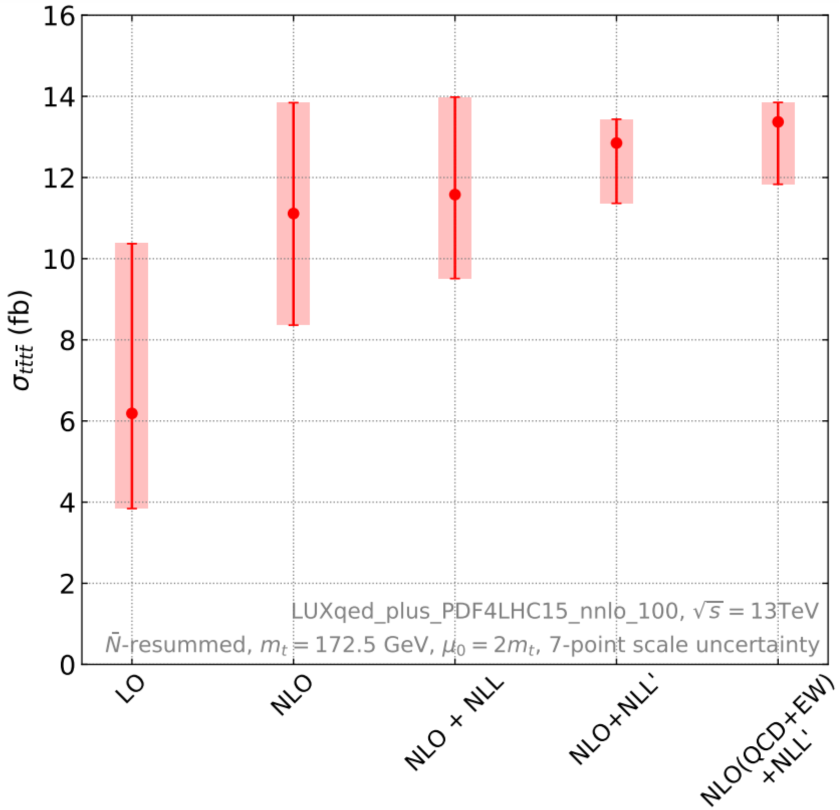
Soft gluon resummation for the production of four top quarks at the LHC

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster

(based on the paper **arXiv:2212.03259**)



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WHY 4 TOP?

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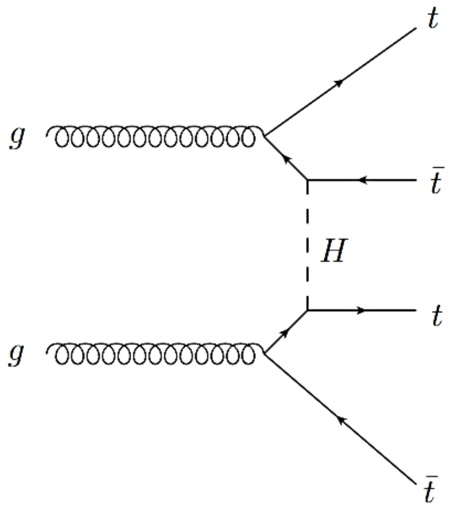

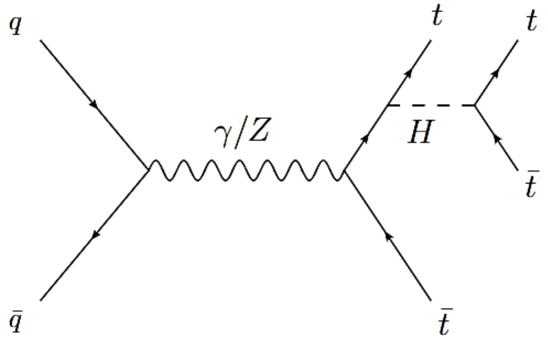

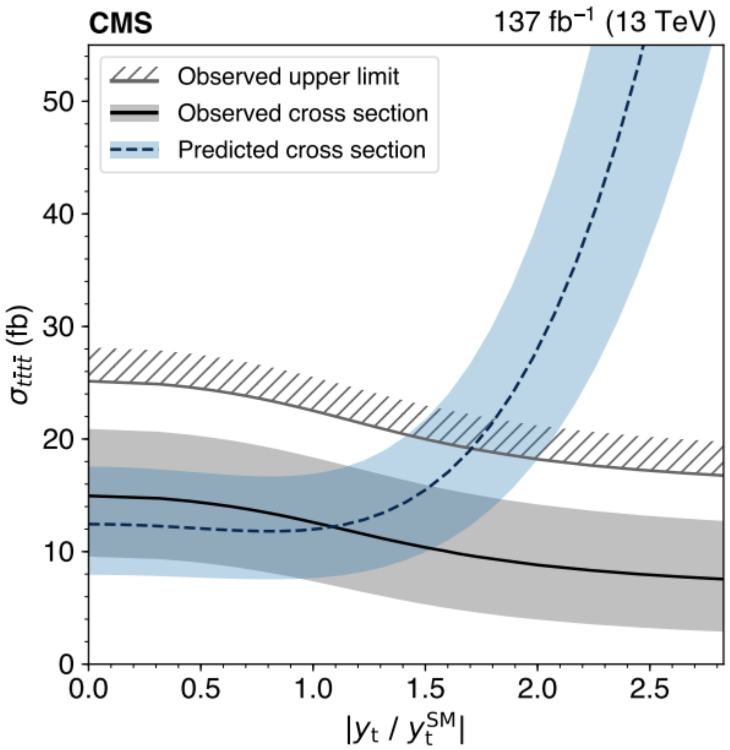

[Eur. Phys. J. C (2020) 80, 75]

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- Sensitive to the Yukawa coupling
- Sensitive to new physics (gluinos, scalar gluons, heavy scalar bosons, ...)
- Constrains SMEFT coefficients: e.g. four-fermion operator

WHY 4 TOP?

- Measured at the LHC
  - ATLAS [Eur. Phys. J. C (2020) 80, 1085; JHEP 11 (2021), 118; Phys. Rev. D 99, 052009 (2019), arXiv:2303.15061]
  - CMS [Eur. Phys. J. C (2020) 80, 75; JHEP 11 (2019), 082, arXiv:2303.03864, CMS-PAS-TOP-22-013]

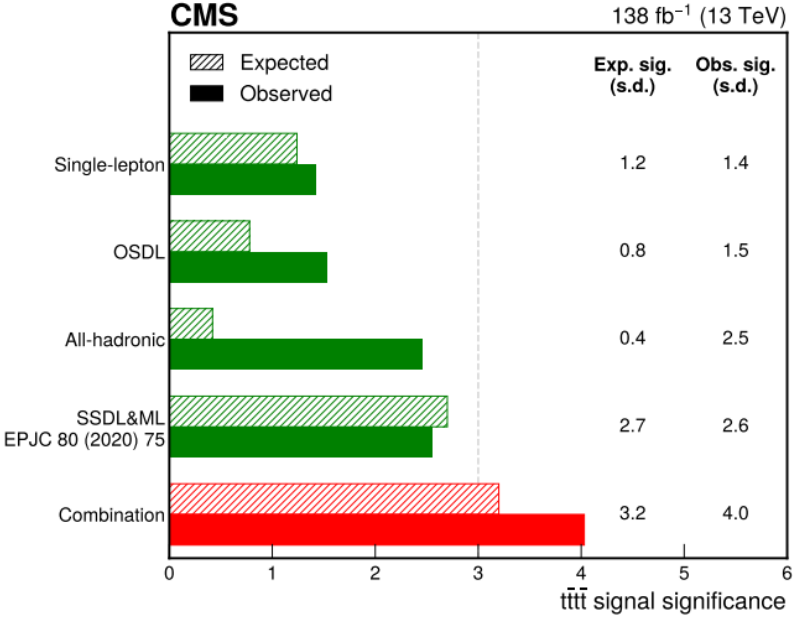
WHY 4 TOP?

