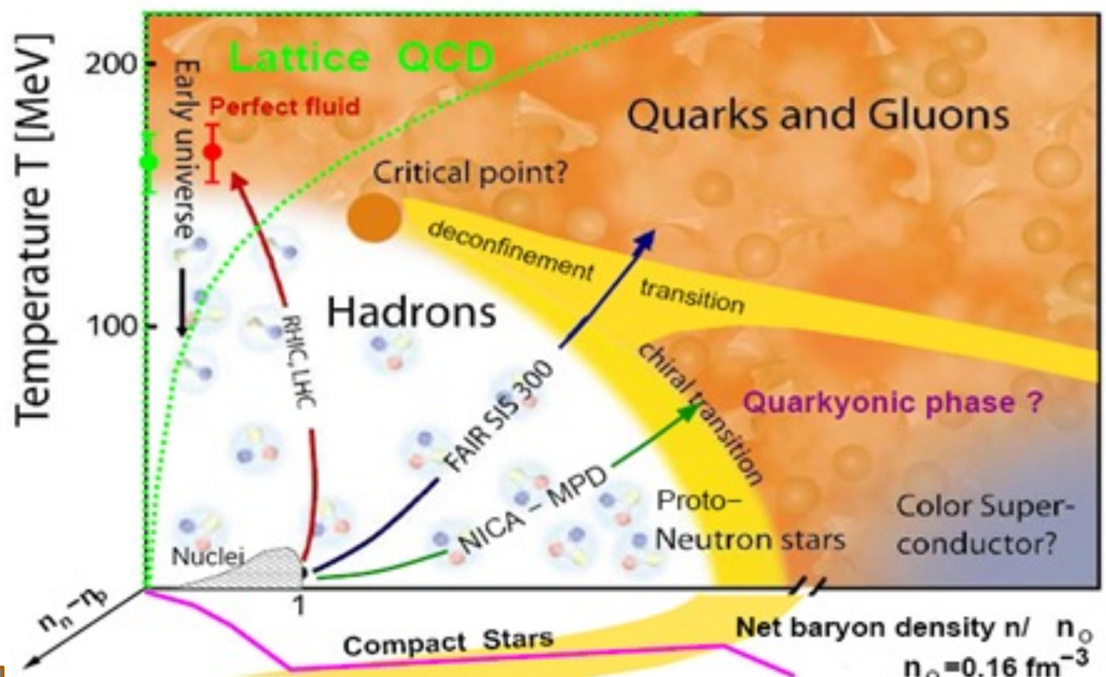
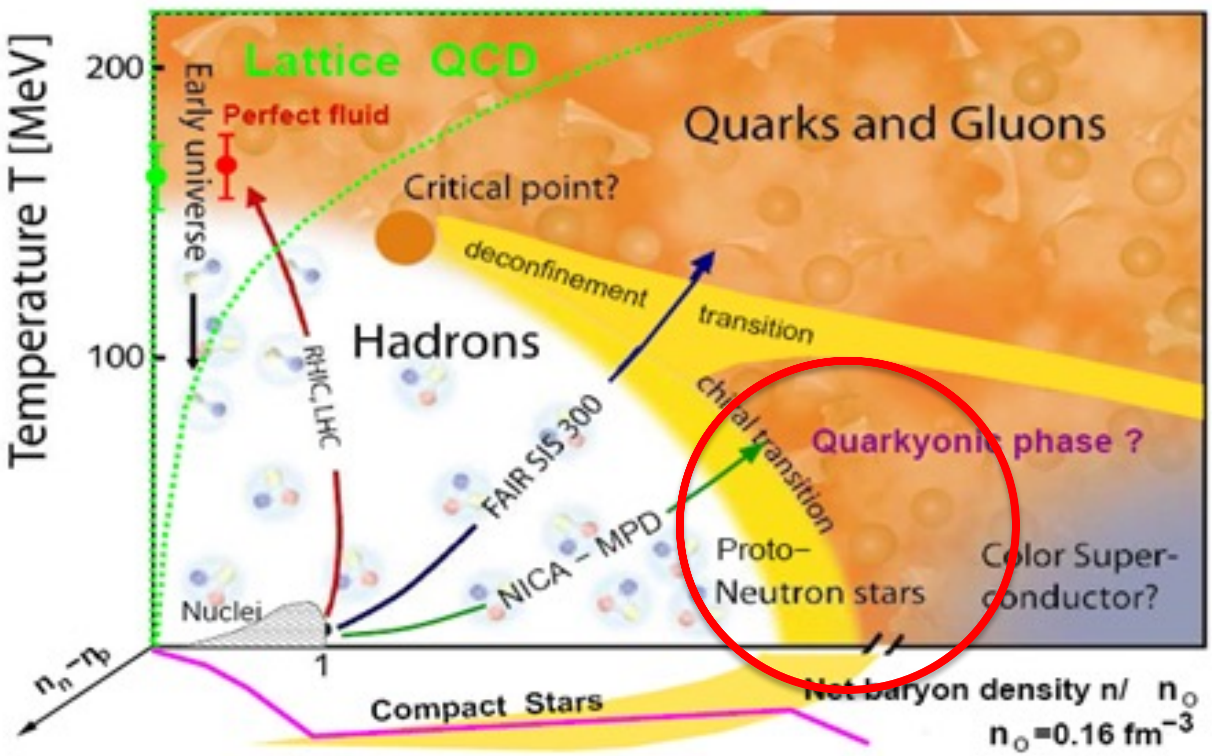
Topology and Robustness of a Quark Matter Phase Candidate for Magnetar's Cores

Vivian de la Incera

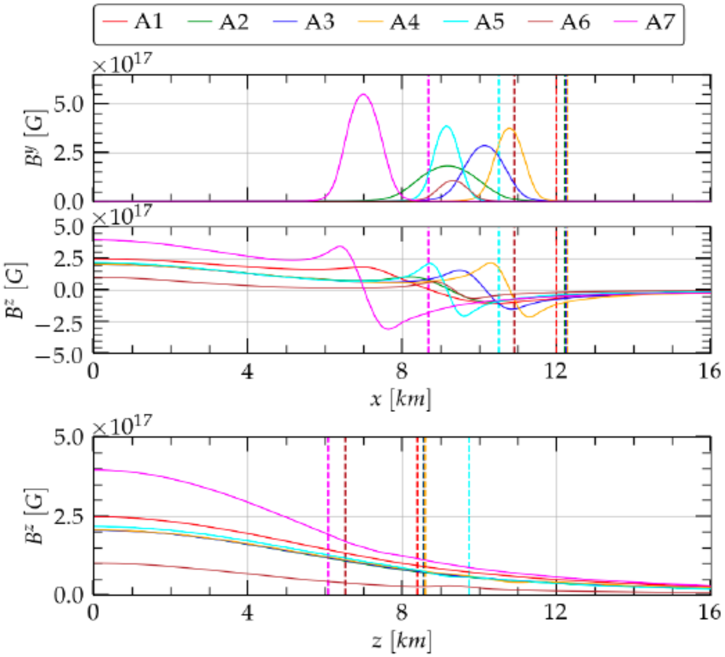


XQCD 2023, Coimbra, July 26-28, 2023

Single-Modulated (Chiral) Inhomogeneous Phases



- ✓ Critical temperature to erase the inhomogeneous condensate at each density
- ✓ Does it have the Landau-Pierls instability of single modulated phases?
- ✓ Relevance of Fluctuations with increasing T



