Impact of top mass on top differential distributions

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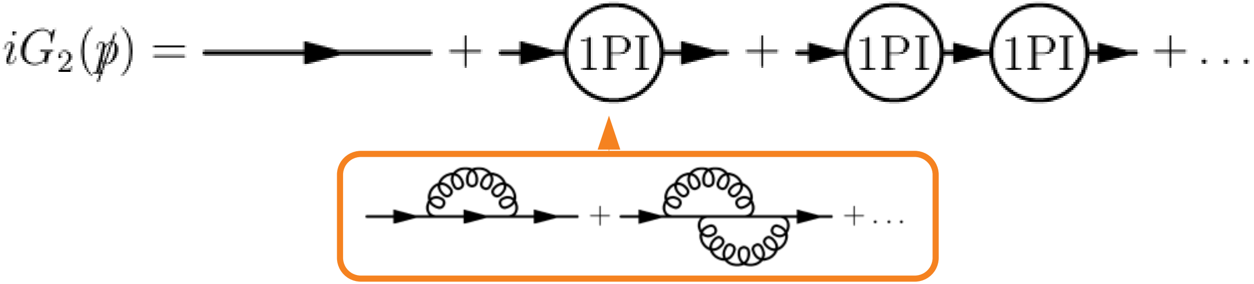
DESY.

Quark masses

- Quarks are not observable as free particles, masses defined formally via renormalization



- The **pole mass** is the pQCD analogue to the mass of a free particle with the propagator
  - Suffers from an inherent theoretical IR uncertainty of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
    - Avoided in **short distance** schemes, such as $\overline{\text{MS}}$ and MSR

- Theoretically well-defined masses can be extracted from *cross section measurements*

  - Here, behavior of single-differential $t\bar{t}$ production cross-sections will be examined w.r.t. separate scales μ_r, μ_r, *R (or* μ_m*)* – *Dedicated scale for mass renormalization*

The running top quark mass

- The pole and $\overline{\text{MS}}$ masses are related by

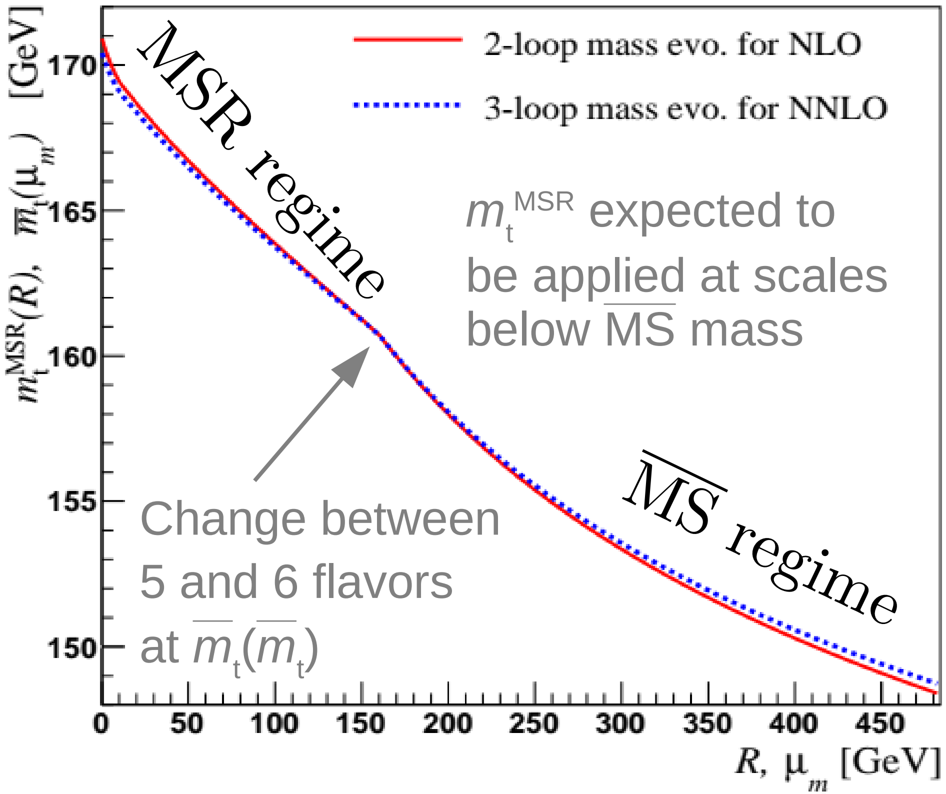
$$m_t^{\text{pole}} = \overline{m}_t(\mu_m) + \overline{m}_t(\mu_m) \sum_{n=1} \frac{\alpha_S(\mu_m)^n}{\pi^n} d_n(\mu_m)$$

