ttH production at NNLO

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**Introduction**

- $t\bar{t}H$ production → ‘direct’ measurement of the top Yukawa coupling

- Observed 5 years ago by LHC collaborations

- Current experimental uncertainties at $O(20\%)$ level

- Experimental precision expected to go down to $O(2\%)$ at HL-LHC

- Precise theoretical predictions are needed to match it!
Theoretical status

NLO QCD

[Beenakker at al.; 0107081, 0211352], [Reina and Dawson; 0107101], [Reina, Dawson and Wackeroth; 0109066], [Dawson at al.; 0211438], [Dawson at al.; 0305087]

NLO EW

[Frixione et al.; 1407.0823, 1504.03446], [Zhang et al.; 1407.1110]

Soft-gluon resummation

[Kulesza et al.; 1509.02780, 1704.03363], [Broggio et al.; 1510.01914], [Broggio et al.; 1611.00049], [Broggio et al.; 1907.04343], [Ju and Yang; 1904.08744], [Kulesza et al.; 2001.03031]

NLO with off-shell effects

[Denner and Feger; 1506.07448], [Denner et al.; 1612.07138]

NLO QCD + PS

[Frederix et al.; 1104.5613], [Garzelli et al.; 1108.0387], [Hartanto et al.; 1501.04498]

- Current perturbative uncertainties: O(10%)
- NNLO in QCD needed to reduce them!

Challenges in NNLO calculation:

Subtraction of infrared divergencies

Two-loop scattering amplitudes
Infrared subtraction

- We use the $q_T$-subtraction method, originally developed for colour singlet
  [Catani, Grazzini; hep-ph/0703012]

- Extended to heavy-quark production: additional soft divergencies from FS emissions
  [Catani, Devoto, Grazzini, JM; 2301.11786]

Used for $t\bar{t}$, $b\bar{b}$, both at NNLO and NNLO+PS

**Infrared subtraction**

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- Extended to heavy-quark production: additional soft divergencies from FS emissions
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- Further extension needed to deal with heavy-quark + colourless
  Used for $t\bar{t}$, $b\bar{b}$, both at NNLO and NNLO+PS

- Remove back-to-back constraint for heavy quarks

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**Soft function for Heavy quark production in ARbitrary Kinematics**

[Devoto, JM; in preparation]

Already applied to $t\bar{t}H$ and $b\bar{b}W$

[this talk] [Buonocore, JM, et al.; 2212.04954]
Two-loop corrections: soft Higgs emission

- $2 \to 3$ at 2 loops with 3 external masses $\to$ beyond current capabilities

  Need to rely on some approximation

- We have derived a factorization formula valid in the limit in which the Higgs is soft

\[
\begin{align*}
\text{soft} \quad H & \\ \sim & \\ F(\alpha_s, \frac{m_t}{\mu_R}) J(p_H) \times \end{align*}
\]
Two-loop corrections: soft Higgs emission

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\[
1 + \frac{\alpha_s}{2\pi} (-4) + \left( \frac{\alpha_s}{2\pi} \right)^2 \left( -\frac{89}{3} - \frac{46}{3} \log \frac{\mu_R^2}{m_t^2} \right) + \ldots
\]

Derived from soft limit of scalar heavy quark form factor, alternatively using Higgs low energy theorems


- Higgs soft current is ‘abelian’, no higher-order corrections apart from normalization

- This formula can serve as a non-trivial cross check to future Higgs+HQ loop calculations

Known at two loops from top-pair production at NNLO

[Baernreuther, Czakon, Fiedler; 1312.6279]
Two-loop corrections: soft Higgs emission

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Derived from soft limit of scalar heavy quark form factor, alternatively using Higgs low energy theorems

- We can use the formula to approximate the only unknown contribution to $t\bar{t}H$ at NNLO:

\[
H^{(2)} = \frac{2\text{Re} \left( M^{(2)}_{\text{fin}} M^{(0)*}\right)}{|M^{(0)}|^2}
\]