$$\boxed{\sigma_{t\bar{t}t\bar{t}}^{\text{SM, NLO (QCD+EW)}} = 12.0 \pm 2.4 \text{ fb}}$$

[Frederix, Pagani, Zaro (2017)]

CMS

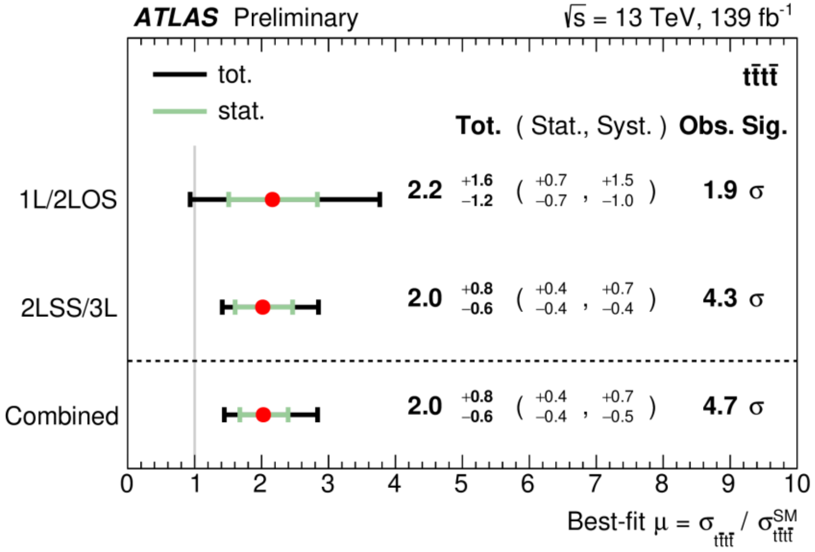
[arXiv:2303.03864]



$$\sigma_{t\bar{t}t\bar{t}}^{\text{CMS}} = 17 \pm 4(\text{stat.}) \pm 3(\text{syst.}) \text{ fb}$$

ATLAS

[JHEP 11 (2021), 118]



$$\sigma_{t\bar{t}t\bar{t}}^{\text{ATLAS}} = 24 \pm 5(\text{stat.})^{+5}_{-4}(\text{syst.}) \text{ fb}$$

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- Theoretical predictions for total rate: NLO (QCD + EW) available

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GOAL: extend the precision of theoretical predictions beyond NLO for $pp \rightarrow t\bar{t}t\bar{t}$ by means of **resummation** techniques

RESUMMATION

- Perturbative expansion cross-section

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \ldots \quad \text{with} \quad c_n = f_n + \sum_{k=0}^{2n} d_{nk} L^k \quad , \quad L^k = \left[\frac{\ln^k(1 - M^2/s)}{1 - M^2/s}\right]_+$$

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- **Soft Gluon Resummation**
  - Systematic treatment to all orders: resummation
  - Relies on ME and Phase space factorization → Mellin space $L := \left[\frac{\ln(1 - M^2/s)}{1 - M^2/s}\right]_+ \quad \to \quad \tilde{L} := \ln N$

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LL

$\alpha_s^n \ln^{2n} N$

NLL

$\alpha_s^n \ln^{2n-1} N$

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Finite contributions

LL $\alpha_s^n \ln^{2n} N$

NLL $\alpha_s^n \ln^{2n-1} N$

RESUMMATION

- Resummed partonic cross section for $pp \to t\bar{t}t\bar{t}$ in Mellin space

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}} = \text{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}} \, \mathbf{S}_{ij\to t\bar{t}t\bar{t}} \, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right] \; \Delta_i \, \Delta_j$$

RESUMMATION

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Incoming jet funcions

(soft-)collinear enhancements

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Hard function

constant contributions as $N \to \infty$

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Soft function

soft wide-angle enhancements

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\,\boldsymbol{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

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Soft anomalous dimension (SAD) matrix

COLOUR STRUCTURE

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right)\enclose{circle}{\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus\boxed{(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

6-dimensional colour space

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \boxed{\mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}}$

14-dimensional colour space

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Diagonal SAD matrix

$$\longrightarrow \quad \bar{\mathbf{U}}_R\,\mathbf{S}_R\,\mathbf{U}_R = \mathbf{S}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

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$$c_1^{q\bar{q}} = \frac{1}{\sqrt{N_c^3}}\delta_{c_1c_3}\delta_{c_2c_4}\delta_{c_6c_8}$$

$$c_2^{q\bar{q}} = \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}\delta_{c_1c_3}t^{a_1}_{c_2c_4}t^{a_1}_{c_6c_8}$$

$$c_3^{q\bar{q}} = \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}\delta_{c_2c_4}t^{a_1}_{c_6c_8}$$

$$c_4^{q\bar{q}} = \frac{1}{T_R\sqrt{N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}t^{a_1}_{c_2c_4}\delta_{c_6c_8}$$

$$c_5^{q\bar{q}} = \frac{\sqrt{N_c}}{T_R^2\sqrt{2(N_c^4-5N_c^2+4)}}t^{a_1}_{c_1c_3}d^{a_1a_2b_3}t^{a_2}_{c_2c_4}t^{b_3}_{c_6c_8}$$

$$c_6^{q\bar{q}} = \frac{1}{T_R^2\sqrt{2N_c(N_c^2-1)}}t^{a_1}_{c_1c_3}if^{a_1a_2b_3}t^{a_2}_{c_2c_4}t^{b_3}_{c_6c_8}\,,$$

[Keppeler, Sjodahl (2012)]

6x6 SAD matrix

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,1} = -\,C_F\,(L_{\beta_{34}}+L_{\beta_{56}}+2)$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,2} = \frac{\sqrt{N_c^2-1}}{2N_c}\,(L_{\beta_{35}}+L_{\beta_{46}}-L_{\beta_{36}}-L_{\beta_{45}})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,3} = \frac{\sqrt{N_c^2-1}}{2N_c}\,(T_{15}-T_{16}-T_{25}+T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,4} = \frac{\sqrt{N_c^2-1}}{2N_c}\,(T_{13}-T_{14}-T_{23}+T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,5} = 0$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},1\,6} = 0$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,2} = -\,2C_F+\frac{1}{2N_c}\,(L_{\beta_{34}}+L_{\beta_{56}})-\frac{1}{N_c}\,(L_{\beta_{35}}+L_{\beta_{46}})-\frac{N_c^2-2}{2N_c}\,(L_{\beta_{36}}+L_{\beta_{45}})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,3} = \frac{1}{2N_c}\,(T_{13}-T_{14}-T_{23}+T_{24})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,4} = \frac{1}{2N_c}\,(T_{15}-T_{16}-T_{25}+T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}\,(T_{13}-T_{14}+T_{15}-T_{16}-T_{23}+T_{24}-T_{25}+T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},2\,6} = \frac{1}{2\sqrt{2}}\,(-T_{13}-T_{14}+T_{15}+T_{16}+T_{23}+T_{24}-T_{25}-T_{26})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},3\,3} = -\,\frac{N_c^2-2}{2N_c}-C_FL_{\beta_{34}}+\frac{1}{2N_c}L_{\beta_{56}}+\frac{N_c^2-2}{2N_c}T_{15}+\frac{1}{N_c}T_{16}+\frac{N_c^2-2}{2N_c}T_{26}+\frac{1}{N_c}T_{25}$$

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$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},4\,4} = -\,C_F+\frac{1}{2N_c}-C_FL_{\beta_{56}}+\frac{1}{2N_c}L_{\beta_{34}}+\frac{N_c^2-2}{2N_c}\,(T_{13}+T_{24})+\frac{1}{N_c}\,(T_{14}+T_{23})$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},4\,5} = \frac{\sqrt{N_c^2-4}}{2\sqrt{2}\,N_c}\,(L_{\beta_{35}}-L_{\beta_{36}}-L_{\beta_{45}}+L_{\beta_{46}}+T_{15}-T_{16}-T_{25}+T_{26})$$

