Neutron Star Radial Oscillations with Delta Baryons

Ishfaq Ahmad Rather

Centre for Astrophysics and Gravitation-CENTRA,

Instituto Superior Tecnico-IST, University of Lisbon, Portugal

Collaborators: Kauan D. Marquez, Grigoris Panotopoulos, and Ilídio Lopes

19th International Conference on QCD in Extreme Conditions (XQCD 2023)
University of Coimbra

26 July 2023

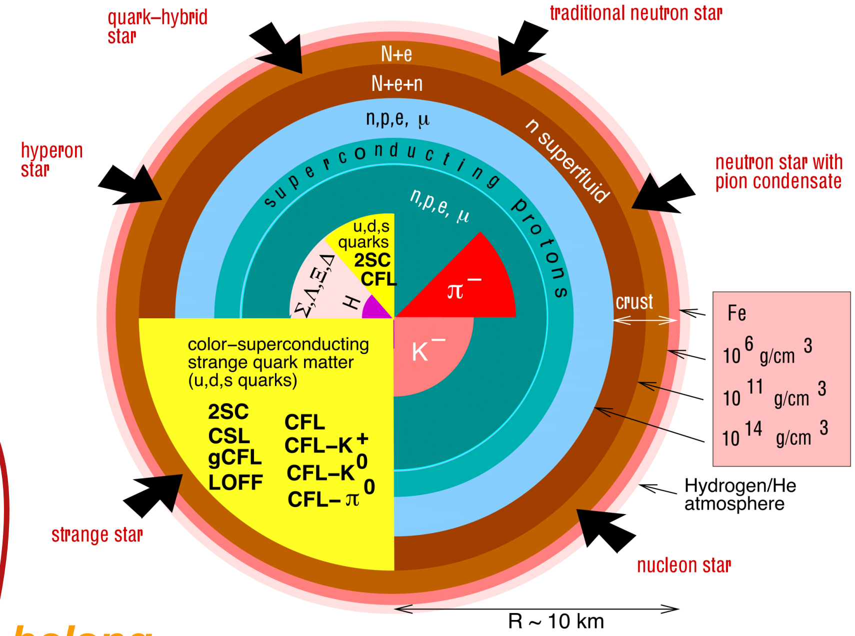
Contents

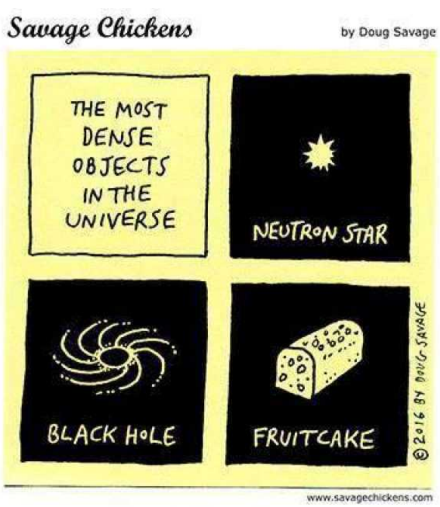
Table of Contents

What is a Neutron star?

- **For astronomers**: dim, small stars, visible as radio-pulsars, X-ray and γ-ray sources, or the compact binary companion of some other big star.
- **For nuclear physicists**: The largest and most neutron rich nucleus an in the Universe ($A \approx 1057$, $Z \approx 1056$).
- **For particle physicists**: A strong neutrino (and sterile neutrinos, axions ?) source at birth (SN 1987a). Probably the only place where deconfined quark matter can exist
- **For relativists**: Almost a black hole (≈ 3 Schwarzschild radii).
- **For plasma physicists**. Plasma under the most extreme conditions in a pulsar magnetosphere.
- **For everybody else**: surely you can find your favorite physics in a neutron star (superconductivity, magneto-elasticity ...)

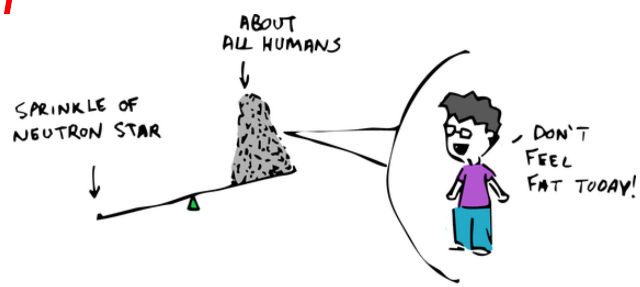
We belong Somewhere here

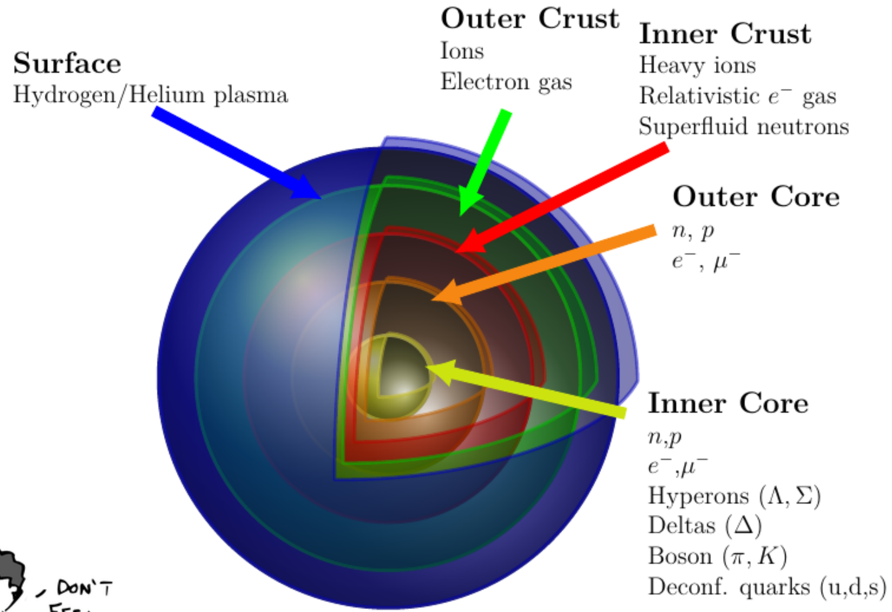




Neutron stars

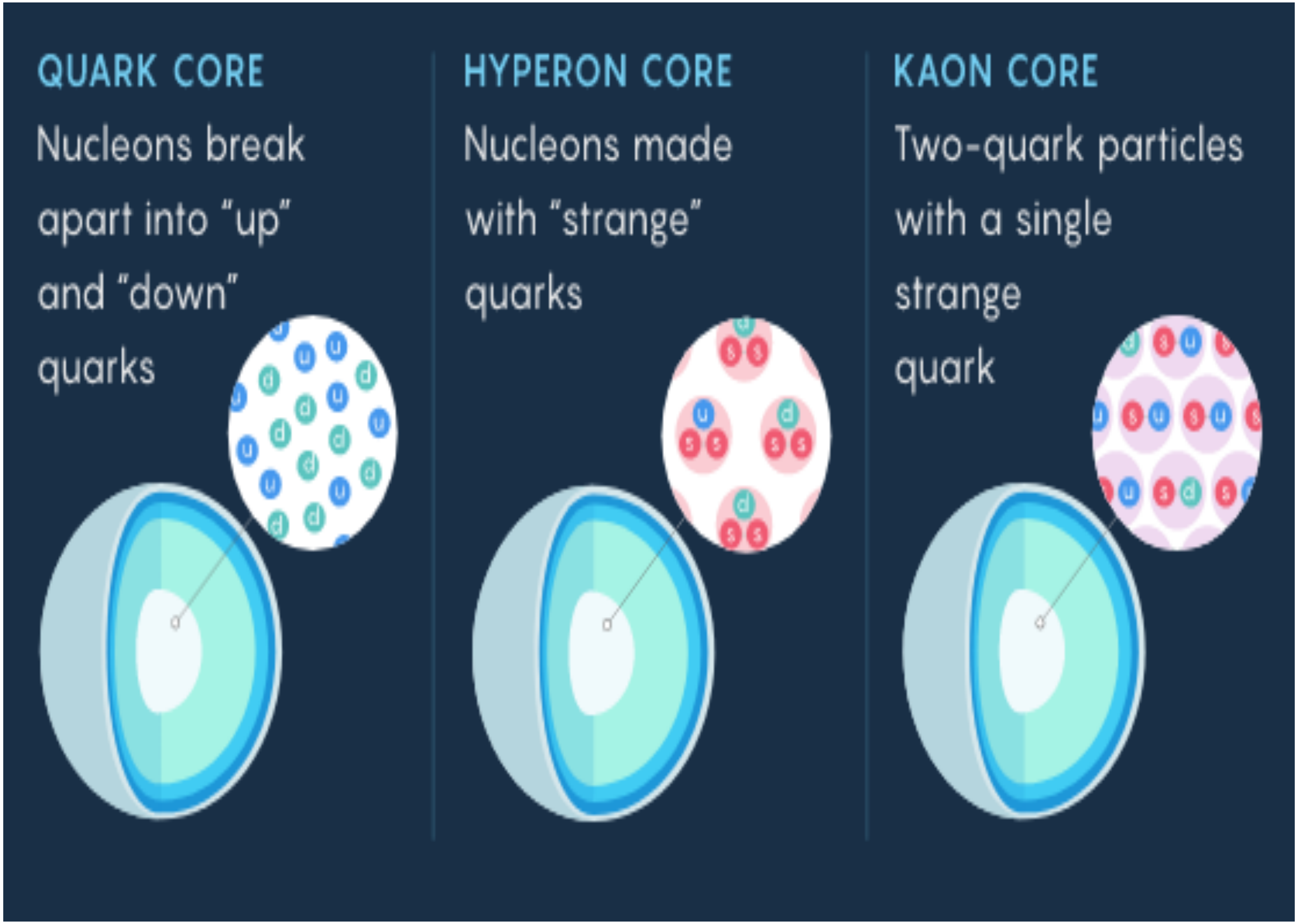
- Formed in: Type II, Ib or Ic SN (remnant of M ≈ 8-30 $M_\odot$)
- Mass: M ≈ 2 $M_\odot$ **(or even more?)**
- Radius: R ≈ 10-13 km
- Density: $\rho \approx 10^{14} - 10^{15}\ g/cm^3$
- Magnetic Field: $B \approx 10^{15} - 10^{18}\ G$





- NS has 5 major regions:
- The Atmosphere & The Envelope:shaping the emergent photon spectrum.
- The Crust:extending about 1-2 km, primarily contains nuclei.
- The Inner & Outer Cores: contains 99% of stars mass.

Exotic Matter



- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.

- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.
- It was found that the Δ baryons could be present only at densities $\approx 10\ \rho_0$ inside the NSs. N. K. Glendenning, Astrophys. J. 293, 470 (1985).

- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.
- It was found that the Δ baryons could be present only at densities $\approx 10\ \rho_0$ inside the NSs. N. K. Glendenning, Astrophys. J. 293, 470 (1985).
- With proper couplings between Δ baryons and nucleons, they might be present inside the NSs and that they could in fact make up a large fraction of the baryons in NS matter K. D. Marquez, *et al.*, PRC 106, 055801 (2022), L. L. Lopes *et al.*, PRD 107, 036011 (2023).

- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.
- It was found that the Δ baryons could be present only at densities $\approx 10\ \rho_0$ inside the NSs. N. K. Glendenning, Astrophys. J. 293, 470 (1985).
- With proper couplings between Δ baryons and nucleons, they might be present inside the NSs and that they could in fact make up a large fraction of the baryons in NS matter K. D. Marquez, *et al.*, PRC 106, 055801 (2022), L. L. Lopes *et al.*, PRD 107, 036011 (2023).
- Δ baryons can exist inside NSs at almost the same density range as the hyperons ($\approx$ 2-3 ρ_0)

$$m_N < m_\Delta(1232MeV) < m_\Xi.$$

- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.
- It was found that the Δ baryons could be present only at densities $\approx 10\ \rho_0$ inside the NSs. N. K. Glendenning, Astrophys. J. 293, 470 (1985).
- With proper couplings between Δ baryons and nucleons, they might be present inside the NSs and that they could in fact make up a large fraction of the baryons in NS matter K. D. Marquez, *et al.*, PRC 106, 055801 (2022), L. L. Lopes *et al.*, PRD 107, 036011 (2023).
- Δ baryons can exist inside NSs at almost the same density range as the hyperons ($\approx$ 2-3 ρ_0)

$$m_N < m_\Delta(1232MeV) < m_\Xi.$$

- In this work, The spin-3/2 baryons are described by the Rarita Schwinger Lagrangian density.

- Several exotic degrees of freedom such as hyperons, kaons, and delta baryons (Δ) are present in the NS core.
- It was found that the Δ baryons could be present only at densities $\approx 10\ \rho_0$ inside the NSs. N. K. Glendenning, Astrophys. J. 293, 470 (1985).
- With proper couplings between Δ baryons and nucleons, they might be present inside the NSs and that they could in fact make up a large fraction of the baryons in NS matter K. D. Marquez, *et al.*, PRC 106, 055801 (2022), L. L. Lopes *et al.*, PRD 107, 036011 (2023).
- Δ baryons can exist inside NSs at almost the same density range as the hyperons ($\approx$ 2-3 ρ_0)

$$m_N < m_\Delta(1232MeV) < m_\Xi.$$

- In this work, The spin-3/2 baryons are described by the Rarita Schwinger Lagrangian density.

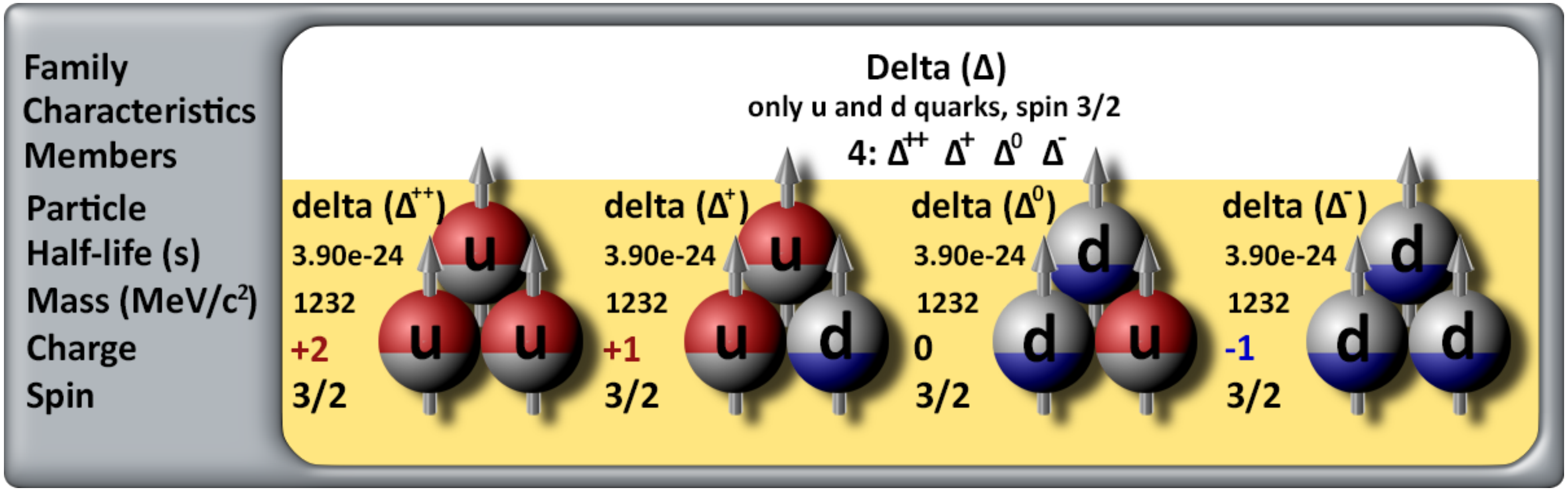


Table of Contents

- The hadronic matter composing the neutron stars is described within a relativistic mean-field approach with density-dependent couplings (DD-RMF).

- The hadronic matter composing the neutron stars is described within a relativistic mean-field approach with density-dependent couplings (DD-RMF).
- This type of model is shown to be very consistent in the description of nuclear matter experimental properties and also holds when astrophysical constraints are imposed.

- The hadronic matter composing the neutron stars is described within a relativistic mean-field approach with density-dependent couplings (DD-RMF).
- This type of model is shown to be very consistent in the description of nuclear matter experimental properties and also holds when astrophysical constraints are imposed.
-

Spin-1/2

$$
\begin{aligned}
\mathcal{L}_{\rm RMF} = & \sum_{b\in H} \bar{\psi}_b \Big[i\gamma^\mu \partial_\mu - \gamma^0 \left(g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b} I_{3b}\rho_{03} \right) - \left(m_b - g_{\sigma b}\sigma_0 \right) \Big] \psi_b \\
& - \frac{i}{2} \sum_{b\in\Delta} \bar{\psi}_{b\mu} \Big[\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\nu\partial_\rho - \gamma^0 \left(g_{\omega b}\omega_0 + g_{\rho b} I_{3b}\rho_{03} \right) - \left(m_b - g_{\sigma b}\sigma_0 \right) \varsigma^{\mu\lambda} \Big] \psi_{b\nu} \\
& + \sum_{\lambda} \bar{\psi}_\lambda \left(i\gamma^\mu\partial_\mu - m_\lambda \right) \psi_\lambda - \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2
\end{aligned}
\qquad (1)
$$

Spin-3/2

Leptons

Pure mesonic contribution

I. A. Rather et al., Phys. Rev. D 107, 123022 (2023).

- The density-dependent coupling constants are represented as:

$$g_i(n_B) = g_i(n_0)f_i(x)$$

$$g_{\rho b}(n_B) = g_{ib}(n_0)\exp\left[-a_\rho(\eta-1)\right],$$

For rho meson

$$f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}, i = \sigma, \omega.\phi$$

$$\varepsilon_B = \sum_b \frac{\gamma_b}{2\pi^2}\int_0^{k_{Fb}} dk k^2\sqrt{k^2+m_b^{*2}} + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_{F\lambda}} dk k^2\sqrt{k^2+m_\lambda^2}$$

$$+\frac{m_\sigma^2}{2}\sigma_0^2 + \frac{m_\omega^2}{2}\omega_0^2 + \frac{m_\phi^2}{2}\phi_0^2 + \frac{m_\rho^2}{2}\rho_{03}^2.$$

$$P = \sum_i \mu_i n_i - \epsilon + n_B\Sigma^r,$$

Rearrangement term

$$\Sigma^r = \sum_b\left[\frac{\partial g_{\omega b}}{\partial n_b}\omega_0 n_b + \frac{\partial g_{\rho b}}{\partial n_b}\rho_{03} I_{3b} n_b + \frac{\partial g_{\phi b}}{\partial n_b}\phi_0 n_b - \frac{\partial g_{\sigma b}}{\partial n_b}\sigma_0 n_b^s\right]$$

S. Typel *et al.*, Nucl. Phys. A 656, 331 (1999).

DD-RMF Parameter set

Table: DDME2 parameters (top) and its predictions to the nuclear matter at saturation density (bottom) G. A. Lalazissis, *et al.*, Phys. Rev. C 71, 024312 (2005).

i	m_i(MeV)	a_i	b_i	c_i	d_i	$g_{iN}(n_0)$
σ	550.1238	1.3881	1.0943	1.7057	0.4421	10.5396
ω	783	1.3892	0.9240	1.4620	0.4775	13.0189
ρ	763	0.5647	—	—	—	7.3672

Quantity	Constraints	This model
n_0 (fm^{-3})	0.148–0.170	0.152
$-B/A$ (MeV)	15.8–16.5	16.4
K_0 (MeV)	220–260	252
S_0 (MeV)	31.2–35.0	32.3
L_0 (MeV)	38–67	51

Table: Baryon-meson coupling constants χ_{ib} at α_v = 1.0.

b	$\chi_{\omega b}$	$\chi_{\sigma b}$	$I_{3b}\chi_{\rho b}$	$\chi_{\phi b}$
Λ	2/3	0.611	0	0.471
$\Sigma^-, \Sigma^0, \Sigma^+$	2/3	0.467	$-1, 0, 1$	-0.471
Ξ^-, Ξ^0	1/3	0.284	$-1/2, 1/2$	-0.314
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	1	1.053	$-3/2, -1/2, 1/2, 3/2$	0

The coupling constant at other values of α_v can be found here Phys. Rev. D 107, 036011 (2023).

