Topological structure of the QCD vacuum at finite temperature

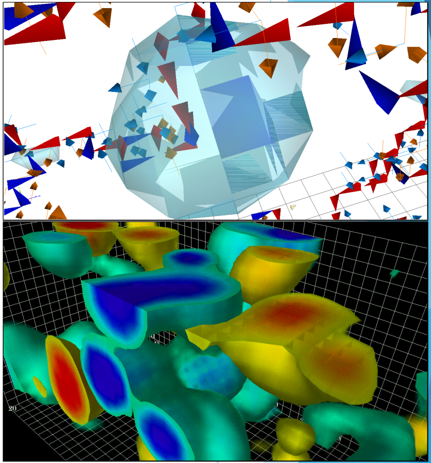
Waseem Kamleh

Collaborators
Derek Leinweber, Jackson Mickley

19th International Conference on QCD in Extreme Conditions (XQCD)
University of Coimbra, Portugal, 26-28 July 2023

QCD vacuum structure

- QCD vacuum state is non-trivial
  - Non-perturbative description
  - Lattice QCD
- Key *emergent* features of QCD
  - Dynamical mass generation
  - Confinement transition
- Structure of the QCD vacuum
  - Center vortices
  - Topological charge density

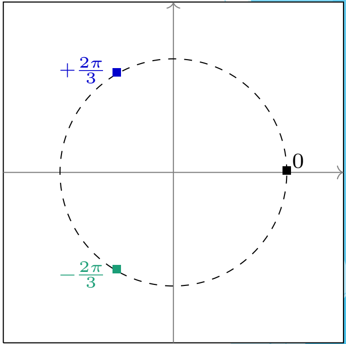


Centre group of SU(3)

Centre elements commute with every group element,

$$z = \exp(\frac{2\pi i}{3} m) I, \quad m \in \{-1, 0, 1\} \simeq \mathbb{Z}_3$$

Each of the three centre phases corresponds to a centre element of SU(3)

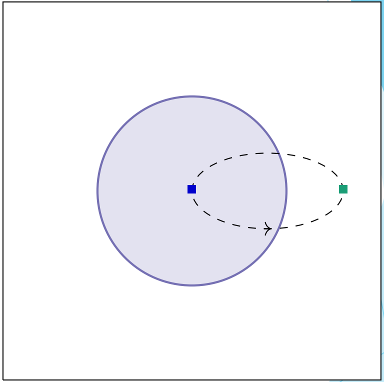


Centre vortices

In D dimensions, a (thin) *centre vortex* has dimension $D-2$.

In 4-dimensional space-time, thin vortices are surface-like.

A Wilson loop $W(C)$ along a curve $C = \partial A$ is topologically linked if the vortex pierces the enclosed area A only once.



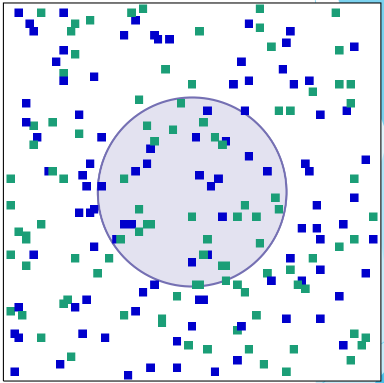
Confinement

The piercing vortex generates a non-trival centre phase z,

$$W(C) \to zW(C)$$

If centre vortices percolate through a volume with density ρ, this gives rise to an area law for the Wilson loop

$$W(C) = e^{-2\rho A}$$



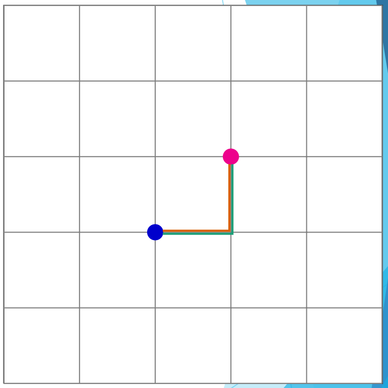
Lattice QCD

Discretise space-time onto 4D hypercube

Gauge field $U_\mu(x) \in SU(3)$ becomes unitary

$32^3 \times N_t$ (periodic) lattice volume

Pure gauge (PG), $a = 0.100$ fm



Identifying centre vortices

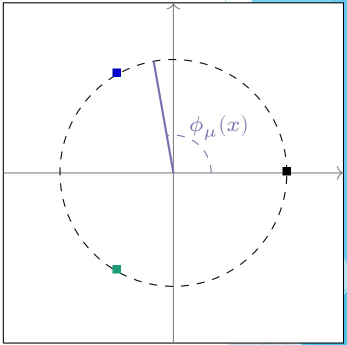
Transform to Maximal Centre Gauge

$$\sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}[U_\mu^\Omega(x) Z_\mu^\dagger(x)] \to \text{ Max}$$

$\Omega(x)$ maximises overlap with centre elements.

Project onto $\mathbb{Z}_3$ by choosing closest centre element to the phase of

$$\frac{1}{3}\,\mathrm{Tr}\,U_\mu^\Omega(x) = r_\mu(x)\exp(i\phi_\mu(x))$$



Identifying centre vortices

The centre vortex field lives on the dual lattice,

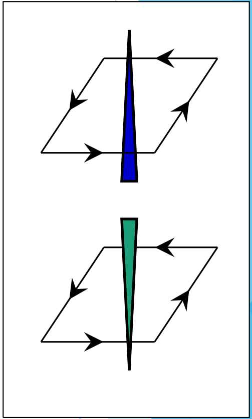
$$\bar{x} = x + \frac{a}{2}(\hat{\mu} + \hat{\nu} - \hat{\lambda} - \hat{\kappa})$$

The centre flux $m_{\kappa\lambda}(\bar{x})$ through an elementary plaquette is

$$P_{\mu\nu}(x) = \exp\left(\frac{\pi i}{3}\,\epsilon_{\kappa\lambda\mu\nu} m_{\kappa\lambda}(\bar{x})\right)$$

Centre-projected plaquette is pierced by a (P-)vortex if

$$\begin{aligned} P_{\mu\nu}(x) &= Z_{\mu}(x) Z_{\nu}(x+\mu) Z_{\mu}^{\dagger}(x+\nu) Z_{\nu}^{\dagger}(x) \\ &= \exp\left(\frac{\pm 2\pi i}{3}\right) I \end{aligned}$$



Centre vortices on the lattice

Untouched configurations

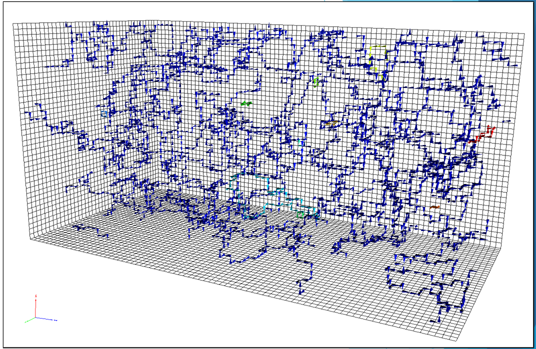
$$U_\mu(x)$$

Vortex-only configurations

$$Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]\mathrm{I}$$

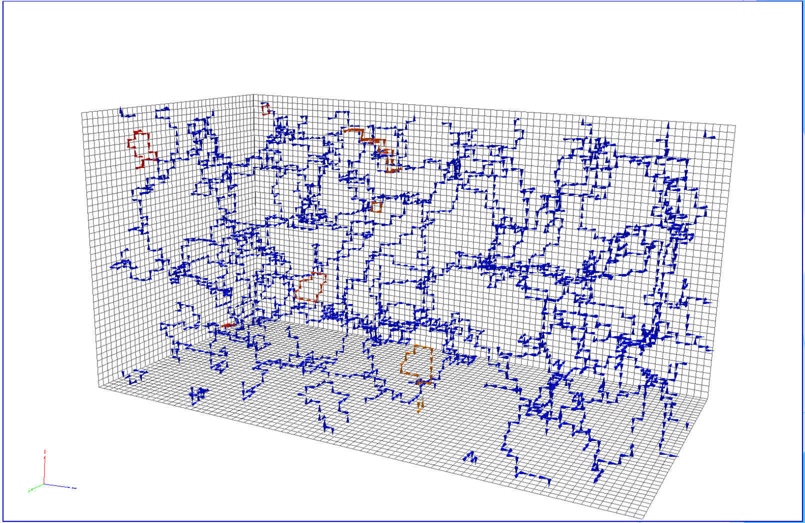
Vortex removed configurations

$$R_\mu(x) = Z_\mu^\dagger(x) U_\mu^\Omega(x)$$

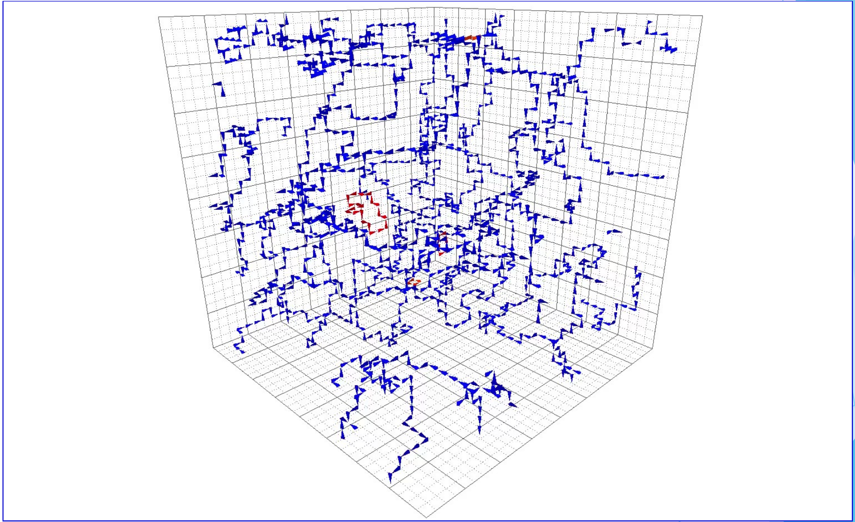


Visualization of center vortex structure
J.C. Biddle, WK, D.B. Leinweber
Phys. Rev. D 102 (2020) 3, 034504