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$$\begin{aligned}\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},5\,5} = {} & -\,C_F+\frac{1}{2N_c}+\frac{1}{2N_c}\left(L_{\beta_{34}}-3L_{\beta_{35}}-3L_{\beta_{46}}+L_{\beta_{56}}-\frac{N_c^2-6}{2}L_{\beta_{36}}-\frac{N_c^2-6}{2}L_{\beta_{45}}\right)\\ & +\frac{3}{2N_c}\,(T_{14}+T_{16}+T_{23}+T_{25})+\frac{N_c^2-6}{4N_c}\,(T_{13}+T_{15}+T_{24}+T_{26})\end{aligned}$$

$$\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},5\,6} = \frac{\sqrt{N_c^2-4}}{4}\,(-L_{\beta_{36}}+L_{\beta_{45}}-T_{13}+T_{15}+T_{24}-T_{26})$$

$$\begin{aligned}\bar{\Gamma}^{(1)}_{q\bar{q}\to 4\mathrm{top},6\,6} = {} & -\,C_F+\frac{1}{2N_c}-\frac{N_c^2-2}{4N_c}\,(L_{\beta_{36}}+L_{\beta_{45}})+\frac{1}{2N_c}\,(L_{\beta_{34}}-L_{\beta_{35}}-L_{\beta_{46}}+L_{\beta_{56}})\\ & +\frac{N_c^2-2}{4N_c}\,(T_{13}+T_{15}+T_{24}+T_{26})+\frac{1}{2N_c}\,(T_{14}+T_{16}+T_{23}+T_{25})\end{aligned}$$

SAD matrix diagonal at threshold

COLOUR STRUCTURE

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,, \qquad c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}}\delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}}\delta_{c_2c_4} d_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,, \qquad c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}}\delta_{c_2c_4} i f_{a_1a_2b_1} t^{b_1}_{c_6c_8}\,,$$

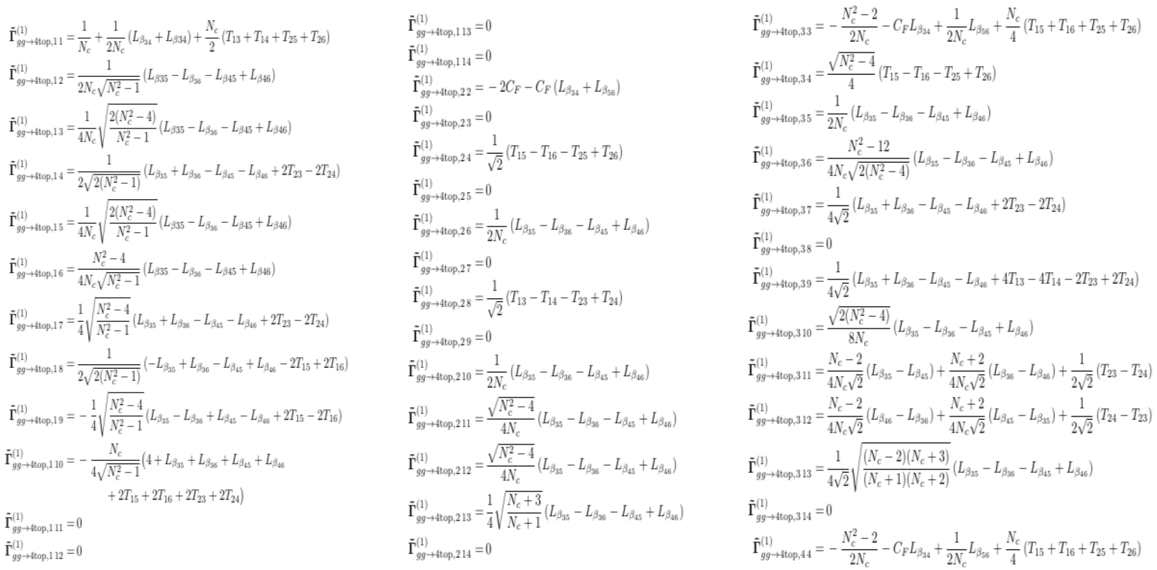
$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2}\delta_{c_6c_8}\,, \qquad c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} i f_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$



$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},11} = \frac{1}{N_c} + \frac{1}{2N_c}\left(L_{\beta_{34}} + L_{\beta 34}\right) + \frac{N_c}{2}\left(T_{13}+T_{14}+T_{25}+T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},12} = \frac{1}{2N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},13} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},14} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},15} = \frac{1}{4N_c}\sqrt{\frac{2(N_c^2-4)}{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},16} = \frac{N_c^2-4}{4N_c\sqrt{N_c^2-1}}\left(L_{\beta 35} - L_{\beta_{36}} - L_{\beta 45} + L_{\beta 46}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},17} = \frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},18} = \frac{1}{2\sqrt{2(N_c^2-1)}}\left(-L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}} - 2T_{15} + 2T_{16}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},19} = -\frac{1}{4}\sqrt{\frac{N_c^2-4}{N_c^2-1}}\left(L_{\beta_{35}} - L_{\beta_{36}} + L_{\beta_{45}} - L_{\beta_{46}} + 2T_{15} - 2T_{16}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,10} = -\frac{N_c}{4\sqrt{N_c^2-1}}\Big(4 + L_{\beta_{35}} + L_{\beta_{36}} + L_{\beta_{45}} + L_{\beta_{46}} + 2T_{15} + 2T_{16} + 2T_{23} + 2T_{24}\Big)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,11} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,12} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,13} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},1\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},22} = -2C_F - C_F\left(L_{\beta_{34}} + L_{\beta_{56}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},23} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},24} = \frac{1}{\sqrt{2}}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},25} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},26} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},27} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},28} = \frac{1}{\sqrt{2}}\left(T_{13} - T_{14} - T_{23} + T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},29} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,10} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,11} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,12} = \frac{\sqrt{N_c^2-4}}{4N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,13} = \frac{1}{4}\sqrt{\frac{N_c+3}{N_c+1}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},2\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},33} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15}+T_{16}+T_{25}+T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},34} = \frac{\sqrt{N_c^2-4}}{4}\left(T_{15} - T_{16} - T_{25} + T_{26}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},35} = \frac{1}{2N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},36} = \frac{N_c^2-12}{4N_c\sqrt{2(N_c^2-4)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},37} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 2T_{23} - 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},38} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},39} = \frac{1}{4\sqrt{2}}\left(L_{\beta_{35}} + L_{\beta_{36}} - L_{\beta_{45}} - L_{\beta_{46}} + 4T_{13} - 4T_{14} - 2T_{23} + 2T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,10} = \frac{\sqrt{2(N_c^2-4)}}{8N_c}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,11} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{35}} - L_{\beta_{45}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{36}} - L_{\beta_{46}}\right) + \frac{1}{2\sqrt{2}}\left(T_{23} - T_{24}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,12} = \frac{N_c-2}{4N_c\sqrt{2}}\left(L_{\beta_{46}} - L_{\beta_{36}}\right) + \frac{N_c+2}{4N_c\sqrt{2}}\left(L_{\beta_{45}} - L_{\beta_{35}}\right) + \frac{1}{2\sqrt{2}}\left(T_{24} - T_{23}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,13} = \frac{1}{4\sqrt{2}}\sqrt{\frac{(N_c-2)(N_c+3)}{(N_c+1)(N_c+2)}}\left(L_{\beta_{35}} - L_{\beta_{36}} - L_{\beta_{45}} + L_{\beta_{46}}\right)$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},3\,14} = 0$$
$$\tilde{\Gamma}^{(1)}_{gg\to 4\text{top},44} = -\frac{N_c^2-2}{2N_c} - C_F L_{\beta_{34}} + \frac{1}{2N_c}L_{\beta_{56}} + \frac{N_c}{4}\left(T_{15}+T_{16}+T_{25}+T_{26}\right)$$

SAD matrix not
diagonal at threshold

+ 65 more components!

