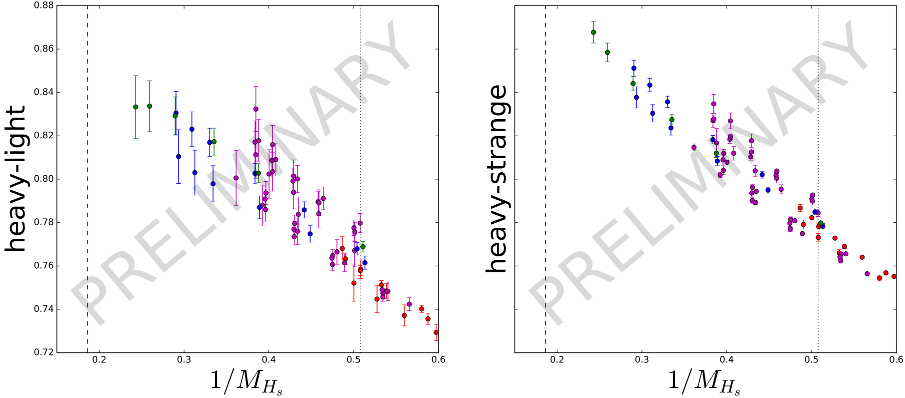
RBC/UKQCD/JLQCD: preliminary data – ratios



- Mild behaviour with $1/M_{H_s}$
- Data a lot closer to physical b-quark mass
- Precision on individual datapoints $\lesssim O(1\%)$.
- Starting to do global fits

RBC/UKQCD/JLQCD: preliminary data – bag parameters

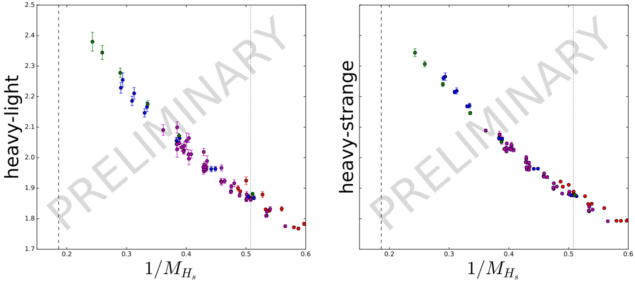
renormalised bag parameters B_1 (VV+AA)



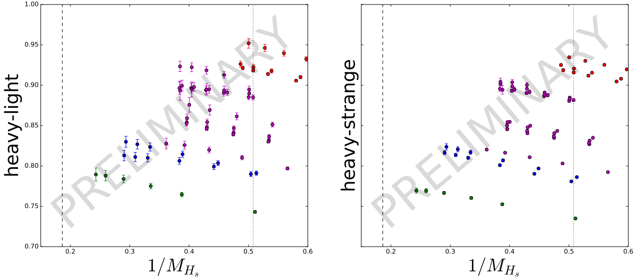
- a^{-1}, M_π, m_s, V effects not taken into account yet. . .
- . . . but seem to be mild
- strong $1/M_{Hs}$ behaviour but similar for all ensembles

RBC/UKQCD/JLQCD: preliminary data – bag parameters

renormalised bag parameters B_2 (VV-AA)



renormalised bag parameters B_3 (SS-PP)



- Mild discretisation effects
- Seemingly mild behaviour with a, M_π, m_s, V
- strong $1/M_{Hs}$ dependence

- Large discretisation effects
- More noticeable chiral effects
- mild $1/M_{Hs}$ dependence

Summary and Outlook

Literature Summary

- Complementary lattice results:
  - ensembles
  - light quark actions
  - heavy quark treatment
  - renormalisation
- Physical pion mass results
- Results for full operator basis
- First result (ratios only) w/o effective action for the b-quark

RBC/UKQCD/JLQCD

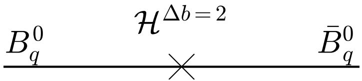
- Huge data set (15 ensembles)
- Target all 5+5 bag parameters, decay constants, ratios
- Good HQ reach $\Rightarrow$ no effective action prescription for b-quark
- χ-symmetry $\Rightarrow$ continuum-like block-diagonal renormalisation
- Control all systematics (data)