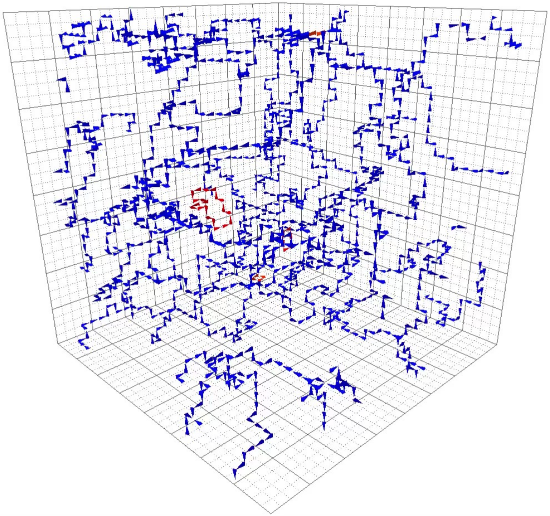
$N_t = 64$



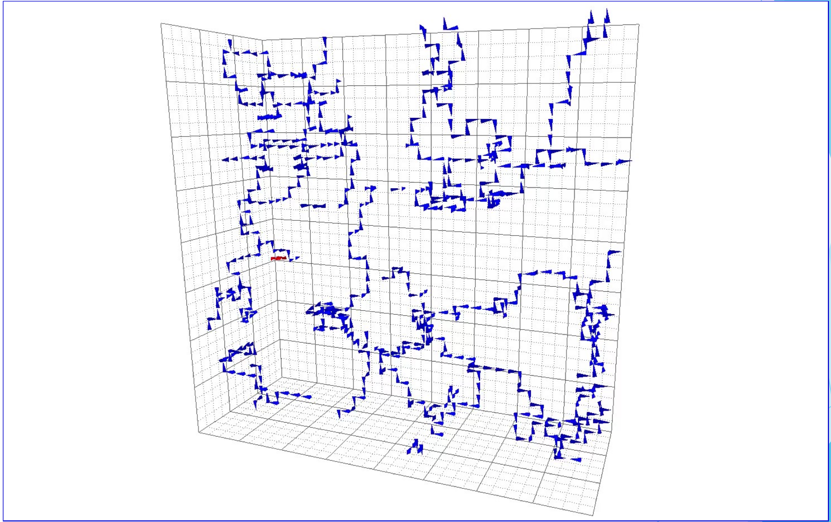
$N_t = 12$



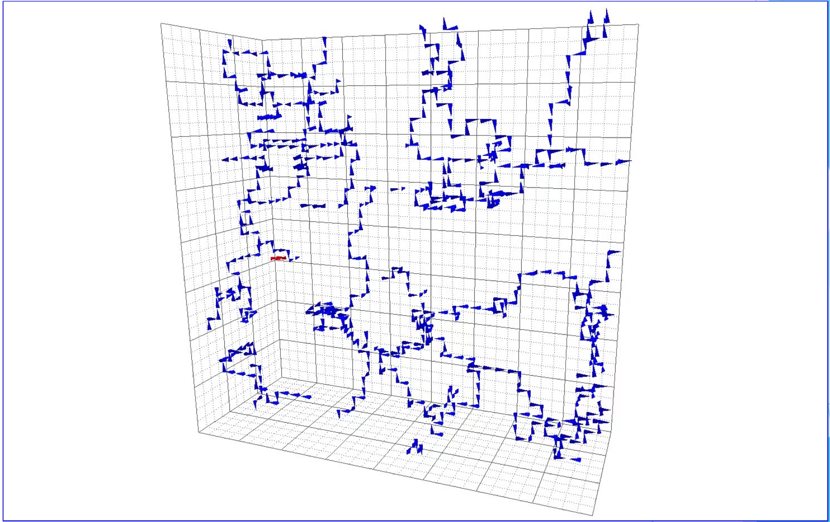
$N_t = 12$



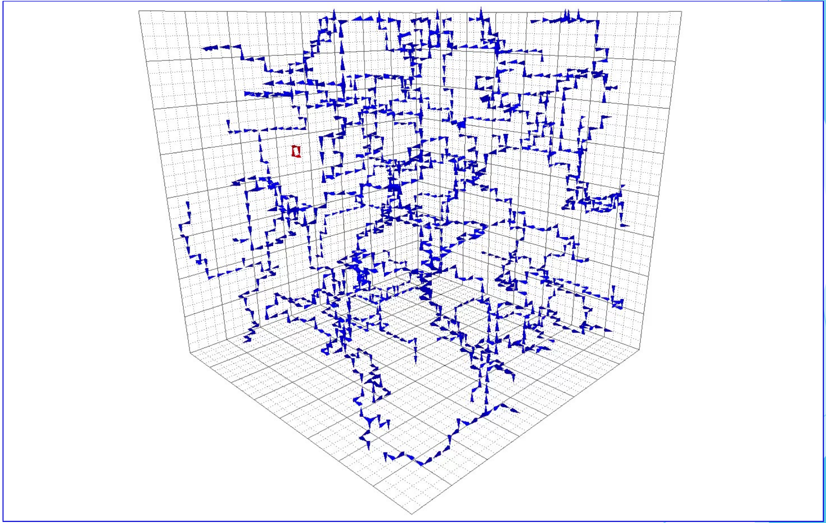
$N_t = 12$



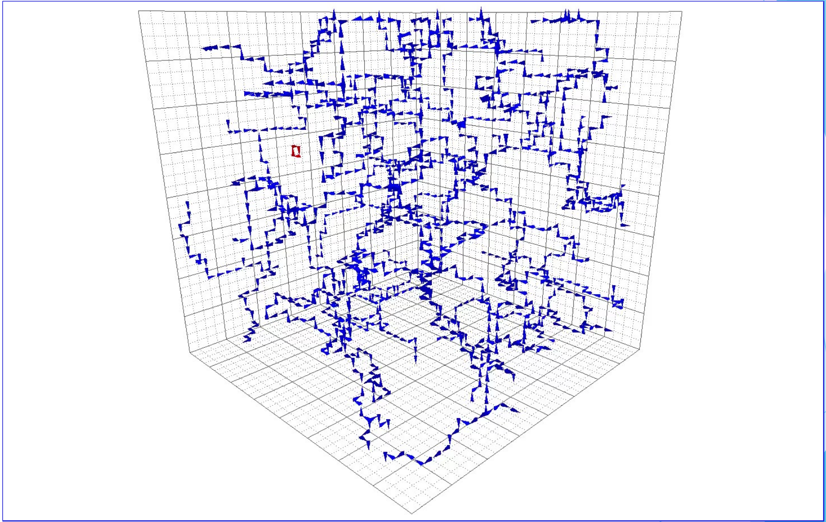
$N_t = 12$



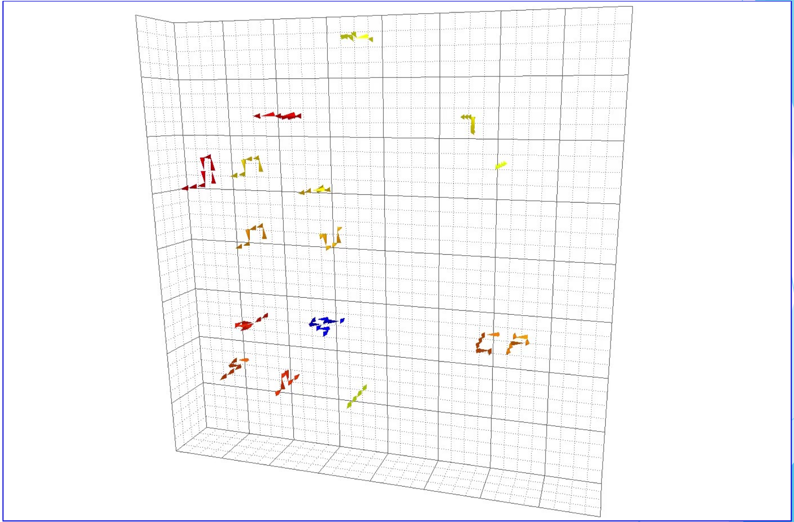
$N_t = 4$



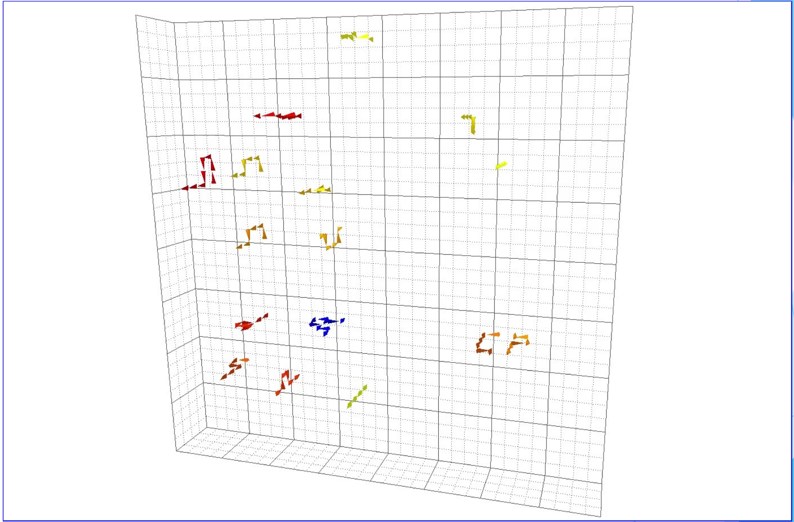
$N_t = 4$



$N_t = 4$

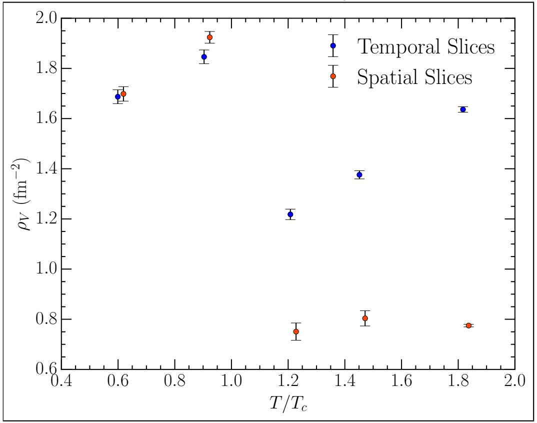


$N_t = 4$

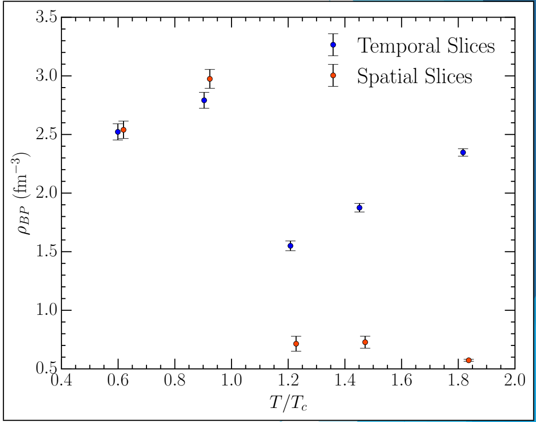


Densities

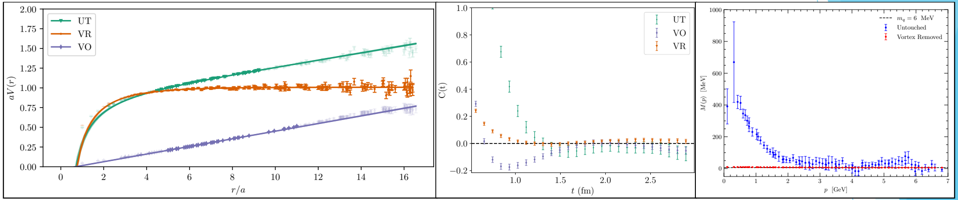
Vortex density



Branching point density



Centre vortices at zero temperature



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Centre vortex removal restores chiral symmetry
A. Trewartha, WK, D.B. Leinweber, J. Phys. G 44 (2017) 12, 125002

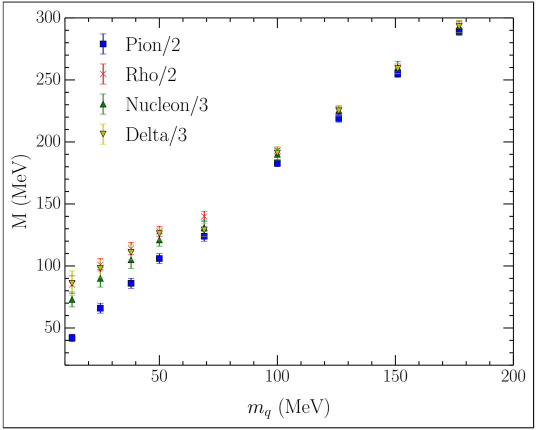
Gluon propagator on a center-vortex background
J.C. Biddle, WK, D.B. Leinweber, Phys. Rev. D 98 (2018) 9, 094504

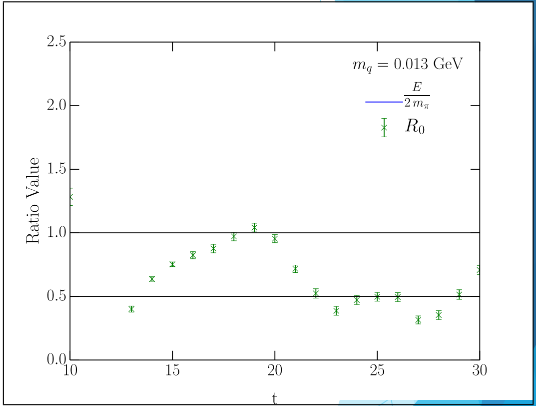
Smoothing algorithms for projected center-vortex gauge fields
A. Virgili, WK, D.B. Leinweber, Phys.Rev.D 106 (2022) 1, 014505