[Keppeler, Sjodahl (2012)]

COLOUR STRUCTURE

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8} \otimes \mathbf{8} = \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \longrightarrow \mathbf{0} \oplus \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27} = \boxed{\mathbf{0} \oplus (2 \times \mathbf{1}) \oplus (2 \times \mathbf{8}) \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}}$

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,, \qquad c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}}\,\delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}}\delta_{c_2c_4} d_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,, \qquad c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}}\delta_{c_2c_4} i f_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2}\delta_{c_6c_8}\,, \qquad c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} i f_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,, \qquad c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,, \qquad c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$

[Keppeler, Sjodahl (2012)]

SAD matrix not diagonal at threshold

$$\bar{c}_1^{gg} = \frac{3\sqrt{3}}{8}c_1^{gg} + \frac{3}{10}\sqrt{\frac{3}{2}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{3}{2}}c_{10}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{11}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{12}^{gg} + \frac{7}{40}c_{13}^{gg}\,,$$

$$\bar{c}_2^{gg} = -\frac{\sqrt{5}}{4}c_1^{gg} + \sqrt{\frac{2}{5}}c_6^{gg} - \frac{1}{2\sqrt{2}}c_{11}^{gg} - \frac{1}{2\sqrt{2}}c_{12}^{gg} + \frac{1}{4}\sqrt{\frac{3}{5}}c_{13}^{gg}\,,$$

$$\bar{c}_3^{gg} = -\frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_5^{gg} = -\frac{1}{2\sqrt{2}}c_1^{gg} - \frac{1}{2}c_6^{gg} - \frac{1}{2}c_{10}^{gg} + \frac{1}{2}\sqrt{\frac{3}{2}}c_{13}^{gg}\,,$$

$$\bar{c}_6^{gg} = -\frac{1}{2}\sqrt{\frac{5}{14}}c_1^{gg} + \frac{3}{2\sqrt{35}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{5}{7}}c_{10}^{gg} + \frac{2}{\sqrt{7}}c_{12}^{gg} - \frac{3}{2}\sqrt{\frac{3}{70}}c_{13}^{gg}\,,$$

$$\bar{c}_7^{gg} = -\frac{1}{2\sqrt{7}}c_1^{gg} + \frac{3}{5\sqrt{14}}c_6^{gg} - \frac{1}{\sqrt{14}}c_{10}^{gg} + \sqrt{\frac{7}{10}}c_{11}^{gg} - \frac{3}{\sqrt{70}}c_{12}^{gg} - \frac{3}{10}\sqrt{\frac{3}{7}}c_{13}^{gg}\,,$$

$$\bar{c}_8^{gg} = \frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_{13}^{gg} = \frac{1}{8}c_1^{gg} + \frac{1}{2\sqrt{2}}c_6^{gg} + \frac{1}{2\sqrt{2}}c_{10}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{11}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{12}^{gg} + \frac{3\sqrt{3}}{8}c_{13}^{gg}\,,$$

$$\bar{c}_4^{gg} = c_{14}^{gg}\,, \quad \bar{c}_9^{gg} = c_8^{gg}\,, \quad \bar{c}_{10}^{gg} = c_5^{gg}\,, \quad \bar{c}_{11}^{gg} = c_4^{gg}\,, \quad \bar{c}_{12}^{gg} = c_3^{gg}\,, \quad \bar{c}_{14}^{gg} = c_2^{gg}\,.$$

SAD matrix diagonal at threshold

SAD MATRIX

- One-loop SAD matrix needed at NLL

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right)\boxed{\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Diagonal SAD matrix

$$\longrightarrow \quad \bar{\mathbf{U}}_R\, \mathbf{S}_R\, \mathbf{U}_R = \mathbf{S}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0}\ln(1-2\lambda)\right]$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus\boxed{(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \boxed{\mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}}$

SAD MATRIX

- One-loop SAD matrix needed at NLL

Diagonal SAD matrix

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2, \mu_F^2, \mu_R^2) = \mathrm{Pexp}\left[\int_{\mu}^{M/\bar{N}} \frac{dq}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right] \longrightarrow \bar{\mathbf{U}}_R\, \mathbf{S}_R\, \mathbf{U}_R = \mathbf{S}_R \exp\left[\frac{2\,\mathrm{Re}\left(\mathbf{\Gamma}_R^{(1)}\right)}{2\pi b_0} \ln(1-2\lambda)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s\right) = \left(\frac{\alpha_s}{\pi}\right) \boxed{\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}}} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

with $\lambda = \alpha_s b_0 \ln(\bar{N})$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus\boxed{(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0, 0, -3, -3, -3, -3\Big)$$

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \boxed{\mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}}$

$$2\mathrm{Re}\left[\mathbf{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\Big)$$

SAD MATRIX

- Quadratic Casimir Invariants $N_c = 3$

$$\begin{aligned} C_2(\mathbf{1}) &= 0 \\ C_2(\mathbf{8}_{(S/A)}) &= 3 \\ C_2(\mathbf{10},\overline{\mathbf{10}}) &= 6 \\ C_2(\mathbf{27}) &= 8 \\ C_2(\mathbf{0}) &= 4 \end{aligned}$$

- $q\bar{q} \to t\bar{t}t\bar{t}:\ \mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus\boxed{(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$

$$2\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,q\bar{q}\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(0,0,-3,-3,-3,-3\Big)$$

- $gg \to t\bar{t}t\bar{t}:\ \mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \boxed{\mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}}$

$$2\mathrm{Re}\left[\boldsymbol{\Gamma}^{(1)}_{R,gg\to t\bar{t}t\bar{t}}\right] = \mathrm{diag}\Big(-8,-6,-6,-4,-3,-3,-3,-3,-3,-3,-3,-3,0,0\Big)$$

RESUMMATION: NLL and NLL'

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots$$

$$\mathbf{S} = \mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{S}^{(1)} + \ldots$$

- **NLL** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)}\right]$$

- **NLL'** accuracy: exponential functions at NLL together with

$$\mathrm{Tr}\left[\mathbf{HS}\right] = \mathrm{Tr}\left[\mathbf{H}^{(0)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)}\mathbf{S}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(0)}\mathbf{S}^{(1)}\right]$$

MATCHING

$$\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLL}(')}(\tau) = \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2)\, \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)$$

- Combination fixed-order + resummation → **Matching**

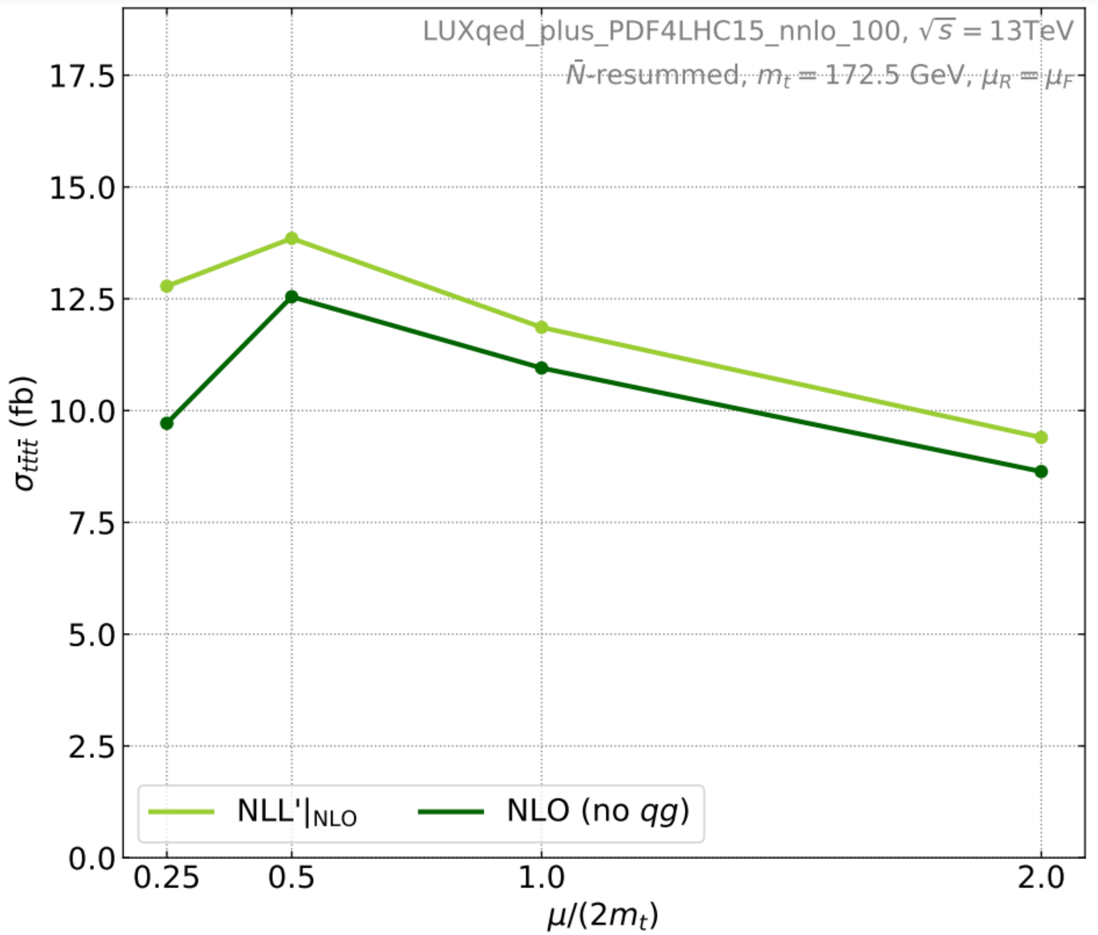
$$\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLO+NLL}(')}(\tau) = \boxed{\sigma_{t\bar{t}t\bar{t}}^{\mathrm{NLO}}(\tau)} + \int_{\mathcal{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N}\, f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \underbrace{\left[\hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)\big|_{\mathrm{NLO}}\right]}_{\text{avoid double counting with NLO!}}$$