Table of Contents

Radial displacement perturbation

$$\xi'(r) = -\frac{1}{r}\left(3\xi + \frac{\eta}{\gamma}\right) - \frac{P'(r)}{P+\mathcal{E}}\xi(r), \tag{2}$$

Radial pressure perturbation

$$\eta'(r) = \xi\left[\omega^2 r(1+\mathcal{E}/P)e^{\lambda-\nu} - \frac{4P'(r)}{P} - 8\pi(P+\mathcal{E})re^{\lambda} + \frac{r(P'(r))^2}{P(P+\mathcal{E})}\right] + \eta\left[-\frac{\mathcal{E}P'(r)}{P(P+\mathcal{E})} - 4\pi(P+\mathcal{E})re^{\lambda}\right], \tag{3}$$

D. Gondek et al., Astron. Astrophys. 325, 217 (1997).

Boundary conditions: at the center where *r* = 0 requires that

$$\eta = -3\gamma\xi \tag{4}$$

Eq. (3) must be finite at the surface where *r* = *R* and hence

$$\eta = \xi\left[-4 + (1-2M/R)^{-1}\left(-\frac{M}{R} - \frac{\omega^2 R^3}{M}\right)\right] \tag{5}$$

The frequencies are computed by

$$\nu = \frac{\bar{\omega}}{2\pi}\ (kHz), \tag{6}$$

Table of Contents

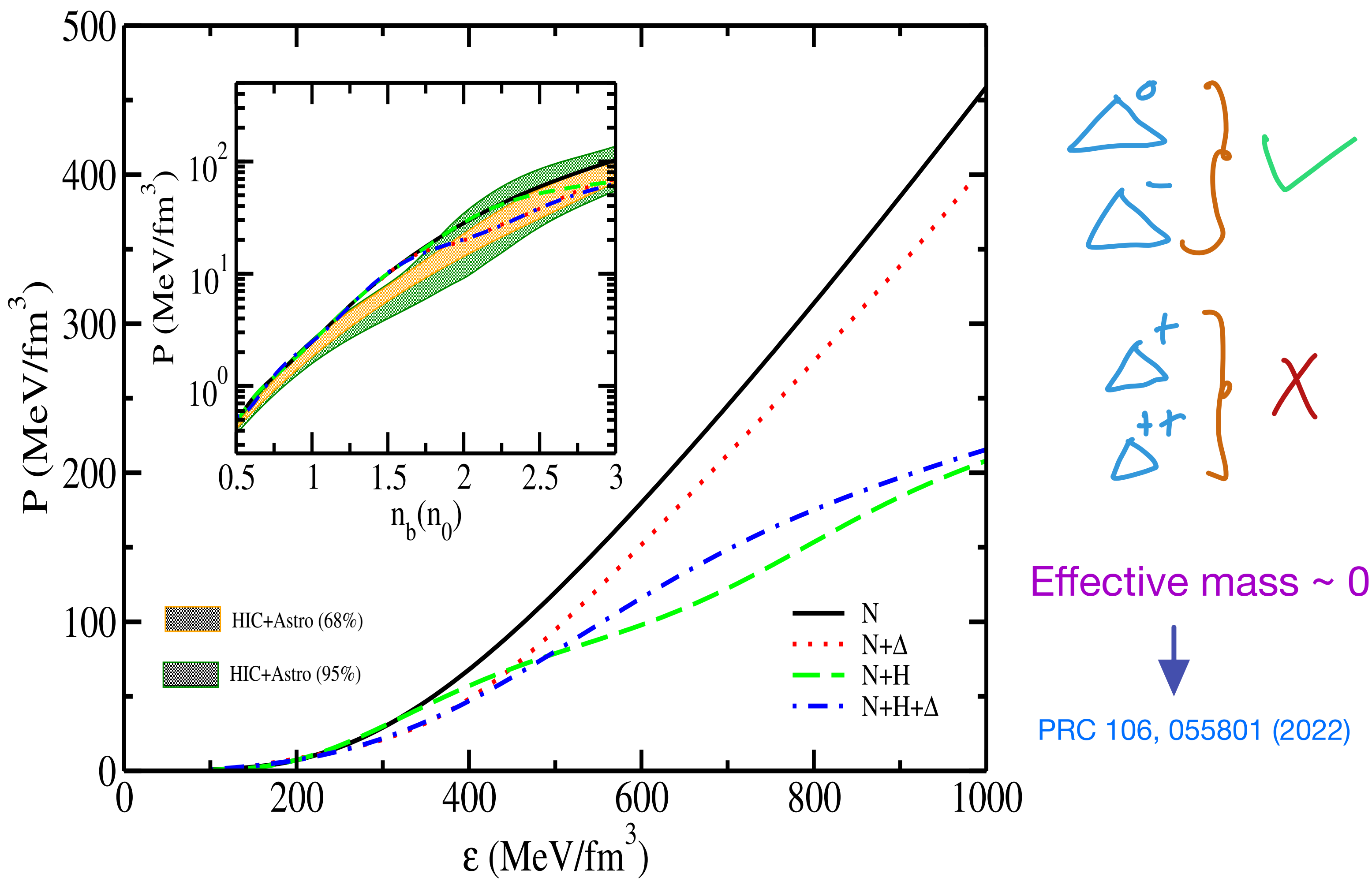
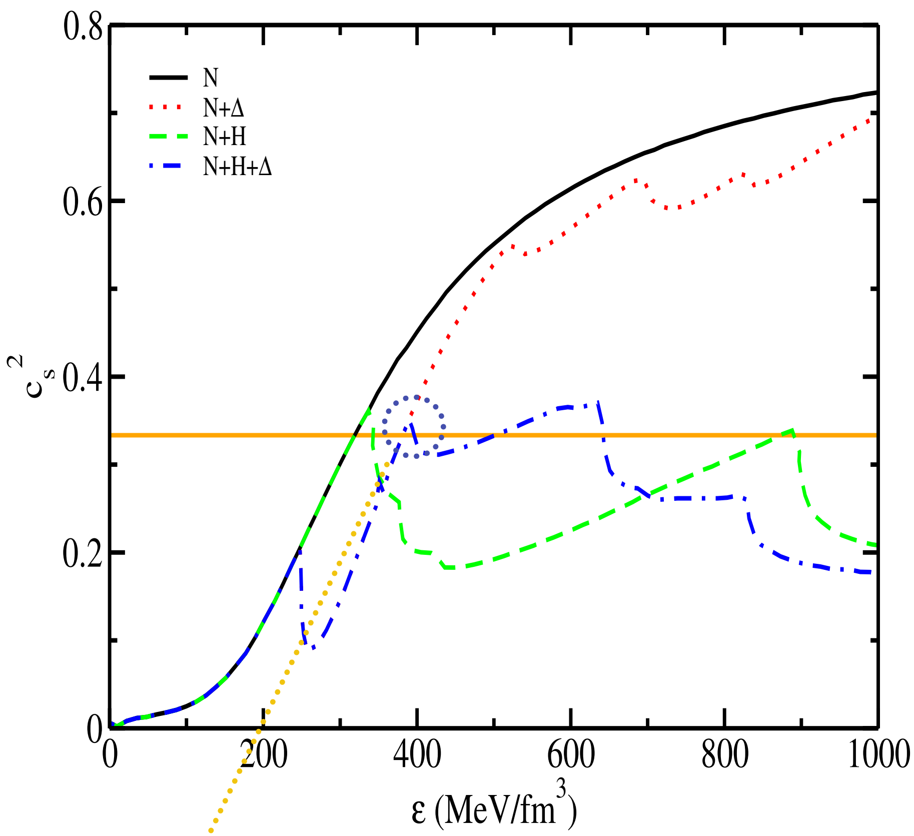


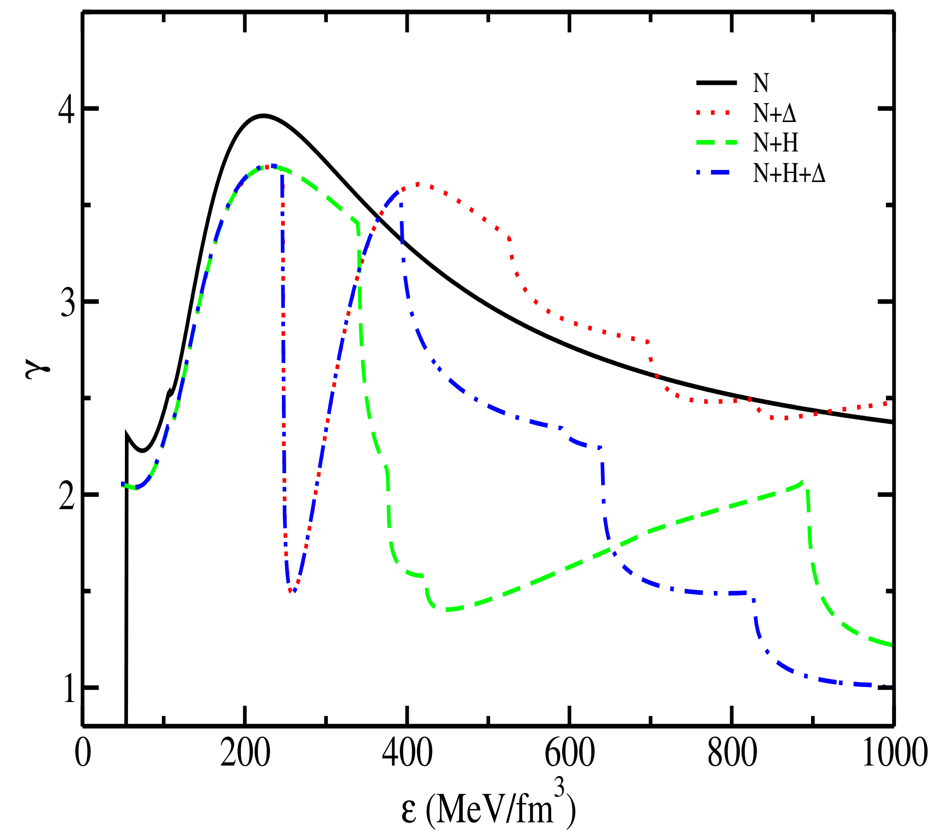
Figure: Equation of State



(a) Speed of sound

New particles

$$c_s^2 = \left(\frac{\partial P}{\partial \varepsilon}\right) c^2$$



(b) Adiabatic index

$$\gamma = \left(1 + \frac{\varepsilon}{P}\right) c_s^2$$

Max mass for N+H and N+H+D. < 2 Mo

Ruled out ??