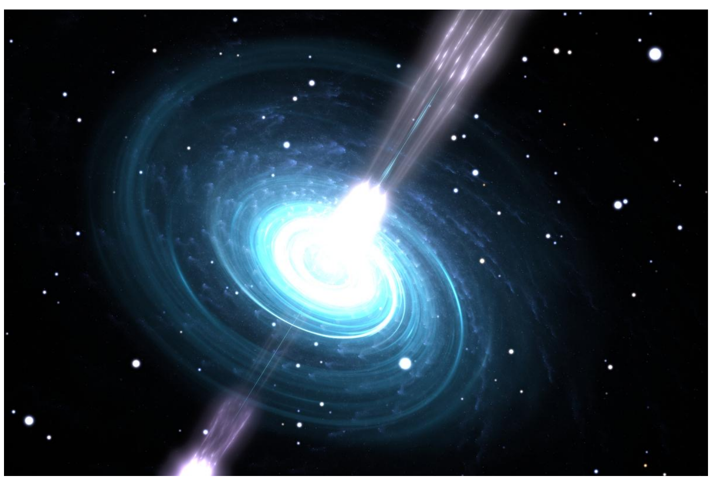
GRMHD simulations of magnetars' field evolution leads to several times 10^{17} G for B_z at the core

Tsokaros, Ruiz, Shapiro, & Uryu, PRL 128, 2022

Magnetars:

Core: $B < 8 \times 10^{18}$ G

Cardall, Prakash, and Lattimer, ApJ 554, 2001



Magnetars

surface: $B \sim 10^{15}$G

core $B \sim 10^{17}$- 10^{18} G

$$l_m = 7 \times 10^9 (n_e/n_s)/B^{*2} fm, \qquad B^* = B/B_e^c, \qquad R \gg l_m$$

B can be considered uniform and constant as far as effects on the EOS

Broderick, Prakash, and Lattimer, ApJ 537, 2000

Magnetic Dual Chiral Density Wave Model

2-flavor NJL model at finite baryon density and with magnetic field B|| z

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} + iQA_{\mu}) + \gamma_0\mu]\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2].$$

It favors the formation of a single-modulated inhomogeneous chiral condensate

$$\langle\bar{\psi}\psi\rangle = \Delta\cos q_{\mu}x^{\mu}, \qquad \langle\bar{\psi}i\tau_3\gamma_5\psi\rangle = \Delta\sin q_{\mu}x^{\mu} \qquad q^{\mu} = (0,0,0,q)$$

Mean-field Lagrangian

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} - i\mu\delta_{\mu 0} + iQA_{\mu} - i\tau_3\gamma_5\delta_{\mu 3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

$$E_0 = \epsilon\sqrt{m^2 + k_3^2} + b, \quad \epsilon = \pm,$$ $b=q/2$, LLL mode is Asymmetric!

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1,2,3,\ldots$$

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

Nontrivial Topology of the MDCDW Phase

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega^f_{vac} = \frac{1}{4\sqrt{\pi}} \frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\ell\xi\epsilon} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^{3/2}} e^{-s(E_\ell)^2}$$

Come from the LLL

$$\Omega^f_{anom} = -\frac{N_c\,|e_f B|}{(2\pi)^2} 2b\mu$$

Anomalous. Leads to a quark number proportional to the Atiyah-Singer invariant

$$\Omega^f_{\mu} = -\frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\xi,\ell>0} [(\mu - E_\ell)\theta(\mu - E_\ell)]\Big|_{\epsilon=+} + \Omega^{f,LLL}_{\mu}$$

$$\Omega^f_T = -\frac{N_c\,|e_f B|}{(2\pi)^2} \frac{1}{\beta} \int_{-\infty}^{+\infty} dk \sum_{\ell\xi\epsilon} \ln\left(1 + e^{-\beta|E_\ell - \mu|}\right)$$

$$\Omega^{f,LLL}_{\mu} = -\frac{1}{2} \frac{N_c\,|e_f B|}{(2\pi)^2} \int_{-\infty}^{+\infty} dk \sum_{\epsilon} (|E_0 - \mu| - |E_0|)_{reg}\,,$$

Nontrivial Topology of the MDCDW Phase

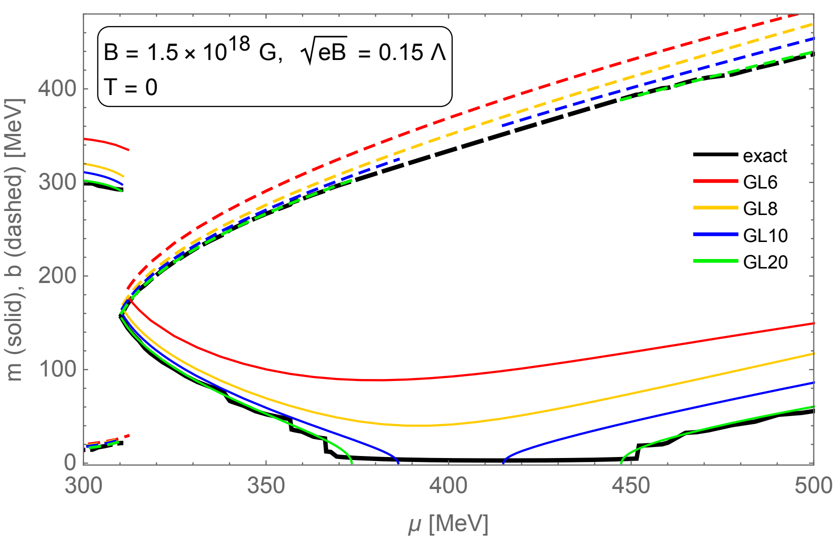
Topology emerges due to the LLL spectral asymmetry

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\Omega^f_{anom} = -\frac{N_c|e_f B|}{(2\pi)^2} q\mu$$

$$\rho^A_B = 3\frac{|e|}{4\pi^2} qB$$

Anomalous baryon number density



The anomaly makes the MDCDW energetically favored over the homogeneous condensate.

Solution exists even at low μ

Condensate never disappears!

Gyory and VI, PRD 106, 016011, 2022

Improved GL Expansion & Origin of the beta (odd in b) terms

$$\Omega = \alpha_{2,0}m^2 + \boxed{\beta_{3,1}bm^2} + \alpha_{4,0}m^4 + \alpha_{4,2}b^2m^2 + \boxed{\beta_{5,1}bm^4}$$
$$\boxed{+\,\beta_{5,3}b^3}m^2 + \alpha_{6,0}m^6 + \alpha_{6,2}b^2m^4 + \alpha_{6,4}b^4m^2 + \ldots$$

Found analytical expressions for the coefficients as functions of μ, T, and B that allowed quick calculation of arbitrarily high-order coefficients and led to much higher precision results than in previous works.

$$\alpha_{n_b+2,n_b} \sim \frac{\delta_{0,n_b}}{4G} + \sum_{j=0,2,4,\ldots} |eB|^j \frac{B_j}{j!} \cdot \frac{1+2^j}{2\pi^2 3^{j-1}} \cdot \frac{1}{(n_b-1)!!} I_{n_b+2j-2}(\mu,T)$$

$$I_0(\mu,T) = -\frac{\gamma}{2} - \left\{ \ln\left(\frac{4\pi T}{\Lambda}\right) + \mathrm{Re}\left[\psi\left(\frac{1}{2} + i\frac{\mu}{2\pi T}\right)\right]\right\} \qquad I_{-2}(\mu,T) = -\frac{1}{4}\Lambda^2 + \frac{1}{2}\mu^2 + \frac{\pi^2}{3}T^2$$