QCD-only NLO and
QCD + EW NLO

RESULTS

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$

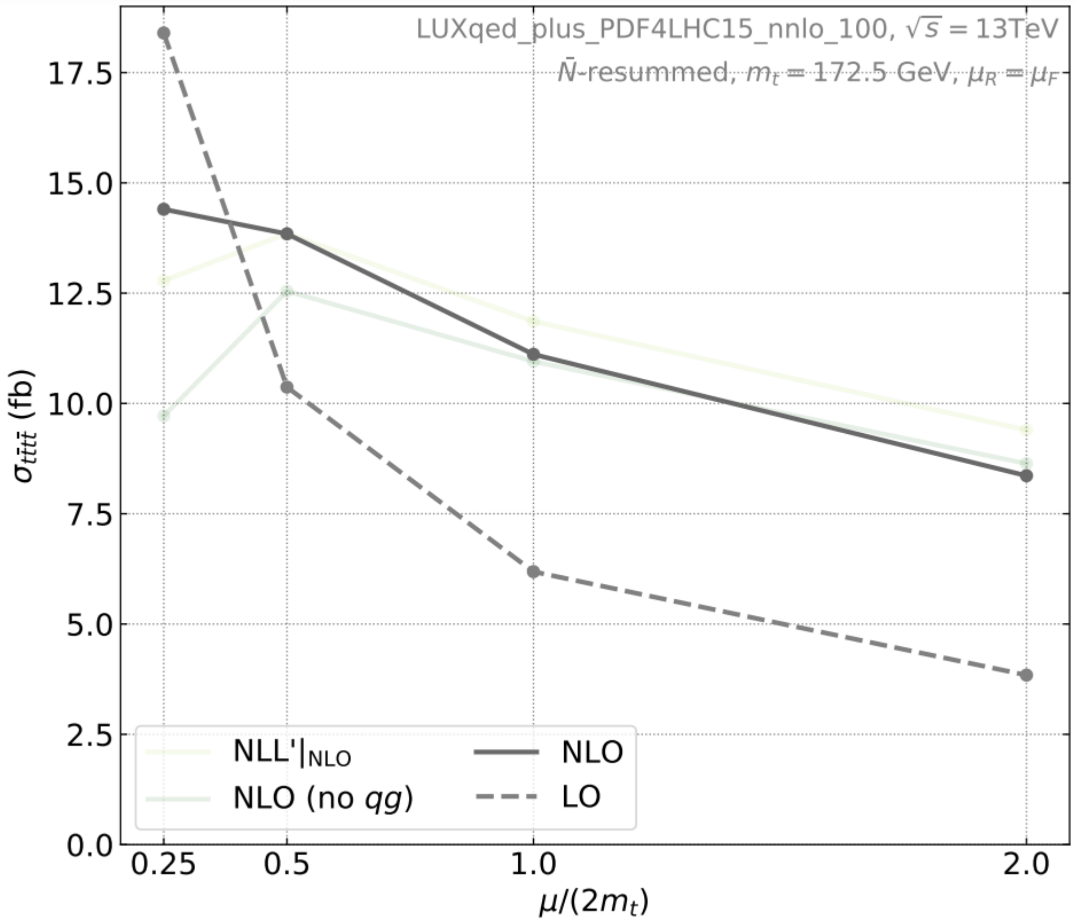
- Comparison expanded resummed cross section (NLL’|NLO) against NLO (no qg)



Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD



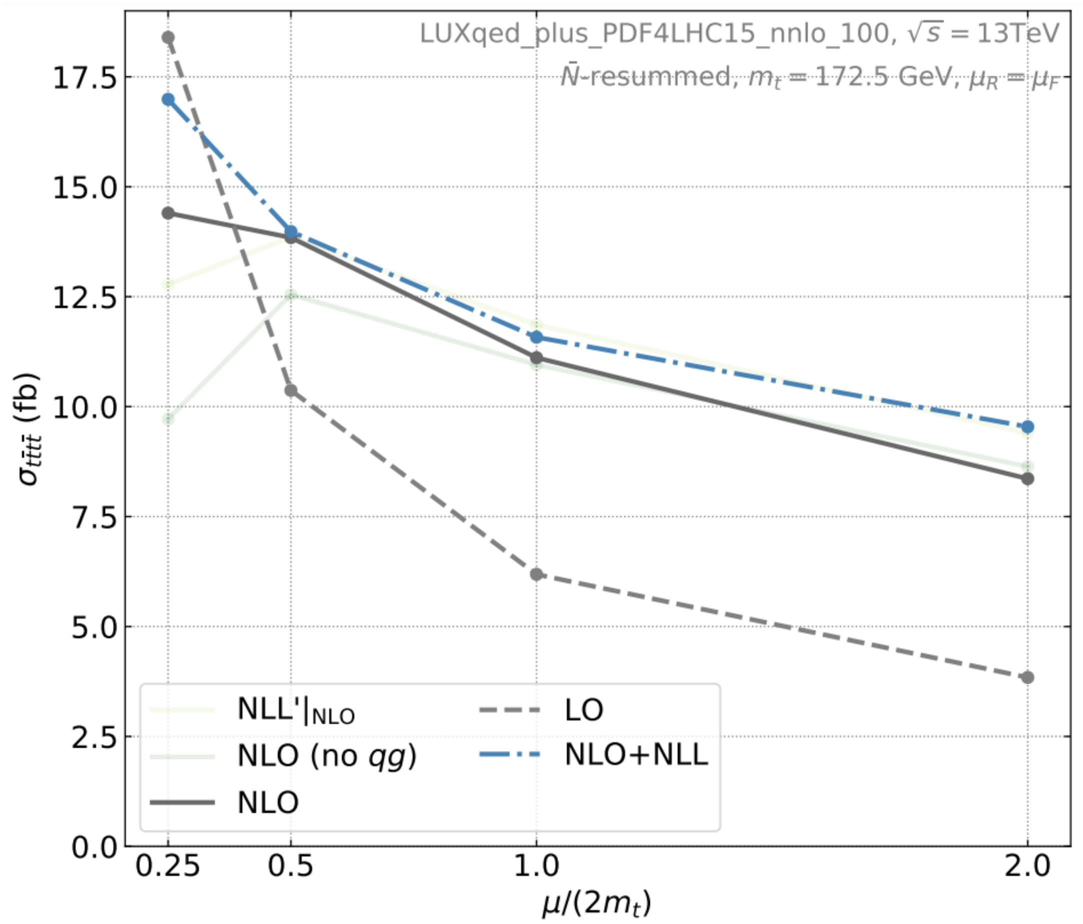
NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

