QCD EoS for isospin asymmetric matter

Aritra Bandyopadhyay

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

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In collaboration with A.Ayala, R.L.S.Farias, L.A.Hernandez and J.L.Hernandez

Talk prepared for XQCD 23.

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Outline

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Why study isospin asymmetric matter

- Isospin density $n_I \equiv n_u - n_d$
- Consequences of isospin asymmetric matter :
  1. Excess of neutrons over protons
  2. Excess of π_- over π_+
- Relevance?

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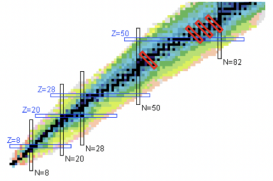
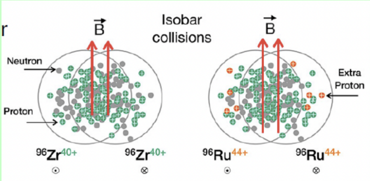
  1. Heavy Ion collisions, specifically RHIC isobar program





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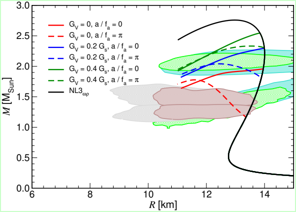
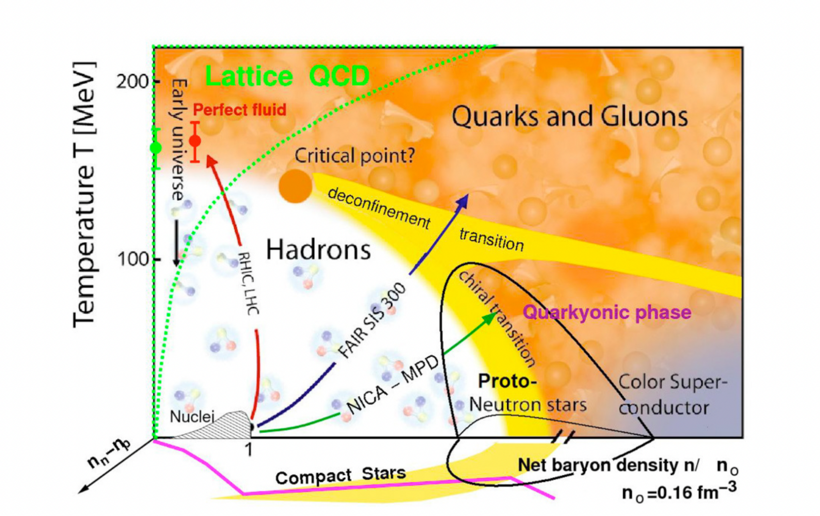
- Relevance?
  1. Heavy Ion collisions, specifically RHIC isobar program
  2. Neutron stars : interior and composition

Georgia Tech (Caltech Media Assets)

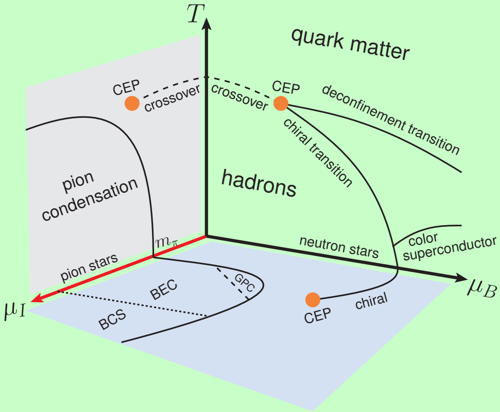


Lopes, Farias, Dexheimer, Bandyopadhyay, Ramos; Phys.Rev.D 106 (2022) 12, L121301