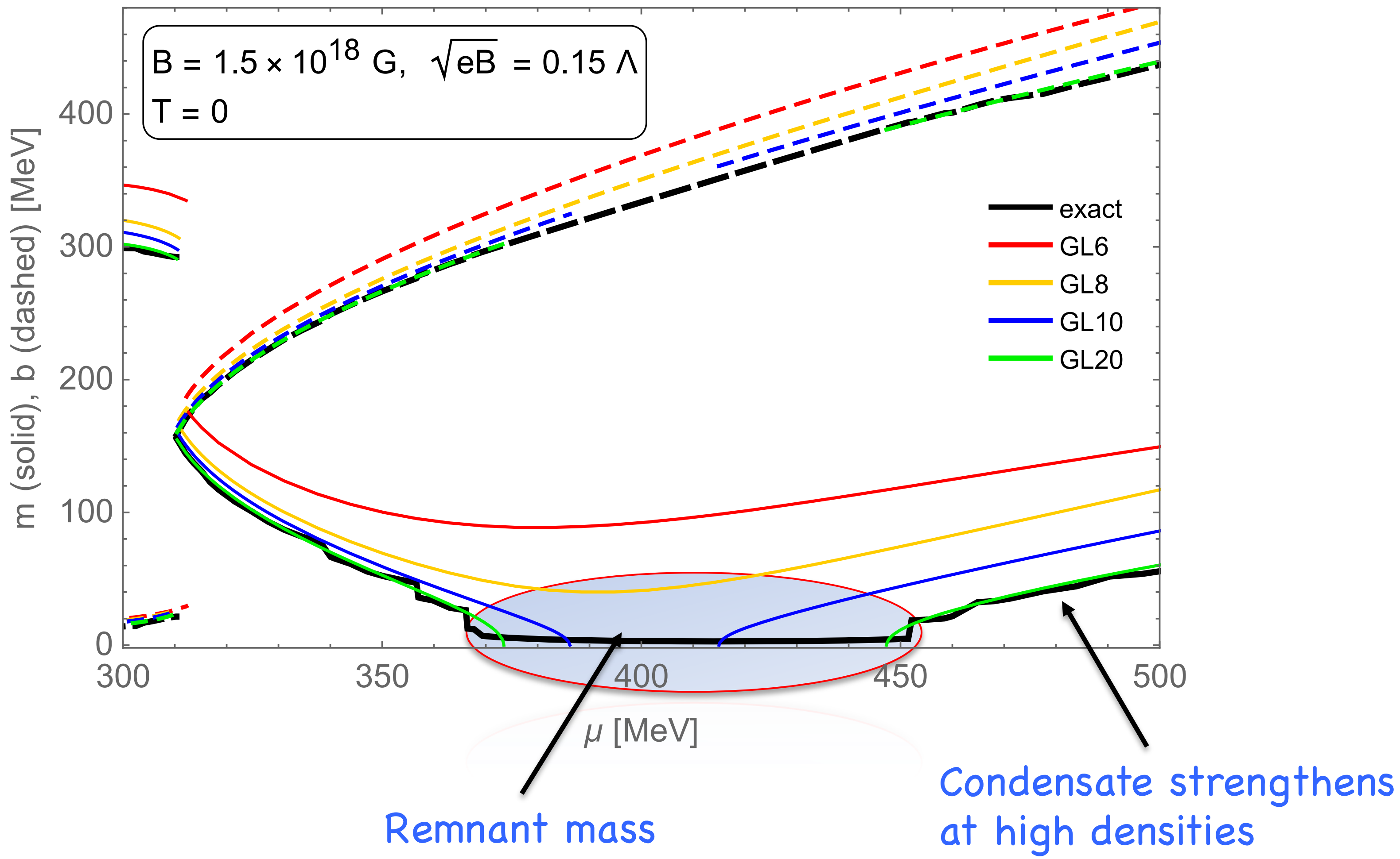
$$I_{p>0}(\mu,T) = -\frac{1}{p}\left(\frac{i\sqrt{2}}{\Lambda}\right)^p - \frac{1}{p!!}\left\{\frac{1}{(2\pi T)^p}\mathrm{Re}\left[(-i)^p\psi^{(p)}\left(\frac{1}{2} + i\frac{\mu}{2\pi T}\right)\right]\right\}$$

$$\beta_{n_b+2,n_b} = \frac{3|eB|}{(2\pi)^2} \cdot \begin{cases} \frac{1}{n_b!}\frac{1}{(2\pi T)^{n_b}}\mathrm{Re}\left[(-i)^{n_b}\psi^{(n_b)}\left(\frac{1}{2} + i\frac{\mu}{2\pi T}\right)\right] & T > 0, \\ -\frac{1}{n_b\mu^{n_b}} & T = 0, \end{cases}$$