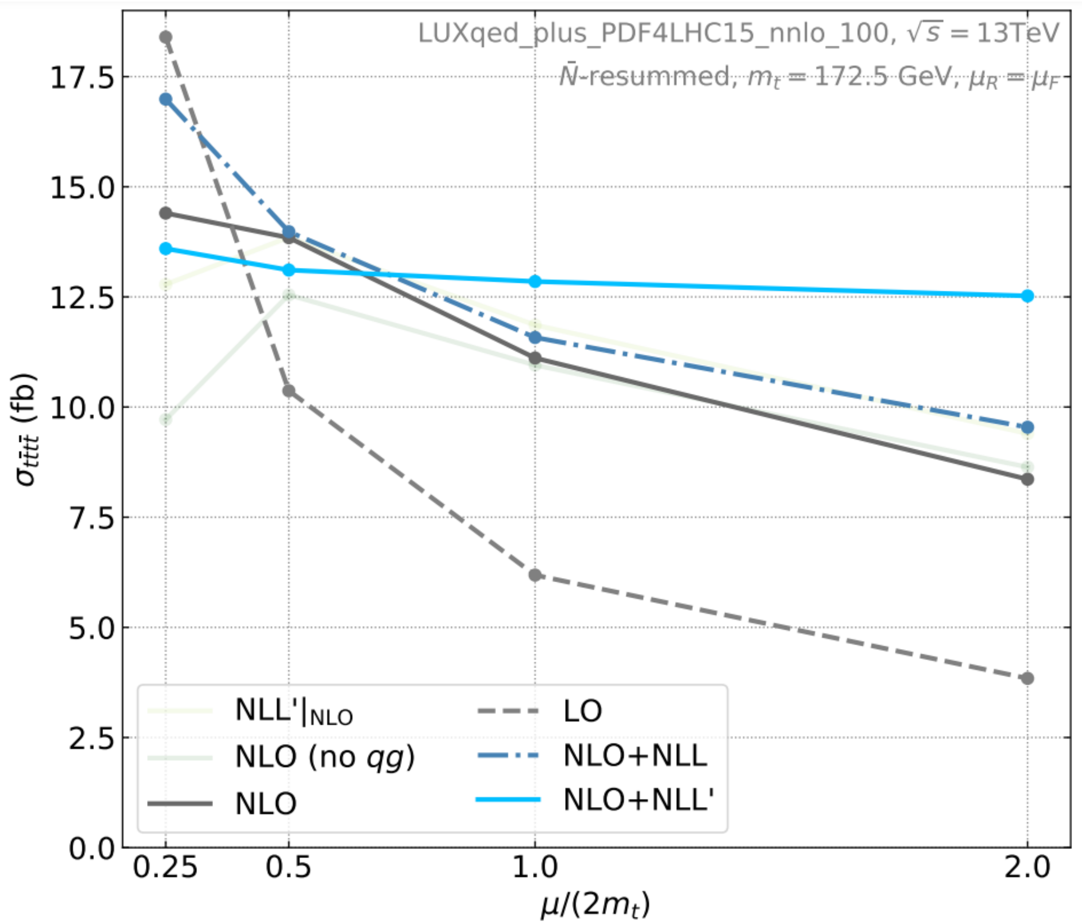


Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL
- Upgrade to NLL’

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



Fixed-order results obtained with
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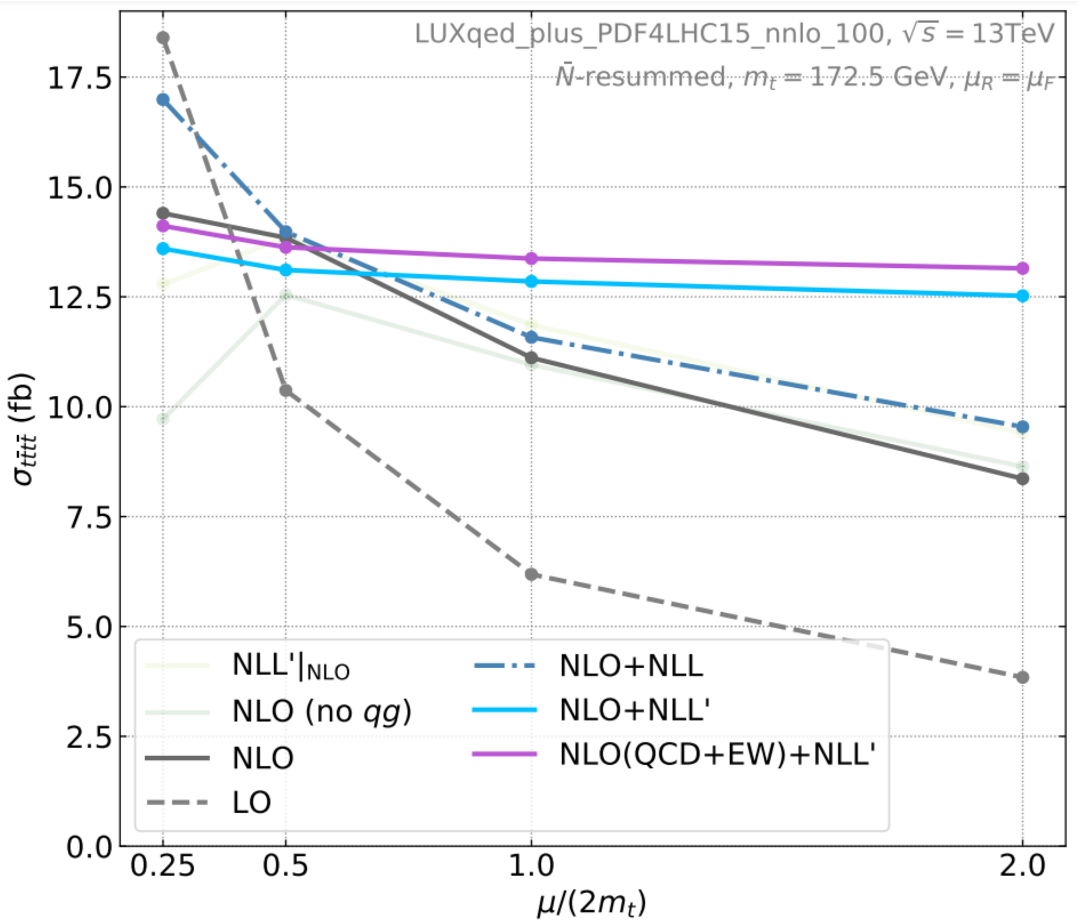
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS

- $\sqrt{S} = 13$ TeV, $\mu_R = \mu_F$
- Fixed order QCD
- Exponentials at NLL
- Upgrade to NLL’
- Match to NLO (QCD+EW)