QCD Phase Diagram



QCD Phase Diagram



QCD Phase Diagram with μ_I

The sign problem

- The partition function calculation in Lattice QCD produces a fermion functional determinant :
  $\mathrm{Det}M = \mathrm{Det}\left(\not{D} + m + \mu\gamma_0\right)$
- Considering a complex value of μ if one takes the determinant of both sides of the identity
  $\gamma_5\left(\not{D} + m + \mu\gamma_0\right)\gamma_5 = \left(\not{D} + m - \mu^*\gamma_0\right)^\dagger$
  one obtains
  $\mathrm{Det}\left(\not{D} + m + \mu\gamma_0\right) = \left[\mathrm{Det}\left(\not{D} + m - \mu^*\gamma_0\right)\right]^*$
- Unless $\mu = 0, \mathcal{I}$; $\mathrm{Det}M$ is not real $\rightarrow$ Sign problem.
- So, for real μ it is not possible to carry out the direct sampling on a finite density ensemble by Monte Carlo methods.

No sign problem for finite isospin density

- So, we need real $\mathrm{Det}M$ (not necessarily positive if we are dealing with even number of flavors) which can correspond to the identity: $M^\dagger = PMP^{-1}$.
  E.g. for $\mu = 0$ and $P = \gamma_5$, $\gamma_5 \left(\not{D} + m\right) \gamma_5^{-1} = \left(\not{D} + m\right)^\dagger$
- Now for 2 flavor QCD with finite isospin density, M has a block diagonal structure

$$M(\mu_I) = \begin{pmatrix} L(\mu_I) & 0 \\ 0 & L(-\mu_I) \end{pmatrix}$$

$L(\mu_I)$ being the Dirac operator for one flavor with μ_I.

No sign problem for finite isospin density

In presence of finite Baryonic and Isospin chemical potentials, the quark chemical potentials can be expressed as :

$$\mu_u = \frac{\mu_B}{3} + \mu_I$$
$$\mu_d = \frac{\mu_B}{3} - \mu_I$$

When we have vanishing Baryonic chemical potential :

$$\mu_u = \mu_I$$
$$\mu_d = -\mu_I$$

No sign problem for finite isospin density

$$M(\mu_I) = \begin{pmatrix} L(\mu_I) & 0 \\ 0 & L(-\mu_I) \end{pmatrix}$$

- $L(\mu_I)$ satisfies $L^\dagger(\mu_I) = \gamma_5 L(-\mu_I)\gamma_5$.
- Hence, the positivity condition is satisfied by setting

$$P = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}$$

- So we have $\mathrm{Det} M(\mu_I) = |\mathrm{Det} L|^2 \geq 0$.
- $\mathrm{Det} M$ is real for QCD at non-zero isospin density $\rightarrow$ No sign problem.

Outline

LSMq Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - ig\bar{\psi}\gamma^5 \vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma,$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices, $\psi_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}$ is a $SU(2)_{L,R}$ doublet, σ is a real scalar field and $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ is a triplet of real scalar fields. π_3 corresponds to the neutral pion (π_0) whereas the charged ones are represented by the combinations

$$\pi_- = \frac{1}{\sqrt{2}}(\pi_1 + i\pi_2), \quad \pi_+ = \frac{1}{\sqrt{2}}(\pi_1 - i\pi_2).$$

The parameters a^2, λ and g are real and positive definite.

LSMq Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu \partial_\mu \psi - ig\bar{\psi}\gamma^5 \vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma,$$

can be written in terms of charged and neutral pion degrees of freedom as

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi_0)^2] + \partial_\mu \pi_- \partial^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2) + a^2 \pi_- \pi_+$$
$$- \frac{\lambda}{4}(\sigma^4 + 4\sigma^2 \pi_- \pi_+ + 2\sigma^2 \pi_0^2 + 4\pi_-^2 \pi_+^2 + 4\pi_- \pi_+ \pi_0^2 + \pi_0^4)$$
$$+ i\bar{\psi}\partial\!\!\!/\psi - g\bar{\psi}\psi\sigma - ig\bar{\psi}\gamma^5(\tau_+ \pi_+ + \tau_- \pi_- + \tau_3 \pi_0)\psi,$$

where we introduced the combination of Pauli matrices

$$\tau_+ = \frac{1}{\sqrt{2}}(\tau_1 + i\tau_2), \quad \tau_- = \frac{1}{\sqrt{2}}(\tau_1 - i\tau_2).$$