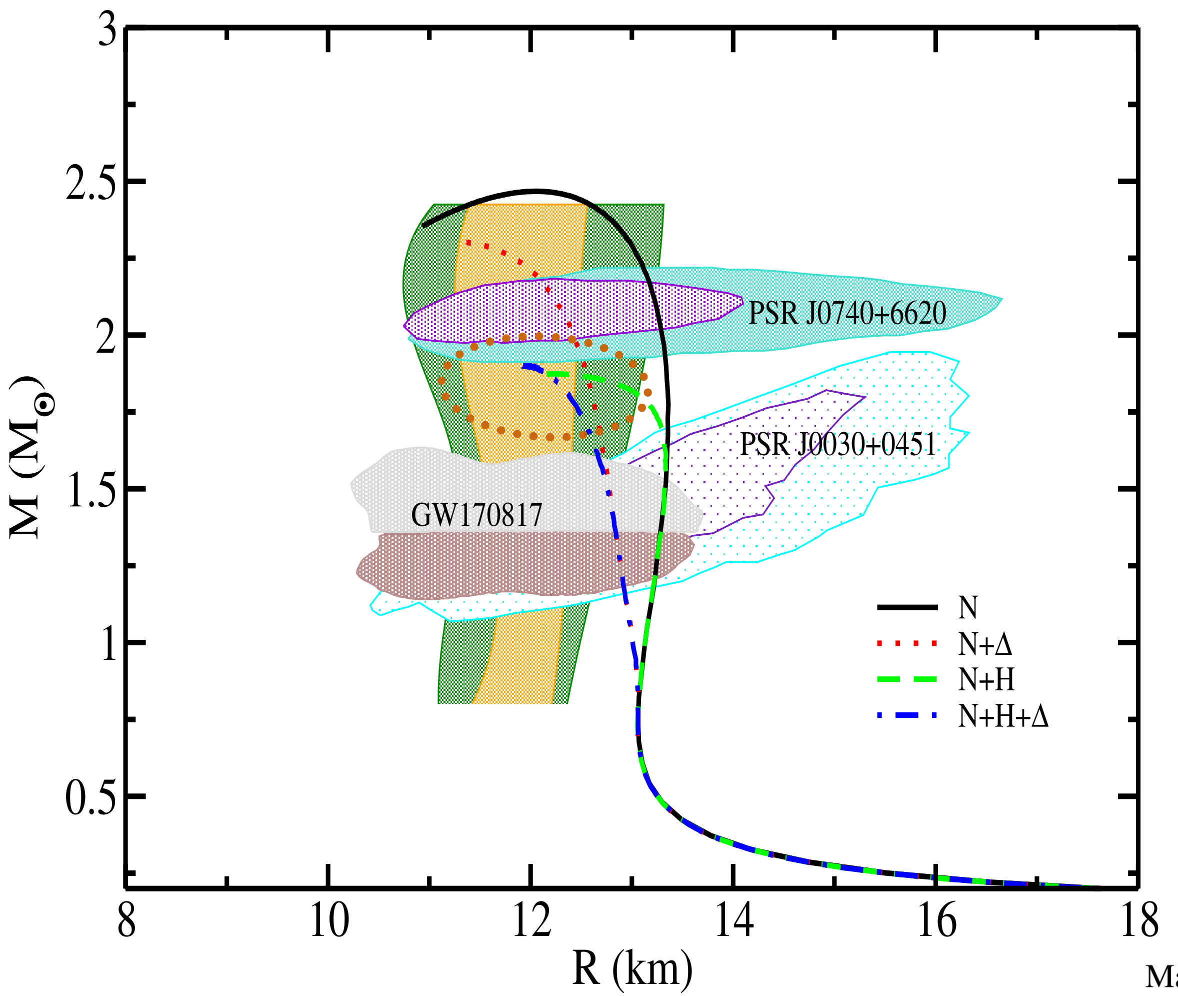
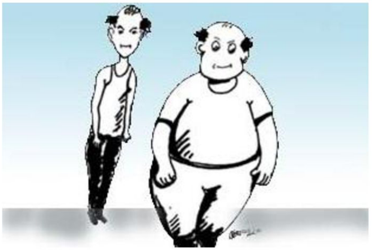


Figure: Mass-Radius Profile

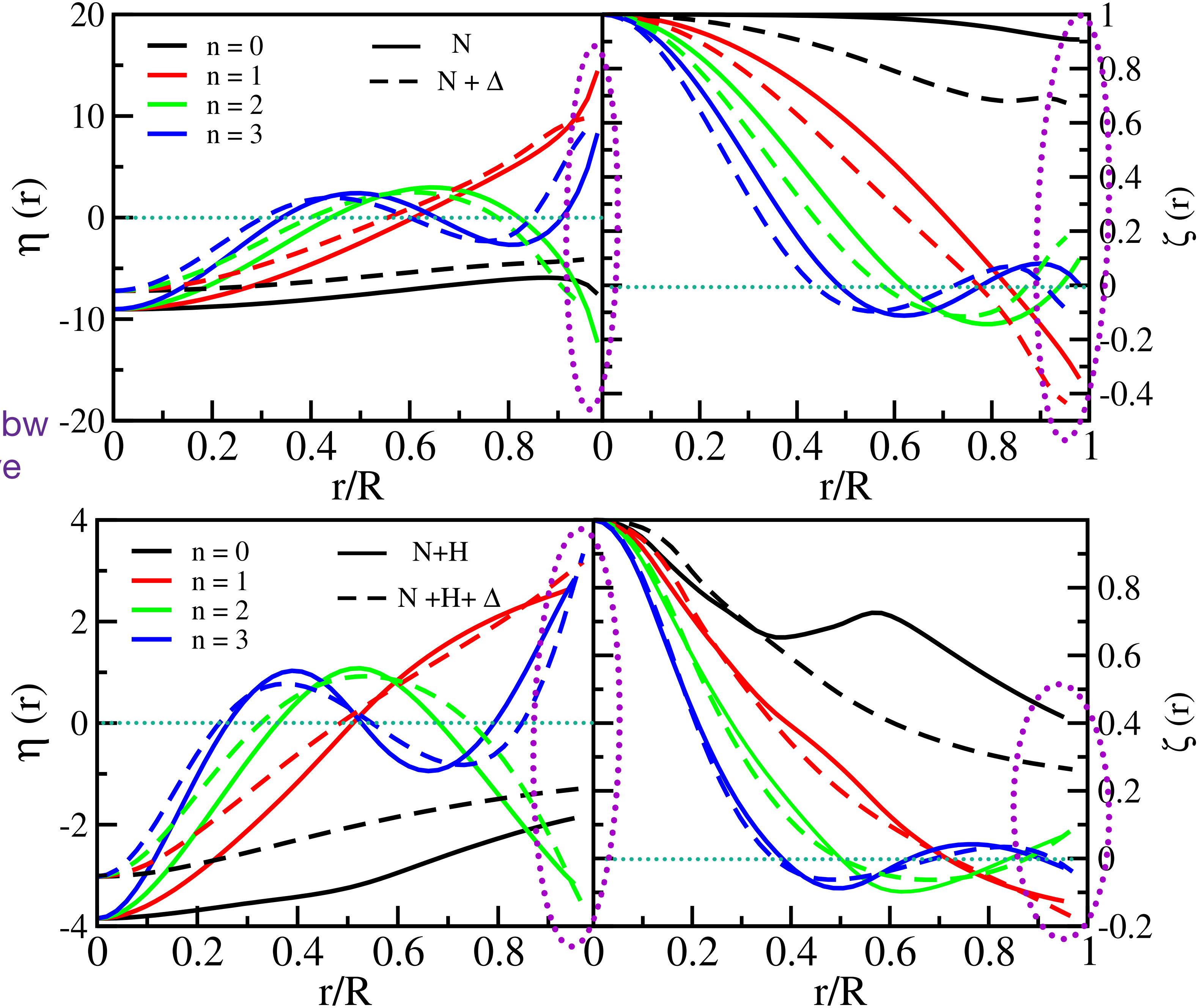
Ishfaq A. Rather *et al.*, PRD, 107, 123022 **(2023)**.

