Propagation of gauge fields in hot and dense plasmas at higher orders

Andreas Ekstedt

DESY
II. Institut für Theoretische Physik

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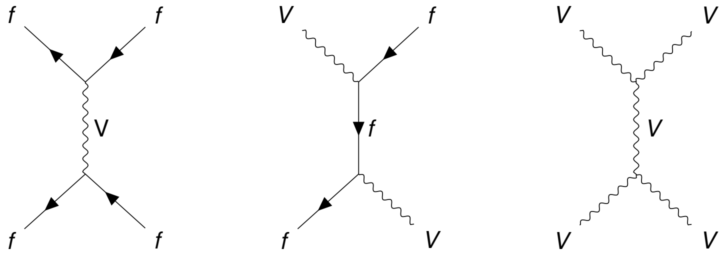
Where do hard-thermal loops show up?

Describes the effective propagation of *soft* fields $k^0, \vec{k} \sim gT$

Soft momentum transfers dominate collisions:

$$\partial_t f + \vec{v}\cdot\vec{\nabla} f + \vec{F}\cdot\vec{\nabla}^p f = -C[f] \sim \sigma$$

Typically $f \sim f_{\text{eq}} + \delta C^{-1}$source



Cross-section set by the screening: $\sigma \sim g^4 \int d\hat{t} \frac{1}{\hat{t} - \Sigma_{\text{HTL}}} \sim g^4 \log \Sigma_{\text{HTL}}$

Equilibrium distributions depend on **asymptotic** masses: $E^2 \approx \vec{p}^2 + m_0^2 + m_\infty^2$.

Hard thermal loops—EFT for soft modes $k^0, \vec{k} \sim gT$

Integrate out hard modes $E \sim T$ like for a normal EFT

Gives an effective (**non-local**) self-energy $\Pi^{\mu\nu}(K)$

Match to this effective theory at some scale $\mu \sim T$

Coefficients in the EFT are independent of μ

There are also (sub-leading) dimensional-reduction like contributions

$\rightarrow$ Effective gauge/scalar couplings etc: $g^2_{\mathrm{3d}}(\mu), \ldots$

A brief overview

Diagrammatic approaches (imaginary time):

Fermion self-energy	Weldon 1982
Vector self-energy and higher-point functions	Braaten, Pisarski 1989
Gluon damping rate	Braaten, Pisarski 1990
Soft contributions	Rebhan et.al 1994
Two-loop QCD pressure	Andersen et.al 2002

Kinetic-theory:

Hard thermal loops	Blaizot, Iancu 1993; Frenkel, Taylor 1992
Transport coefficients	Jeon, Yaffe 1996
Colour conductivity	Bödeker 1999; Arnold et.al 1998

Recent results from diagrammatics

- Power corrections — Manuel et.al 2017
- Two-loop QED at finite temperature — Carignano et.al 2019
- Fermion-mass corrections — Comadran, Manuel 2021
- Two-loop QCD at finite density — Säppi, Seppänen et.al 2023
- Dense N^3LO pressure — Kneur, Fernandez; Seppänen,Säppi et.al

Advantages with using diagrams at two-loops:

- Can crosscheck against $k^0 \to 0$ results diagram-by-diagram
- General gauge-fixing is (in principle) straightforward

Cons :

- Way too many diagrams $\to$ Automatization
- The physics is not transparent
- Higher-point functions are not fun to calculate

Recent results from kinetics

"NLO" shear viscosity in QCD — Ghiglieri et.al 2018

Power corrections from transport equations — Manuel, Carignano 2021

Shear Viscosity in Dense QCD — Danhoni, Moore 2022

Two-loop vector self energies — Ekstedt, 2023

Advantages with using kinetic theory:

Very compact result $\rightarrow$ Pen-and-paper level

The physics is transparent

Higher-point functions come for **free**

Cons :

Quite annoying in general gauges $\rightarrow$ Easy in Feynman gauge

No partial crosschecks with $k^0 \rightarrow 0$ results

Hard-thermal loop a'la Blaizot & Iancu

Long-distance fields behave classically:

$$\partial_\mu F^{\nu\mu} = \langle j_R^\nu \rangle \approx \# e \int_p v^\nu \left[f^+(p) - f^-(p) \right] \equiv \Pi^{\mu\nu} A_\mu$$