LSMq Lagrangian

$$
\begin{aligned}
\mathcal{L} = &\frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi_0)^2] + \partial_\mu \pi_- \partial^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2) + a^2 \pi_- \pi_+ \\
&- \frac{\lambda}{4}(\sigma^4 + 4\sigma^2 \pi_- \pi_+ + 2\sigma^2 \pi_0^2 + 4\pi_-^2 \pi_+^2 + 4\pi_- \pi_+ \pi_0^2 + \pi_0^4) \\
&+ i\bar{\psi}\partial\!\!\!/ \psi - g\bar{\psi}\psi\sigma - ig\bar{\psi}\gamma^5(\tau_+\pi_+ + \tau_-\pi_- + \tau_3\pi_0)\psi,
\end{aligned}
$$

The Lagrangian possesses the following symmetries:

- A $SU(N_c)$ global color symmetry,
- A $SU(2)_L \times SU(2)_R$ chiral symmetry,
- A $U(1)_B$ symmetry. The sub-index of the latter emphasizes that the conserved charge is the baryon number B.

LSMq + finite isospin

1. A conserved isospin charge multiplied by the isospin chemical potential $\rightarrow \bar{\psi}\mu_I\tau_3\gamma_0\psi$.
2. For the charged pions $\rightarrow$ ordinary derivative becomes a covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i\mu_I\delta^0_\mu, \quad \partial^\mu \rightarrow D^\mu = \partial^\mu - i\mu_I\delta^\mu_0,$$

3. To include a finite vacuum pion mass, m_0, we add an explicit symmetry breaking term $\rightarrow h(\sigma + v)$. v is the non-vanishing vacuum expectation value of σ (SCSB).

LSMq + finite isospin

before

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi_0)^2] + \partial_\mu \pi_- \partial^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2) + a^2 \pi_- \pi_+$$
$$- \frac{\lambda}{4}(\sigma^4 + 4\sigma^2 \pi_- \pi_+ + 2\sigma^2 \pi_0^2 + 4\pi_-^2 \pi_+^2 + 4\pi_- \pi_+ \pi_0^2 + \pi_0^4)$$
$$+ i\bar{\psi}\partial\!\!\!/ \psi - g\bar{\psi}\psi\sigma - ig\bar{\psi}\gamma^5(\tau_+\pi_+ + \tau_-\pi_- + \tau_3\pi_0)\psi,$$

after

$$\mathcal{L}' = \frac{1}{2}[(\partial_\mu \sigma)^2 + (\partial_\mu \pi_0)^2] + D_\mu \pi_- D^\mu \pi_+ + \frac{a^2}{2}(\sigma^2 + \pi_0^2) + a^2 \pi_- \pi_+$$
$$- \frac{\lambda}{4}(\sigma^4 + 4\sigma^2 \pi_- \pi_+ + 2\sigma^2 \pi_0^2 + 4\pi_-^2 \pi_+^2 + 4\pi_- \pi_+ \pi_0^2 + \pi_0^4) + h(\sigma + v)$$
$$+ i\bar{\psi}\partial\!\!\!/ \psi - g\bar{\psi}\psi\sigma + \bar{\psi}\mu_I \tau_3 \gamma_0 \psi - ig\bar{\psi}\gamma^5(\tau_+\pi_+ + \tau_-\pi_- + \tau_3\pi_0)\psi.$$

LSMq + finite isospin

- Symmetry structures :
  $U(1)_B \times SU(2)_L \times SU(2)_R \to U(1)_B \times U(1)_{I_3}$.
- Ansatz for further simplifications in the pseudoscalar channels

$$\langle \bar{\psi} i\gamma_5 \tau_3 \psi \rangle = 0,$$
$$\langle \bar{u} i\gamma_5 d \rangle = \langle \bar{d} i\gamma_5 u \rangle^* \neq 0.$$

- Breaks the residual $U(1)_{I_3}$ symmetry $\to$ BEC.
- The charged pion fields can be referred from their condensates

$$\pi_+ \to \pi_+ + \frac{\Delta}{\sqrt{2}} e^{i\theta}, \quad \pi_- \to \pi_- + \frac{\Delta}{\sqrt{2}} e^{-i\theta}.$$

- θ indicates the direction of the $U(1)_{I_3}$ symmetry breaking.
  We take $\theta = \pi$ for definitiveness.