Mass versus radius relation



- For objects made of normal matter, radius tends to increase with mass

Difference bw consecutive modes



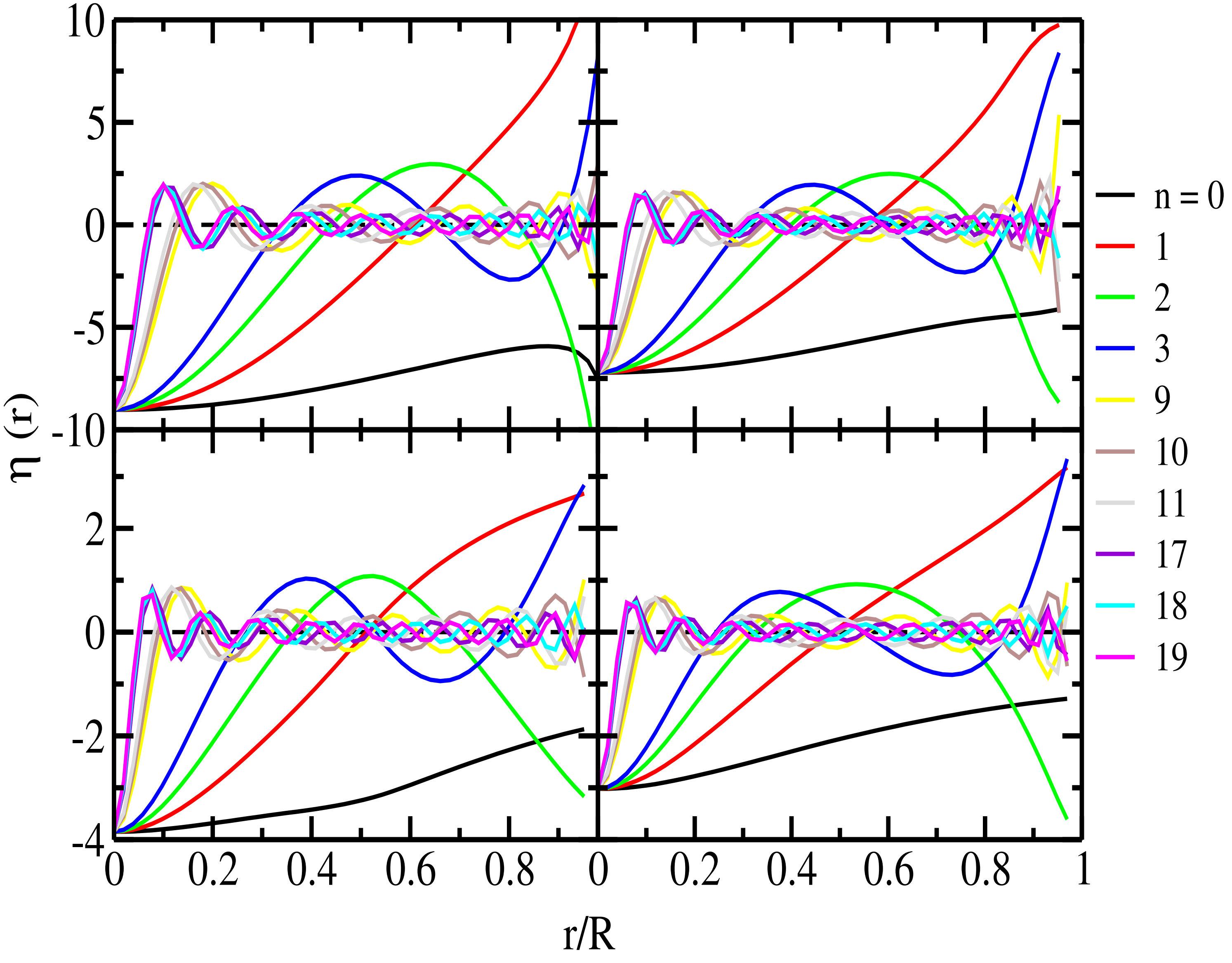


Figure: The radial pressure perturbation

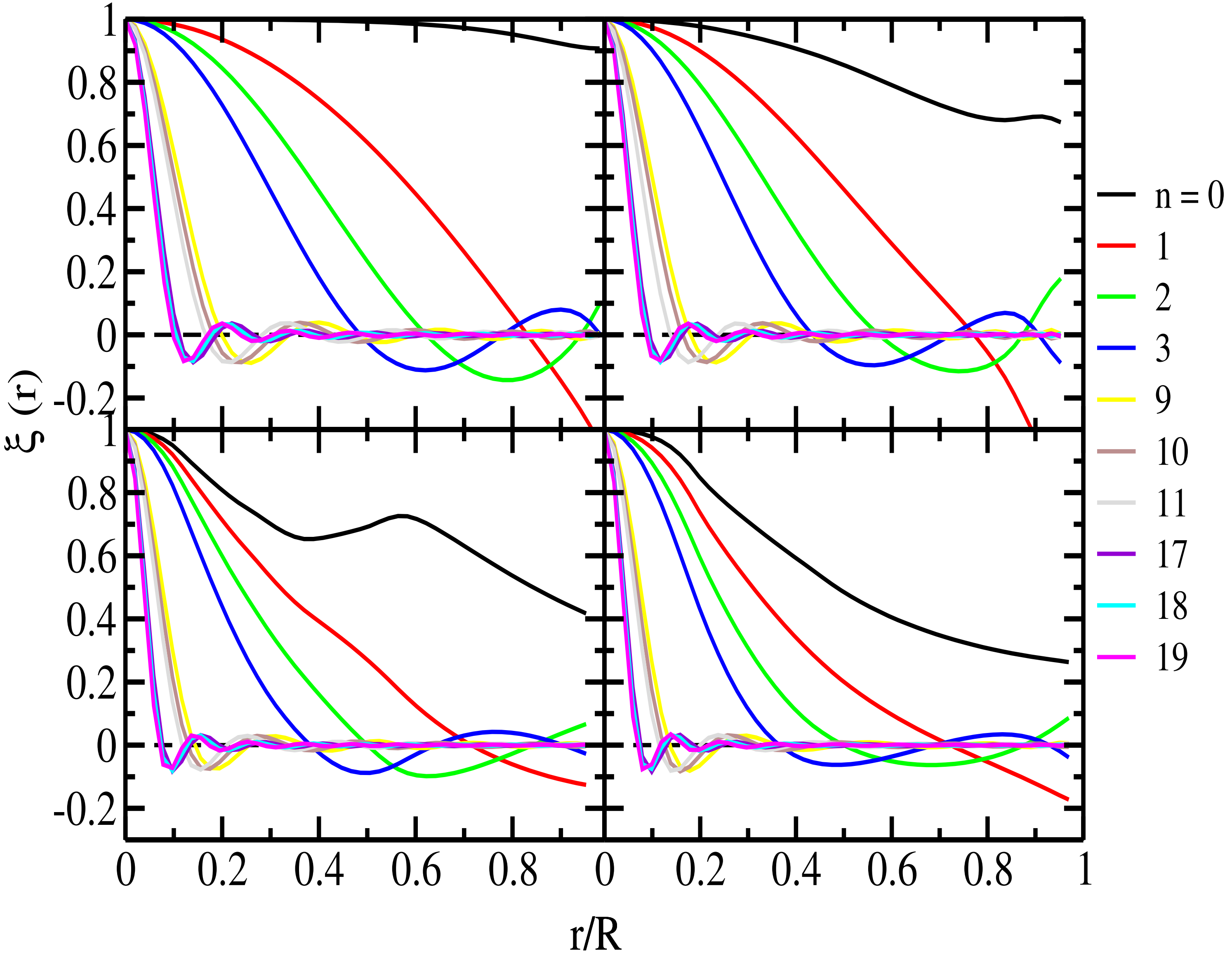


Figure: The radial displacement perturbation

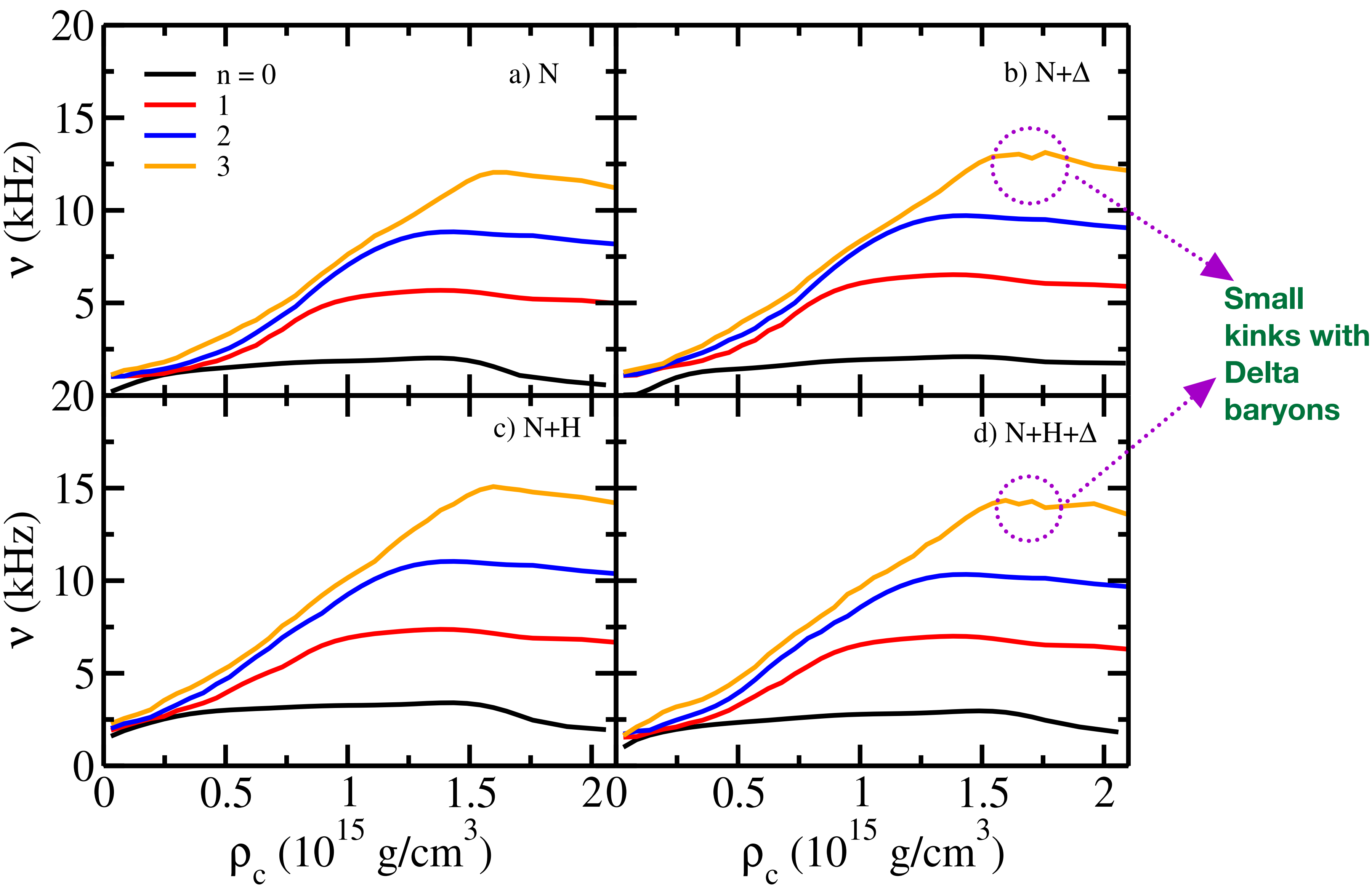


Figure: Frequency vs Density

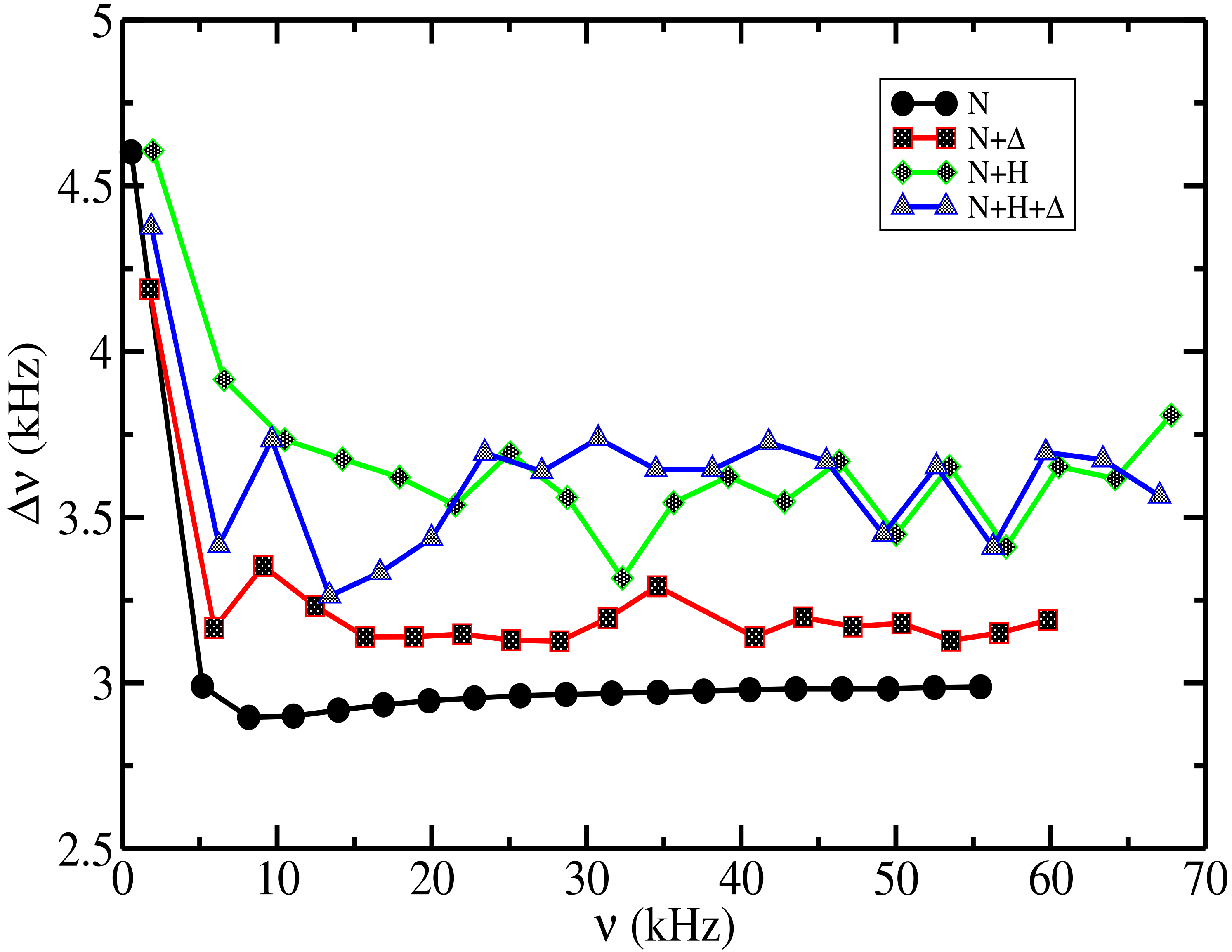


Figure: Frequency difference between consecutive modes

Table: 20 lowest order radial oscillation frequencies, ν in (kHz) for different EoSs considered calculated at the maximum mass of the corresponding star.