- $\overline{\text{MS}}$ mass has issues at the $t\bar{t}$ production threshold, unlike the pole mass

- **The MSR mass**: a mass renormalization scheme to bridge $\overline{\text{MS}}$ and pole masses

$$m_t^{\text{pole}} = m_t^{\text{MSR}} + R \sum_{n=1} \frac{\alpha_S(R)^n}{\pi^n} d_n^{\text{MSR}}(R)$$

- ***The behavior of the mass renormalization scale R is studied here for the first time***



The single-differential $t\bar{t}$ cross section at NLO

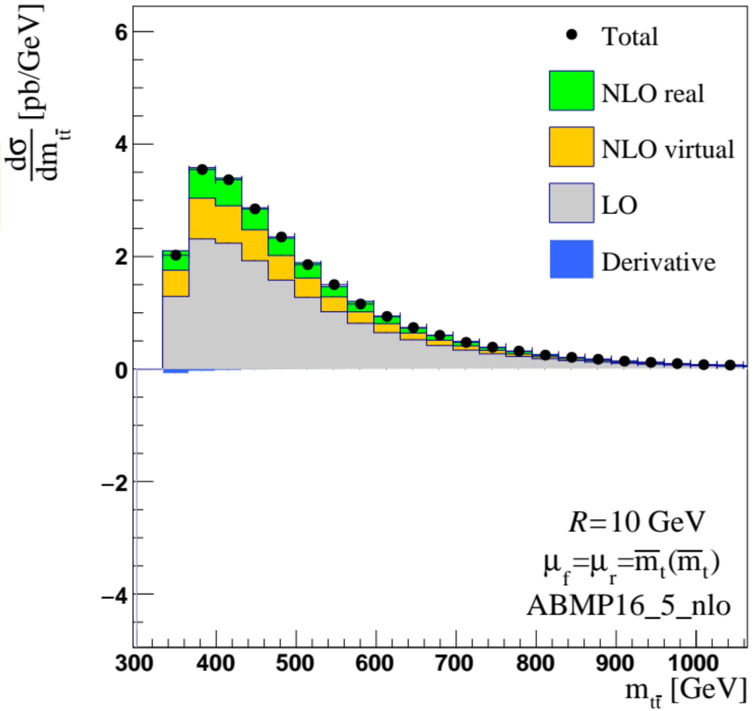
- In the MSR scheme, the cross section is divided into LO NLO and derivative terms

$$\frac{d\sigma}{dm_{t\bar{t}}} = a_S(\mu_r)^2 \frac{d\sigma^{(0)}}{dm_{t\bar{t}}}(m_t^{MSR}(R), \mu_r) + a_S(\mu_r)^3 \frac{d\sigma^{(1)}}{dm_{t\bar{t}}}(m_t^{MSR}(R), \mu_r)$$
$$+ a_S(\mu_r)^3 d_1 R \frac{d}{dm_t}\left(\frac{d\sigma^{(0)}(m_t, \mu_r)}{dm_{t\bar{t}}}\right)\Bigg|_{m_t = m_t^{MSR}(R)}$$

- Implemented into the MCFM v6.8 Monte Carlo
  - Also antiquark rapidity and p_T distributions available
  - Focus here on pair invariant mass distribution

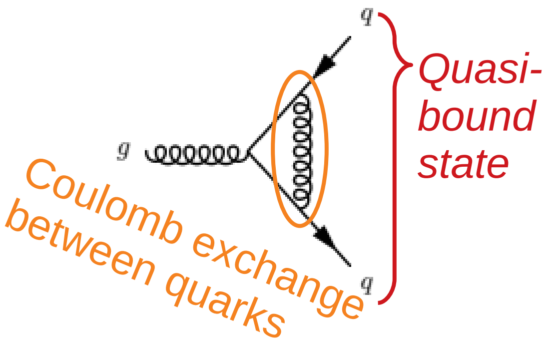
Validated against:

- Inclusive $t\bar{t}$ cross section implemented into HATHOR
- External differential computation translating pole scheme results to MSR



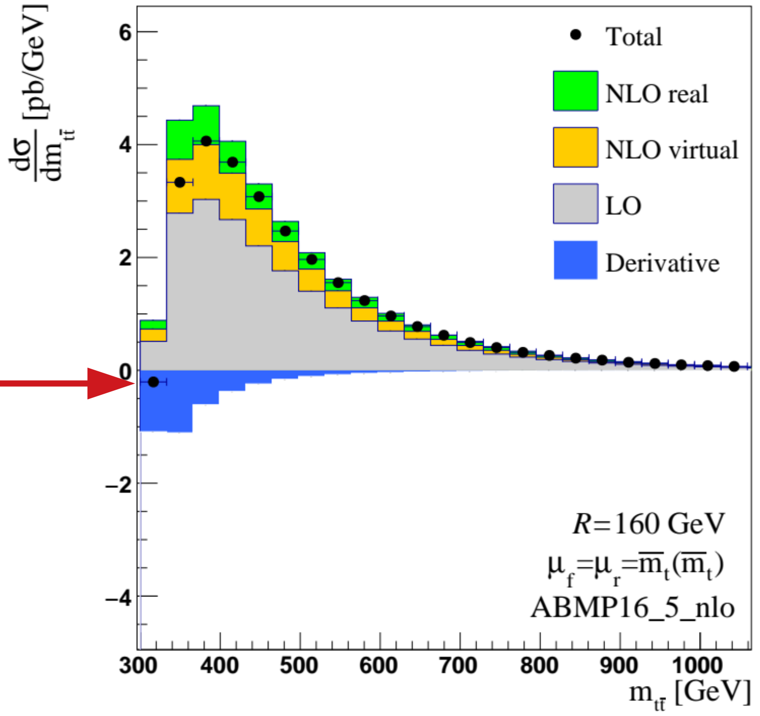
The single-differential $t\bar{t}$ cross section at NLO

Known issue
Fixed-order pQCD does not account for Coulomb effects at production threshold!