Static quark potential from centre vortices in the presence of dynamical fermions
J.C. Biddle, WK, D.B. Leinweber, arXiv:2206.00844

Evidence that centre vortices drive dynamical mass generation in QCD
WK, D.B. Leinweber, A. Virgili, arXiv:2305:18690

Centre vortices at zero temperature





Centre vortex removal restores chiral symmetry

A. Trewartha, WK, D.B. Leinweber, J. Phys. G 44 (2017) 12, 125002

Overlap quark propagator - zero temperature

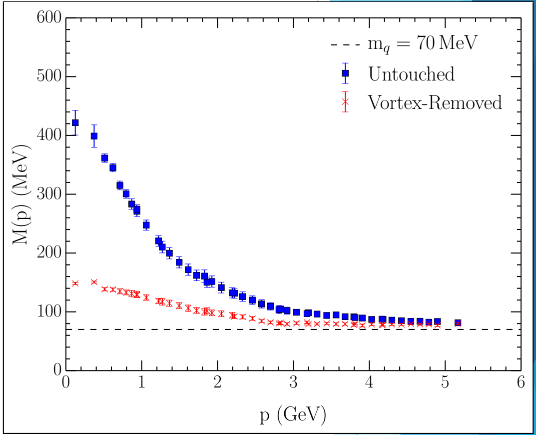
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

$M(p)$ is the mass function

$Z(p)$ is the renormalisation function

Landau gauge

Overlap fermions



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - zero temperature

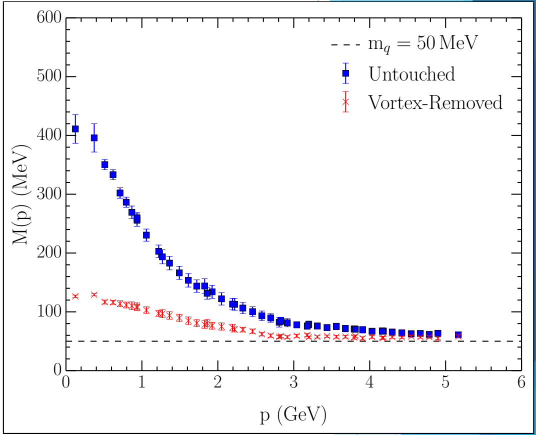
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Dynamical mass generation

Increases as m_q decreases

Dynamical chiral symmetry breaking

Reduced by vortex removal



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - zero temperature

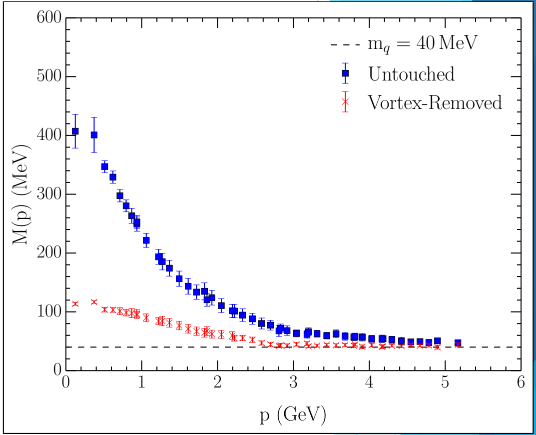
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Dynamical mass generation