One loop effective potential

In the condensed phase the tree-level potential can be written as

$$V_{\rm tree} = -\frac{a^2}{2}\left(v^2+\Delta^2\right) + \frac{\lambda}{4}\left(v^2+\Delta^2\right)^2 - \frac{1}{2}\mu_I^2\Delta^2 - hv.$$

The fermion contribution to the one-loop effective potential becomes

$$\sum_{f=u,d} V_f^1 = iV^{-1}\ln(\mathcal{Z}_f^1) = iV^{-1}\ln\left(\det\left(S_{\rm mf}^{-1}\right)\right)$$
$$= -2N_c\int\frac{d^3k}{(2\pi)^3}\left[E_\Delta^u + E_\Delta^d\right],$$

with

$$E_\Delta^{u[d]} \quad = \quad \left\{\left(\sqrt{k^2+m_f^2} + [-]\mu_I\right)^2 + g^2\Delta^2\right\}^{1/2},$$

One loop effective potential

- V_f^1 is ultraviolet divergent which depends on μ_I.
- To identify the divergent terms, we expand the fermion energies in powers of $\mu_I^2/[g^2(v^2+\Delta^2)]$

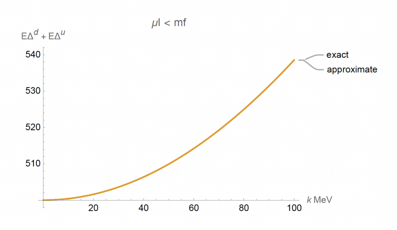
$$\sum_{f=u,d} V_f^1 = -2N_c \int \frac{d^3k}{(2\pi)^3}\left(2\sqrt{k^2+m_f^2+g^2\Delta^2} + \frac{\mu_I^2 g^2 \Delta^2}{(k^2+m_f^2+g^2\Delta^2)^{3/2}}\right)$$

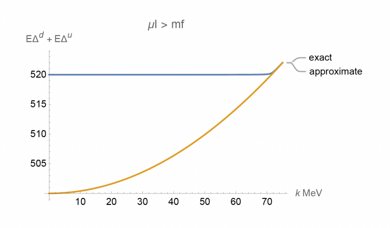
- Using dimensional regularization in the $\overline{\text{MS}}$ scheme,

$$\sum_{f=u,d} V_f^1 = 2N_c \frac{g^4\left(v^2+\Delta^2\right)^2}{(4\pi)^2}\left[\frac{1}{\epsilon}+\frac{3}{2}+\ln\left(\frac{\Lambda^2/g^2}{v^2+\Delta^2}\right)\right]$$
$$-2N_c\frac{g^2\mu_I^2\Delta^2}{(4\pi)^2}\left[\frac{1}{\epsilon}+\ln\left(\frac{\Lambda^2/g^2}{v^2+\Delta^2}\right)\right],$$

where Λ is the dimensional regularization ultraviolet scale and the limit $\epsilon \to 0$ is to be understood.

Two distinct regimes : $\mu_I < m_f$ and $\mu_I > m_f$





We focus on the $\mu_I < m_f$ regime.