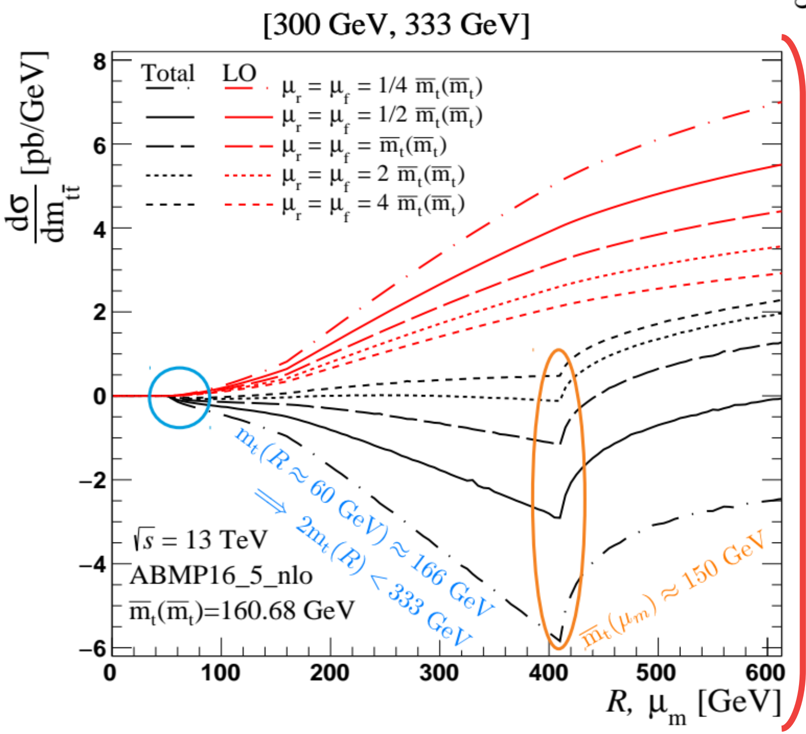
- **Multiscale problem**

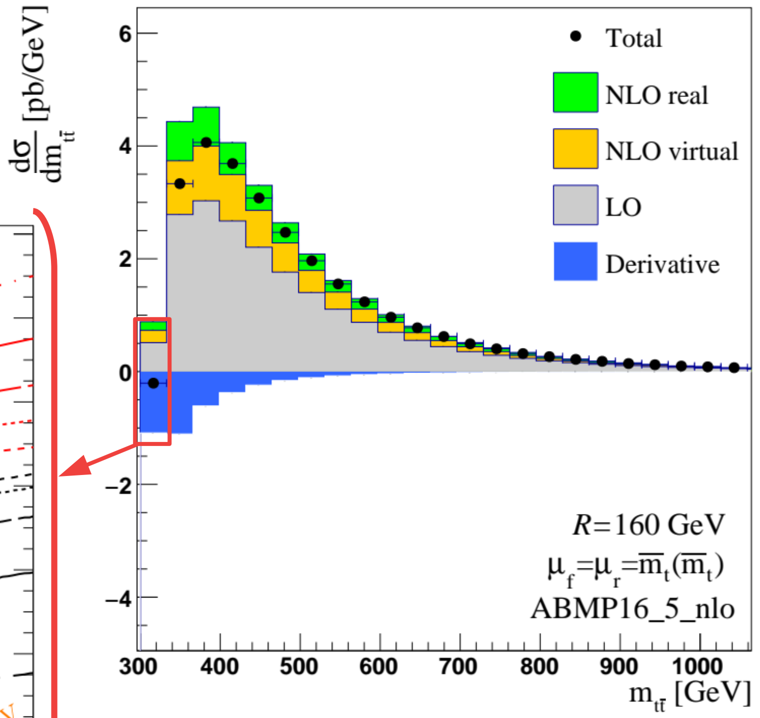
  close to the pair production threshold, should be treated in non-relativistic QCD: $v \ll 1 \Rightarrow m_t \gg p \sim m_t v \gg E_K \sim \frac{1}{2} m_t v^2$
- Perturbative expansion in coupling breaks down
- Some theory work for corrections exists, but no computation is publicly available yet… so let's see what we can say about stability in fixed pQCD



The single-differential $t\bar{t}$ cross section at NLO

- At the production threshold the derivative decreases faster as a function of R than other contributions increase
- As R increases, the total cross section decreases in the first bin, but much slower in the MSR regime
- At μ_m > 410 GeV, $\overline{m}_t(\mu_m)$ gets small, artificially pushing the threshold towards lower $m_{t\bar{t}}$