Increases as m_q decreases

Dynamical chiral symmetry breaking

Reduced by vortex removal



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - zero temperature

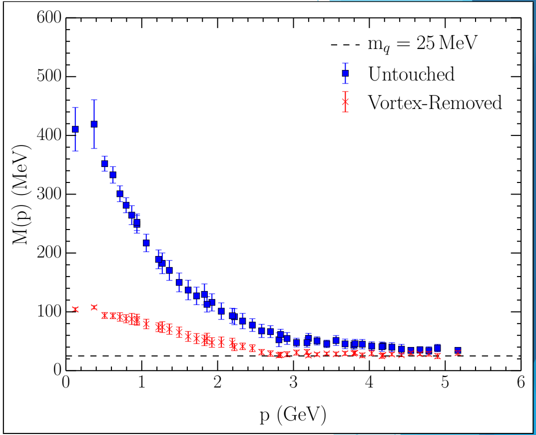
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Dynamical mass generation

Increases as m_q decreases

Dynamical chiral symmetry breaking

Reduced by vortex removal



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - zero temperature

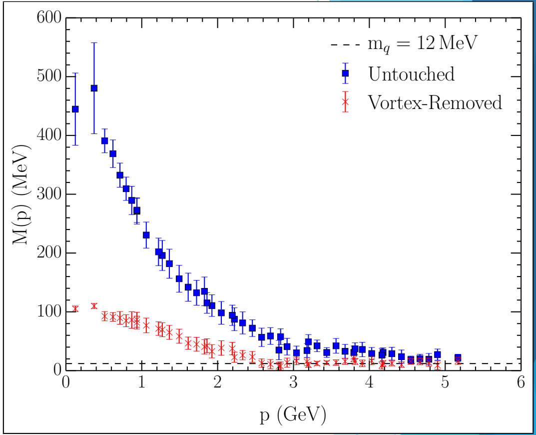
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Dynamical mass generation

Increases as m_q decreases

Dynamical chiral symmetry breaking

Reduced by vortex removal



Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

Overlap quark propagator - zero temperature

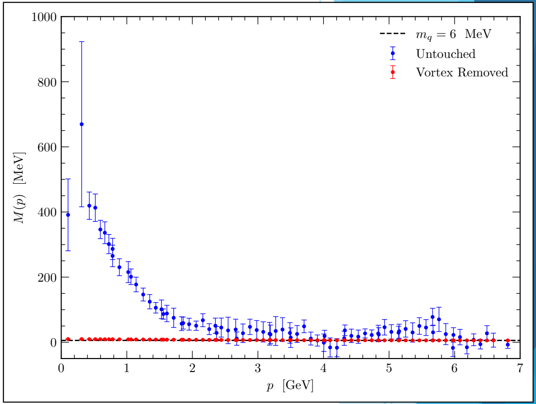
$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}$$

Dynamical mass generation

Increases as m_q decreases

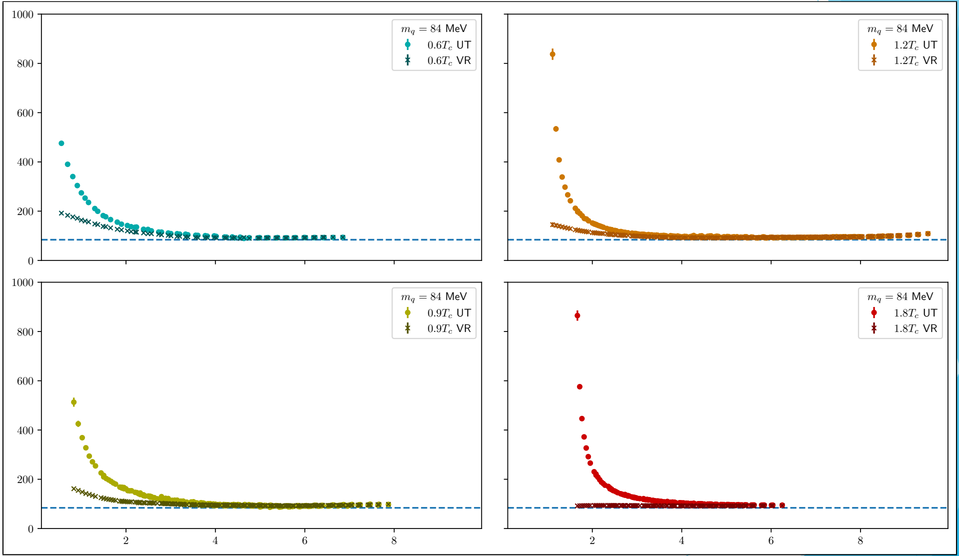
Dynamical chiral symmetry breaking

Eliminated in dynamical QCD

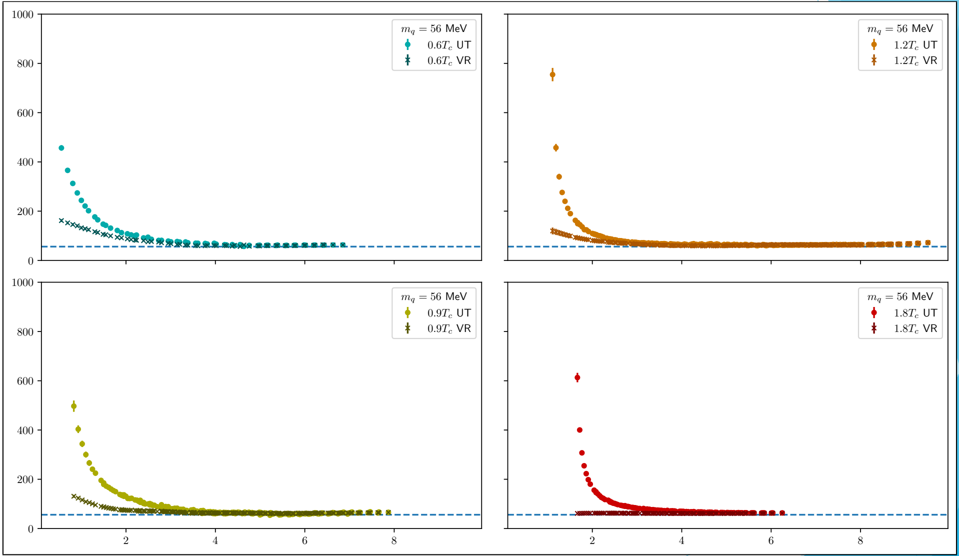


Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory
A. Trewartha, WK, D.B. Leinweber, Phys. Lett. B 747 (2015) 373-377