Vacuum Stability

- Term proportional to $\mu_I^2\Delta^2 \rightarrow$ same structure present in the tree-level potential and divergent term.
- Introduce counter-terms that respect the structure of the tree-level potential and determine them by accounting for the stability conditions

$$\frac{\partial V_{\text{tree}}}{\partial v} = \left[\lambda v^3 - (a^2 - \lambda\Delta^2)v - h\right]\Big|_{v_0,\,\Delta_0} = 0$$

$$\frac{\partial V_{\text{tree}}}{\partial \Delta} = \left[\lambda\Delta^2 - (\mu_I^2 - \lambda v^2 + a^2)\right]\Big|_{v_0,\,\Delta_0} = 0.$$

- $\mu_I^2 > \lambda v^2 - a^2 = m_0^2 \rightarrow$ Condensed phase.
- Simultaneous solutions (classical solution)

$$v_0 = \frac{h}{\mu_I^2}, \quad \Delta_0 = \sqrt{\frac{\mu_I^2}{\lambda} - \frac{h^2}{\mu_I^4} + \frac{a^2}{\lambda}}.$$

One loop effective potential with counterterms

$$V_{\text{eff}} = V_{\text{tree}} + \sum_{f=u,d} V_f^1 - \frac{\delta\lambda}{4}(v^2+\Delta^2)^2 + \frac{\delta a}{2}(v^2+\Delta^2) + \frac{\delta}{2}\Delta^2\mu_I^2$$

- The counter-terms $\delta\lambda$ and δ are determined from the *gap equations*

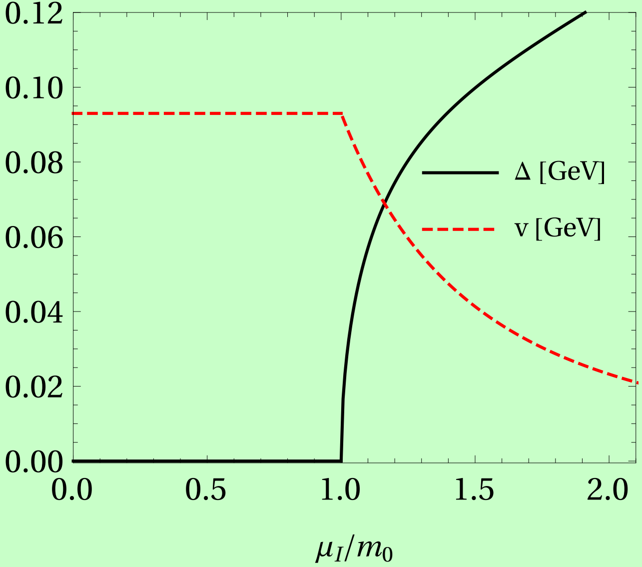
$$\left.\frac{\partial V_{\text{eff}}}{\partial v}\right|_{v_0,\,\Delta_0} = 0, \quad \left.\frac{\partial V_{\text{eff}}}{\partial \Delta}\right|_{v_0,\,\Delta_0} = 0.$$

- These conditions suffice to absorb the infinities.
- The counter-term δa is determined by requiring that the slope of V_{eff} vanishes at $\mu_I = m_0$,

$$\left.\frac{\partial V_{\text{eff}}}{\partial \mu_I}\right|_{\mu_I=m_0} = 0,$$

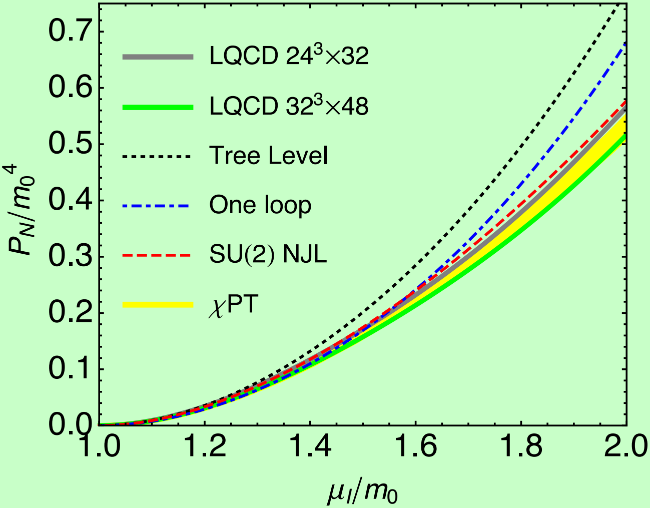
  or in other words, that the transition from the non-condensed to the condensed phase be smooth.

