QCD with fundamental and adjoint fermions

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Motivation

- Strongly coupled theories are hard! Confinement, chiral symmetry breaking.
- Exploring the territory of strong dynamics is desirable.
- QCD(F) and SYM (1 adj Weyl) are the most understood: chiral perturbation theory, $\mathbb{R}^3 \times \mathbb{S}^1$ setup, holomorphy, 't Hooft anomalies.
- Recently, new tools: **generalized symmetries and their 't Hooft anomalies**.
- **QCD(adj/F) interpolates** between QCD(F) and SYM.

Outline

- The new anomalies of the theory.
- Matching anomalies in the IR.
- IR phase structure.

Symmetries and anomalies

- Theory: $SU(N)$ $\overbrace{\text{YM}+\underbrace{\lambda}_{\text{massless}}}^{\text{SYM}} + \underbrace{\psi + \tilde{\psi}}_{m \ll \Lambda}$

	$SU(N)$	$U(1)_q$	$\mathbb{Z}_{2N}^{\chi}$
ψ	$\square$	$+1$	0
$\tilde{\psi}$	$\overline{\square}$	-1	0
λ	adj	0	1

Symmetries and anomalies

- Faithful Global symmetry is

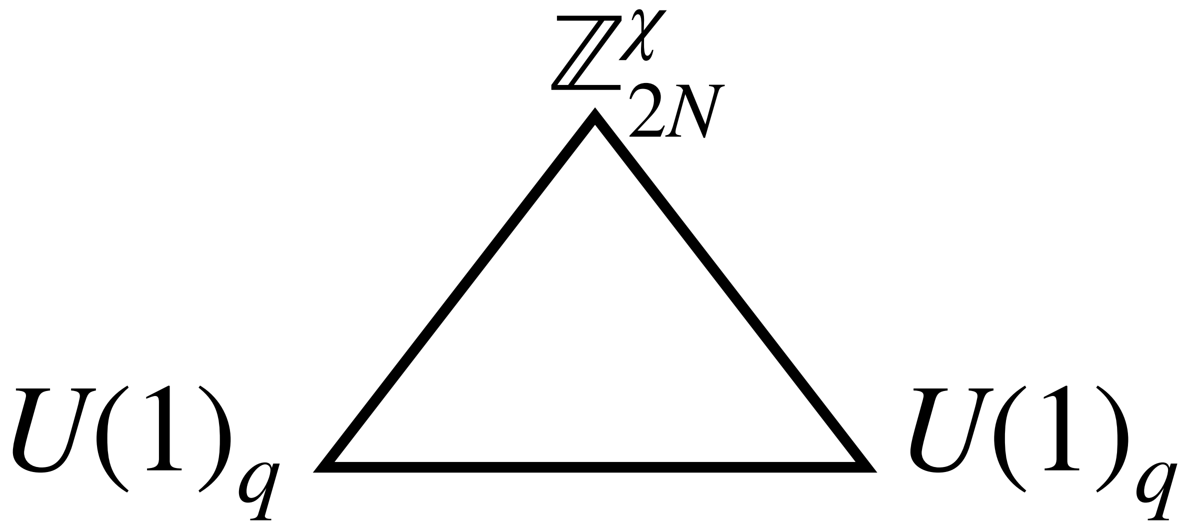
$$G^{g} = \frac{U(1)_q}{\mathbb{Z}_N} \times \mathbb{Z}^{\chi}_{2N} \equiv \underbrace{U(1)_B \times}_{\text{smallest baryon has charge } +1} \mathbb{Z}^{\chi}_{2N} \,.$$

- **Notice**: unlike SYM, there is **no** $\mathbb{Z}_N^{(1)}$ 1-form symmetry.
- There are two traditional 't Hooft anomalies:
- $\mathbb{Z}^{\chi}_{2N} - [\text{grav}] \,:\, \mathscr{Z} \longrightarrow \mathscr{Z} \exp[-i\frac{2\pi}{2N} 2(N^2-1)] \,.$
- $[\mathbb{Z}_N]^3$ (Ibanez-Ross) anomaly $= N^2 - 1 \bmod 2N$.

Symmetries and anomalies

- Yet, there is a new mixed anomaly: $\mathbb{Z}_{2N}^{\chi} - U(1)_B$.
- This is surprising!