Fast $\vec{p} \sim T$ modes behave like particles ($f^\pm = f_{\text{eq}} + \delta f^\pm$):

$$\partial_t f^\pm + \vec{v} \cdot \vec{\nabla} f^\pm \mp e(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{\nabla}^p f = 0 + \mathscr{O}(g^4)$$

One finds

$$\langle j_R^\nu \rangle \sim -e^2 T^2 \int \frac{d\Omega_v}{4\pi} \frac{V^\nu \vec{v} \cdot \vec{E}}{V \cdot K}$$ In non-abelian theories $V \cdot K \to iV \cdot D[K]^{ab}$

The physical picture:

A hard particle interacts with the electric field and travels for

$$t \sim \left[(p^0 + k^0) - \left| \vec{p} + \vec{k} \right| \right]^{-1} \sim (-k^0 + \vec{v} \cdot \vec{k})^{-1} \equiv (V \cdot K)^{-1}$$

$\rightarrow$Its change in velocity is $T\delta v \sim t(e\vec{E})$

$\rightarrow$The **generated** current is $\vec{j} \sim en\delta v \sim t\left(T^2 e^2 \vec{E}\right)$

Using kinetic theory at two-loops:

1, Consider self-energy corrections to the Boltzmann equation (not **pleasant**)

2, Treat it like any other background-field calculation Abbott 1981

For the effective potential: $\Delta = \frac{1}{K^2+M^2+\lambda\phi^2(x)} \to V_{\text{eff}}(\phi(x)) \sim m_{\text{eff}}^2\phi^2(x) + \ldots$

For hard thermal loops: $\Delta^{rr}(P) \sim 2\pi\delta(P^2)f(\vec{p};\vec{E}(x)) \to \langle j_R^{\nu}\rangle \sim \Pi_{\text{HTL}}^{\mu\nu}A_{\mu} + \ldots$

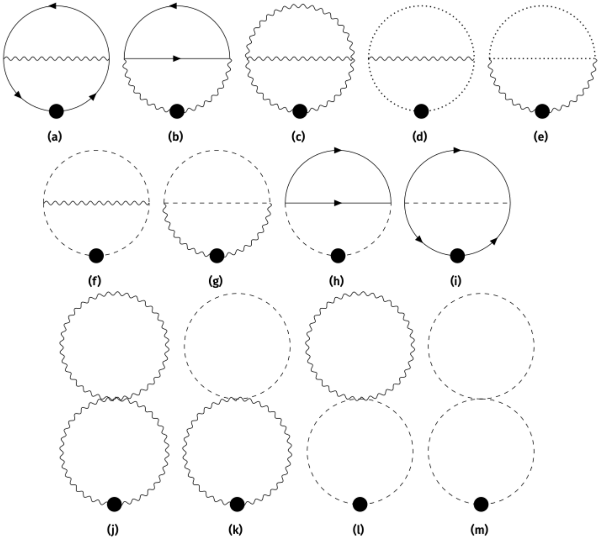
In practice:

Calculate the two-loop contribution to $\langle j_R^{\nu}\rangle$

Use resummed distribution functions in all propagators

Identify the self-energy $\langle j_R^{\nu}\rangle_{\text{2-loop}} = \Pi_{\text{2-loop}}^{\mu\nu}A_{\mu} + \ldots$

Two-loop diagrams after all?



Only need 1-point correlators

Most terms cancel for a given diagram

A few lines with pen-and-paper

Same terms always appear

Many diagrams are zero in Feynman gauge

What goes on physically?

In QED:

$$\Pi^{\mu\nu}_{\text{1-loop}} = -\frac{e^2 T^2}{3} \int \frac{d\Omega_v}{4\pi} \left[k^0 \frac{V^\mu V^\nu}{V\cdot K} + \delta^\mu_0 \delta^\nu_0 \right], \quad \vec{v} = \frac{\vec{p}}{p},$$

$$\Pi^{\mu\nu}_{\text{2-loop}} = -\frac{e^4 T^2}{8\pi^2} \int \frac{d\Omega_v}{4\pi} \left\{ V^\mu V^\nu \left[\frac{(k^0)^2}{(V\cdot K)^2} - \frac{2k^0}{V\cdot K} \right] + \left[V^\mu \delta^\nu_0 + \delta^\mu_0 V^\nu \right] \frac{k^0}{V\cdot K} - \delta^\mu_0 \delta^\nu_0 \right\}$$