Condensates



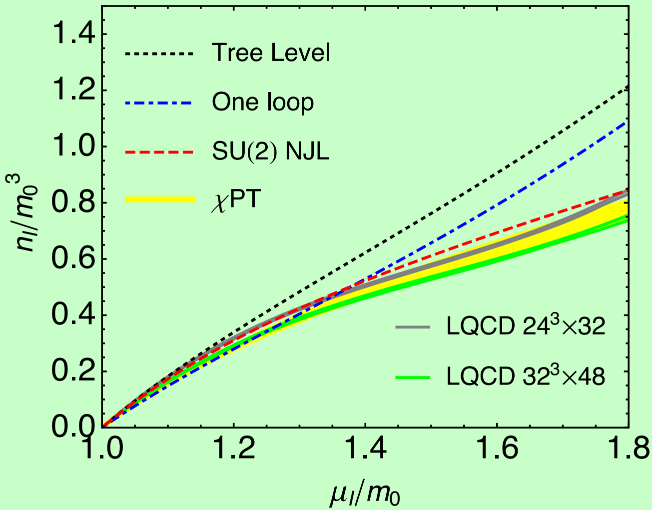
Outline

Pressure ($P = -V_{\rm eff}$)



NJL : Avancini, Bandyopadhyay, Duarte, Farias; Phys. Rev. D. 100, 116002 (2019)
χPT : Adhikari, Andersen, Kneschke; Eur. Phys. J. C 79, 874 (2019)
LQCD : Brandt, Cuteri, Endrödi; Pos LATTICE2022, 144 (2023)

Isospin density ($n_I = dP/d\mu_I$)



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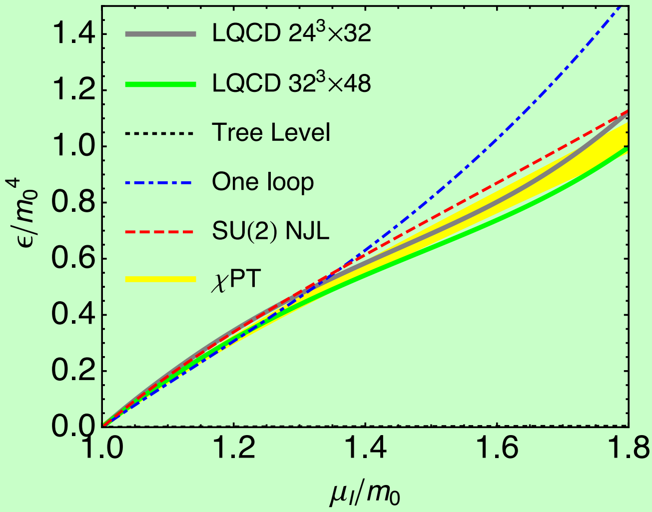
Energy density ($\epsilon = -P + n_I \mu_I$)



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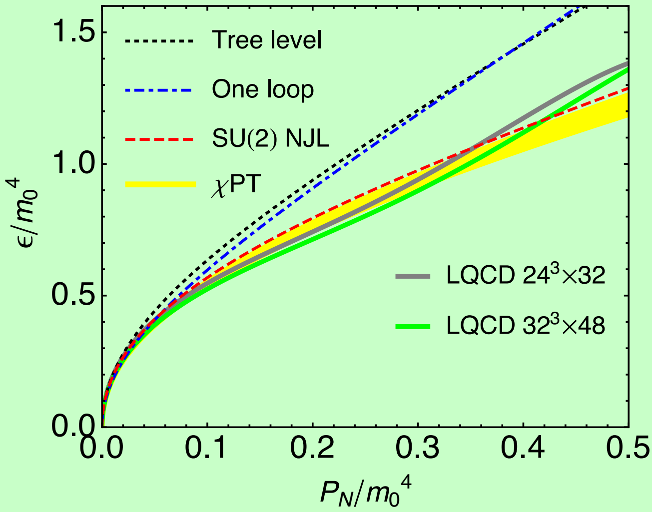
EoS