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$



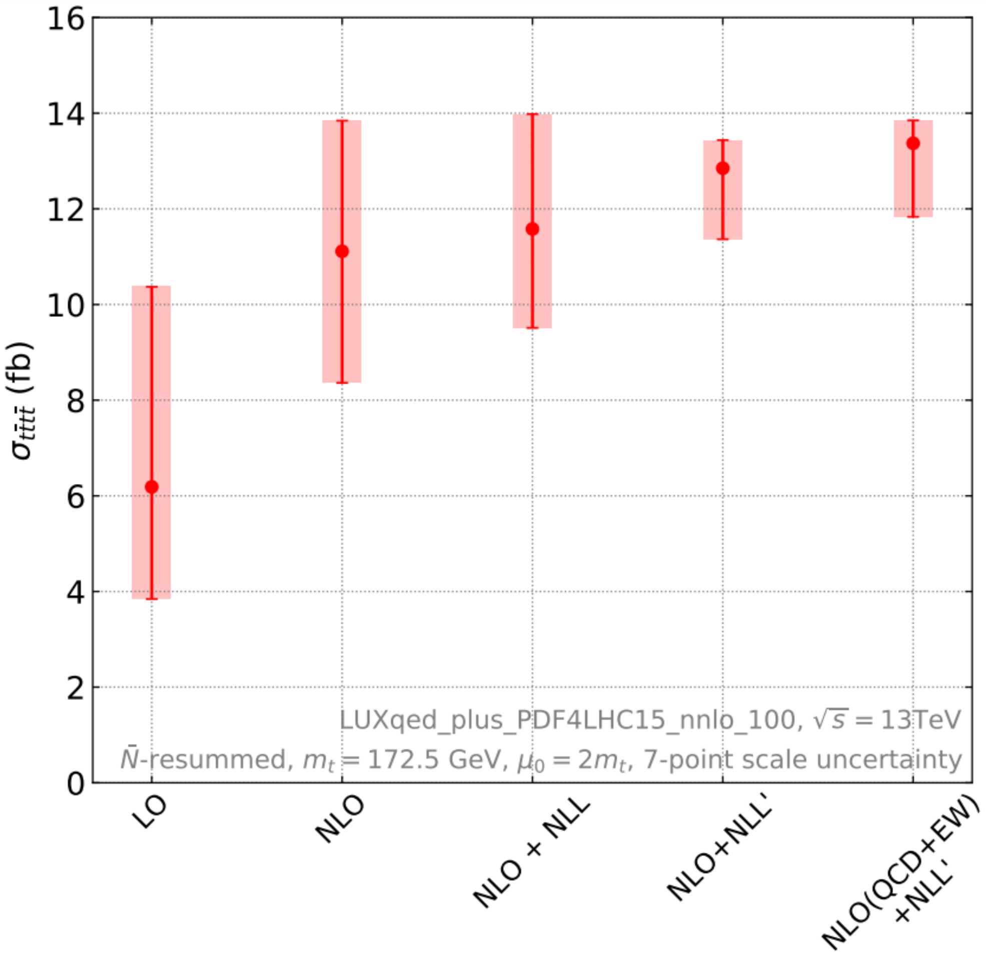
Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$11.00(2)^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46(2)^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL$'$	$12.73(2)^{+4.1\%}_{-11.8\%}$	1.16
NLO (QCD+EW)	$11.64(2)^{+23.2\%}_{-22.8\%}$	
NLO (QCD+EW)+NLL$'$	$13.37(2)^{+3.6\%}_{-11.4\%}$	1.15

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

PDF error: $\pm$ 6.9%



Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

RESULTS for $\sqrt{S} = 13.6$ TeV

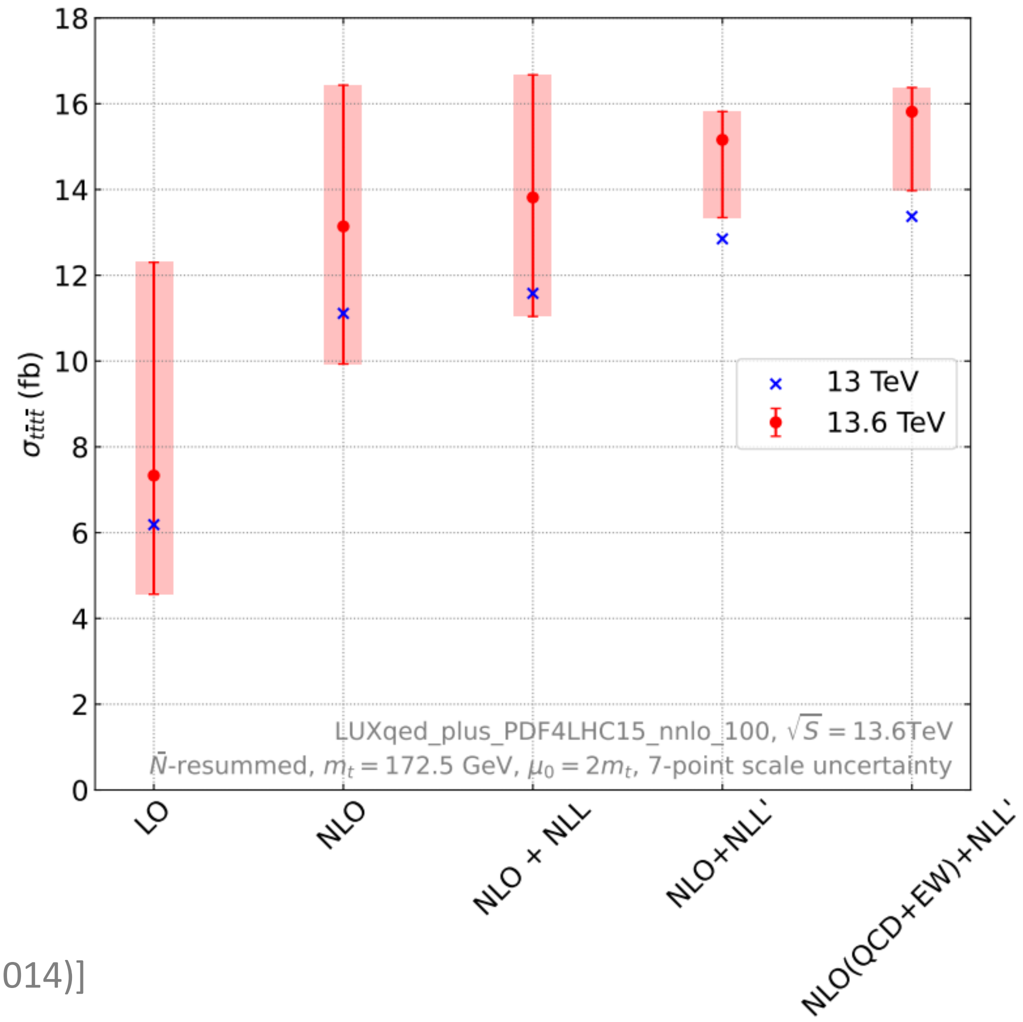
- 7-point scale variation

	$\sigma_{t\bar{t}t\bar{t}}$ [fb]	K-factor
NLO	$13.14(2)^{+25.1\%}_{-24.4\%}$	
NLO+NLL	$13.81(2)^{+20.7\%}_{-20.1\%}$	1.05
NLO+NLL$'$	$15.16(2)^{+4.3\%}_{-11.9\%}$	1.15
NLO (QCD+EW)	$13.80(2)^{+22.9\%}_{-22.6\%}$	
NLO (QCD+EW)+NLL$'$	$15.81(2)^{+3.6\%}_{-11.6\%}$	1.14