All β are negative, thus the LLL topology always favors the inhomogeneity

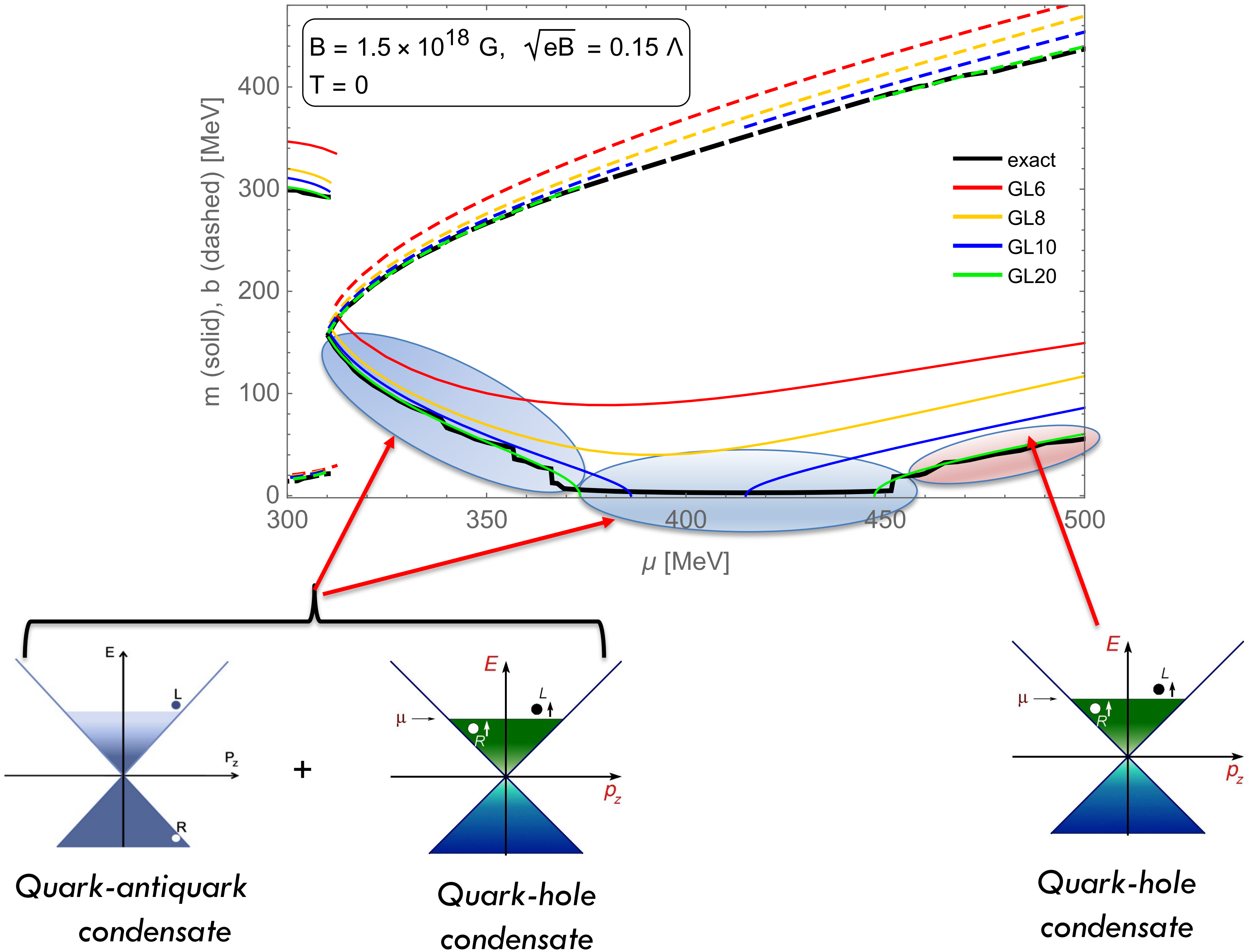
Gyory and VI, PRD 106, 016011, 2022

MDCDW Resilience



Gyory and VI, PRD 106, 016011, 2022

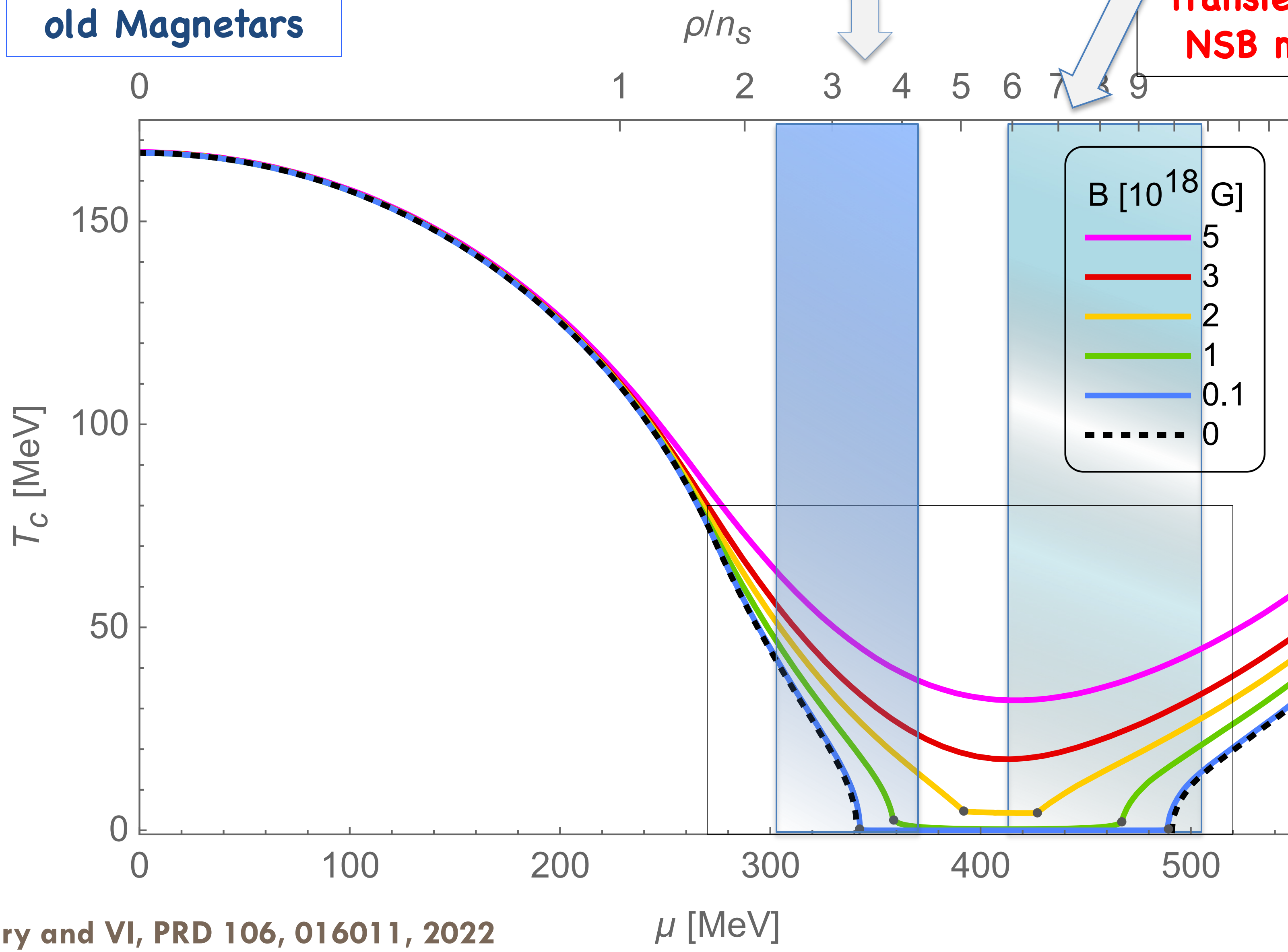
Pairing Mechanisms' Roles in the Condensate Behavior



Relevance for NS

Relevant for old Magnetars

Relevant for transient NS in NSB mergers



Gyory and VI, PRD 106, 016011, 2022

Goldstone Bosons in the MDCDW Phase

$M_0(z) = m e^{iqz}$ with m and q solutions of the stationary equations: $\frac{\partial \Omega}{\partial m} = 0, \quad \frac{\partial \Omega}{\partial q} = 0$

Two global continuous symmetries broken: translation and chiral

$$M(x) = M_0\big(z + u(x)\big)e^{i\pi} = M_0 e^{i(qu+\pi)}$$

Translational and chiral symmetries are locked, just like in the zero-B case.
Only one independent Goldstone boson

To go beyond mean-field we study the effect of low-energy phonon fluctuations u(x) on the condensate $M_0(z)$

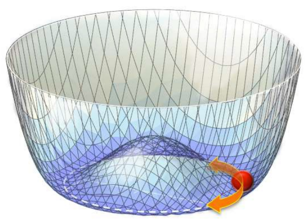
Low-Energy Theory of Fluctuations

We start with the generalized GL expansion in powers of the order parameter

and its derivatives with symmetry $U_V(1)\times U_A(1)\times SO(2)\times R^3$

$$\mathcal{F} = a_{2,0}|M|^2 - i\frac{b_{3,1}}{2}[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M] + a_{4,0}|M|^4 + a_{4,2}^{(0)}|\nabla M|^2$$
$$+ a_{4,2}^{(1)}(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) - i\frac{b_{5,1}}{2}|M|^2[M^*(\hat{B}\cdot\nabla M) - (\hat{B}\cdot\nabla M^*)M]$$
$$+ \frac{ib_{5,3}}{2}[(\nabla^2 M^*)\hat{B}\cdot\nabla M - \hat{B}\cdot\nabla M^*(\nabla^2 M)] + a_{6,0}|M|^6 + a_{6,2}^{(0)}|M|^2|\nabla M|^2$$
$$+ a_{6,2}^{(1)}|M|^2(\hat{B}\cdot\nabla M^*)(\hat{B}\cdot\nabla M) + a_{6,4}|\nabla^2 M|^2 + \cdots .$$

$$M(x) = \sigma(x) + i\pi(x) \quad \sigma = -2G\bar{\psi}\psi \quad \pi = -2G\bar{\psi}i\gamma^5\tau_3\psi,$$