Nodes	EoS			
	N	N+Δ	N+H	N+H+Δ
0	0.571	1.578	1.977	1.865
1	5.173	5.891	6.582	6.241
2	8.163	9.118	10.497	9.656
3	11.060	12.379	14.231	13.390
4	13.960	15.674	17.907	16.652
5	16.878	18.984	21.527	19.986
6	19.812	22.316	25.064	23.422
7	22.758	25.649	28.758	27.116
8	25.713	29.011	32.317	30.754
9	28.674	32.358	35.633	34.492
10	31.639	35.732	39.177	38.135
11	34.609	39.089	42.800	41.779
12	37.580	42.457	46.347	45.505
13	40.556	45.822	50.016	49.174
14	43.535	49.189	53.464	52.622
15	46.518	52.556	57.117	56.275
16	49.500	55.933	60.527	59.685
17	52.483	59.308	64.180	63.380
18	55.469	62.688	67.796	67.054
19	58.457	66.061	71.603	70.618

Summary and Conclusion

- We studied the 20 lowest eigenfrequencies and corresponding oscillation functions of Δ-admixed nuclear (NΔ) and hyperonic matter (NHΔ) by solving the Sturm-Liouville boundary value problem and also verifying its validity.
- The spin-$3/2$ baryons (Δs) are described using the Rarita-Schwinger Lagrangian density.
- While the addition of Δ baryons to nucleonic matter softens the EoS, it gets stiffer for the hyperonic matter.
- Due to the onset of the Δ^-, we observe a significant decrease in the value of the adiabatic index γ followed by a rapid increase. This increases even more at intermediate densities than it does for the pure nucleonic case, a behavior not seen in the hyperonic case.

- We investigated the radial displacement perturbation profile $\xi(r)$ and pressure perturbation profile $\eta(r)$ with Δ-admixed matter and found that they oscillate with exactly n nodes for the n^{th} mode for all cases.
- The lowest modes show a smooth drop in their profiles while the higher modes depict small oscillations which become large for higher modes.
- For Δ-admixed nucleonic matter, small kinks are present for $n = 0$ mode. For hyperonic matter, the kink at the same node is found to be large and present at a small radius.
- These kinks in $\xi(r)$ correspond to the emergence of new exotic particles which provides a discontinuity in the adiabatic index γ.

HESS J1731-347 EoS

- According to a recent analysis of the supernova remnant HESS J1731-347, the central compact object (CCO) mass and radius are of $M = 0.77^{+0.20}_{-0.17}\ M_{\odot}$ and $R = 10.4^{+0.86}_{-0.78}$ km, respectively. V.Doroshenko *et al.*, 2022, Nature Astronomy, 6, 1444.

HESS J1731-347 EoS

- According to a recent analysis of the supernova remnant HESS J1731-347, the central compact object (CCO) mass and radius are of $M = 0.77^{+0.20}_{-0.17}\ M_\odot$ and $R = 10.4^{+0.86}_{-0.78}$ km, respectively. V.Doroshenko *et al.*, 2022, Nature Astronomy, 6, 1444.
- This estimate of maximum mass, $M = 0.77\ M_\odot$, is intriguing. It is possible that the compact object in HESS J1731-347 is an exotic object rather than a neutron star J. J. Li *et al.*, 2023 (arXiv:2306.14185), V.Sagun *et al.*, 2023 (arXiv:2306.12326)

HESS J1731-347 EoS

- According to a recent analysis of the supernova remnant HESS J1731-347, the central compact object (CCO) mass and radius are of $M = 0.77^{+0.20}_{-0.17}\ M_\odot$ and $R = 10.4^{+0.86}_{-0.78}$ km, respectively. V.Doroshenko *et al.*, 2022, Nature Astronomy, 6, 1444.
- This estimate of maximum mass, $M = 0.77\ M_\odot$, is intriguing. It is possible that the compact object in HESS J1731-347 is an exotic object rather than a neutron star J. J. Li *et al.*, 2023 (arXiv:2306.14185), V.Sagun *et al.*, 2023 (arXiv:2306.12326)
- We investigate the peculiar nature of strange stars to understand the EoS governing the intriguing central compact object (CCO) within the supernova remnant HESS J1731-347,

HESS J1731-347 EoS

- According to a recent analysis of the supernova remnant HESS J1731-347, the central compact object (CCO) mass and radius are of $M = 0.77^{+0.20}_{-0.17}\ M_\odot$ and $R = 10.4^{+0.86}_{-0.78}$ km, respectively. V.Doroshenko *et al.*, 2022, Nature Astronomy, 6, 1444.
- This estimate of maximum mass, $M = 0.77\ M_\odot$, is intriguing. It is possible that the compact object in HESS J1731-347 is an exotic object rather than a neutron star J. J. Li *et al.*, 2023 (arXiv:2306.14185), V.Sagun *et al.*, 2023 (arXiv:2306.12326)
- We investigate the peculiar nature of strange stars to understand the EoS governing the intriguing central compact object (CCO) within the supernova remnant HESS J1731-347,
- Additionally, we compare the radial oscillations of two models (vBag and CFL) to determine the frequency of the HESS J1731-347 compact object at its maximum mass I. A. Rather, arXiv:2307.03703 (2023) .

- For vBag model:
  B = 55 $\mathrm{MeV/fm^3}$, K_ν = 9 $\mathrm{GeV^{-2}}$ **(vBag1)**,
  B = 55 $\mathrm{MeV/fm^3}$, K_ν = 6 $\mathrm{GeV^{-2}}$ **(vBag2)**

Vector coupling

Bag constant

Vector interaction enhanced Bag Model

T. Klahn *et al.* The Astrophys. J., 810, 134 (2015), W. Wei *et al.*, As- trophys. J., 887, 151 (2019)

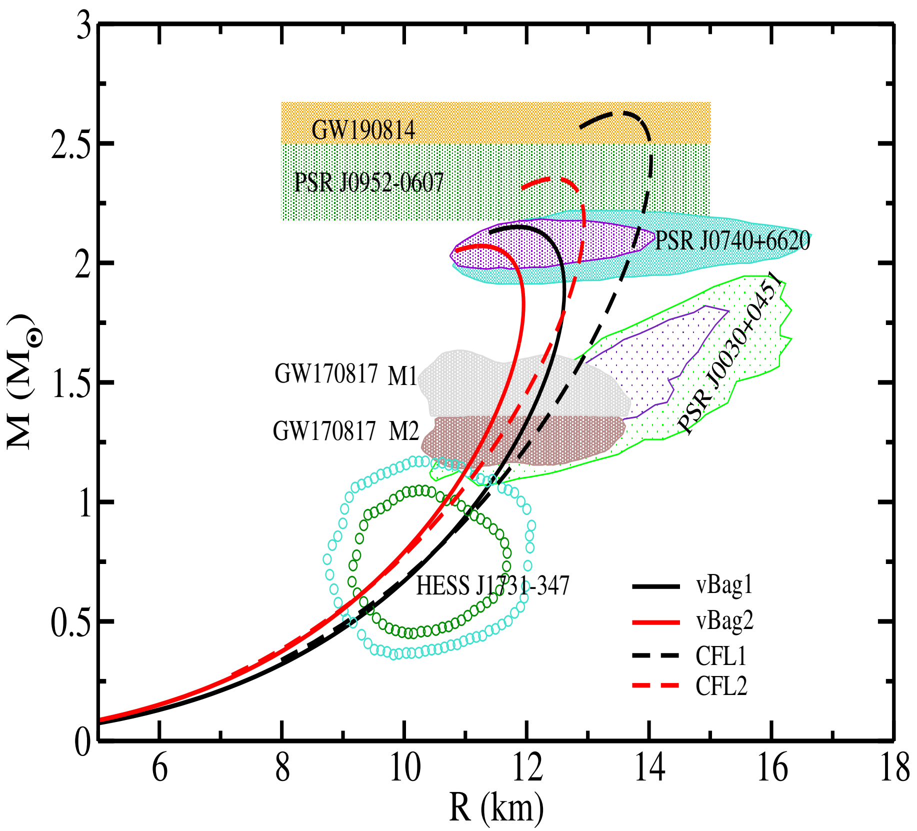
- **For vBag model:**
  $B = 55\ \mathrm{MeV/fm^3}$, $K_\nu = 9\ \mathrm{GeV^{-2}}$ **(vBag1)**,
  $B = 55\ \mathrm{MeV/fm^3}$, $K_\nu = 6\ \mathrm{GeV^{-2}}$ **(vBag2)**

  T. Klahn *et al.* The Astrophys. J., 810, 134 (2015), W. Wei *et al.*, As- trophys. J., 887, 151 (2019)

- **For CFL model:** **Superconducting gap parameter**
  $B = 60\ \mathrm{MeV/fm^3}$, $\Delta = 150\ \mathrm{MeV}$, $m_s = 150\ \mathrm{MeV}$ **(CFL1)**,
  $B = 60\ \mathrm{MeV/fm^3}$, $\Delta = 100\ \mathrm{MeV}$, $m_s = 0\ \mathrm{MeV}$ **(CFL2)**

  C. V. Flores *et al.*, Phys. Rev. C, 95, 025808 (2017)

EoS	M_{max} ($M_\odot$)	R_{max} (km)	$R_{1.4M_\odot}$ (km)	$R_{0.77M_\odot}$ (km)	$\Lambda_{1.4M_\odot}$
vBag1	2.15	11.76	12.15	10.43	158
vBag2	2.06	11.23	11.55	9.90	119
CFL1	2.62	13.50	12.47	10.47	484
CFL2	2.36	12.39	11.86	10.05	212



