Multiple Parton Interactions: Theory Progress

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LHCP 2023
May 25th 2023
Protons are ‘bags’ of quarks and gluons – multiple interactions likely!

‘Minimum bias’
‘Underlying event’
Ubiquitous!

‘Primary process’
Additional scatters

Double hard parton scattering (DPS). Rarer, but measured in several channels.

Crudest model for DPS: ‘Pocket formula’

\[
\sigma_{\text{eff}} \sim (\text{proton radius})^2 \sim 60\text{mb}
\]
POCKET FORMULA APPROACH

Crudest description of Underlying Event:

\[ \frac{1}{\sigma_{eff}^n} \]

Roughly speaking, approach taken by most Monte Carlo event generators
EXPERIMENTAL DATA ON $\sigma_{eff}$

Some experimental extractions of $\sigma_{eff}$:

- $\sigma_{eff,DPS} \ll 60 \text{mb}$!
- $\sigma_{eff}$ with quarkonium $< \sigma_{eff}$ with high-$p_T$ jets/EW bosons

Measurement in triple $J/\psi$. Process receives contributions from triple parton scattering (TPS)!

Taken from Nature Phys. 19 (2023) 3, 338-350

What is missing in simple pocket formula approach?

Perturbative correlations

Favours small separation $y$ – reduces $\sigma_{eff,DPS}$
See e.g. Blok et al., Eur.Phys.J.C 74 (2014) 2926

Issue with overlap between DPS and loop corrections to single scattering.
Now solved.
BEYOND THE POCKET FORMULA

What is missing in simple pocket formula approach?

Non-perturbative correlations

Correlations in spin and colour between partons

E.g. two quarks may prefer to have their spins aligned ↑↑, or anti-aligned ↑↓

All of these correlations are intrinsically interesting – aspect of proton structure not accessible via single scattering

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
MEASURING CORRELATIONS

Effects of these correlations will be more prominent at HL-LHC where we accumulate more statistics, and study final states in more detail.

E.g. same-sign WW

\[ \mathcal{A} = \frac{l^+}{l^-} \]

\[ \frac{|\eta_1|}{\eta_2} > 0 > 0.6 > 1.2 \]

\[ \begin{array}{ccc}
A & 0.07 & 0.11 \\
\sigma [fb] & 0.51 & 0.29 & 0.13
\end{array} \]

Spin correlations: ~10% effect

Valence number constraints + pQCD correlations

\[ \mathcal{A} \text{ values of a few per cent will be measurable at HL-LHC!} \]
**NEW TOOLS**

dShower:
New Monte Carlo simulation of double scattering
- Proper account of *perturbative correlations*
- Non-perturbative correlations via input double parton densities

Methodology developed to combine single and double scattering in shower without double counting

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At $\mu_y$, partons overlap, & can merge

$\mu_y \sim 1/y$

$Q_0$

$Q$

$Q_h$

$1/Q$

$1/Q_h$

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JG, Cabouat, Ostrolenk, JHEP 11 (2019) 061

JG, Cabouat, JHEP 10 (2020) 012
ChiliPDF: Evolution + interpolation of double parton densities via Chebyshev polynomials. Accurate interpolation even with relatively few points.

Cross-checked against existing DOVE code

Diehl, Nagar, Piöbl, Tackmann, arXiv:2305.04845

Diehl, JG, Stirling, JHEP 03 (2010) 005
Diehl, JG, Schönwald JHEP 1706 (2017) 083
PROGRESS: NONPERTURBATIVE
Double parton densities should obey momentum and valence number constraints.

\[ \int d^2y \, dx_2 \, x_2 = (1 - x_1) \]

Sum rules for double parton densities:

Pythia MPI model:

Sequence of interactions ordered in hardness.

Number/momentum constraints taken account of by adjusting PDFs after each interaction.

JG. Stirling, JHEP 03 (2010) 005

Sjostrand, Skands, JHEP 03 (2004) 053 ,
How well do Pythia double parton distributions satisfy sum rules?

Momentum sum rule \((j_1 = u)\). Should = 1.

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<th>(x_1)</th>
<th>0.979</th>
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\(\bar{u}u\) number sum rule. Should = 3.

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Sum rules mostly satisfied at 10-25% level

Some larger deviations, particularly at large \(x\)

Derived sum rules for triple parton distributions and checked Pythia model.

Similar picture.
LATTICE CALCULATION OF DPDS

Calculation of lowest Mellin moment:

\[ I(y^2) = \int dx_1 dx_2 \]

Bali et al., JHEP 09 (2021) 106

In principle possible to access \( x_1, x_2 \) dependence of double parton distributions on lattice via quasi-PDF approach.

Zhang, arXiv:2304.12481
Jaarsma, Rahn, Waalewijn, arXiv:2305.09716
PROGRESS: PERTURBATIVE
Significant progress towards NLO computations for double scattering:

Perturbative correlations computed at NLO.

(Double) DGLAP evolution evaluated beyond LO for first time in ChiliPDF

No full NLO predictions of DPS yet…
MASS EFFECTS IN PERTURBATIVE SPLITTING

Two schemes developed to include heavy quarks in perturbative splitting, “massless” and “massive” schemes.

Predictions of schemes compared at level of DPS luminosities

\[ n'_F = n_F + 1 \]

Diehl, Nagar, Plößl, arXiv:2212.07736
COLOUR CORRELATIONS

Colour correlations: decrease rapidly with scale, could be relevant at intermediate scale.

Exception: interplay between perturbative and colour correlations for coloured particle production. Few % effect.

Blok, Mehl, arXiv:2210.13282

Colour correlations in perturbative splitting (Diehl, JG, Plößl, JHEP 08 (2021) 040), and DGLAP evolution (Diehl, Fabry, Vladimirov, JHEP 05 (2023) 067), computed at NLO
Double scattering data can be a valuable input to models of nuclear structure.


Two choices of parameters with $N_q = 3$ and $N_q = 7$ give agreement with DIS data.

In Blok, Segev, Strikman Eur.Phys.J.C 83 (2023) 5, 415, $\sigma_{eff}$ values are computed from this model:

$N_q = 3: \sigma_{eff} = 10.5 \text{ mb}$

$N_q = 7: \sigma_{eff} = 17 \text{ mb}$

Tension with data (esp. when including PT correlations)
SUMMARY

• Spectrum of multiple parton interactions: **underlying event/minimum bias** (soft) and **double parton scattering** (hard).
• Simplest approach to multiple parton interactions is **pocket formula**: ignore all correlations.
• Misses nonperturbative and perturbative correlations, also in spin and colour.
• Existing data shows tension with pocket formula predictions, and future statistics should allow a more detailed probe of correlations → motivation to improve theory predictions and tools.
• New tools: dShower, ChiliPDF
• Improvements to understanding/modelling of nonperturbative correlations: **sum rules, lattice calculations**.
• Improvements to description of perturbative correlations: **higher orders, mass effects**.
• Improvements to theoretical description of **colour correlations**.