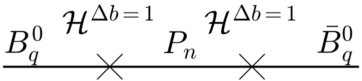
CKM Status and Future

- $|V_{td}|$ and $|V_{ts}|$ known at $\approx 2.5\%$, $|V_{td}/V_{ts}|$ at $\approx 1.5\%$
- Uncertainty theory dominated - work is ongoing
- RBC/UKQCD/JLQCD: Aim to reduce uncertainties with **all DWF** approach (data generated using Grid and Hadrons [Grid, Hadrons])

ADDITIONAL SLIDES

Neutral $B_{(s)}$ meson mixing - theory





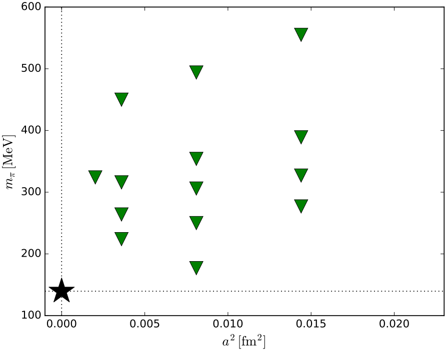
$$\left\langle B_q^0 \right| \mathcal{H}_{\mathrm{eff}} \left| \bar{B}_q^0 \right\rangle \propto \underbrace{\left\langle B_q^0 \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}_q^0 \right\rangle}_{\text{Short distance}} + \underbrace{\sum_n \frac{\left\langle B_q^0 \right| \mathcal{H}^{\Delta b=1} \left| n \right\rangle \left\langle n \right| \mathcal{H}^{\Delta b=1} \left| \bar{B}_q^0 \right\rangle}{E_n - M_{B_q}}}_{\text{Long distance}}$$

$$\text{short distance} \propto \left| \sum_{q'=u,c,t} \frac{m_{q'}^2}{M_W^2} V_{q'b} V_{q'q}^* \right|^2 \approx \frac{m_t^4}{M_W^4} \left| V_{tb} V_{tq}^* \right|^2$$

SD: Top enhanced: $m_t^2 V_{tb} V_{tq}{}^* \gg m_c^2 V_{cb} V_{cq}^* \gg m_u^2 V_{ub} V_{uq}^*$
LD: Only m_c, m_u in intermediate states: no top + CKM suppressed
$\Rightarrow$ **Short distance dominated**.

FNAL/MILC: set-up [1602.03560]

- $N_f = 2 + 1$ flavours of staggered quarks (asqtad) in the sea
- 4 lattice spacings, pion masses from $177 - 555\,\mathrm{MeV}$
- valence light & strange: asqtad
- Fermilab method for the b-quark
- *mostly non-perturbative* 1-loop lattice perturbation theory
- Computation of $f_{B_q}\sqrt{\hat{B}_{B_q}}$ and ξ
- f_{B_q} taken from the PDG average to access to $\hat{B}_{B_q}$
- all 5 operators for B_d and B_s



$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 227.7(9.5)\,\mathrm{MeV}$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 274.6(8.4)\,\mathrm{MeV}$$

$$\xi = 1.206(18)$$

FNAL/MILC: error budget [%] [1602.03560]