Hard particles pick up thermal masses $m^2_\infty \sim e^2 T^2$, so they move slower:

$V\cdot K = -k^0 + \vec{v}\cdot\vec{k} \to -k^0 + \vec{p}\cdot\vec{k}\left(\vec{p}^2 + m^2_\infty\right)^{-1/2}$

Essentially $\Pi^{\mu\nu}_{\text{2-loop}} = m^2_\infty \frac{d}{dm^2_\infty} \Pi^{\mu\nu}_{\text{1-loop}}\Big|_{m^2_\infty = 0}$

The result can be written down for a generic theory:

$$\mathscr{L}_{\text{int}} = -\frac{1}{4!}\lambda^{ijkm}R_iR_jR_kR_m - \frac{1}{2}(Y^{iIJ}R_i\psi_I\psi_J + h.c) + g^{a,I}_{J}A^a_\mu\psi^{\dagger,J}\overline{\sigma}^\mu\psi_I - g^a_{jk}A^a_\mu R_j\partial^\mu R_k - \frac{1}{2}g^a_{jn}g^b_{kn}A^a_\mu A^{\mu,b}R_jR_k - g^{abc}A^{\mu,a}A^{\nu,b}\partial_\mu A^c_\nu - \frac{1}{4}g^{abe}g^{cde}A^{\mu a}A^{\nu b}A^c_\mu A^d_\nu + g^{abc}A^a_\mu\eta^b\partial^\mu\overline{\eta}^c.$$

The result is some combination of the Lorentz structures:

$$\Pi^{\mu\nu}_1 = \int\frac{d\Omega_v}{4\pi}\left(k^0\frac{V^\mu V^\nu}{V\cdot K} + \delta^\mu_0\delta^\nu_0\right),$$

$$\Pi^{\mu\nu}_2 = \int\frac{d\Omega_v}{4\pi}\left\{V^\mu V^\nu\left[\frac{(k^0)^2}{(V\cdot K)^2} - \frac{2k^0}{V\cdot K}\right] + \left[V^\mu\delta^\nu_0 + \delta^\mu_0 V^\nu\right]\frac{k^0}{V\cdot K} - \delta^\mu_0\delta^\nu_0\right\},$$

$$\Pi^{\mu\nu}_3 = \int\frac{d\Omega_v}{4\pi}\left[\frac{k^0V^\mu}{V\cdot K} + \delta^\mu_0\right]\int\frac{d\Omega_w}{4\pi}\left[\frac{k^0W^\nu}{W\cdot K} + \delta^\nu_0\right]$$ Only at finite chemical potential

Asymptotic masses at higher orders (hard-scale contribution)

$$\mathrm{Im}\Delta^T(K) = 2\pi\rho^{\mathrm{res}}(k)\delta(k^0-\omega(k)) + \rho^{\mathrm{disc}}(k)\theta(k-k^0)$$

Problems at two-loops: $\omega^2(k) = \omega^2(k)_{\mathrm{LO}} + \omega^2(k)_{\mathrm{NLO}}$ Seppänen, Säppi et.al 2022

$$\omega^2(k)_{\mathrm{LO}} \sim \vec{k}^2 + g^2T^2, \quad \rho^{\mathrm{res}}_{\mathrm{LO}} \sim \frac{1}{2k}$$

$$\omega^2(k)_{\mathrm{NLO}} \sim g^4T^2\left(1+\log k^2\right), \quad \rho^{\mathrm{res}}_{\mathrm{NLO}} \sim \frac{\log k}{2k}$$

The expansion breaks down

Things go wrong when $K^2 \to 0$: Lebedev,Smilga 1990

$$\left(-k^0 + \vec{p}\cdot\vec{k}\left(\vec{p}^2+m_\infty^2\right)^{-1/2}\right)^{-1} = \underbrace{\frac{1}{V\cdot K}}_{\text{1-loop}} + \underbrace{\frac{km_\infty^2}{2p^2(V\cdot K)^2}}_{\text{2-loop}} + \ldots$$