Gaiotto, Kapustin, Komargodski, Seiberg 2017
Tanizaki 2018
M.A., Poppitz 2019
M.A., Lohitsiri, Sulejmanpasic 2023



$= 0$

- The anomaly is tied to the global structure of $\dfrac{U(1)_q}{\mathbb{Z}_N} \times \mathbb{Z}_{2N}^{\chi}$.

Symmetries and anomalies

- UV or IR (RG invariant) are two ways to see the anomaly.
- UV: put the theory on $\mathbb{T}^4$ and turn on fluxes in the centers of $SU(N)$ and $U(1)_q$.
- Cocyle conditions

$$\psi: \quad \underbrace{e^{i\frac{2\pi}{N}}}_{\mathbb{Z}_N \subset SU(N)} \times \underbrace{e^{-i\frac{2\pi}{N}}}_{U(1)_q} = 1\,, \quad \tilde{\psi}: \quad \underbrace{e^{-i\frac{2\pi}{N}}}_{\mathbb{Z}_N \subset SU(N)} \times \underbrace{e^{i\frac{2\pi}{N}}}_{U(1)_q} = 1\,, \quad \lambda: \quad \underbrace{e^{iN\frac{2\pi}{N}}}_{\mathbb{Z}_N \subset SU(N)} = 1\,.$$

- Topological charges: $Q_c = -\dfrac{1}{N}\,, Q_u = \dfrac{1}{N^2}\,.$

Symmetries and anomalies

- Under the action of $\mathbb{Z}^{\chi}_{2N}$: $\quad \mathcal{Z} \longrightarrow \mathcal{Z} \exp\left[i\frac{2\pi}{2N} \underbrace{(2N)}_{T_{adj}} \underbrace{\frac{-1}{N}}_{Q_c} \right] = e^{-i\frac{2\pi}{N}}\mathcal{Z}$.
- We can show equiv. $\mathbb{Z}^{\chi}_{2N}$: $\quad \mathcal{Z} \longrightarrow \mathcal{Z} \exp\left[-i\frac{2\pi}{N} \underbrace{\int \frac{F_B \wedge F_B}{8\pi^2}}_{\in\mathbb{Z}} \right]$.
- The phase $e^{-i\frac{2\pi}{N}}$ is the $\mathbb{Z}^{\chi}_{2N} - U(1)_B$ anomaly.

Symmetries and anomalies

- There is another new anomaly.
- $\mathbb{Z}_2^F \subset U(1)_B \times \mathbb{Z}_{2N}^\chi : (\lambda, \psi, \tilde{\psi}) \to -(\lambda, \psi, \tilde{\psi})$.
- We can put the theory on a non-spin spacetime: $\text{Spin}-\mathbb{Z}_{2N}^\chi = (\text{Spin}(4) \times \mathbb{Z}_{2N}^\chi)/\mathbb{Z}_2$
- $\Omega_5^{\text{Spin}-\mathbb{Z}_{2N}^\chi} \neq 0$ ➡ nonperturbative anomaly of $\mathbb{Z}_{2N}^\chi$. Hsieh, 2018; M.A., Lohitsiri, Sulejmanpasic 2023

Symmetries and anomalies

- In the **massless** ψ limit, $\mathbb{Z}_{2N}^{\chi}$ is enhanced to $U(1)_\chi$

	$SU(N)$	$U(1)_q$	$U(1)_\chi$
ψ	$\square$	$+1$	$-N$
$\tilde{\psi}$	$\overline{\square}$	-1	$-N$
λ	adj	0	$+1$