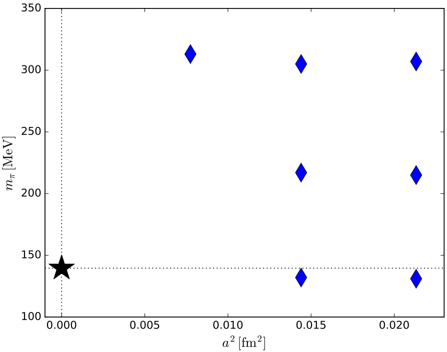
	statistics	inputs	κ tuning	matching	chiral	LQ disc	HQ disc	fit total
$\langle\mathcal{O}_1^d\rangle$	4.2	0.4	2.1	3.2	2.3	0.6	4.6	7.7
$\langle\mathcal{O}_2^d\rangle$	4.6	0.3	1.1	3.7	2.6	0.6	4.6	8.0
$\langle\mathcal{O}_3^d\rangle$	8.7	0.2	2.1	12.6	4.8	1.2	9.9	19.0
$\langle\mathcal{O}_4^d\rangle$	3.7	0.4	1.7	2.2	1.9	0.5	3.9	6.4
$\langle\mathcal{O}_5^d\rangle$	4.7	0.5	2.5	4.7	2.7	0.8	4.9	9.1
$\langle\mathcal{O}_1^s\rangle$	2.9	0.4	1.5	2.1	1.6	0.4	3.2	5.4
$\langle\mathcal{O}_2^s\rangle$	3.1	0.3	0.8	2.5	1.6	0.4	3.1	5.5
$\langle\mathcal{O}_3^s\rangle$	5.9	0.3	1.4	8.6	3.0	0.7	6.9	13.0
$\langle\mathcal{O}_4^s\rangle$	2.7	0.4	1.2	1.6	1.3	0.3	2.9	4.8
$\langle\mathcal{O}_5^s\rangle$	3.4	0.4	1.8	3.4	1.9	0.5	3.6	6.7
ξ	0.8	0.4	0.3	0.5	0.4	0.1	0.7	1.4

Uncertainty dominated by chiral-continuum limit fit, in particular

- statistical
- heavy quark discretisation errors
- matching

HPQCD: set-up [1907.01025]

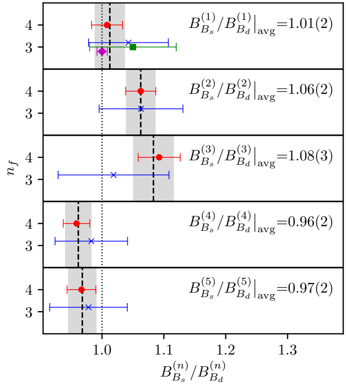
- $N_f = 2+1+1$ flavours of staggered quarks (HISQ) in sea
- light quarks using HISQ in the valence
- 3 lattice spacings, 2 physical pion mass ensembles
- improved nonrelativistic QCD action for the b
- all 5 operators for B_d and B_s
- blinded analysis
- Computation of the $\hat{B}^{(i)}_{B_q}$
- ξ and $f_{B_q}\sqrt{\hat{B}_{B_q}}$ accessed by using decay constants taken from a different computation



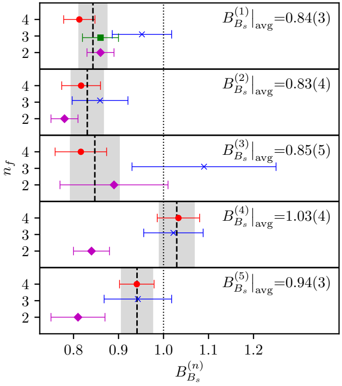
$$\hat{B}^{(1)}_{B_d} = 1.222(61)$$
$$\hat{B}^{(1)}_{B_s} = 1.232(53)$$
$$\hat{B}^{(1)}_{B_s}/\hat{B}^{(1)}_{B_d} = 1.008(25)$$

HPQCD: error budget and results [1907.01025]

HPQCD19, FNAL/MILC16, HPQCD09, RBC/UKQCD18



HPQCD19, FNAL/MILC16, HPQCD09, ETM13



Uncertainty dominated by matching terms α_s^2 and $\alpha_s\Lambda_{QCD}/m_b$.