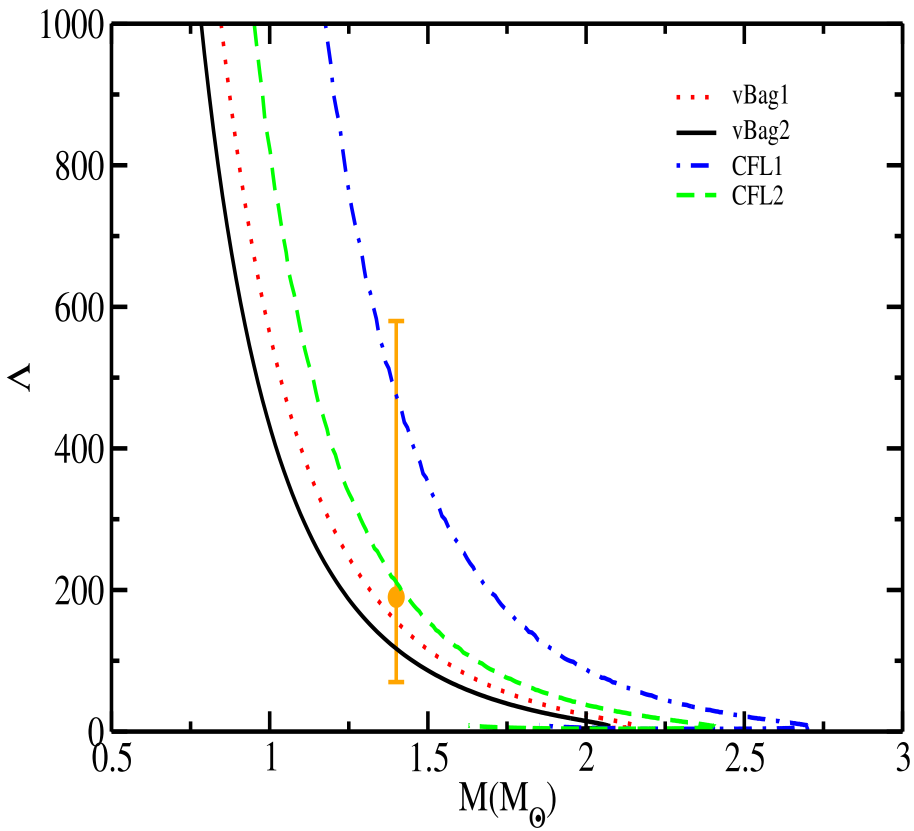


Figure: Mass-Radius and Tidal Deformability Profiles

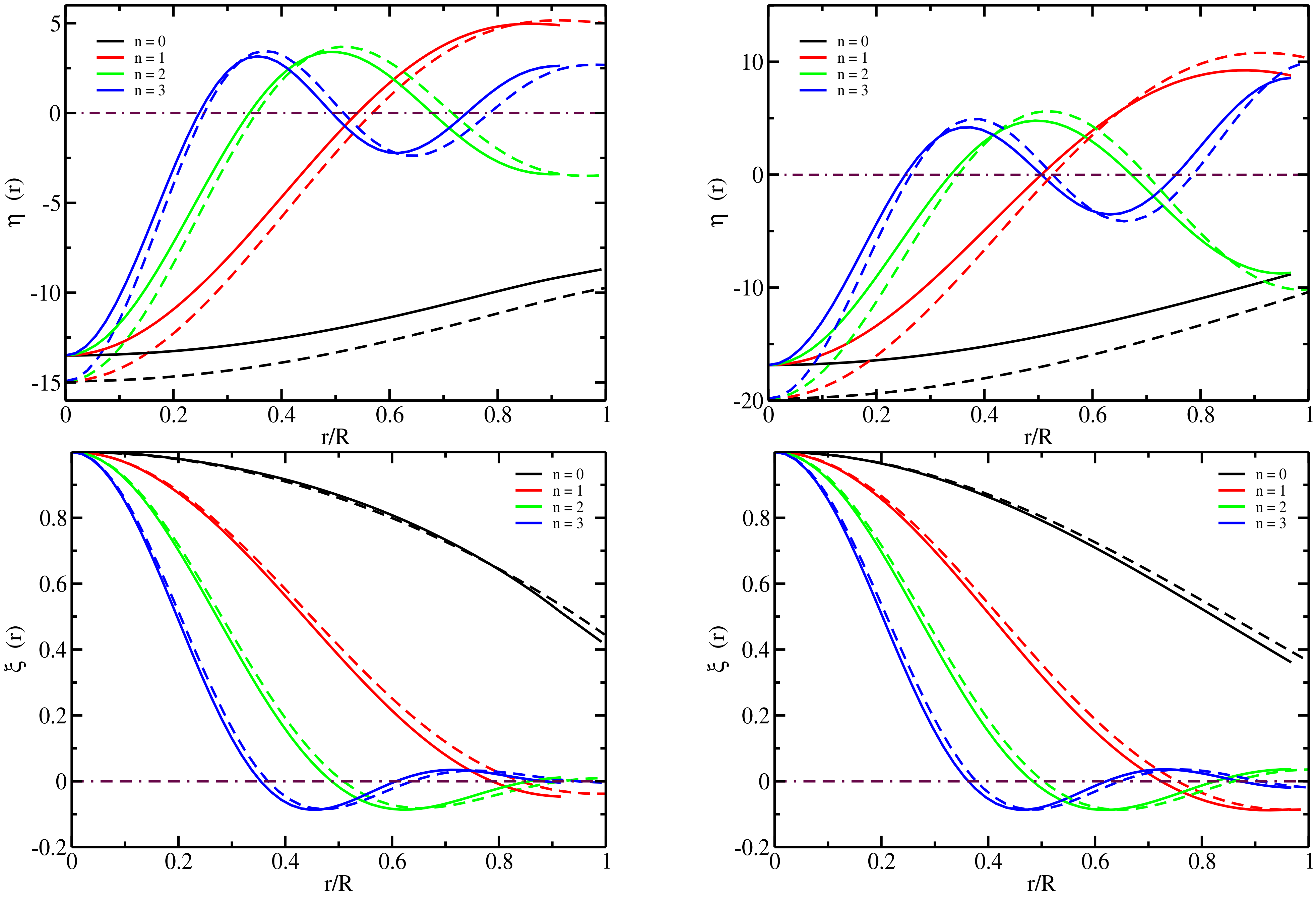
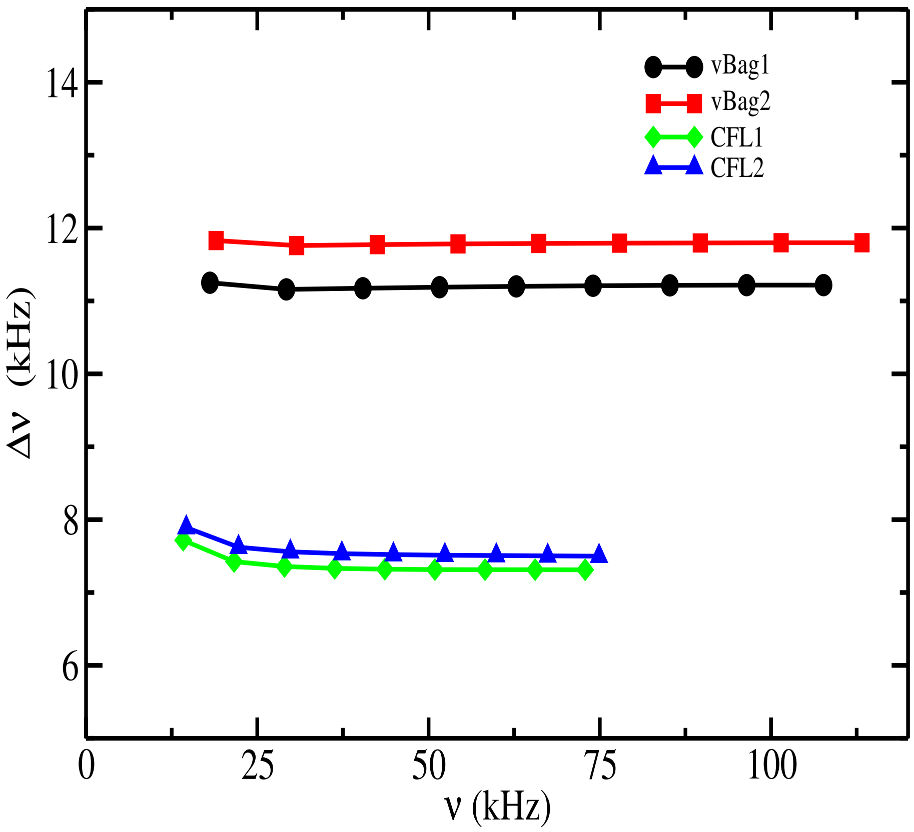


Figure: Radial profiles for vBag (left plots) and CFL (right panels) EoS.



Nodes	EoS			
	vBag1	vBag2	CFL1	CFL2
0	6.83	7.15	6.47	6.74
1	18.08	18.98	14.18	14.63
2	29.24	30.74	21.61	22.25
3	40.41	42.52	28.96	29.81
4	51.60	54.30	36.30	37.35
5	62.81	66.09	43.62	44.87
6	74.01	77.88	50.93	52.38
7	85.23	89.68	58.24	59.89
8	96.44	101.47	65.56	67.39
9	107.66	113.27	72.87	74.89

- The HESS J1731-347 offers crucial information in the density range of 1-2 times the nuclear saturation density, and hence will im- pose some strict constraints on strongly interacting matter.

- The HESS J1731-347 offers crucial information in the density range of 1-2 times the nuclear saturation density, and hence will im- pose some strict constraints on strongly interacting matter.
- By comparing the radial oscillations of the two models and determining the frequencies of the HESS J1731-347 at its maximum mass, the study goes beyond merely outlining the properties of the CCO.

- The HESS J1731-347 offers crucial information in the density range of 1-2 times the nuclear saturation density, and hence will im- pose some strict constraints on strongly interacting matter.
- By comparing the radial oscillations of the two models and determining the frequencies of the HESS J1731-347 at its maximum mass, the study goes beyond merely outlining the properties of the CCO.
- This offers a means to investigate their relationship to the observed mass-radius relationship of the HESS J1731-347 compact object and offers insightful infor- mation on the dynamical behavior of strange stars.

- The HESS J1731-347 offers crucial information in the density range of 1-2 times the nuclear saturation density, and hence will im- pose some strict constraints on strongly interacting matter.
- By comparing the radial oscillations of the two models and determining the frequencies of the HESS J1731-347 at its maximum mass, the study goes beyond merely outlining the properties of the CCO.
- This offers a means to investigate their relationship to the observed mass-radius relationship of the HESS J1731-347 compact object and offers insightful infor- mation on the dynamical behavior of strange stars.
- This high- lights the fundamental characteristics of strange stars and ex- plains their significance in the context of the current under- standing of the HESS J1731-347 compact object by delving into these intricate aspects.

Further Work

- To study the impact of including more exotic particles in the modelling of NSs using E-RMF and pQCD. These include kaons and deconfined QM (with and without mixtures of phases) under work....
- Effect of Dark matter on the radial properties of Hadron-Quark Phase Transition under work....
- To study the oscillations in presence of strong magnetic field.