The single-differential $t\bar{t}$ cross section at NLO

- Threshold effects also visible in the NLO/LO *ratio* near the peak of the distribution
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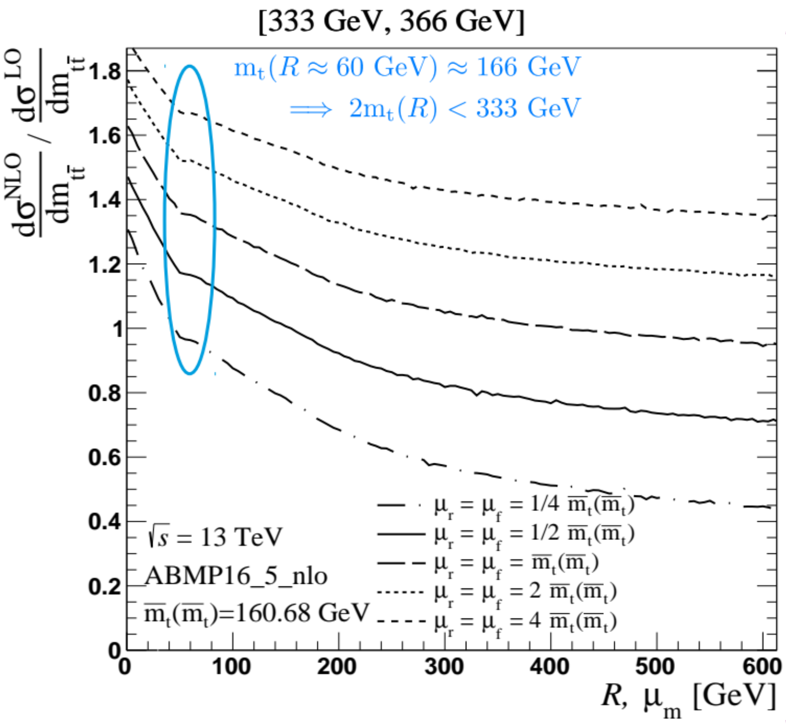
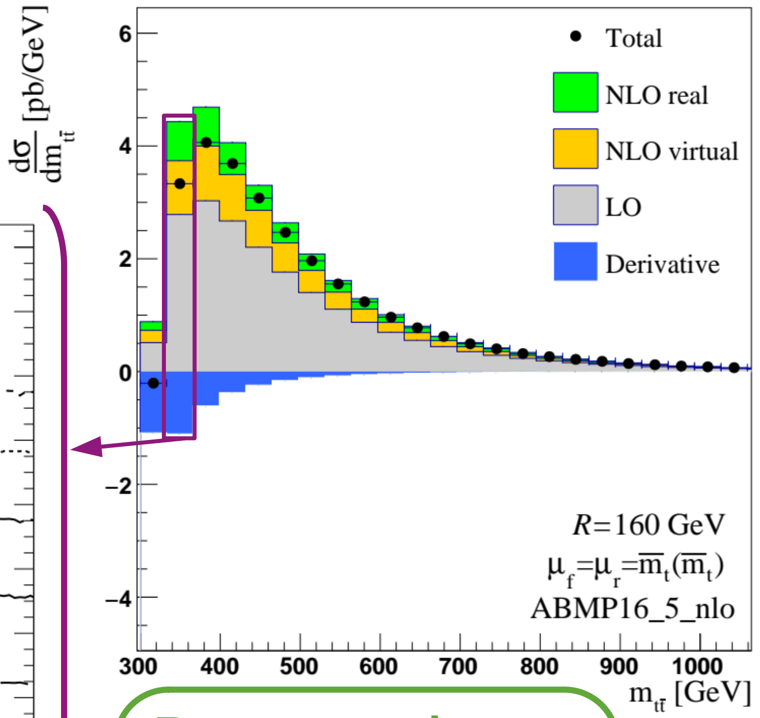
- Most sensitivity to m_t
- Stabilization at $R > 60$ GeV
- With low values of μ_r and μ_r, NLO/LO ratio close to 1 in this region => NLO corrections small





Recommend to set $\mu_r = \mu_r = R = 80$ GeV to increase robustness against scale variations