	$B^{(1)}_{B_s}$	$B^{(2)}_{B_s}$	$B^{(3)}_{B_s}$	$B^{(4)}_{B_s}$	$B^{(5)}_{B_s}$	$B^{(1)}_{B_s}/B^{(1)}_{B_d}$
lattice data	1.4	1.4	1.5	1.6	1.5	1.5
η_i^q	0.0	2.3	2.3	2.1	1.2	0.0
α_s^2 terms	2.1	2.9	5.2	1.9	1.5	0.1
$\alpha_s\Lambda_{\rm QCD}/m_b$ terms	2.9	2.8	2.9	2.8	2.7	0.0
$(a\Lambda_{\rm QCD})^{2n}$ terms	1.8	1.9	2.3	1.5	1.8	0.1
m_l extrapolation	0.4	0.4	0.7	0.5	0.4	1.9
Total	4.3	5.3	7.0	4.6	4.1	2.5

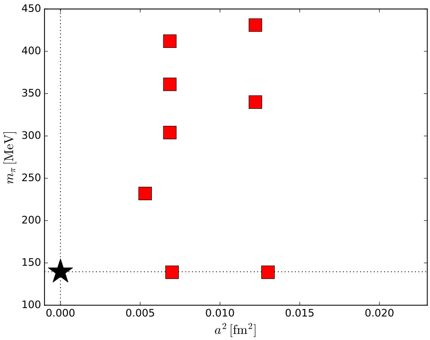
RBC/UKQCD: set-up [1812.08791]

Light and strange

- Unitary light quark mass
- Physical strange quark mass
- DWF parameters same between sea and valence

Heavy: charm and b(eyond)

- Möbius DWF
- $M_5 = 1.0$, $L_s = 12$
- Stout smeared (3 hits, $\rho = 0.1$)
- Many source-sink separations
- Range of heavy quark masses $m_c \lesssim m_h \lesssim m_b/2$



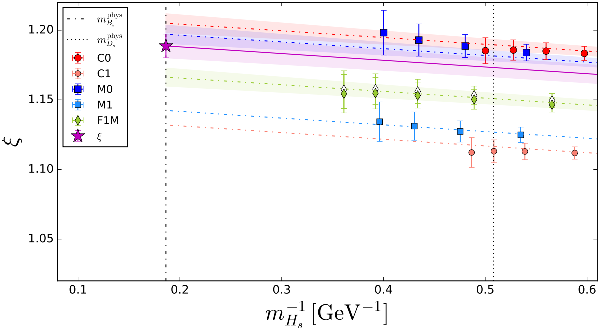
Ensembles

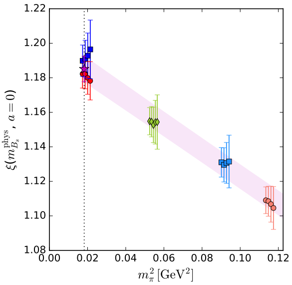
- $N_f = 2+1$ flavours, 3 lattice spacings, 2 physical pion mass ensembles

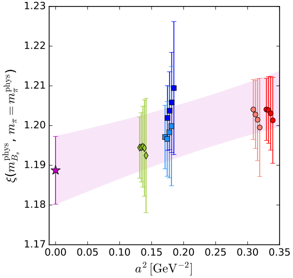
$\Rightarrow$ **All Domain Wall Fermion** mixed action set-up

RBC/UKQCD: strategy [1812.08791]

- Computation of $SU(3)$ breaking ratios f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_{B_d} and ξ
- Renormalisation constants cancel
- Benign behaviour with $1/m_H$
- Simultaneous fit to m_π^2, a^2, $\frac{1}{m_H}$







RBC/UKQCD: error budget [%] [1812.08791]

	f_{D_s}/f_D	f_{B_s}/f_B	ξ	B_{B_s}/B_{B_d}
central	1.1740	1.1949	1.1939	0.9984
stat	0.43%	0.50%	0.56%	0.45%
fit chiral-CL	${}^{+0.31}_{-0.32}\%$	${}^{+0.34}_{-0.54}\%$	${}^{+0.38}_{-0.45}\%$	${}^{+0.42}_{-0.01}\%$
fit heavy mass	${}^{+0.07}_{-0.05}\%$	${}^{+0.00}_{-0.82}\%$	${}^{+0.00}_{-0.87}\%$	${}^{+0.27}_{-0.22}\%$
H.O. heavy	0.00%	0.47%	0.35%	0.21%
H.O. disc.	0.01%	0.01%	0.12%	0.17%
$m_u \neq m_d$	0.08%	0.07%	0.08%	0.01%
finite size	0.18%	0.18%	0.18%	0.18%
total systematic	${}^{+0.38}_{-0.38}\%$	${}^{+0.61}_{-1.38}\%$	${}^{+0.56}_{-1.37}\%$	${}^{+0.66}_{-0.45}\%$
total sys+stat	${}^{+0.58}_{-0.58}\%$	${}^{+0.79}_{-1.47}\%$	${}^{+0.80}_{-1.48}\%$	${}^{+0.80}_{-0.63}\%$