Symmetries and anomalies

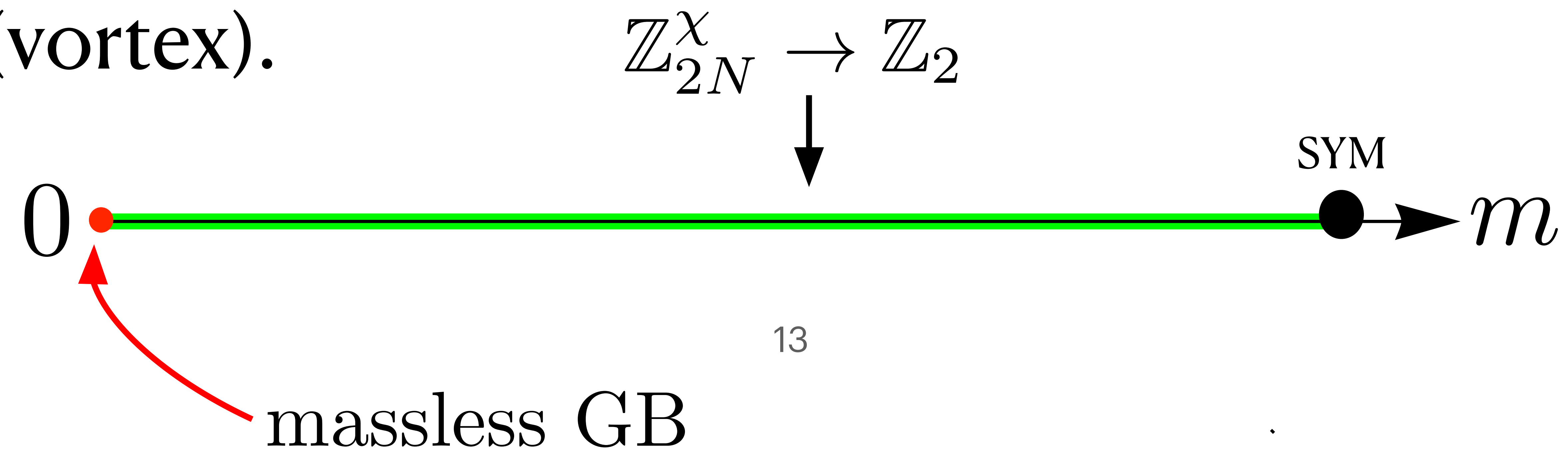
- Traditional 't Hooft anomalies:
  $[U(1)_\chi]^3 = -2N^4 + N^2 - 1\,, \quad U(1)_\chi[U(1)_q]^2 = -2N^2\,, \quad U(1)_\chi - [\text{grav}] = 2(N^2+1)$
- A new anomaly between $\underbrace{U(1)_\chi - [U(1)_B] = -2}_{U(1)_B = U(1)_q/N}$ (turning on fractional fluxes).

IR theory

- 't Hooft anomalies ➡ the IR is not trivially gapped: massless excitations and/or multiple vacua.
- **CFT** (unbroken symmetries) is **unlikely** as we are far from the conformal window.
- **Composite** massless fermions (baryons) could match anomalies (unbroken symmetries).
- The Weingarten theorem **excludes composites**: baryons are heavier than mesons in vector-like theories.

IR theory

- A natural way to match anomalies is by **breaking** $\mathbb{Z}_{2N}^{\chi}$ or $U(1)_{\chi}$. $U(1)_B$ is intact (Vafa-Witten theorem).
- **Massive** case **order parameter** $\langle \mathrm{tr}\lambda\lambda \rangle \neq 0$: $\mathbb{Z}_{2N}^{\chi} \to \mathbb{Z}_2^F$. There are N vacua and DWs.
- In the **massless** case, we may try $\langle \psi\tilde{\psi} \rangle \neq 0$: $U(1)_{\chi} \longrightarrow \mathbb{Z}_{2N}$, which is anomalous.
- The **correct order parameter** is $\langle \mathrm{tr}\lambda\lambda \rangle \neq 0$: $U(1)_{\chi} \to \mathbb{Z}_2^F$. There is a Goldstone boson (vortex).