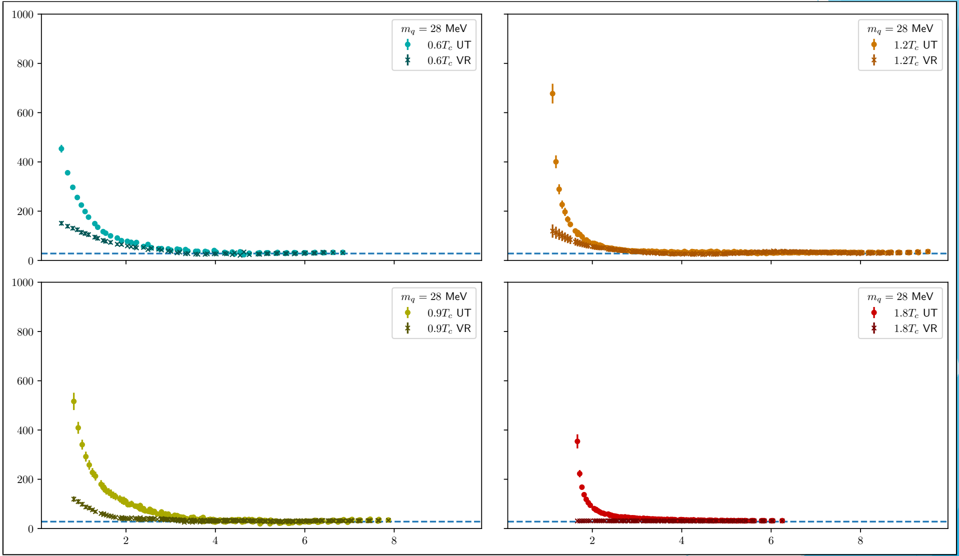
Overlap quark propagator - finite T



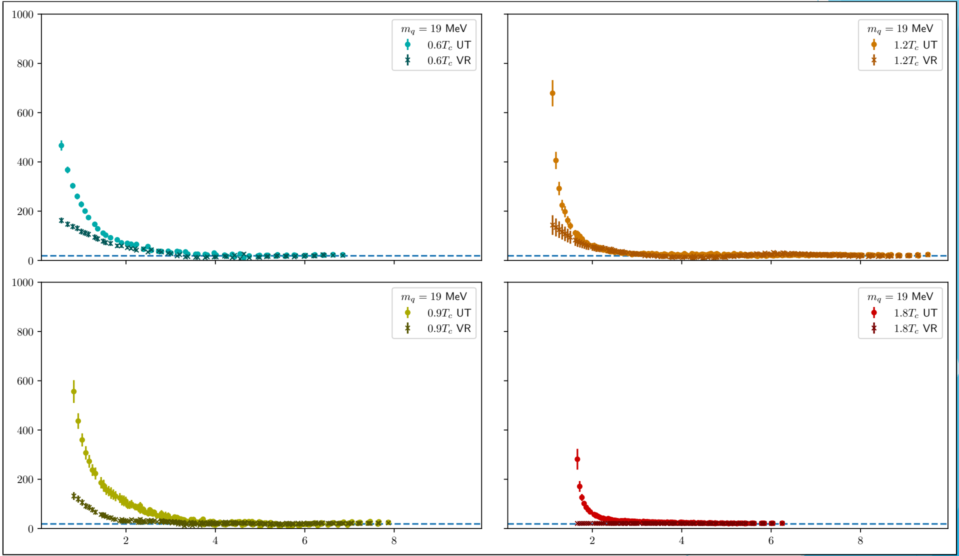
Overlap quark propagator - finite T



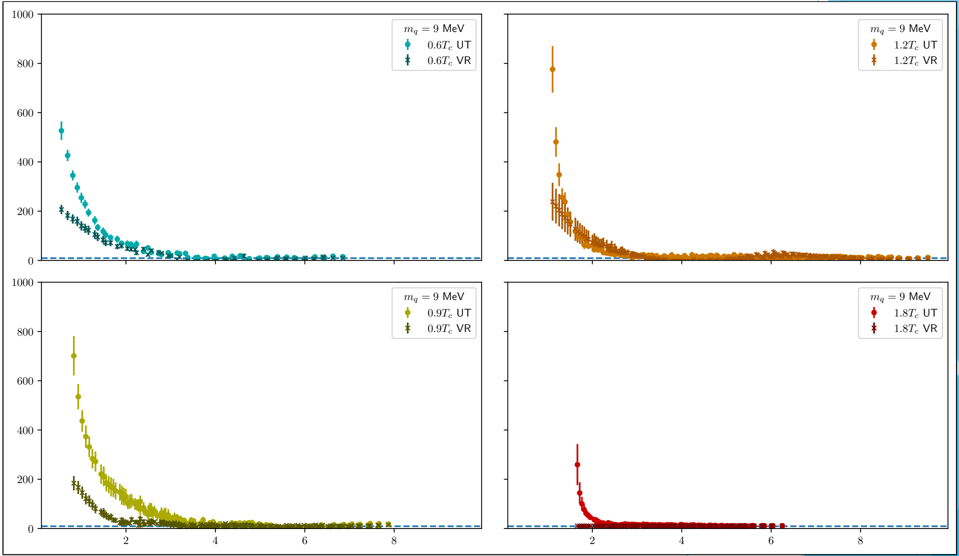
Overlap quark propagator - finite T



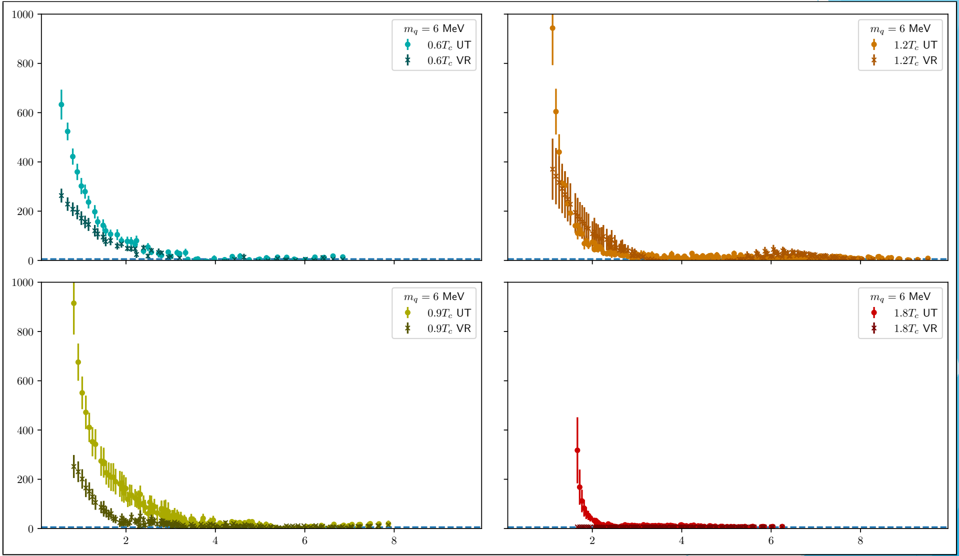
Overlap quark propagator - finite T



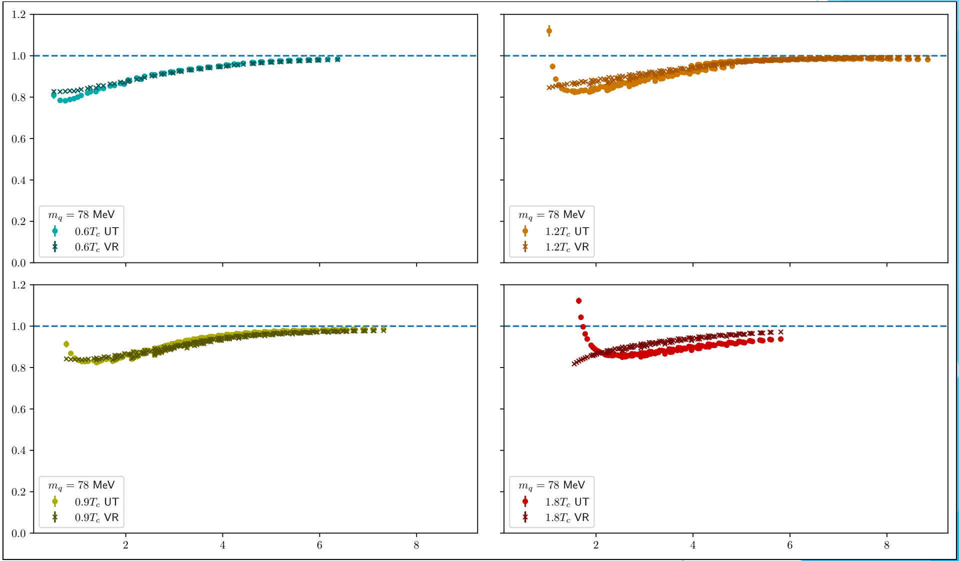
Overlap quark propagator - finite T



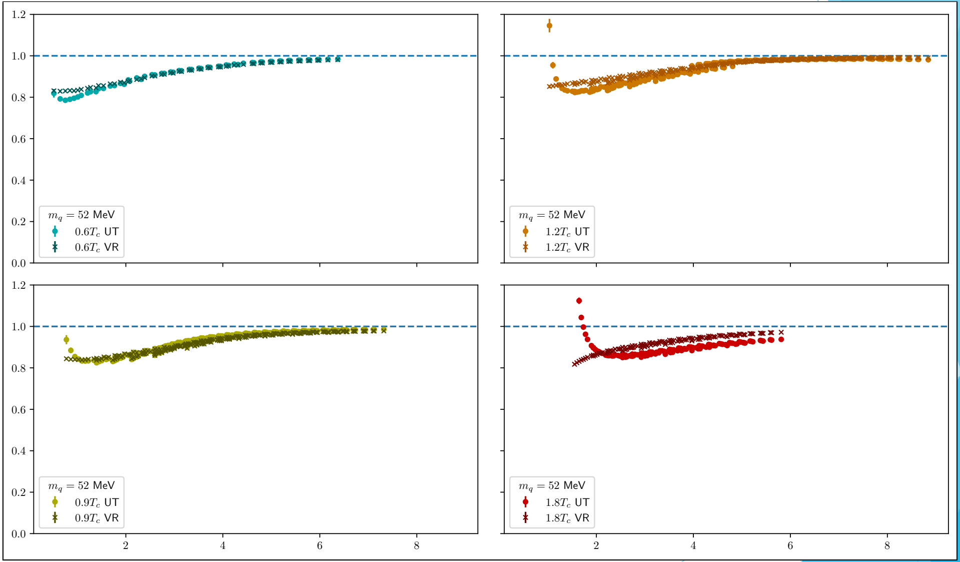
Overlap quark propagator - finite T



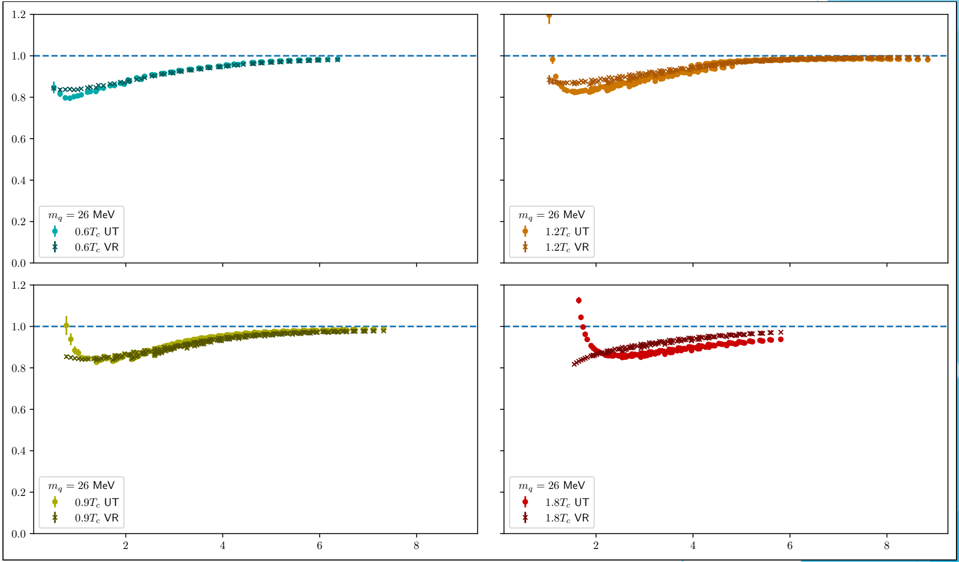
Overlap quark propagator - finite T



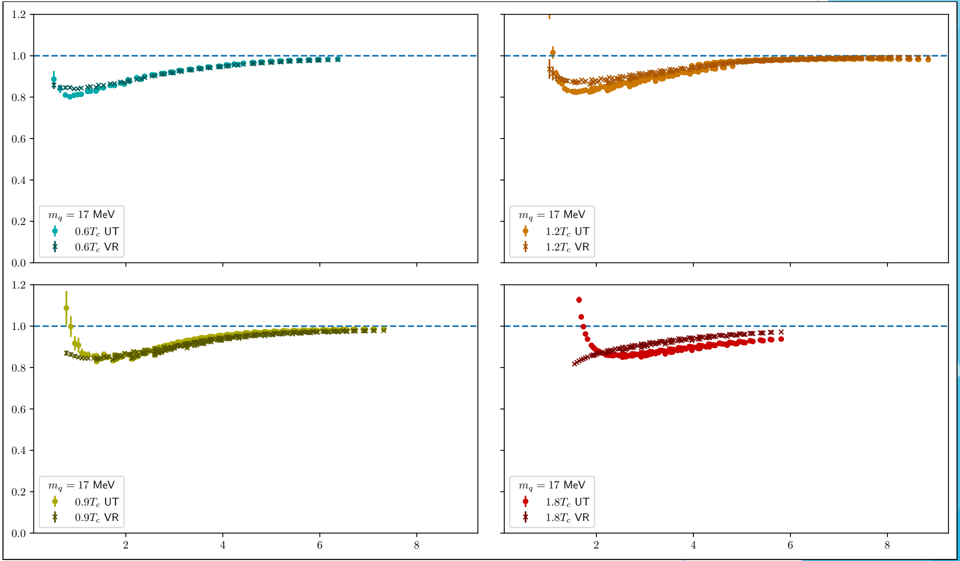
Overlap quark propagator - finite T



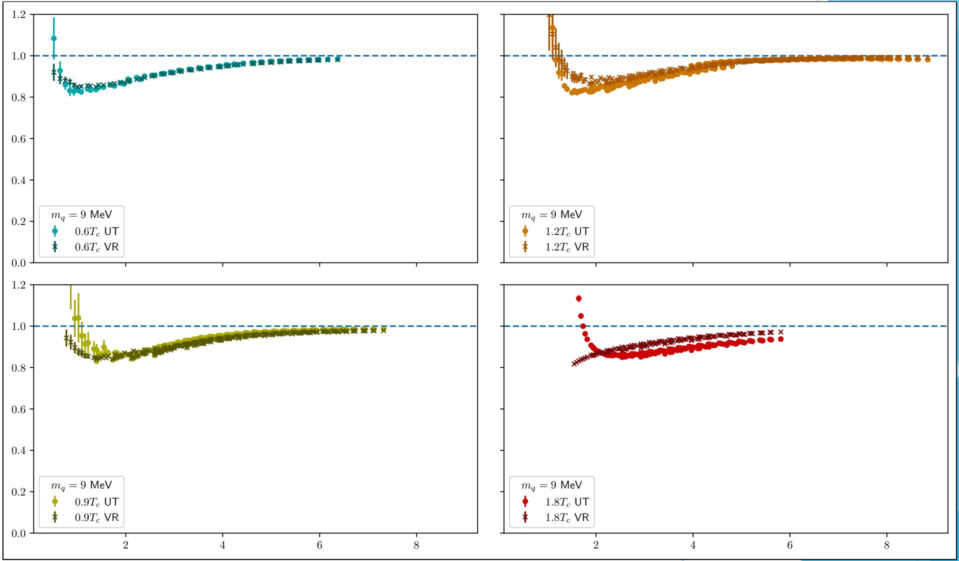
Overlap quark propagator - finite T



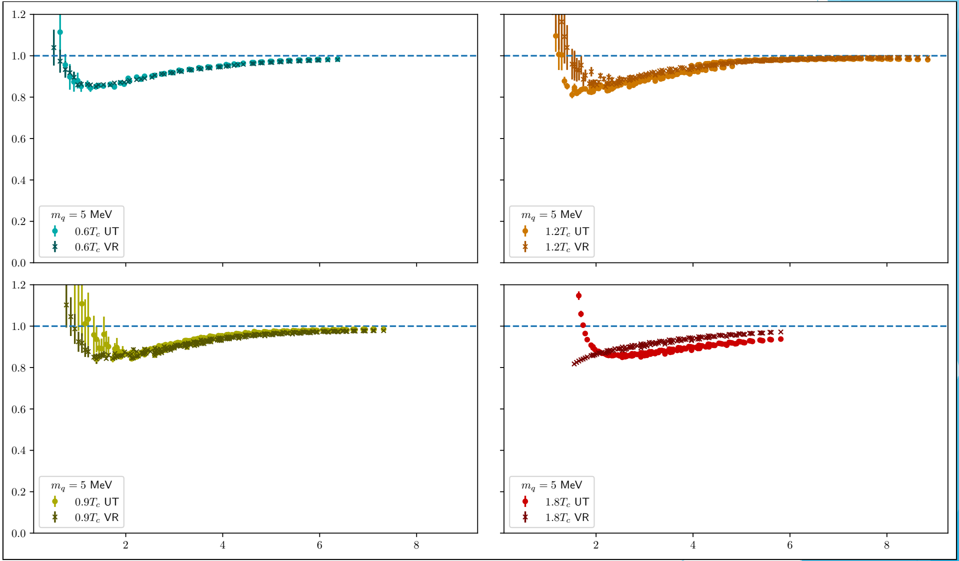
Overlap quark propagator - finite T