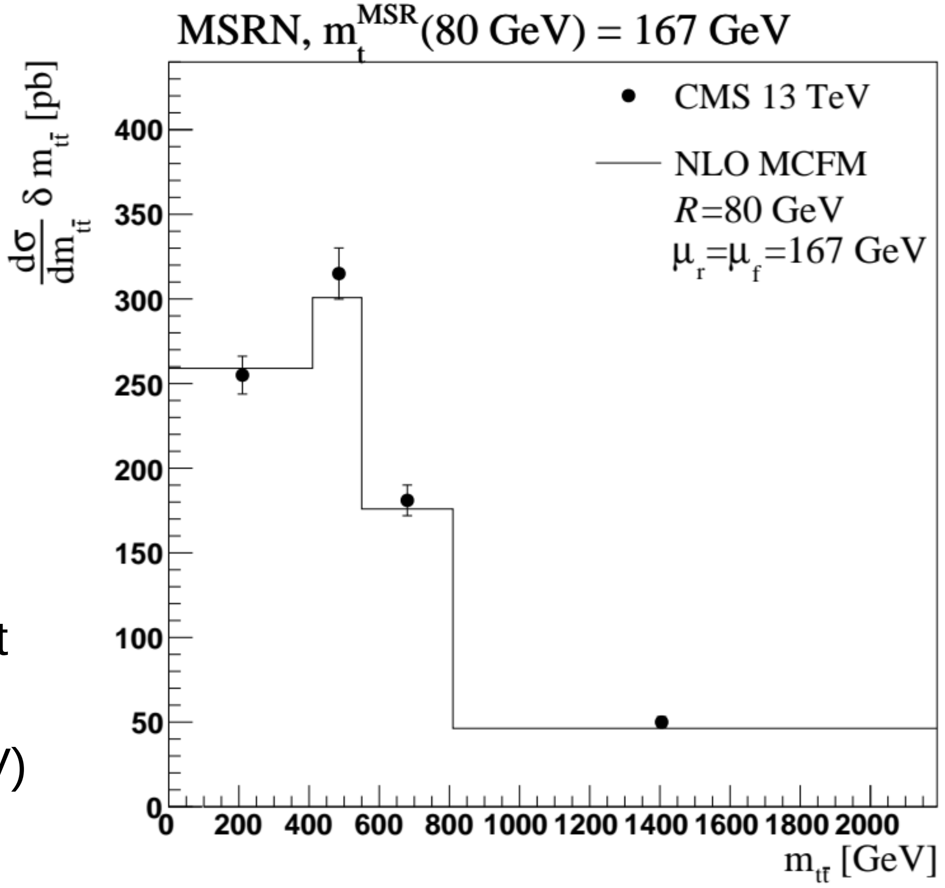
Extraction of the top quark MSR mass

- Using CMS $t\bar{t}$ cross section data measured as a function of $m_{t\bar{t}}$ at $\sqrt{s} = 13$ TeV [doi:10.1016/j.physletb.2020.135263]

- Set R=80 GeV, scan for m_t^{MSR}(80 GeV)
  - For each mass, compute

$$\chi^2 = \sum_{i,j} (\sigma_i^{\mathrm{exp}} - \sigma_i^{\mathrm{th}}) C_{ij}^{-1} (\sigma_j^{\mathrm{exp}} - \sigma_j^{\mathrm{th}})$$

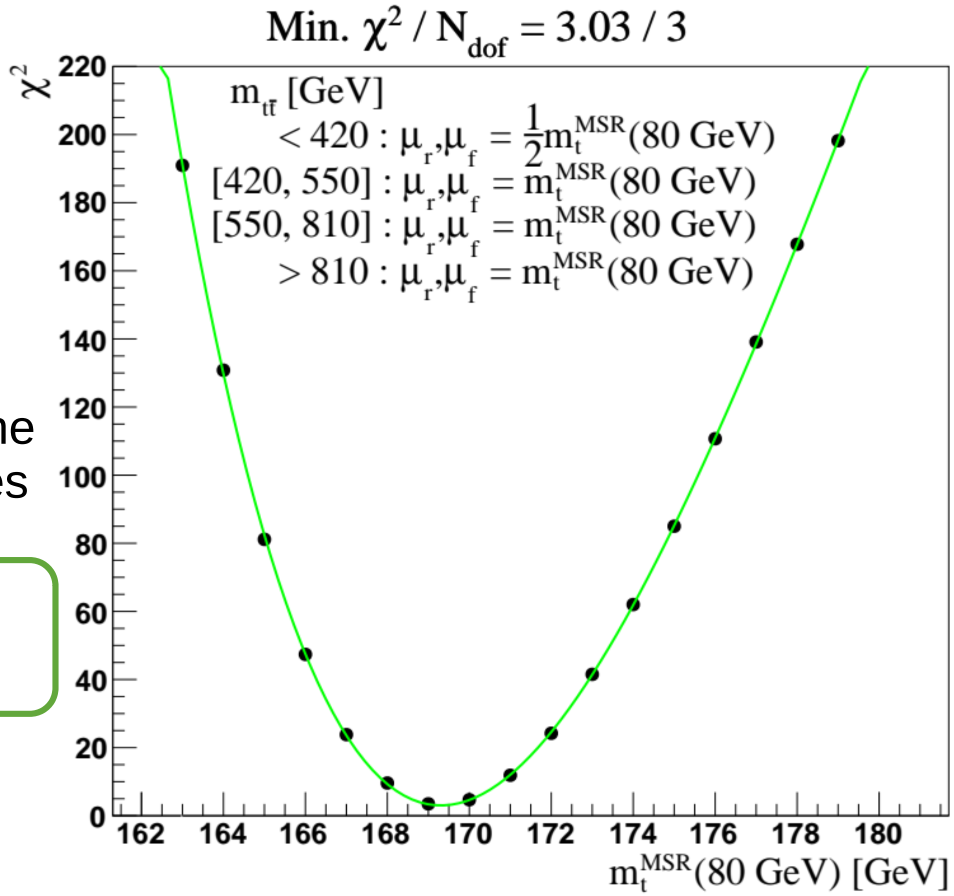
- Examine different scale choice options in different bins, also dynamical scales:
  - For $m_{t\bar{t}} < 420$ GeV, set $\mu_r = \mu_r = ½\, m_t^{MSR}$(80 GeV)
  - For $m_{t\bar{t}} > 420$ GeV, set $\mu_r = \mu_r = m_t^{MSR}$(80 GeV)