Consider the phonon fluctuations of the MDCDW ground state

$$M(x) = M_0\big(z + u(x)\big) = M_0(z) + M_0'(z)u(x) + \tfrac{1}{2}M_0''(z)u^2(x) + \cdots$$

$$M_0(z) = me^{iqz}$$

Ferrer & VI, PRD`2020

Low-energy Theory of Fluctuations in the MDCDW

$$\mathcal{F}^{(N)} = \mathcal{F}_0^{(N)} + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2,$$

θ(x)=qmu(x)

$$E \simeq \sqrt{v_z^2 k_z^2 + v_\perp^2 k_\perp^2},$$

Anisotropic spectrum, linear in each direction.
No soft transverse mode

$$v_z^2 = \sum_{n,n_q}^{N} c_{n,n_q} m^{n-n_q-2} q^{n_q-2} \frac{n_q(n_q-1)}{2}$$

$$v_\perp^2 = \sum_{n,n_q}^{N} c_{n,n_q} m^{n-n_q-2} q^{n_q-2} \left\lfloor \frac{n_q}{2} \right\rfloor .$$

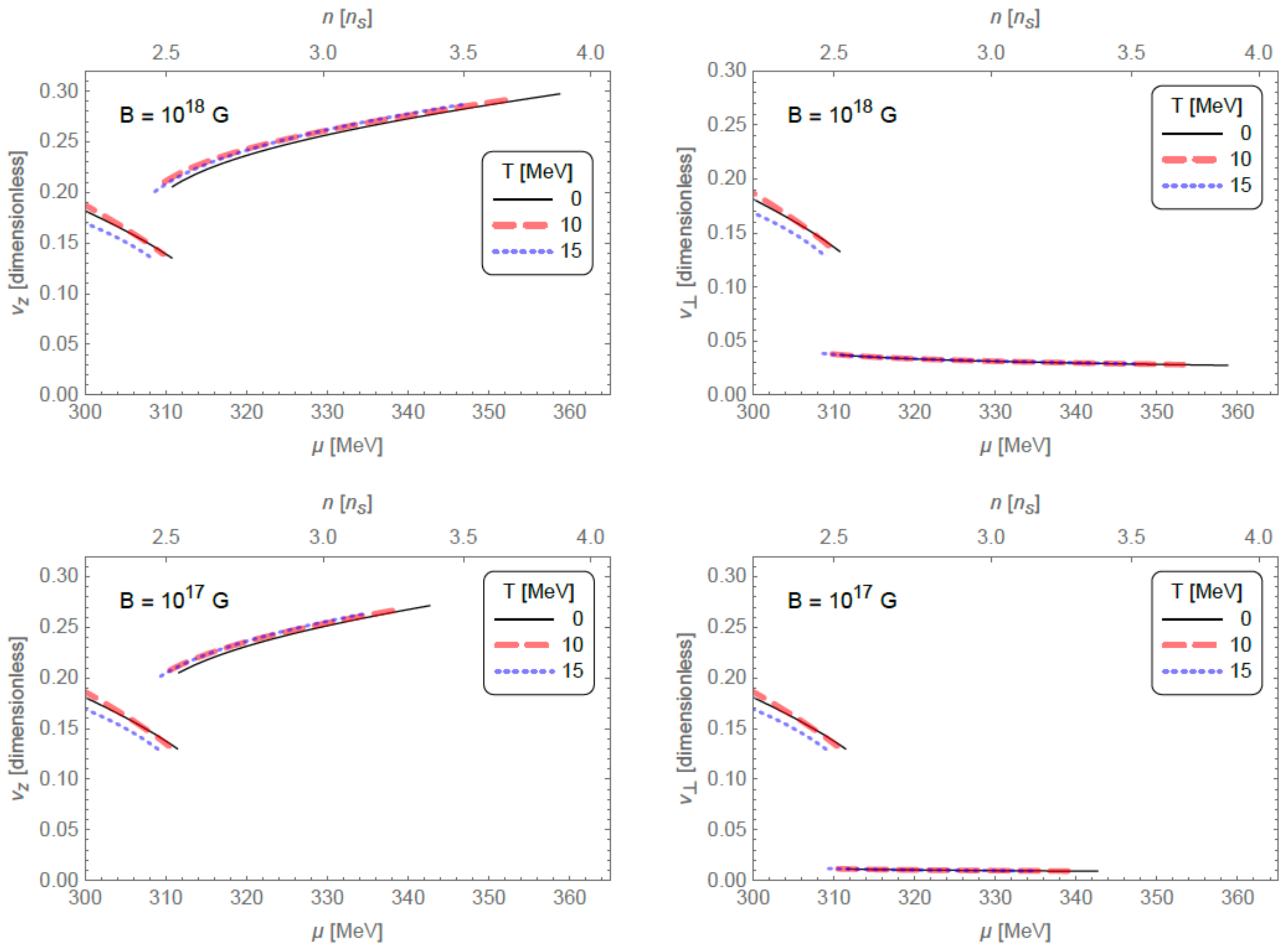
$n = 2, 4, 6, \ldots, \quad n_q = 0, 2, 4, \ldots, n-2$

$n = 3, 5, 7, \ldots, \quad n_q = 1, 3, 5, \ldots, n-2$

c_{n,n_q} denotes a_{n,n_q} for even n, and b_{n,n_q} for odd n

Ferrer, Gyory & VI, arXiv:2307.05621

Group Velocities at B≠0



Ferrer, Gyory & VI, arXiv:2307.05621

Landau-Peierls Instability at B=0

$$\mathcal{F}[M(x)] = \mathcal{F}_0 + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2 + \zeta^2(\partial_z^2\theta + \partial_\perp^2\theta)^2$$

Soft transverse mode

$$\langle u^2\rangle = \frac{1}{(2\pi)^2}\int_{l_\perp^{-1}}^{\infty} dk_\perp k_\perp \int_{-\infty}^{\infty} dk_z \frac{T}{v_z^2k_z^2 + \zeta^2 k_\perp^4}$$

$$\simeq \frac{T}{4\pi v_z\zeta}\ln\left(\frac{v_z l_\perp}{\zeta}\right)$$

Infrared divergent

$$v_\perp^2 = \frac{1}{2m^2q}\frac{\partial\Omega^{(N)}}{\partial q} = 0, \quad \longrightarrow \quad v_\perp^2 = a_{4,2} + m^2 a_{6,2} + 2q^2 a_{6,4} = \frac{1}{2m^2q}\frac{\partial\Omega^{(6)}}{\partial q} = 0,$$

$$\langle M\rangle = m e^{iqz} e^{-\langle (qu)^2\rangle/2} = 0$$

Single-modulation is destroyed by phonon fluctuations at any temperature. The system exhibits the Landau-Peierls instability

Hidaka et.al., PRD 92; Lee et al. PRD 92

Role of Topology in the Lack of Landau-Peierls Instability at B≠0

$$\mathcal{F}^{(N)} = \mathcal{F}_0^{(N)} + v_z^2(\partial_z\theta)^2 + v_\perp^2(\partial_\perp\theta)^2, \qquad \theta = qmu$$

$$\langle M\rangle = me^{iqz}e^{-\langle (qu)^2\rangle/2}$$

$$\langle q^2u^2\rangle = \frac{1}{(2\pi)^2}\int_0^\infty dk_\perp k_\perp \int_{-\infty}^\infty dk_z \frac{T}{m^2(v_z^2k_z^2+v_\perp^2k_\perp^2+\zeta^2k^4)}$$
$$\simeq \frac{T}{4\pi m\sqrt{v_z^2v_\perp^2}}.$$

Infrared Finite. No Landau-Peierls instability.