Uncertainty dominated by

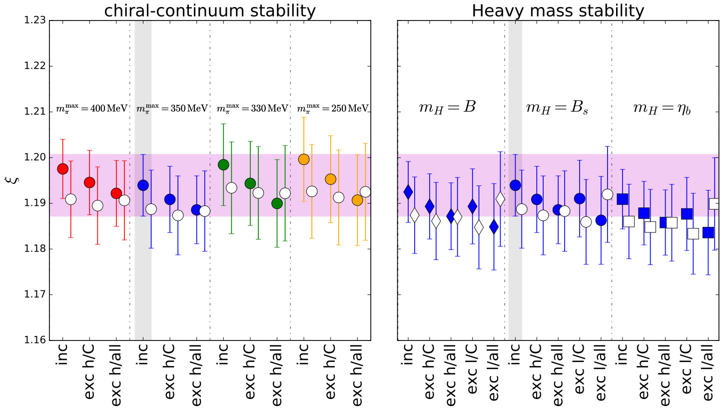
- chiral-continuum fit
- heavy quark extrapolation
- estimates of higher order $1/m_H$ terms

$$f_{D_s}/f_D = 1.1740(51)_{\rm stat}\left({}^{+68}_{-68}\right)_{\rm sys} \qquad B_{B_s}/B_{B_d} = 0.9984(45)_{\rm stat}\left({}^{+80}_{-63}\right)_{\rm sys}$$

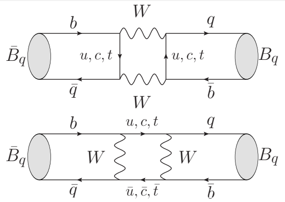
$$f_{B_s}/f_B = 1.1949(60)_{\rm stat}\left({}^{+\ 95}_{-175}\right)_{\rm sys} \qquad \xi = 1.1939(67)_{\rm stat}\left({}^{+\ 95}_{-177}\right)_{\rm sys}$$

RBC/UKQCD: systematic errors [1812.08791]

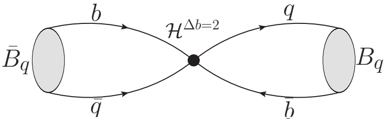
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



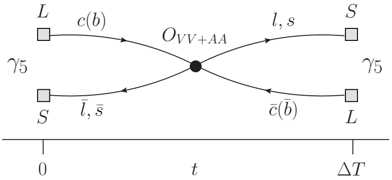
Lattice strategy to simulate meson mixing



$\Rightarrow$



$\Downarrow$ (e.g. for $\mathcal{O}_1$)



3-point correlation functions $\Leftarrow$

$$C_3^{ab,\mathcal{O}}(t;\Delta T) \sim \sum_i \sum_j \frac{Z_a^i Z_b^j \left\langle P_{(j)} \middle| \mathcal{O} \middle| \bar{P}_{(i)} \right\rangle}{4E_{(i)}E_{(j)}} e^{-E_{(j)}t} e^{-E_{(i)}(\Delta T - t)}$$