Extraction of the top quark MSR mass

- **Scale uncertainty**: variations of $\mu_r^{(i)}$, $\mu_f^{(i)}$,
- ***R*-uncertainty**: extracted $m_t^{\mathrm{MSR}}(80\ \mathrm{GeV})$ evolved to reference scales (e.g. R = 1 GeV) for comparison with other results

  Redo fits with R = 60 GeV and 100 GeV, take the difference in masses evolved to reference scales



With dynamical scale

$$m_t^{\mathrm{MSR}}(80\ \mathrm{GeV}) = 169.3 \pm 0.5\,(\mathrm{fit})^{+0.2}_{-0.4}\,(\mu_r,\mu_f)^{+0.2}_{-0.3}\,(R)\ \mathrm{GeV}$$

Without dyn. scale

$$m_t^{\mathrm{MSR}}(80\ \mathrm{GeV}) = 167.7 \pm 0.6\,(\mathrm{fit})^{+0.4}_{-0.6}\,(\mu_r,\mu_f)^{+0.4}_{-0.5}\,(R)\ \mathrm{GeV}$$

Dynamical scales increase precision

Extraction of the top quark MSR mass

- The extracted mass can be evolved to any reference scale:

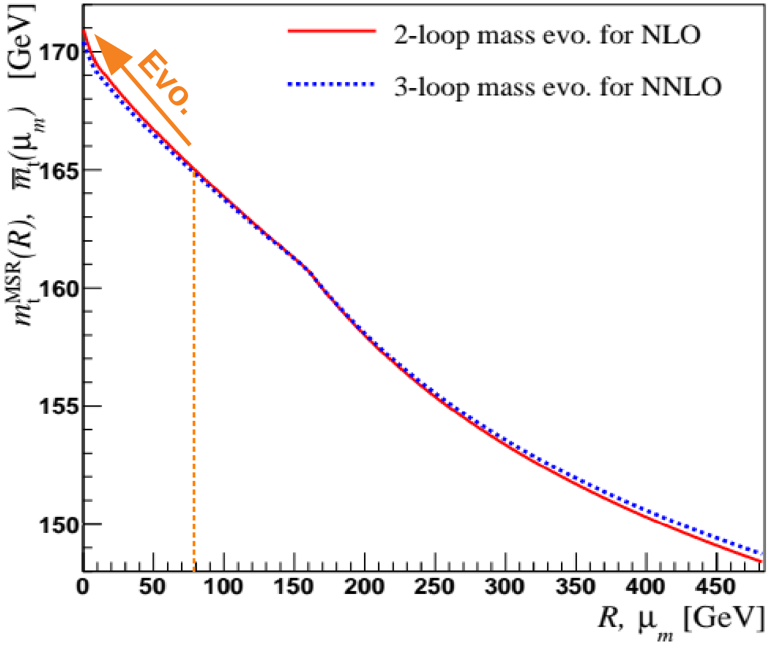
$$m_t^{\mathrm{MSR}}(80\ \mathrm{GeV}) = 169.3 \pm 0.5\,(\mathrm{fit})^{+0.2}_{-0.4}\,(\mu_r,\mu_f)^{+0.2}_{-0.3}\,(R)\ \mathrm{GeV}$$

R slightly above 1 GeV expected to become important checks for future analyses due to stability of $\alpha_s(\mu)$ at higher loop orders

$$m_t^{\mathrm{MSR}}(3\ \mathrm{GeV}) = 174.5 \pm 0.5\,(\mathrm{fit})^{+0.2}_{-0.4}\,(\mu_r,\mu_f)^{+0.2}_{-0.3}\,(R)\ \mathrm{GeV}$$

$$m_t^{\mathrm{MSR}}(1\ \mathrm{GeV}) = 174.8 \pm 0.5\,(\mathrm{fit})^{+0.2}_{-0.4}\,(\mu_r,\mu_f)^{+0.2}_{-0.3}\,(R)\ \mathrm{GeV}$$

