Theoretical models of exotic hadrons

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Conventional and exotic hadrons

- Hadrons are colorless; what types of color singlets should exist?
- Confinement: clue from hadron spectrum?
- Quark model: conventional and exotic hadrons

### Conventional hadrons

- **Baryon:** $q$ $q$ $q$
- **Meson:** $q$ $\bar{q}$

### Exotic hadrons

- **Tetraquark:** $q$ $ar{q}$ $q$ $q$
- **Pentaquark:** $q$ $q$ $q$ $\bar{q}$ $\bar{q}$
- **Hybrid:** $q$
- **Hadronic molecule:** $q$ $q$ $q$ $ar{q}$ $\bar{q}$
- **Glueball:**

**Disclaimer:** NOT a general review; will discuss challenges from my personal point of view.
Hadron spectrum: charm mesons

- Quark model provides qualitative guidance, but the physics is much richer, in particular for energies close to or above thresholds
  - Abundance of new states from peak hunting
    - $b$-hadron ($B, \Lambda_b$) decays
    - $e^+e^-$ collisions
    - Hadron collisions
  - Heavy-ion collisions
- Example: open-charm mesons

Predictions of the $2^-$ states taken from Godfrey, Moats, PRD93(2016)034035
Hadron spectrum: charmonium(-like) mesons

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- Example: charmonium(-like) spectrum
Near-threshold states

- Prominent features: many are narrow and near-threshold; spectrum of explicitly exotic states is emerging

\[ X(3872) [\text{aka } \chi_{c1}(3872)], Z_c(3900)\pm, \]
\[ Z_c(4020)\pm, \ Z_{cs}(3985), \ldots \]


\( P_c \) states: hidden-charm baryon

![Graph showing weighted candidates vs. mass](image)

Data from LHCb, PRL122 (2019) 222001;
Fit from
M.-L. Du, Baru, FKG, Hanhart, Meißner, Oller, Q. Wang,
PRL124 (2020) 072001

\( T_{cc} \): double-charm meson

![Graph showing yield vs. mass](image)

LHCb, Nature Phys. 18 (2022) 751
Models

- **Candidates of hadronic molecules**
- **Other models:** compact multiquark states, hybrids, hadrocharmonia
- >10 reviews in the last 7 years:
  - Y. Yamaguchi et al., *Heavy hadronic molecules with pion exchange and quark core couplings: a guide for practitioners*, JPG 47 (2020) 053001
  - ......

- **And a book:**
Different models predict distinct mass spectra and decay patterns, e.g., charmonium-like.

- **P = + states in a molecular model**
  - N.A. Törnqvist, ZPC61(1994)525;
  - C.-Y. Wong, PRC69(2004)055202;
  - E. Swanson, JPCS9(2005)79;
  - V. Baru et al. PLB763(2016)20; ...

Heavy-quark spin symmetry:

\[ M_{X_2[D^*\bar{D}^*]} - M_{X(3872)} \approx M_{D^*} - M_D \]

\( X_2 \): candidate reported by Belle, 2105.06605

- **P = + states in a tetraquark model**
  \[ \mathcal{H} \approx 2\kappa_{qc}(s_q \cdot s_c + s_{\bar{q}} \cdot s_{\bar{c}}) \]

Spectrum similar with molecular model from fixing \( \kappa_{qc} \) using

\[ M_{Z_c(4020)} - M_{Z_c(3900)} \approx M_{D^*} - M_D \]


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Hadronic molecules & tetraquarks

- Different models predict distinct mass spectra and decay patterns, e.g., charmonium-like

\[ P = - \] states in a molecular model

\[ P = - \] states in a tetraquark model

\[
M = M_{00} + B_{c} \frac{L(L + 1)}{2} \\
+ a [L(L + 1) + S(S + 1) - J(J + 1)] \\
+ \kappa_{cq}[s(s + 1) + \bar{s} (\bar{s} + 1) - 3].
\]

Mol. spectrum using a VMD interaction


M. Cleven, FKG, C. Hanhart, Q. Wang, Q. Zhao, PRD92(2015)014005
using inputs from
To reveal the underlying physics, we need to have a faithful spectrum to start with.

In most cases, resonance parameters are extracted using Breit-Wigner.

- Potentially sizeable corrections due to coupled channels and thresholds

K-matrix formalism with coupled channels.

**Challenges**

F.-K. Guo (ITP, CAS)
In most cases, resonance parameters are extracted using Breit-Wigner BW: for isolated, narrow resonances away from strongly-coupled thresholds

- Unitarity: the same resonance may behave completely different in different processes
- Resonance does not necessarily show up as a peak, may also be a dip

- E.g., $f_0(980)$:

  **peak in**
  $$J/\psi \to \phi \pi^+ \pi^-$$
  **dip in**
  $$J/\psi \to \omega \pi^+ \pi^-$$

Line shapes of the same poles in different processes
X.-K. Dong, FKG, B.-S. Zou, PRL126(2021)152001

$T_{21} \propto \frac{1}{a_{22,\text{eff}}^{-1} - i\sqrt{2\mu_2}E}$

$T_{11} \propto \frac{a_{22}^{-1} - i\sqrt{2\mu_2}E}{a_{22,\text{eff}}^{-1} - i\sqrt{2\mu_2}E}$

BES, PLB607(2005)243

$K\bar{K} \to \pi\pi$

$\pi\pi \to \pi\pi$

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Challenges

- In most cases, resonance parameters are extracted using Breit-Wigner
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- BESIII: narrow $Z_{cs} (3985)$
- LHCb: broad $Z_{cs} (4000)$, and $Z_{cs} (4200)$
- A simultaneous fit to the BESIII and LHCb $Z_{cs}^{\pm}$ data: two virtual states $Z_{cs} (3990, 4110)$

See also Yang et al., PRD103(2021)074029; Baru et al., PRD105(2022)034014

Ortega, Entem, Fernandez, PLB818(2021)136382
Challenges

- A peak is not necessarily due to a resonance
- Triangle singularities
  - on shell and collinear intermediate particles
  - determined by kinematic variables such as masses and energies
  - sensitive to energies and processes

For a review of triangle singularities and threshold cusps, FKG, X.-H. Liu, S. Sakai, PPNP 112 (2020) 103757

\[
\begin{align*}
    m_{A,TS}^2 &\in \left[ (m_R + m_B)^2, (m_R + m_B)^2 + \frac{m_B}{m_C} [ (m_R - m_C)^2 - m_D^2 ] \right] \\
    m_{BC,TS}^2 &\in \left[ (m_B + m_C)^2, (m_B + m_C)^2 + \frac{m_B}{m_R} [ (m_R - m_C)^2 - m_D^2 ] \right]
\end{align*}
\]
Summary and outlook

- Unified description of the “new hadron spectroscopy” is missing
- Precise data and lattice QCD calculations are needed
- Theoretical methods constrained by symmetry, unitarity and analyticity as a bridge
- Looking forward to more discoveries at LHC