$\frac{\partial\Omega}{\partial q} = 0$ leads to $a_{4.2} + a_{6,2}m^2 + 2a_{6.4}q^2 = -b_{3,1}\frac{1}{2q} - b_{5,1}\frac{m^2}{2q} - b_{5,3}\frac{3q}{2} \neq 0$

$$v_\perp^2 = a_{4.2} + m^2a_{6.2} + 2q^2a_{6.4} + qb_{5,3}$$

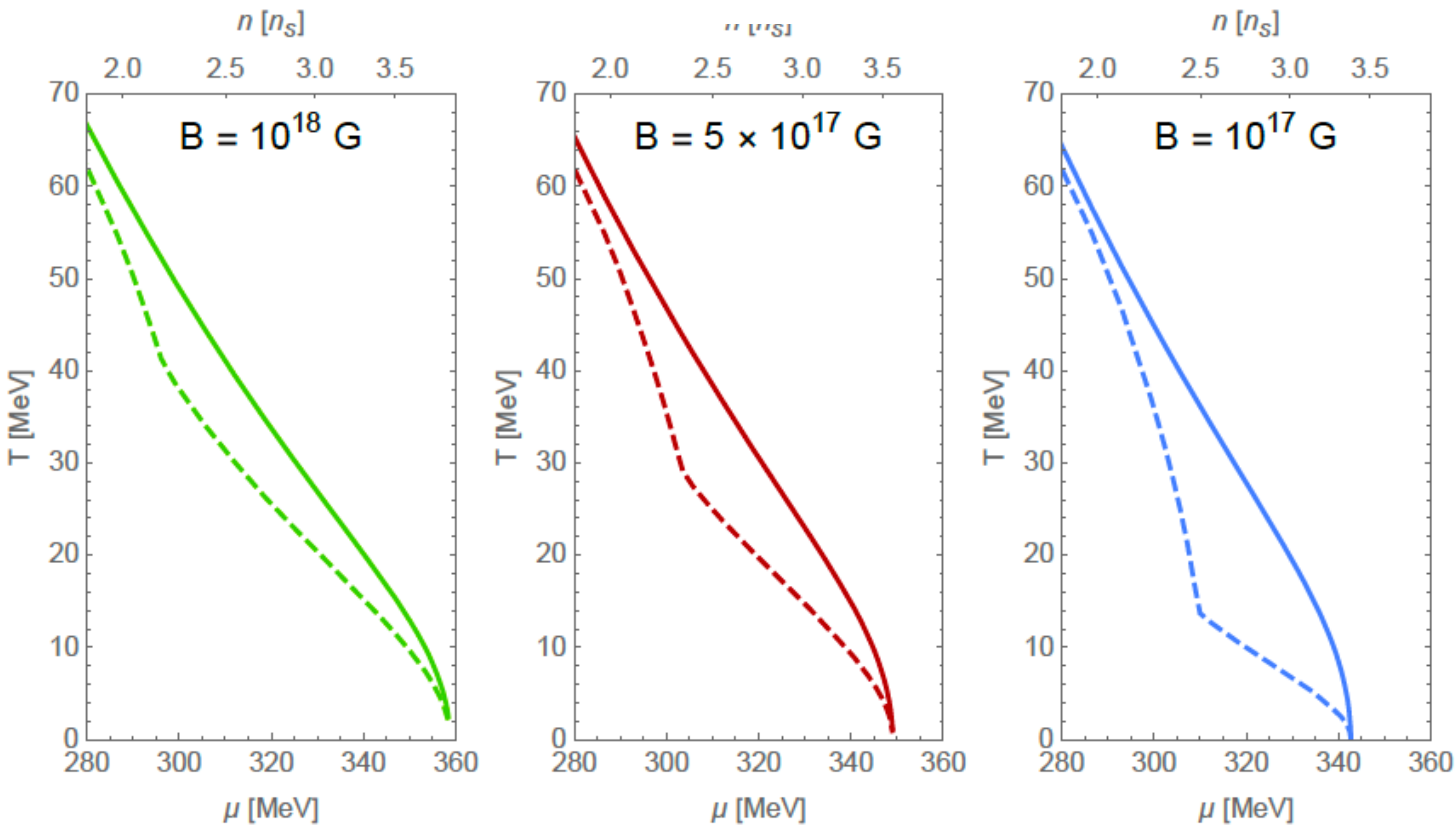
Topology ensures the absence of the LP instability!

Ferrer and VI, PRD 102, 014010, 2020

Threshold and Critical Temperatures

The threshold temperature is the T at which $\langle M \rangle = e^{-1} M_0$

$$T_{\text{thr}} = 16\pi m |v_z||v_\perp|.$$



Ferrer, Gyory & VI, arXiv:2307.05621

Robustness of the MDCDW Phase

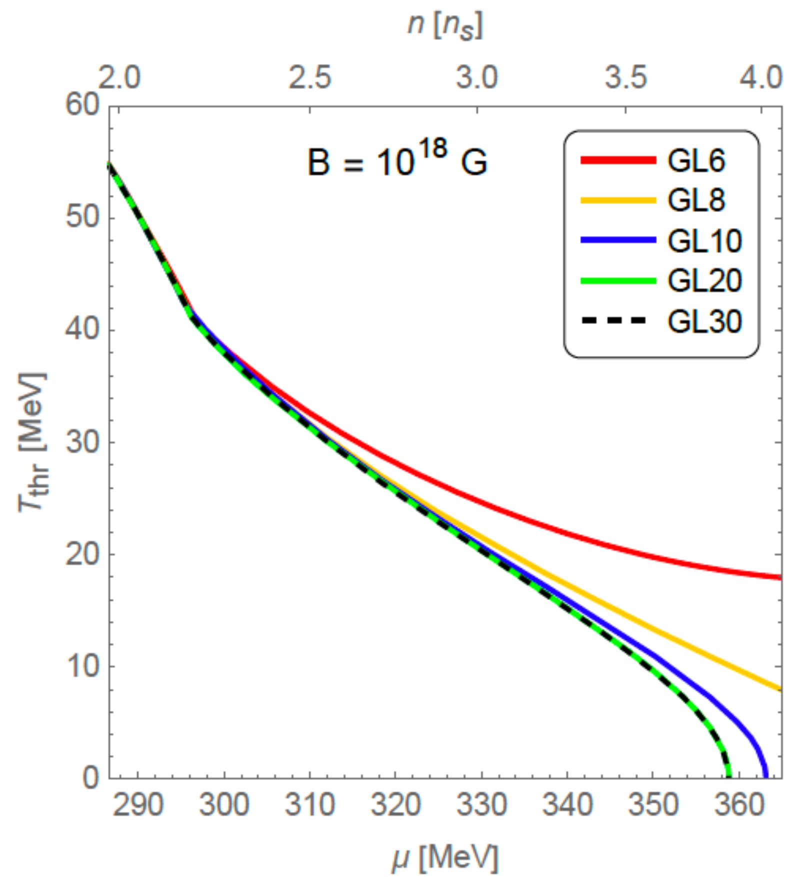
- ✓ Compatible with NS's densities and temperatures
- ✓ Stable against fluctuations at low and high T
- ✓ Consistent with observed NS heat capacity limit
- ✓ Compatible with 2 $M_{\odot}$
- ✓ A candidate for transient NS in BNS Mergers?