At low *R*, the MSR scheme approximates the pole mass



Extraction of the top quark MSR mass

- **Alternatively** compute cross section predictions with R=1 GeV, to extract m_t^{MSR}(1 GeV) instead of evolving m_t^{MSR}(80 GeV) to R=1 GeV afterwards
  - Result $m_{\mathrm{t}}^{\mathrm{MSR}}(1~\mathrm{GeV}) = 170.1 \pm 0.6\,(\mathrm{fit})^{+1.1}_{-0.9}\,(\mu_r, \mu_f)~\mathrm{GeV}$
- This approach has been used in previous extractions of the top quark MSR mass
- The result is significantly lower than the suggested procedure, but agrees with previous results and pole mass results (after translation), e.g. [arXiv:1904.05237]:

$$m_{\mathrm{t}}^{\mathrm{pole}} = 170.5 \pm 0.8~~\mathrm{GeV}$$

- ***Underpins the importance of proper scale setting and procedures in future analyses!***

Take-home-message:

$m^{\mathrm{MSR}}_{\mathrm{t,th}}(80~\mathrm{GeV})$ ↔ $\sigma(m^{\mathrm{MSR}}_{\mathrm{t,th}}(80~\mathrm{GeV}))$ — ***Stability*** *w.r.t. scales*

$m^{\mathrm{MSR}}_{\mathrm{t,th}}(80~\mathrm{GeV})$ → $m^{\mathrm{MSR}}_{\mathrm{t,th}}(1~\mathrm{GeV})$ → $\sigma(m^{\mathrm{MSR}}_{\mathrm{t,th}}(1~\mathrm{GeV}))$ → $m^{\mathrm{MSR}}_{\mathrm{t,extr.}}(1~\mathrm{GeV})$

Conventionally start here

$\sigma(m^{\mathrm{MSR}}_{\mathrm{t,th}}(80~\mathrm{GeV}))$ → $m^{\mathrm{MSR}}_{\mathrm{t,extr.}}(80~\mathrm{GeV})$ → $m^{\mathrm{MSR}}_{\mathrm{t,extr.}}(1~\mathrm{GeV})$

$$m^{\mathrm{MSR}}_{\mathrm{t,extr.}}(1~\mathrm{GeV}) \neq m^{\mathrm{MSR}}_{\mathrm{t,extr.}}(1~\mathrm{GeV})$$

Approaches do not "commute"!

However, stability arguments expected to hold with more complete predictions

Summary and outlook

- First study of *R*-scale behavior + extracting the top quark MSR mass from CMS data at 13 TeV:

$$\mathrm{m_t^{MSR}}(80\ \mathrm{GeV}) = 169.3^{+0.6}_{-0.7}\ \mathrm{GeV} \quad \rightarrow \quad m_{\mathrm{t}}^{\mathrm{MSR}}(1\ \mathrm{GeV}) = 174.8^{+0.6}_{-0.7}\ \mathrm{GeV}$$

  - *Extracting at R=80 GeV and evolving to R=1 GeV =/= extracting at R = 1 GeV!*

- However, our approach is based on stability of cross section predictions, leading to reduced scale uncertainties. Thus, even thought the final word requires Coulomb corrections, the findings are expected to remain valid

Thanks for your attention!

Comparison to previous MSR results

- ATLAS has derived a value for $m_t^{MSR}(R = 1\ \text{GeV})$ [ATL-PHYS-PUB-2021-034]. Their results are however not comparable because:
  - Compares QCD predictions at next-to-leading log to parton shower MC simulations
  - Assuming $m_t^{MC} = 172.5$ GeV.
  - Not based on experimental data and hence not comparable

- Garzelli *et al.* [JHEP 04 (2021) 043] have extracted $m_t^{MSR}(3\ \text{GeV}) = 169.6^{+0.8}_{-1.1}\,(\mu_r, \mu_f)\ \text{GeV}$
  - Some tension to our $m_t^{MSR}(3\ \text{GeV}) = 174.5 \pm 0.5\,(\text{fit})^{+0.2}_{-0.4}\,(\mu_r, \mu_f)^{+0.2}_{-0.3}\,(\mu_r, \mu_f)\ \text{GeV}$
    - Their cross section predictions are computed using $m_t^{MSR}(3\ \text{GeV})$ (not evolving the extracted mass)
    - They simultaneously fit PDFs and α_S, the latter resulting in $\alpha_S(m_Z) = 0.1132^{+0.0023}_{-0.0018}$
      - Two standard deviations away from the ABMP16 fit value at NLO, assumed by us