Obs: approximation used both in numerator and denominator (Born improved)

- Mapping needed from $t\bar{t}H$ to $t\bar{t}$ kinematics: Higgs recoil absorbed in initial state particles

Validation at NLO

<table>
<thead>
<tr>
<th>σ [fb]</th>
<th>13TeV</th>
<th>100TeV</th>
</tr>
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<tbody>
<tr>
<td>gg</td>
<td>261.58</td>
<td>23055</td>
</tr>
<tr>
<td>q̅q</td>
<td>129.47</td>
<td>2323.7</td>
</tr>
<tr>
<td>LO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H(1) exact</td>
<td>88.62</td>
<td>8205</td>
</tr>
<tr>
<td>H(1) approx</td>
<td>61.98</td>
<td>5612</td>
</tr>
<tr>
<td>Difference</td>
<td>30.1%</td>
<td>31.6%</td>
</tr>
</tbody>
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- Deviation w.r.t. exact $H^{(1)}$ contribution is about 30% for gg channel and 5% for q̅q channel
- Quality of approximation independent of c.m. energy
- Better performance in quark channel expected: already at LO Higgs emissions from internal tops in gg channel, which are not captured in the soft limit
- Can we provide precise NNLO predictions with this approximation for $H^{(2)}$?
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<td>H(^{(1)}) exact</td>
<td>88.62</td>
<td>7.826</td>
</tr>
<tr>
<td>H(^{(1)}) approx</td>
<td>61.98</td>
<td>7.413</td>
</tr>
<tr>
<td>Difference</td>
<td>30.1%</td>
<td>5.27%</td>
</tr>
<tr>
<td>H(^{(2)}) approx</td>
<td>-2.980</td>
<td>2.622</td>
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- Deviation w.r.t. exact H\(^{(1)}\) contribution is about **30%** for gg channel and **5%** for q̄q channel.
- Quality of approximation independent of c.m. energy.
- Better performance in quark channel expected: already at LO Higgs emissions from internal tops in gg channel, which are not captured in the soft limit.
- Can we provide **precise NNLO predictions** with this approximation for H\(^{(2)}\)?

**Yes!** Thanks to the small size of the H\(^{(2)}\) contribution to the NNLO cross section.
Uncertainty estimation

How do we estimate the uncertainties of $H^{(2)}$ approx?

- We use the **deviation from the exact result at NLO** as a reference
- We multiply by a **tolerance factor** of 3
- We combine **linearly** the uncertainties of the gg and qq channels

Consistency checks for the uncertainty estimation

- We check the effect of changing the recoil prescription
- We change the subtraction scale $\mu_{IR}$ at which $H^{(2)}$ is defined

Variations that are consistent or smaller than our uncertainty estimation

Final uncertainties: $\pm 15\%$ on $\Delta \sigma_{\text{NNLO}}$ $\pm 0.6\%$ on $\sigma_{\text{NNLO}}$
NNLO results

- Setup: $m_t = 173.3\text{GeV}$, $m_H = 125\text{GeV}$, NNLO NNPDF31 set, $\mu_0 = (2m_t + m_H)/2$

<table>
<thead>
<tr>
<th>$\sigma$ [pb]</th>
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<tr>
<td>$\sigma_{\text{LO}}$</td>
<td>$0.3910 \pm 0.313% \pm 0.222%$</td>
<td>$25.38 \pm 0.211% \pm 0.160%$</td>
</tr>
<tr>
<td>$\sigma_{\text{NLO}}$</td>
<td>$0.4875 \pm 0.56% \pm 0.91%$</td>
<td>$36.43 \pm 0.94% \pm 0.87%$</td>
</tr>
<tr>
<td>$\sigma_{\text{NNLO}}$</td>
<td>$0.5070 (31) \pm 0.9% \pm 3.0%$</td>
<td>$37.20 (25) \pm 0.1% \pm 2.2%$</td>
</tr>
</tbody>
</table>