- arXiv:2307.05621 - E.J. Ferrer, W. Gyory, and VI
- PRD 106, 016011, 2022 - W. Gyory and VI
- Universe 7, 458, 2021 - E.J. Ferrer and VI
- PRD 103, 123013, 2021 - E.J. Ferrer, VI, and P. Sanson
- PRD 102, 014010, 2020 - E.J. Ferrer and VI
- NPB 931,192, 2018 - E.J. Ferrer and VI
- PRD 92, 105018, 2015 - S. Carignano, E.J. Ferrer, VI, L. Paulucci

Conclusions & Outlook:

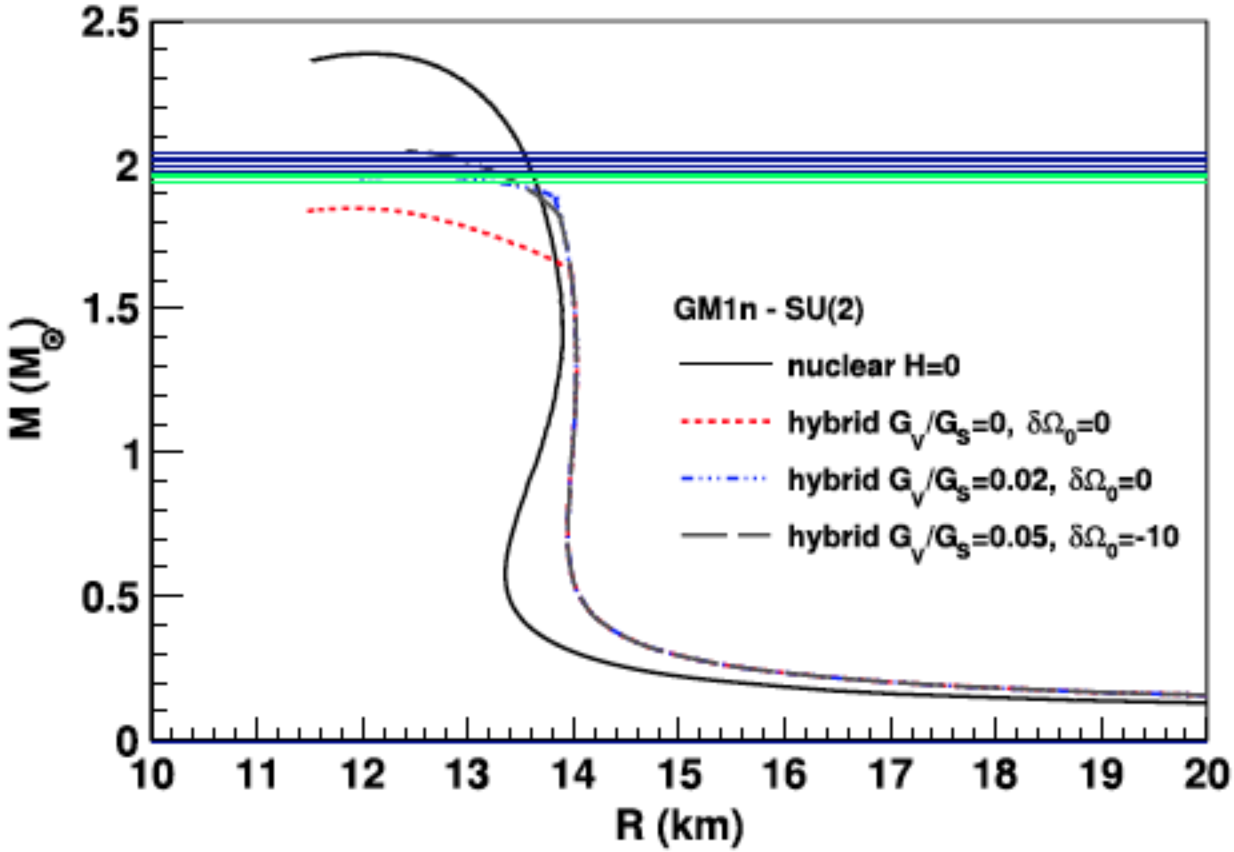
- *B alone is not enough to eliminate the LPI. The fermion dynamics must have nontrivial topology. LLL spectrum here is the same as the DW in* NJL_2
- *Check the MDCDW phase against Tidal Deformability constraints. Is topology important there too?*
- *Compare MDCDW with color superconducting phases. Inhomogeneous ones?*
- *Compatibility with new multimessenger NS observations?*