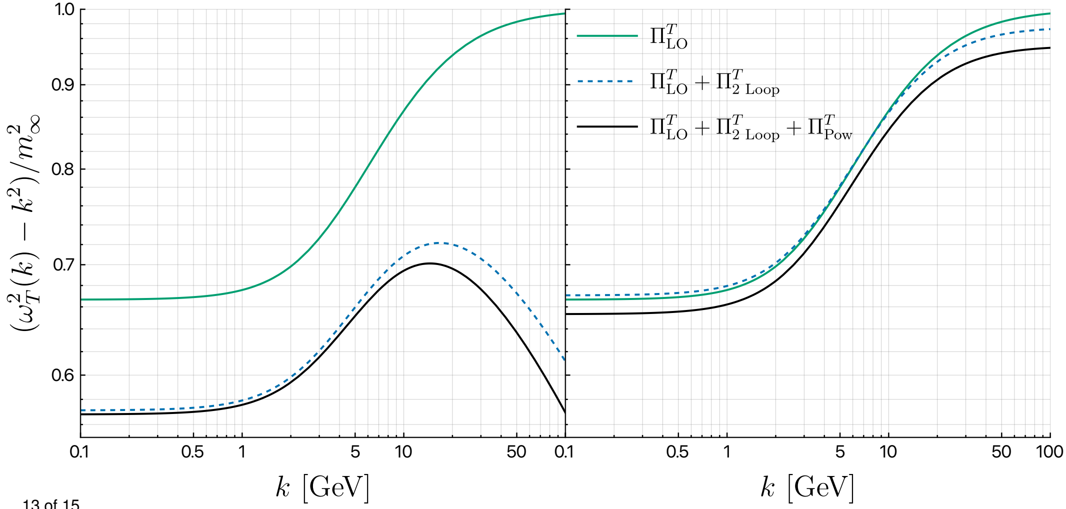
In a strict loop expansion each order is more divergent as $V\cdot K \to 0$

The result is finite if we keep the full propagator **unexpanded**

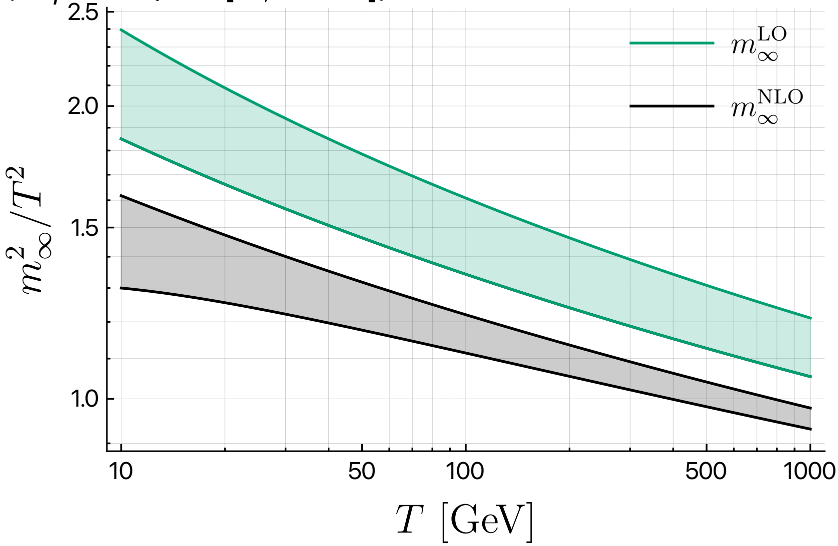
There are also soft contributions to m_∞^2 Caron-Huot 2008;Ghiglieri et.al 2021-2023

Unresummed (left) versus resummed (right)

QED with $\alpha = 0.1$; $T = 10$ GeV



How large are higher-order corrections for QCD?
($N_q = 6,\ \mu \in [T/2, 2T]$)



In summary

A lot of recent progress:

Power-corrections known

Vector two-loop self-energies known

Dense pressure known to (almost) N^3LO

Things to look forward to:

Two-loop self-energies for quarks

Impact of two-loop results on transport coefficients

Calculating the full NLO colour conductivity

Three-loop self-energies

Full 2-loop effective action—gauge/scalar quartics etc