- Effect of NLO corrections is about $+25\%$ at 13TeV and $+44\%$ at 100TeV

- Effect of NNLO corrections is about $+4\%$ at 13TeV and $+2\%$ at 100TeV

- **Strong reduction** of the perturbative uncertainties at NNLO

- Number in parenthesis includes approximation uncertainty, MC integration uncertainty, and systematic uncertainty from subtraction ($q_T \to 0$ extrapolation)
NNLO results

Data from:
[ATLAS 2207.00092]
[CMS 2207.00043]

Bands from symmetrized 7-point scale variation

Dashed band: approximation plus numerical uncertainties

Combination with NLO EW corrections of O(2%) needed for ultimate precision
Future developments

$p p \rightarrow t \bar{t} H @ 13.6 \text{ TeV}$,

$\mu_F = \mu_R = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

**Fully differential NNLO results**

[Catani, JM et al.; in prep.]

**NNLO+PS event generator**

[Wiesemann, JM; in prep.]
Summary

• We have presented the first NNLO calculation for $t\bar{t}H$ production at hadron colliders

• We used the $q_T$-subtraction method, now further extended to deal with heavy quark + colourless final states

• Missing two-loop corrections are estimated via a soft Higgs approx., related uncertainties for $\sigma_{\text{NNLO}}$ at the sub-percent level

• NNLO corrections are moderate, and leading to a significant reduction of the scale uncertainties w.r.t. NLO

• Further studies are underway: fully differential NNLO, NNLO+PS... stay tuned!

Thanks!
Backup slides
\( r_{\text{cut}} \rightarrow 0 \) extrapolation

\[ pp \rightarrow t\bar{t}H \at 13 \text{ TeV}, \mu_F = \frac{2m_t + m_H}{2}, \mu_R = \frac{2m_t + m_H}{2} \]

\[ \Delta \sigma_{\text{NNLO}}^{\text{MATRIX}^{qT}}(r_{\text{cut}} \rightarrow 0) \]

\[ \Delta \sigma_{\text{NNLO}}^{\text{MATRIX}^{qT}}(r_{\text{cut}}) \]

\[ r_{\text{cut}} = \text{cut}_{qT}/m_{t\bar{t}H} \% \]
**More numbers on the soft Higgs approx**

- Soft Higgs approximation at LO:
  
  \[ \text{gg channel: factor } 2.3 \ (2.0) \text{ larger than exact result at } 13 \ (100) \text{ TeV} \]
  
  \[ \text{q\bar{q} channel: factor } 1.11 \ (1.06) \text{ larger than exact result at } 13 \ (100) \text{ TeV} \]

- No Born reweighting at LO → worse performance compared to \( H^{(n)} \)

- The (differential) cross section within the \( q_T \)-subtraction method is

\[
d\sigma = \mathcal{H} \otimes d\sigma_{LO} + \left[ d\sigma_R - d\sigma_{CT} \right] \quad \text{with} \quad \mathcal{H} = H(\mu_{IR})\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}(\mu_{IR})
\]

\[ \text{Independent from subtraction scale} \]

\[
H(\mu_{IR}) = 1 + \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n H^{(n)}(\mu_{IR}) \quad \text{with} \quad H^{(n)}(\mu_{IR}) = \frac{2\text{Re} \left( \mathcal{M}^{(n)}(\mu_{IR}) \mathcal{M}^{(0)*} \right)}{|\mathcal{M}^{(0)}|^2}
\]

\[ \text{Only approximated piece} \]

- Varying \( \mu_{IR} \) by a factor of 2:

  \[ \text{gg channel: } +164\%/-25\% \ (13\text{TeV}) \]
  \[ +142\%/-20\% \ (100\text{TeV}) \]

  \[ \text{q\bar{q} channel: } +4\%/-0\% \ (13\text{TeV}) \]
  \[ +3\%/-0\% \ (100\text{TeV}) \]