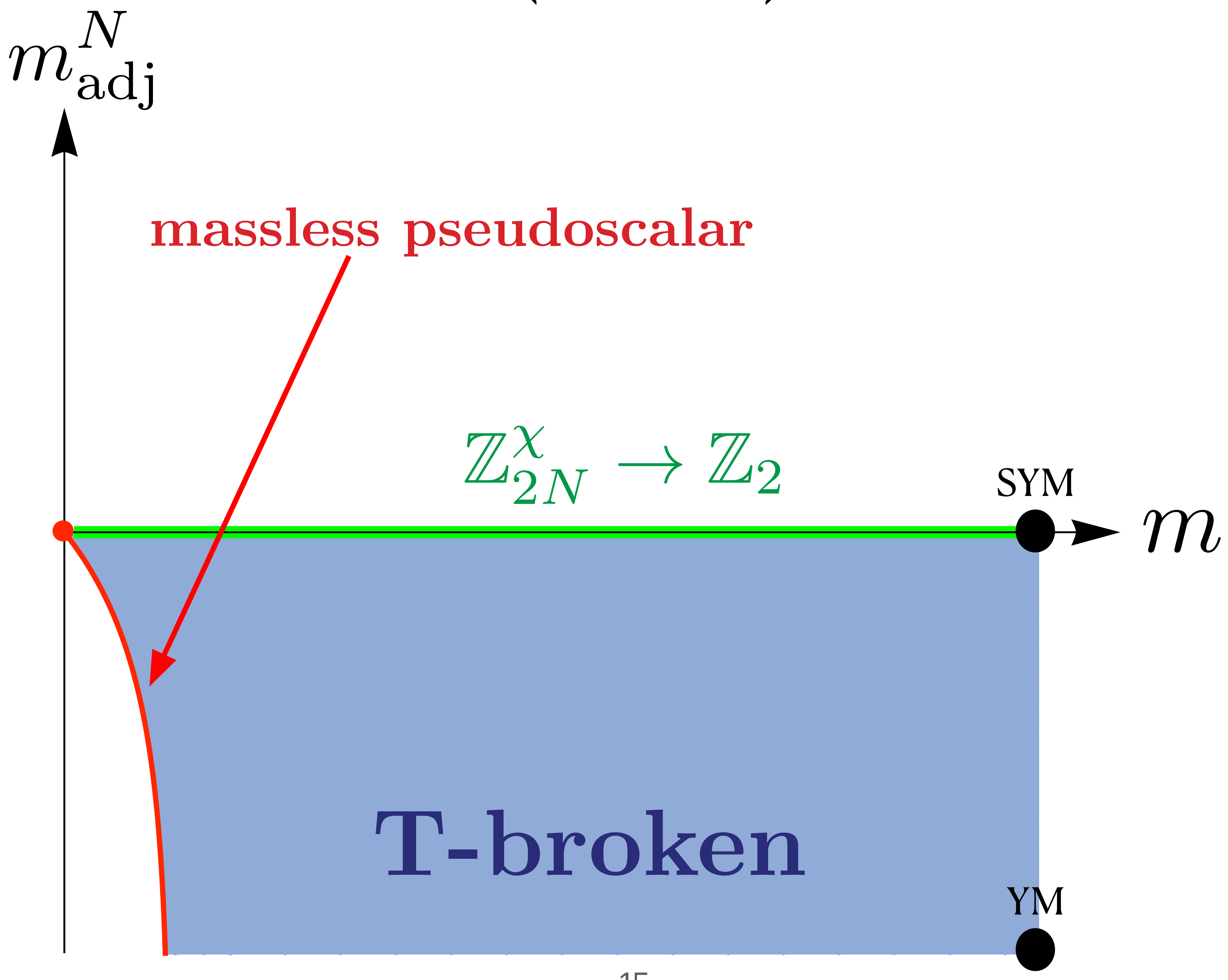


IR theory: adding adjoint mass

- Write down IR EFT.
- Start with massless adj and F: $\mathscr{L} = \Lambda^2(\partial\varphi)^2$.
- Add mass to F: $\mathscr{L} = \Lambda^2(\partial\varphi)^2 - \Lambda^3 m \underbrace{\cos(N\varphi)}_{U(1)_\chi \to \mathbb{Z}_N^\chi}$.
- Add mass to F and adj: $\mathscr{L} = \Lambda^2(\partial\varphi)^2 \underbrace{- \Lambda^3 m \cos(N\varphi) - \Lambda^3 m_{adj} \cos\left(\varphi + \frac{\theta}{N}\right)}_{U(1)_\chi:\quad q(\lambda)=1,\quad q(\psi)=-N}$.