Overlap quark propagator - finite T



Overlap quark propagator - finite T



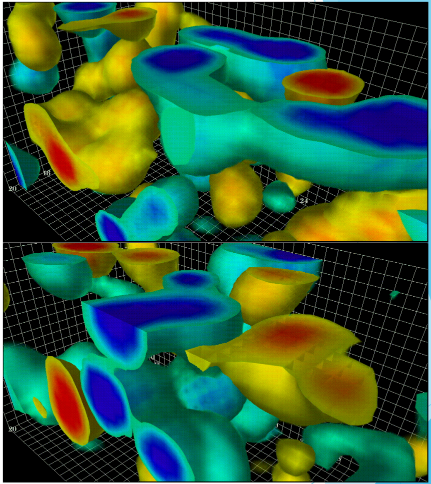
Center vortices and topological structure

Center vortex model originated from t'Hooft (topological) operators.

Smoothing vortex-only field creates topological structures.

These structures resemble instantons with enough smoothing.

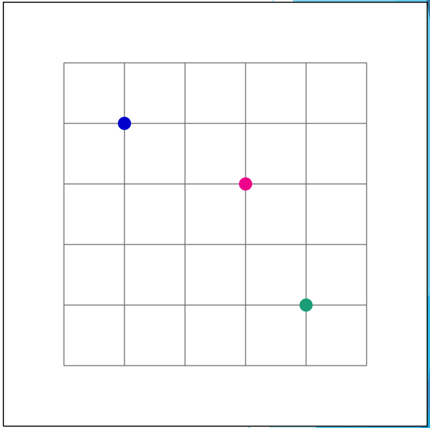
Structures directly from topological charge density?



Connection between center vortices and instantons through gauge-field smoothing
A. Trewartha, WK, D.B. Leinweber, Phys. Rev. D 92 (2015) 7, 074507

Topological object identification

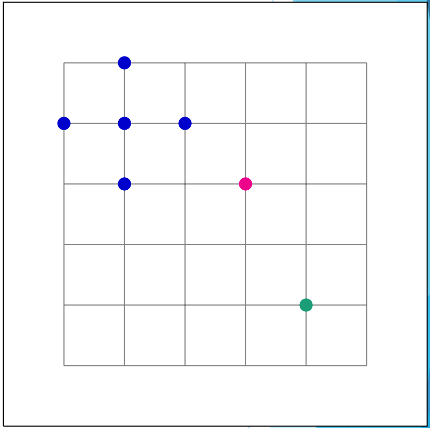
Start with extrema of $q(x)$.



Topological object identification

Start with extrema of $q(x)$.