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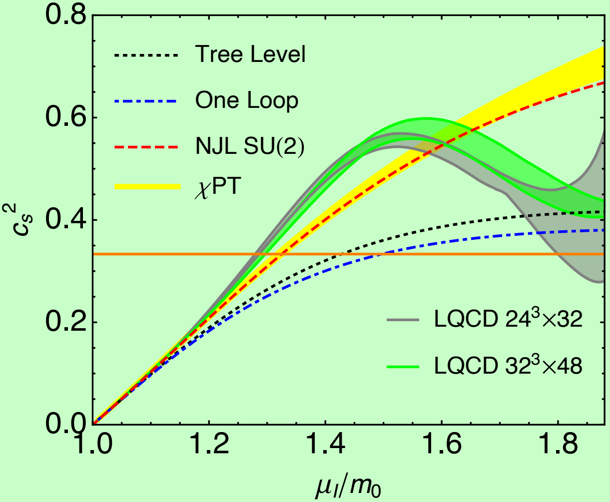
Speed of sound ($c_s^2 = \partial P/\partial \epsilon$)



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Outline

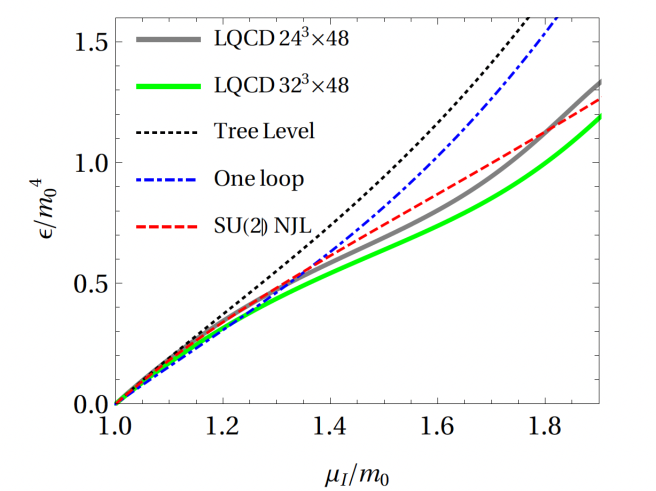
To summarize

- LSMq, with 2 quark flavors, has been used to study the phase structure of isospin asymmetric matter at zero temperature.
- The meson degrees of freedom provide mean field on top of which we include quark fluctuations at one-loop order.
- Appropriate renormalization has been done to absorb UV divergences with the addition of counter-terms that respect the original structure of the theory.
- Two phases in the condensed phase: $\mu_I < m_f$ and $\mu_I > m_f$.
- Evolution of the chiral and isospin condensates as well as the pressure, energy and isospin densities and the sound velocity. Good agreement with LQCD for the studied phase, $\mu_I < m_f$.
- Phase with $\mu_I > m_f$ is work in progress.

Thanksgiving

Thank you for your kind attention.

Energy density ($\epsilon = -P + n_I \mu_I$)



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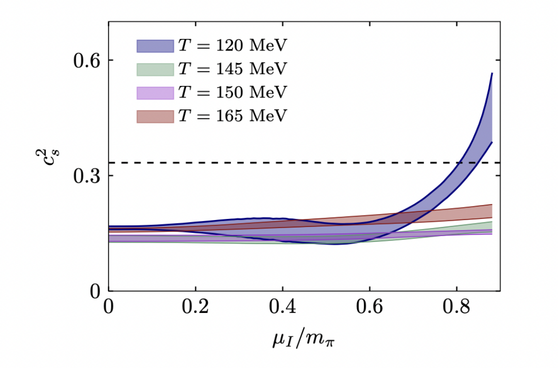
Speed of sound at finite T (LQCD results)



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