IR bulk phase diagram

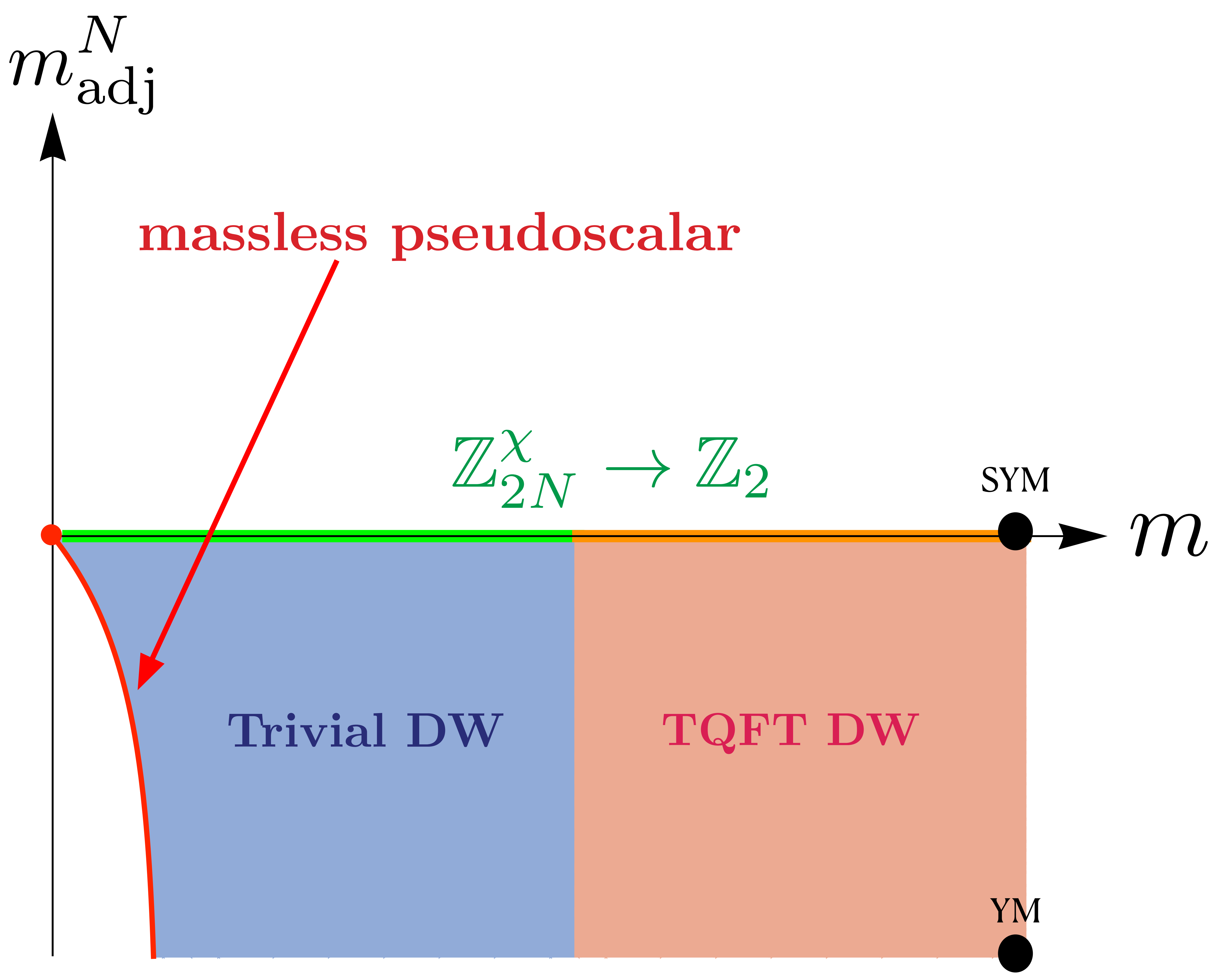
- $V(\varphi) = \Lambda^3 m \cos(N\varphi) + \Lambda^3 m_{adj} \cos\left(\varphi + \frac{\theta}{N}\right)$



Domain walls

- To predict what happens on the DW, we analyze $\mathscr{L} = \Lambda^2(\partial\varphi)^2 - \Lambda^3 m\cos(N\varphi) - \Lambda^3 m_{adj}\cos\left(\varphi + \frac{\theta}{N}\right)$ for large excursions of φ.
- For $|m|, |m_{adj}| \ll \Lambda, \quad \Delta V \ll \Lambda^4$, EFT is robust, and we don't expect anything happens on the wall.
- For $|m| \sim \Lambda$ or $m_{adj} \sim \Lambda$, EFT breaks down since $\Delta V \sim \Lambda^4$. We expect a transition on the DW.

Domain walls



Conclusion

- QCD(F/adj) fits nicely between QCD(F) and SYM.
- Anomalies are very restrictive in the IR.
- Power of EFT to predict the phase diagrams.