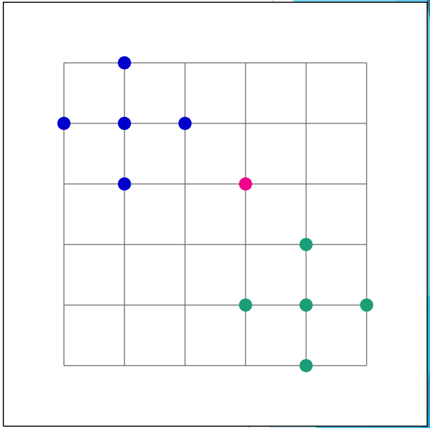
Grow objects in turn one step, monotonically in $q(x)$.



Topological object identification

Start with extrema of $q(x)$.

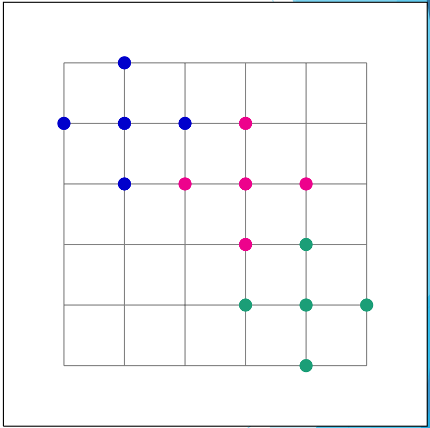
Grow objects in turn one step, monotonically in $q(x)$.



Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

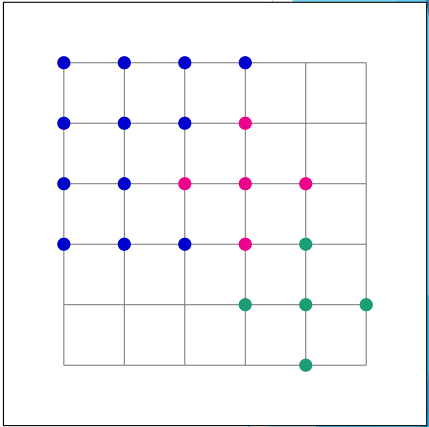


Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

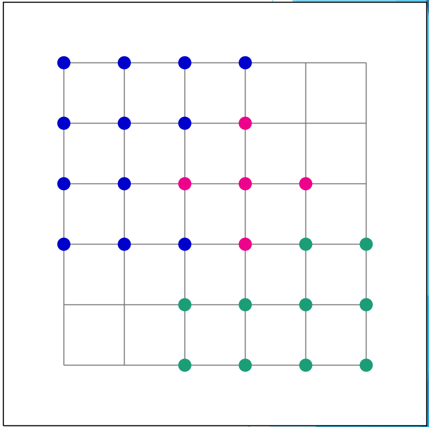


Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

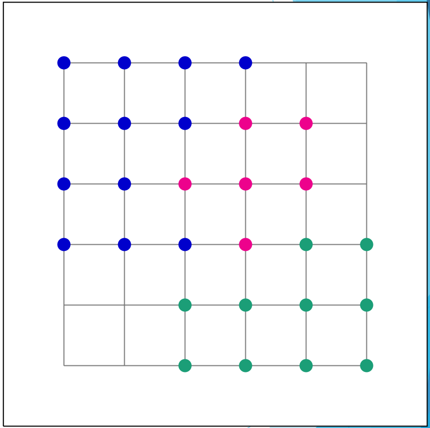


Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.



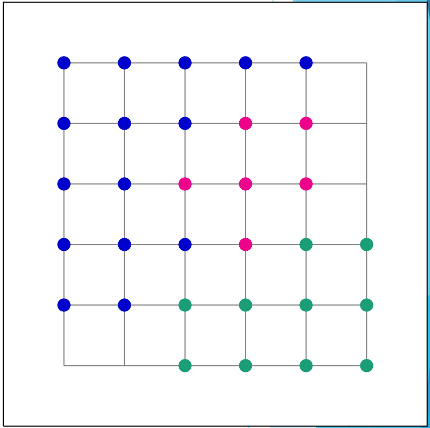
Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.



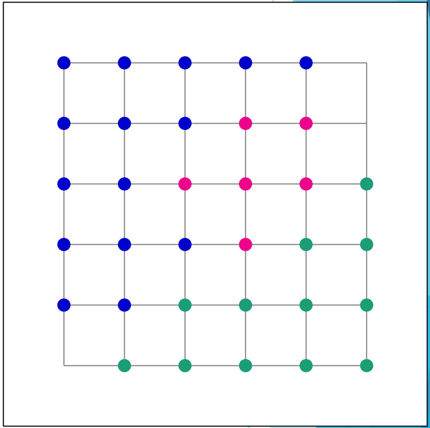
Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.



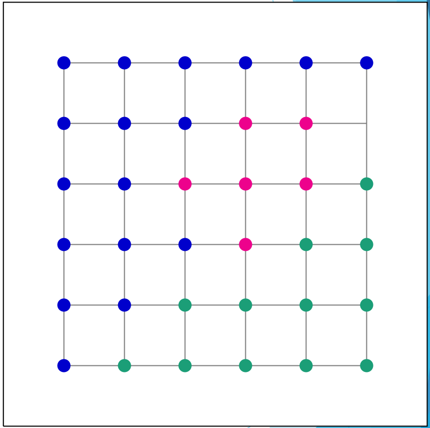
Topological object identification

Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.



Topological object identification

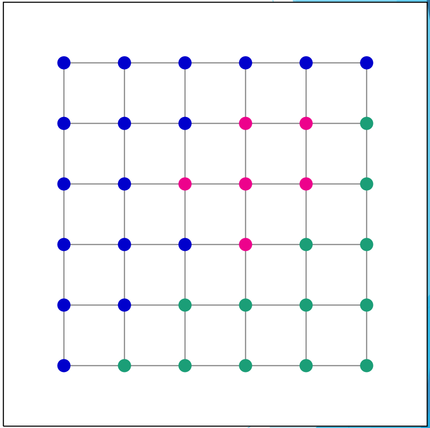
Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.

Filter any objects with radius ≤ 2.



Topological object identification

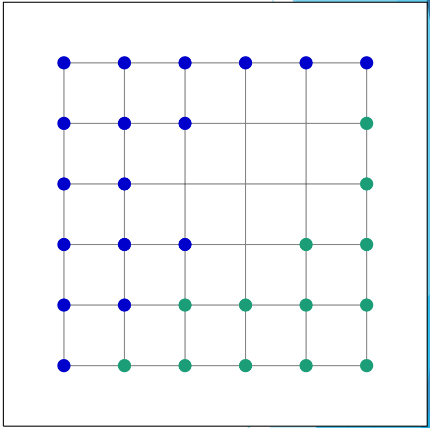
Start with extrema of $q(x)$.

Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.

Filter any objects with radius ≤ 2.



Topological object identification

Start with extrema of $q(x)$.

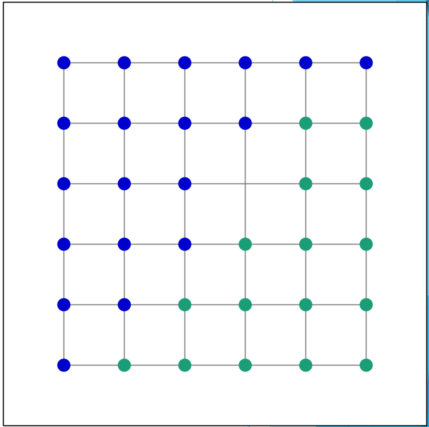
Grow objects in turn one step, monotonically in $q(x)$.

Grow objects in turn one hypercube at a time.

Repeat growth until lattice is filled.

Filter any objects with radius ≤ 2.

Repeat growth until lattice is filled.



Polyakov Loop and confinement

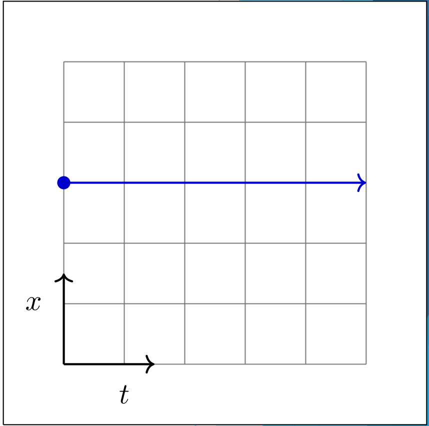
Polyakov loop is given by

$$L(\vec{x}) = \mathrm{Tr} \prod_t U_0(t, \vec{x}) = \rho(\vec{x}) e^{i\phi(\vec{x})}$$

World-line of static quark with free energy F_q,

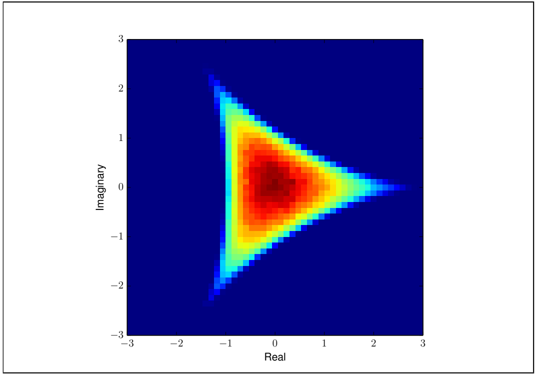
$$\langle L(\vec{x}) \rangle = \exp(-F_q N_t)$$

Confining phase $F_q \to \infty, \langle L(\vec{x}) \rangle \to 0$.
Deconfining phase F_q is finite, $\langle L(\vec{x}) \rangle \neq 0$.
$\langle L(\vec{x}) \rangle$ is an order parameter for confinement.