Auxiliary Slides



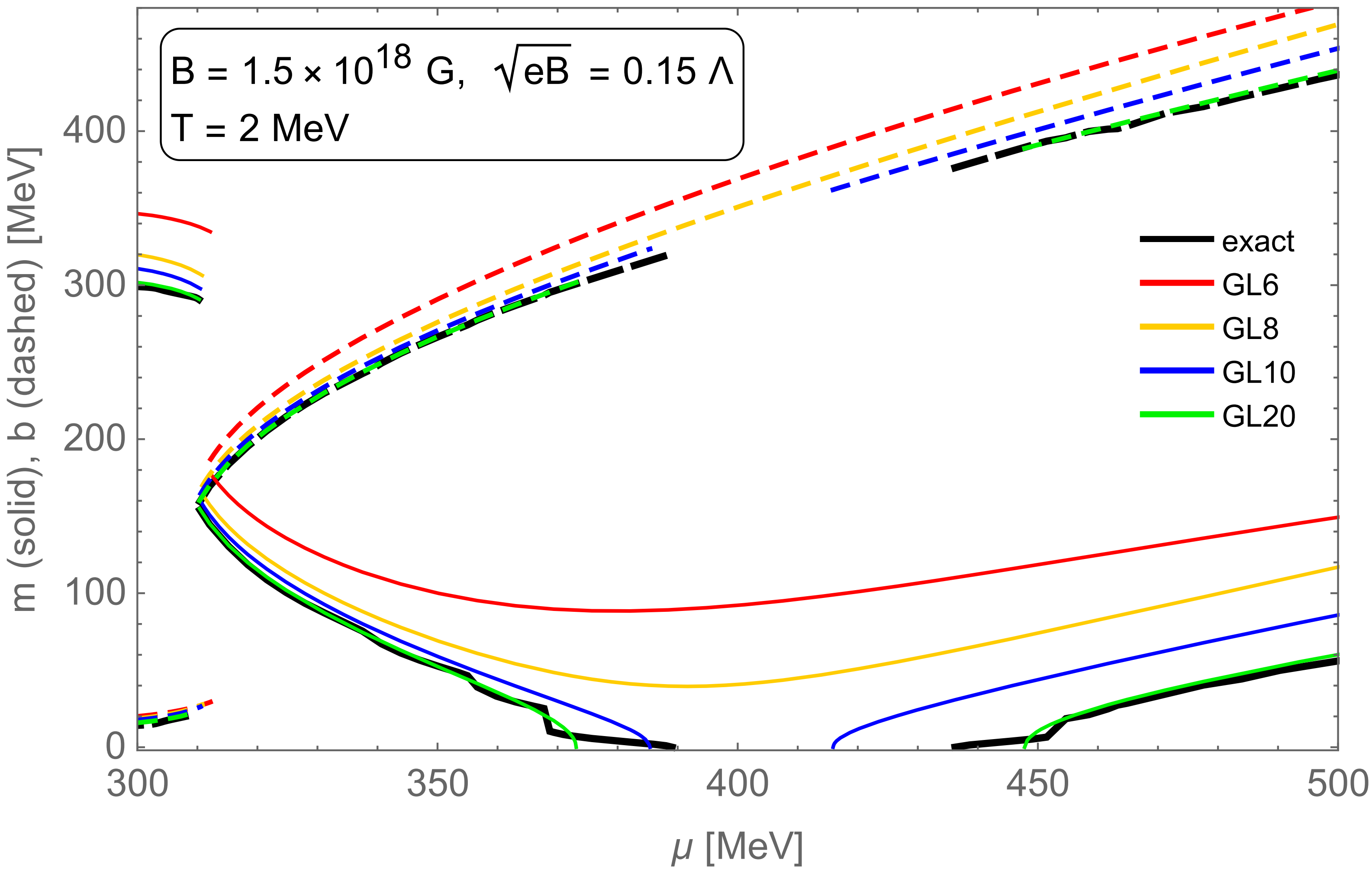
n [n_s]
2.0
2.5
3.0
3.5
4.0
60
50
40
30
20
10
0
B = 10^18 G
GL6
GL8
GL10
GL20
GL30
T_thr [MeV]
290
300
310
320
330
340
350
360
μ [MeV]

Isospin Asymmetric Magnetic DCDW Compatible with 2 $M_\odot$



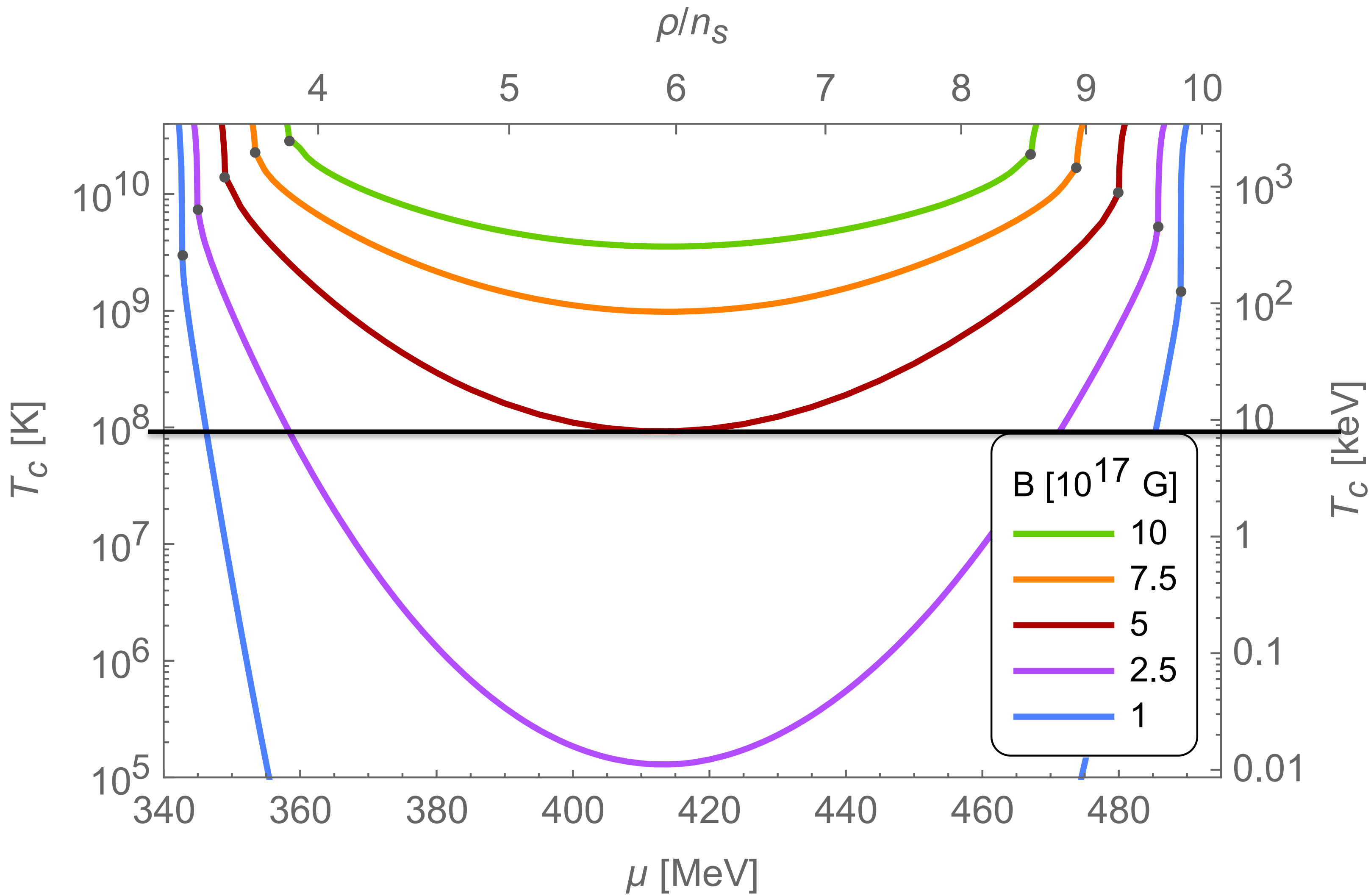
Carignano, Ferrer, VI, Paulucci, Phys.Rev. D92 (2015) no.10, 105018

MDCDW Resilience



Gyory and VI, PRD 106, 016011, 2022

Relevance for NS even in the Remnant Mass Region



GL Expansion & Origin of the beta (odd in b) terms

MDCDW Condensate ansatz $-2G\langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma^5\tau_3\psi\rangle = me^{iqz} = M(z)$

Expanding Ω in powers of m and b=q/2 we obtain

$$\Omega = \alpha_{2,0}m^2 + \boxed{\beta_{3,1}bm^2} + \alpha_{4,0}m^4 + \alpha_{4,2}b^2m^2 + \boxed{\beta_{5,1}bm^4} + \boxed{\beta_{5,3}b^3m^2} + \alpha_{6,0}m^6 + \alpha_{6,2}b^2m^4 + \alpha_{6,4}b^4m^2 + ...$$

Odd in b=q/2 terms!

They come from the antisymmetric part of the LLL asymmetric modes, so are topological in origin.

$$E_0 = \epsilon\sqrt{m^2 + k_3^2} + b, \quad \epsilon = \pm,$$

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, ...$$