ThankYou

For α_v = 0.5

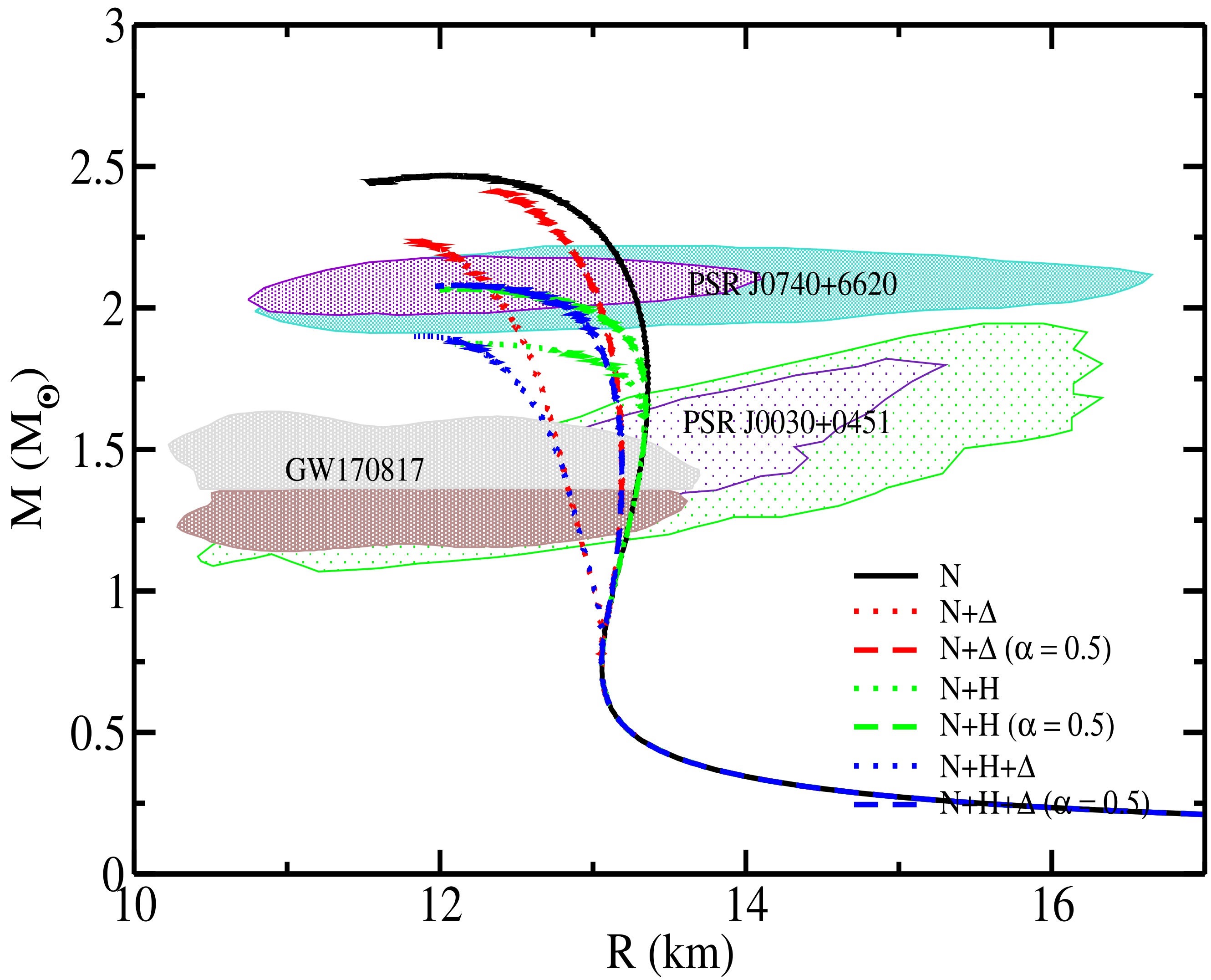
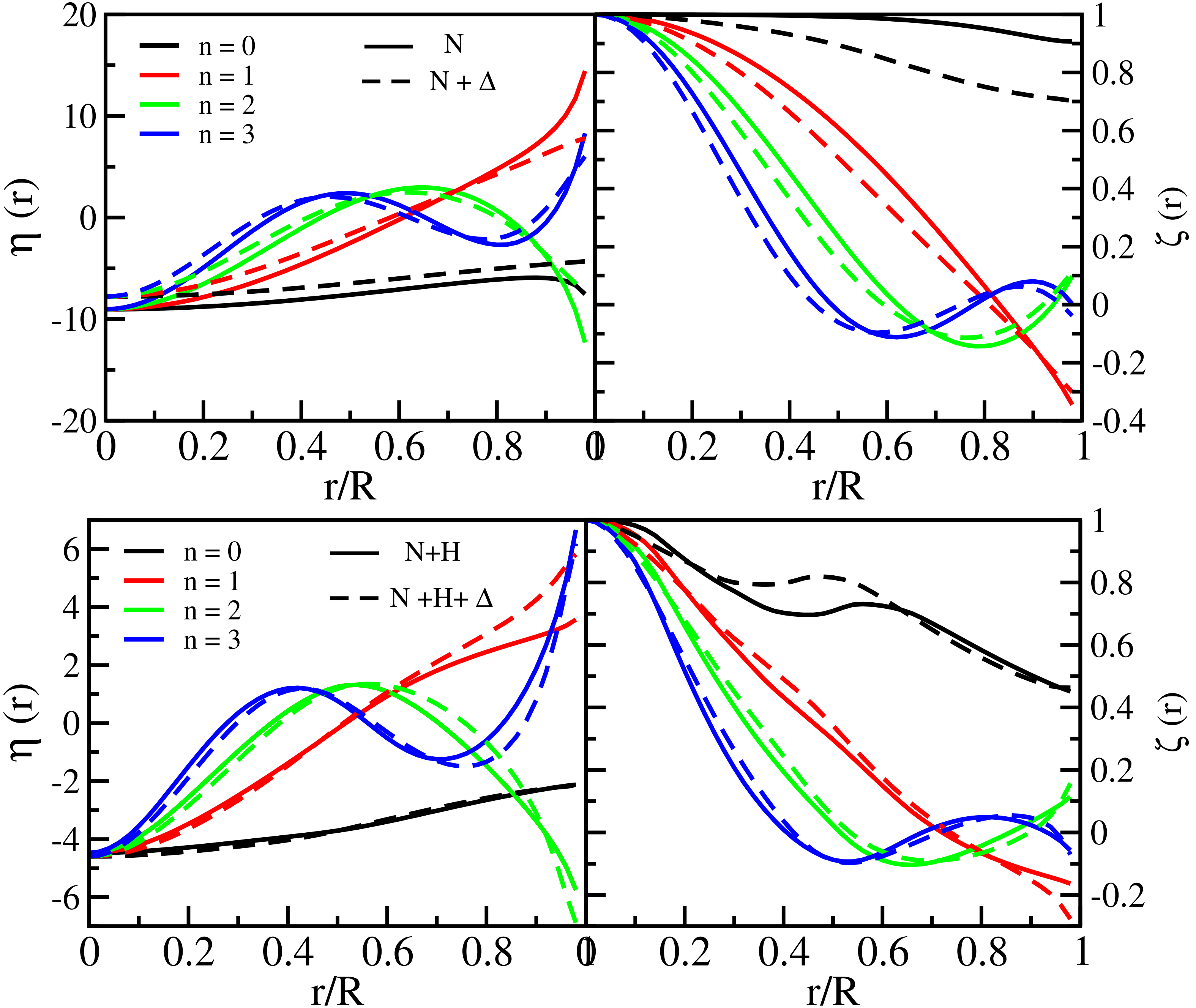


Figure: Mass-Radius Profile at α_v = 1.0 and 0.5


n = 0
n = 1
n = 2
n = 3
N
N + Δ
η (r)
ζ (r)
r/R
n = 0
n = 1
n = 2
n = 3
N+H
N +H+ Δ
η (r)
ζ (r)
r/R

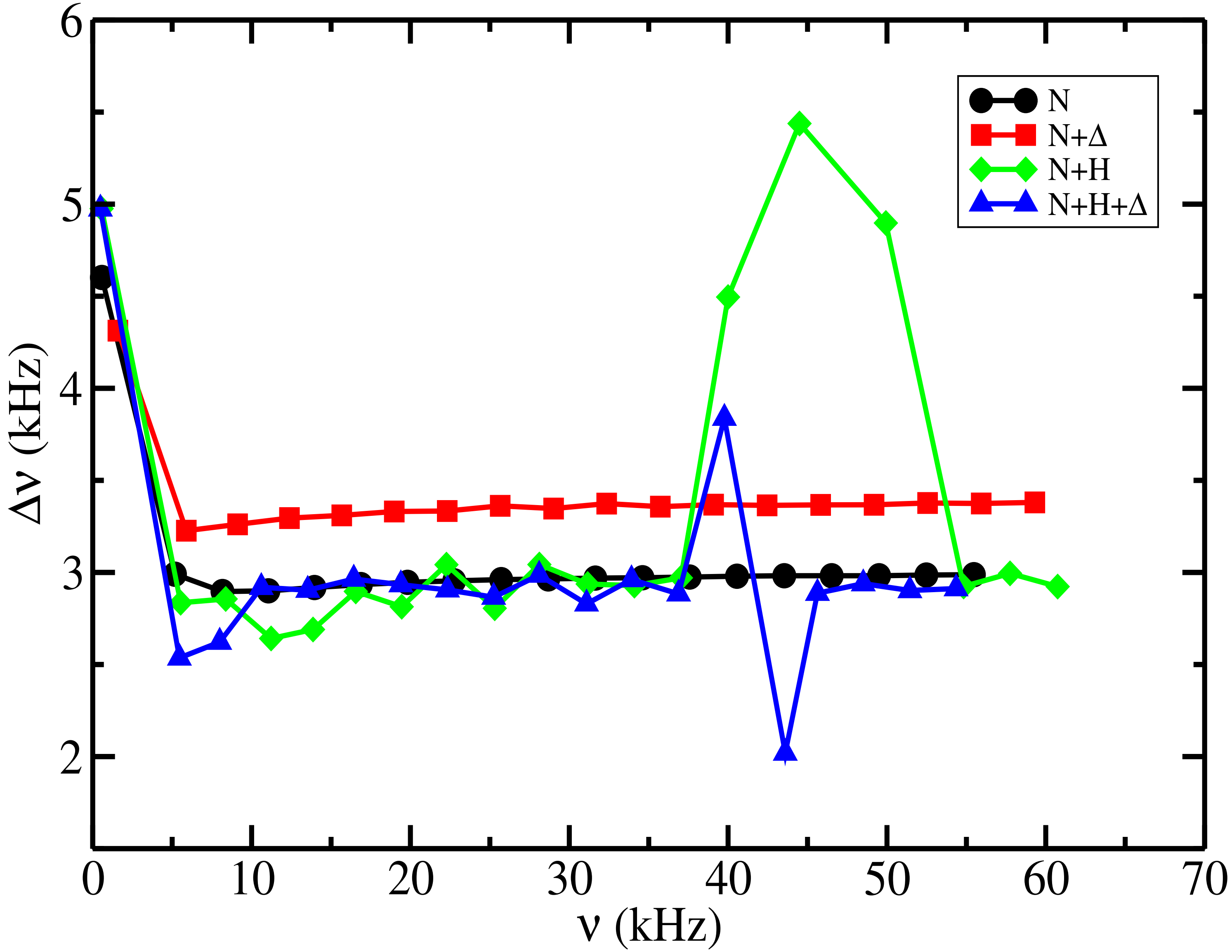


Figure: Frequency difference between consecutive modes

Table: 20 lowest order radial oscillation frequencies, ν in (kHz) for different EoSs at α_v = 0.5

Nodes	EoS			
	N	N+Δ	N+H	N+H+Δ
0	0.571	1.755	0.557	0.477
1	5.173	5.943	5.532	5.451
2	8.163	9.109	8.367	7.988
3	11.060	12.462	11.224	10.609
4	13.960	15.694	13.865	13.529
5	16.878	18.833	16.555	16.432
6	19.812	21.972	19.453	19.397
7	22.758	25.119	22.266	22.331
8	25.713	28.249	25.309	25.236
9	28.674	31.374	28.115	28.102
10	31.639	34.569	31.157	31.088
11	34.609	40.861	34.095	33.917
12	37.580	43.999	37.024	36.879
13	40.556	47.197	39.995	39.763
14	43.535	50.368	44.491	43.601
15	46.518	53.547	49.929	45.620
16	49.500	56.675	54.827	48.620
17	52.483	59.826	57.750	51.446
18	55.469	63.015	60.746	54.348
19	58.457	66.168	63.670	57.261