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

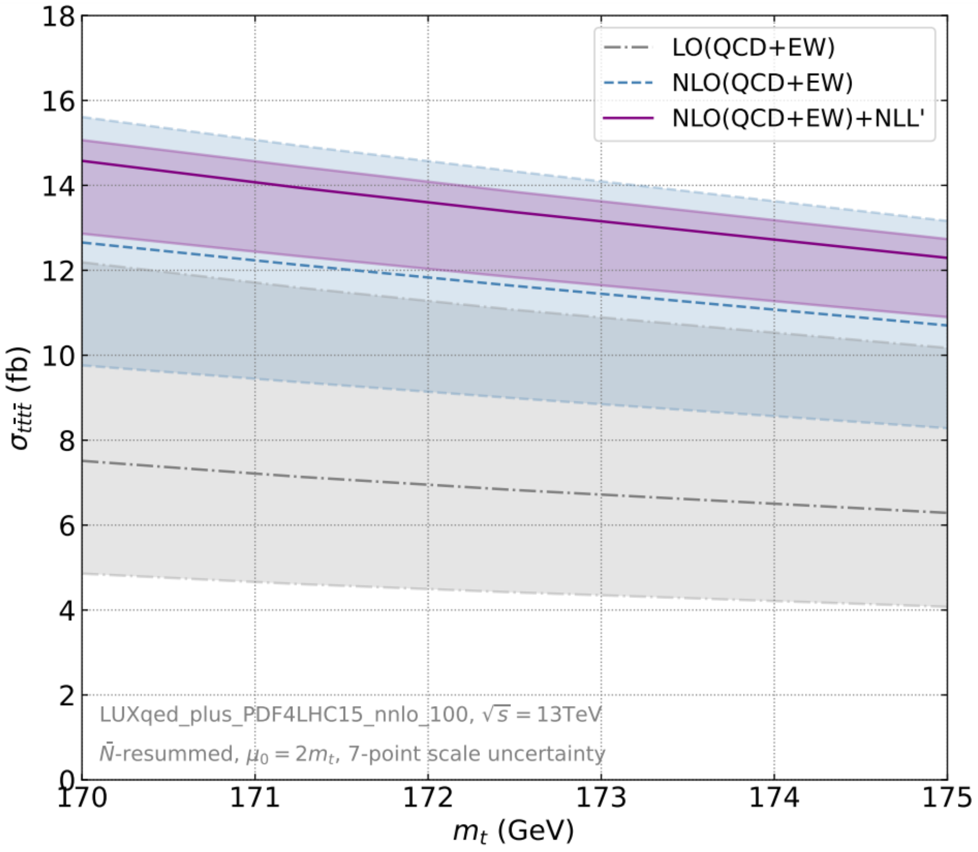
PDF error: $\pm$ 6.7%

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]



TOP MASS DEPENDENCE

- Top mass $\in$ [170,175] GeV

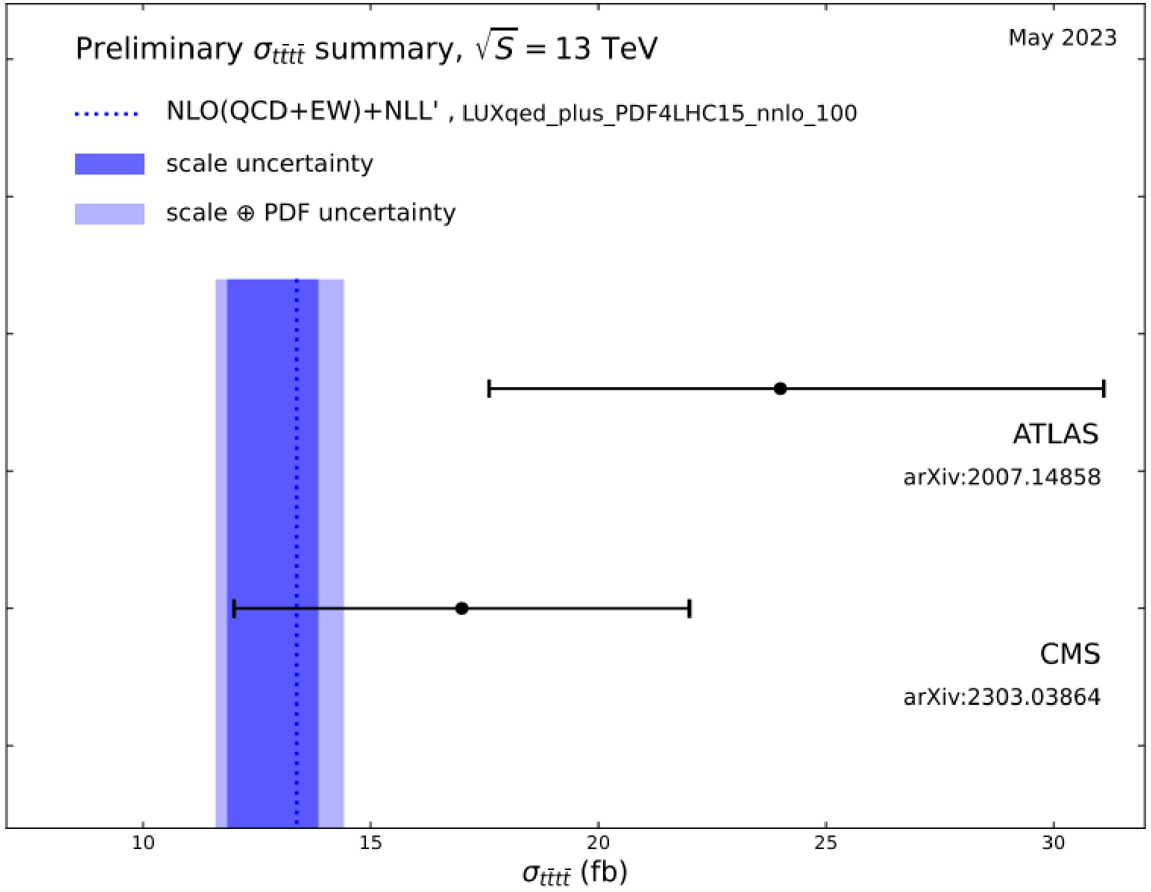


Error bands: 7-point scale uncertainty

NLO (QCD+EW): EW corrections up to $\mathcal{O}(\alpha^2)$

Fixed-order results obtained with
[Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2018)]
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro (2014)]

CONCLUSIONS



- Soft gluon resummation at NLL' accuracy
- Significant reduction of the total scale uncertainty

NLO (QCD + EW) + NLL' at 13 TeV

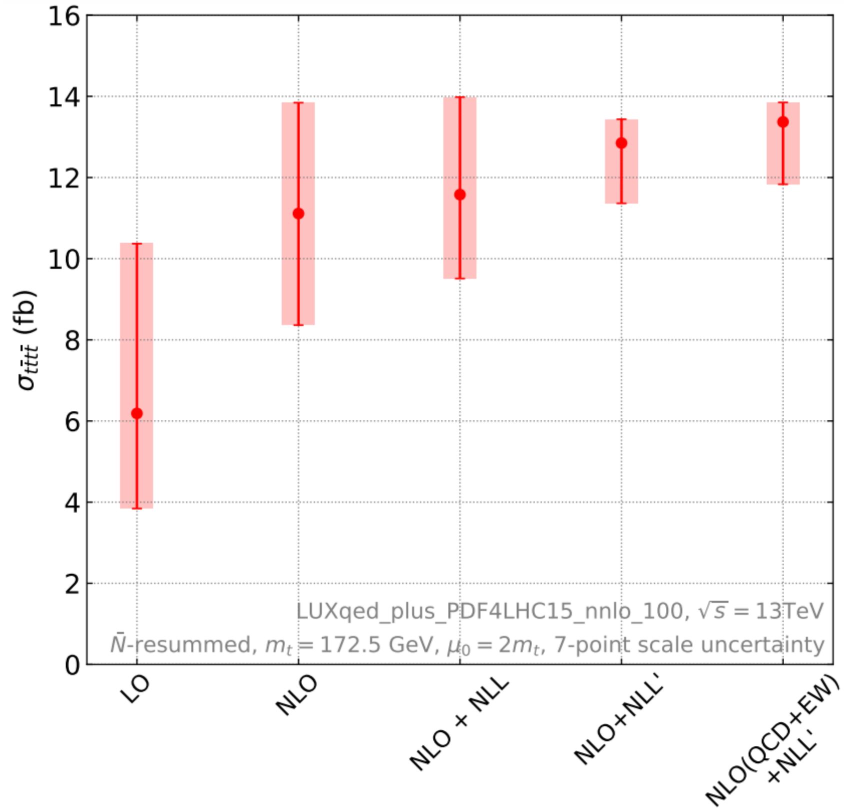
$$13.37\,(2)\,^{+3.6\%}_{-11.4\%}\,^{+6.9\%}_{-6.9\%}\ \text{fb}$$

Soft gluon resummation for the production of four top quarks at the LHC

Laura Moreno Valero

in collaboration with Melissa van Beekveld and Anna Kulesza

Institute for Theoretical Physics, University of Münster



(based on the paper **arXiv:2212.03259**)

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LHCP 2023 Belgrade 22-26 May 2023