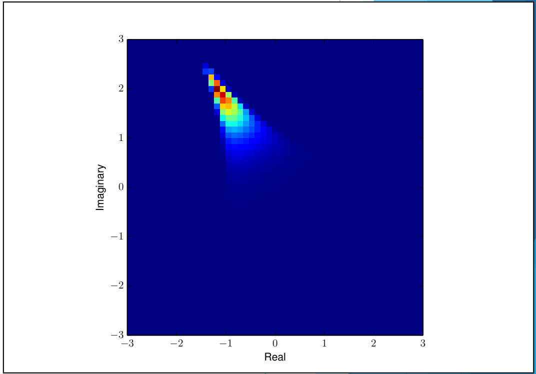


Distribution of $L(\vec{x})$

Below T_c



Above T_c



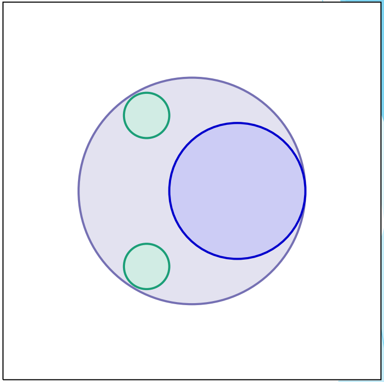
Instanton-dyon model

Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

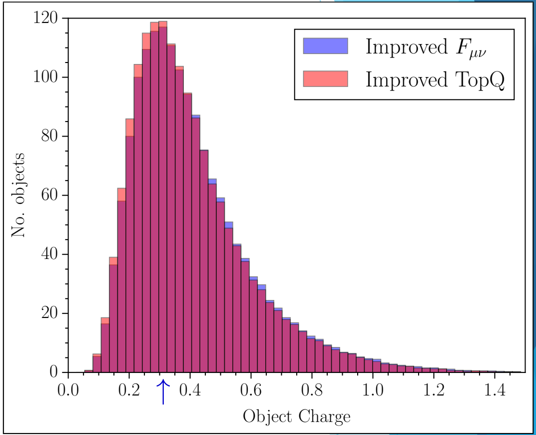
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
64	≈ 0	0.0041	0.3322(1)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

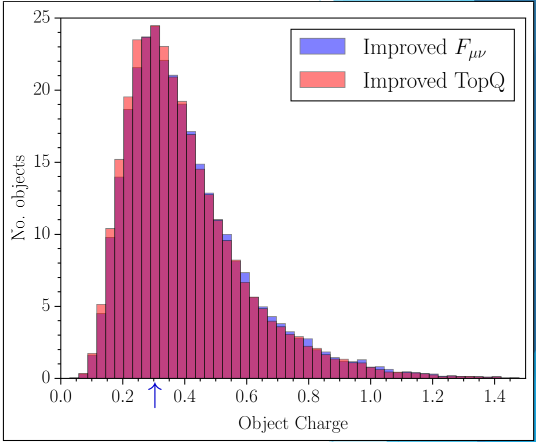
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
12	0.609	0.0189	0.3282(3)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

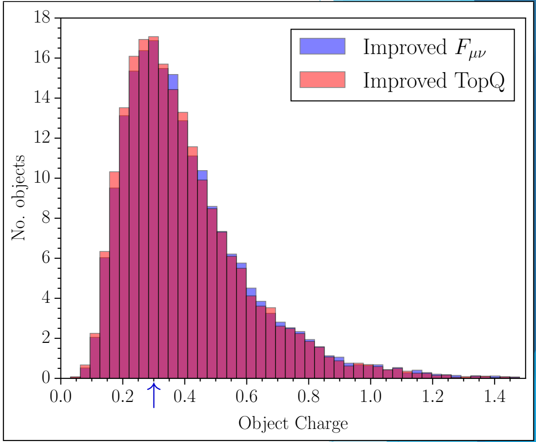
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
8	0.913	0.0327	0.3245(5)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

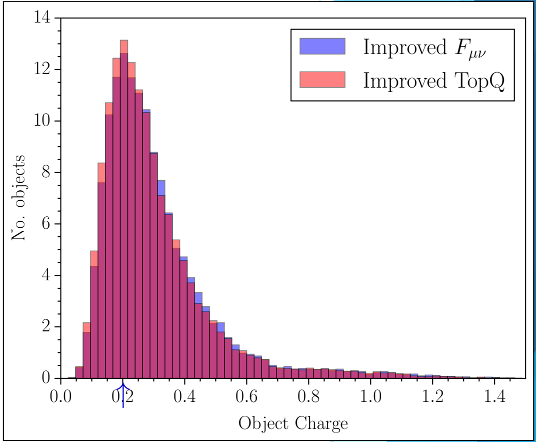
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
6	1.218	0.5263	0.203(4)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

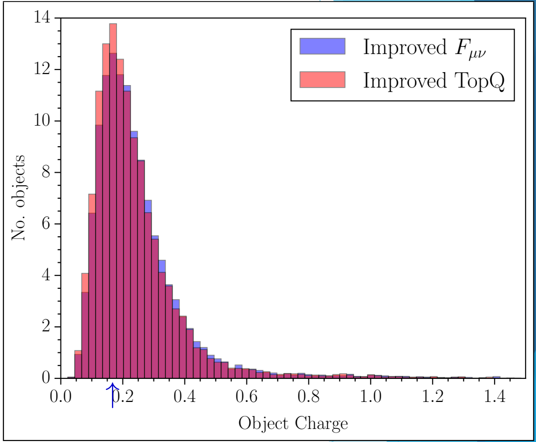
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
5	1.461	0.6450	0.173(5)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Instanton-dyon model

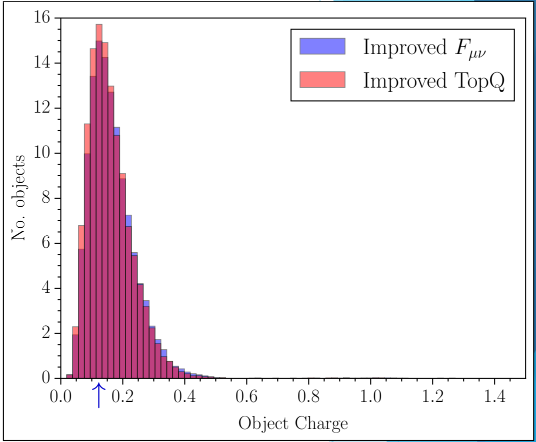
Polyakov loop is related to the holonomy parameter ν via

$$\langle P \rangle = \frac{1}{3}\langle \mathrm{Tr}\, P \rangle = \frac{2}{3}\cos(2\pi\nu) + \frac{1}{3}$$

This predicts the charges of the $N_c = 3$ dyons in $SU(3)$ as

$$Q_1 = Q_2 = \nu, \quad Q_3 = 1 - 2\nu.$$

N_t	T/T_c	$\frac{1}{3}\langle \mathrm{Tr}\, P \rangle$	ν
4	1.827	0.8367	0.1138(2)



D. DeMartini, E. Shuryak, Phys. Rev. D 104 (2021) 5, 054010

Summary

> *It would be tempting to abolish the SU(3) color theory for hadrons altogether, replacing it by a Z(3) theory on a Euclidean lattice and taking the continuum limit close to the critical point. This is presumably not justifiable.*

"On the phase transition towards permanent quark confinement"
G. 't Hooft, 1978

Summary

> *It would be tempting to abolish the SU(3) color theory for hadrons altogether, replacing it by a Z(3) theory on a Euclidean lattice and taking the continuum limit close to the critical point. This is presumably not justifiable.*

"On the phase transition towards permanent quark confinement"
G. 't Hooft, 1978

Centre vortices appear to contain all information necessary to reproduce confinement and DχSB in $\mathrm{SU}(3)$ gauge theory with dynamical fermions (at zero temperature).

Summary

> *It would be tempting to abolish the SU(3) color theory for hadrons altogether, replacing it by a Z(3) theory on a Euclidean lattice and taking the continuum limit close to the critical point. This is presumably not justifiable.*

"On the phase transition towards permanent quark confinement"
G. 't Hooft, 1978

Centre vortices appear to contain all information necessary to reproduce confinement and DχSB in $\mathrm{SU}(3)$ gauge theory with dynamical fermions (at zero temperature).

Percolating centre vortex cluster melts at high temperature, resulting in a change in geometry.

Summary

> *It would be tempting to abolish the SU(3) color theory for hadrons altogether, replacing it by a Z(3) theory on a Euclidean lattice and taking the continuum limit close to the critical point. This is presumably not justifiable.*

"On the phase transition towards permanent quark confinement"
G. 't Hooft, 1978

Centre vortices appear to contain all information necessary to reproduce confinement and DχSB in $\mathrm{SU}(3)$ gauge theory with dynamical fermions (at zero temperature).

Percolating centre vortex cluster melts at high temperature, resulting in a change in geometry.

Topological structure distribution at finite temperature appears to be consistent with an instanton-dyon model.

Thank you!