ΔT separation between source and sink

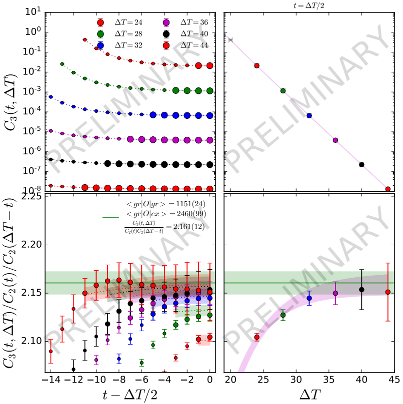
t position of operator $\mathcal{O}$ between source and sink

From correlation functions to matrix elements

$$C_3^{ab,\mathcal{O}}(t;\Delta T) \sim \sum_i \sum_j \frac{Z_a^i Z_b^j \left\langle P_{(j)} \right| \mathcal{O} \left| \bar{P}_{(i)} \right\rangle}{4 E_{(i)} E_{(j)}} e^{-E_{(j)} t} e^{-E_{(i)}(\Delta T - t)}$$

- Simultaneous fits of 2-point functions and 3point functions for multiple ΔT's to extract $\left\langle P_{\mathrm{gr}} \right| \mathcal{O}_{(i)} \left| \bar{P}_{\mathrm{gr}} \right\rangle$ with highest possible confidence and precision.
- Two separate independent analyses.
- Finalising investigation of stability of fits.

expl: heavy-light $\mathcal{O}_2$ for $a^{-1} \sim 3.5\,\mathrm{GeV} \rightarrow$



Non-perturbative renormalisation on the lattice [hep-lat/9411010]

We want

$$\text{Amplitude} = C^{\overline{\text{MS}}}(\mu)\,\langle\mathcal{O}\rangle^{\overline{\text{MS}}}(\mu)$$

- Wilson coefficients (typically) computed in $\overline{\text{MS}}$ at some scale μ.
- Operators $\langle\mathcal{O}\rangle^{\text{bare}}(a)$ computed with lattice regulator a^{-1}.
- Renormalise $\langle\mathcal{O}\rangle^{\text{bare}}(a)$ at scale μ in regularisation independent (RI) scheme, by computing a non-pert. renormalisation factor $Z^{RI}(\mu, a)$.

$$\langle\mathcal{O}\rangle^{RI}(\mu) = \lim_{a^2\to 0} Z^{RI}(\mu, a)\,\langle\mathcal{O}\rangle^{\text{bare}}(a)$$

- Match to preferred scheme (e.g. $\overline{MS}$) using P.T. at μ: $R^{\overline{MS}\leftarrow RI}(\mu)$
- If the operators mix: C and $\langle\mathcal{O}\rangle$ become vectors, R and Z matrices.

$$\text{Amplitude} = C_i^{\overline{\text{MS}}}(\mu) R_{ij}^{\overline{MS}\leftarrow RI}(\mu) \lim_{a\to 0} Z_{jk}^{RI}(\mu, a)\,\langle\mathcal{O}_k\rangle^{\text{bare}}(a)$$

- Chirally symmetric fermions $\Rightarrow$ R and Z are block diagonal.

NPR for Neutral Meson Mixing operators

- Analogous to the neutral kaon mixing case [1708.03552, 1812.04981]
- 5 operators $\mathcal{O}_1 = \mathcal{O}_{VV+AA}$, $\mathcal{O}_2 = \mathcal{O}_{VV-AA}$, $\mathcal{O}_3 = \mathcal{O}_{SS-PP}$, $\mathcal{O}_4 = \mathcal{O}_{SS+PP}$, $\mathcal{O}_5 = \mathcal{O}_{TT}$
- Block diagonal renormalisation matrix (up to $O(am_{\mathrm{res}})$)

$$Z_{ij}^{RI}(\mu, a) = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix}$$

- Generalise idea from 1701.02644 for fully non-perturbative mixed action renormalisation to four quark operators

$$\frac{\mathcal{P}[\Lambda_A](ll)\mathcal{P}[\Lambda_A](hh)}{(\mathcal{P}[\Lambda_A](lh))^2} = \frac{(Z_A^{\mathrm{lh}})^2}{Z_A^{ll} Z_A^{hh}}$$

- Data production of the mixed action NPR in progress.