

Thermal QCD phase transition with dynamical chiral fermions

A. Yu. Kotov

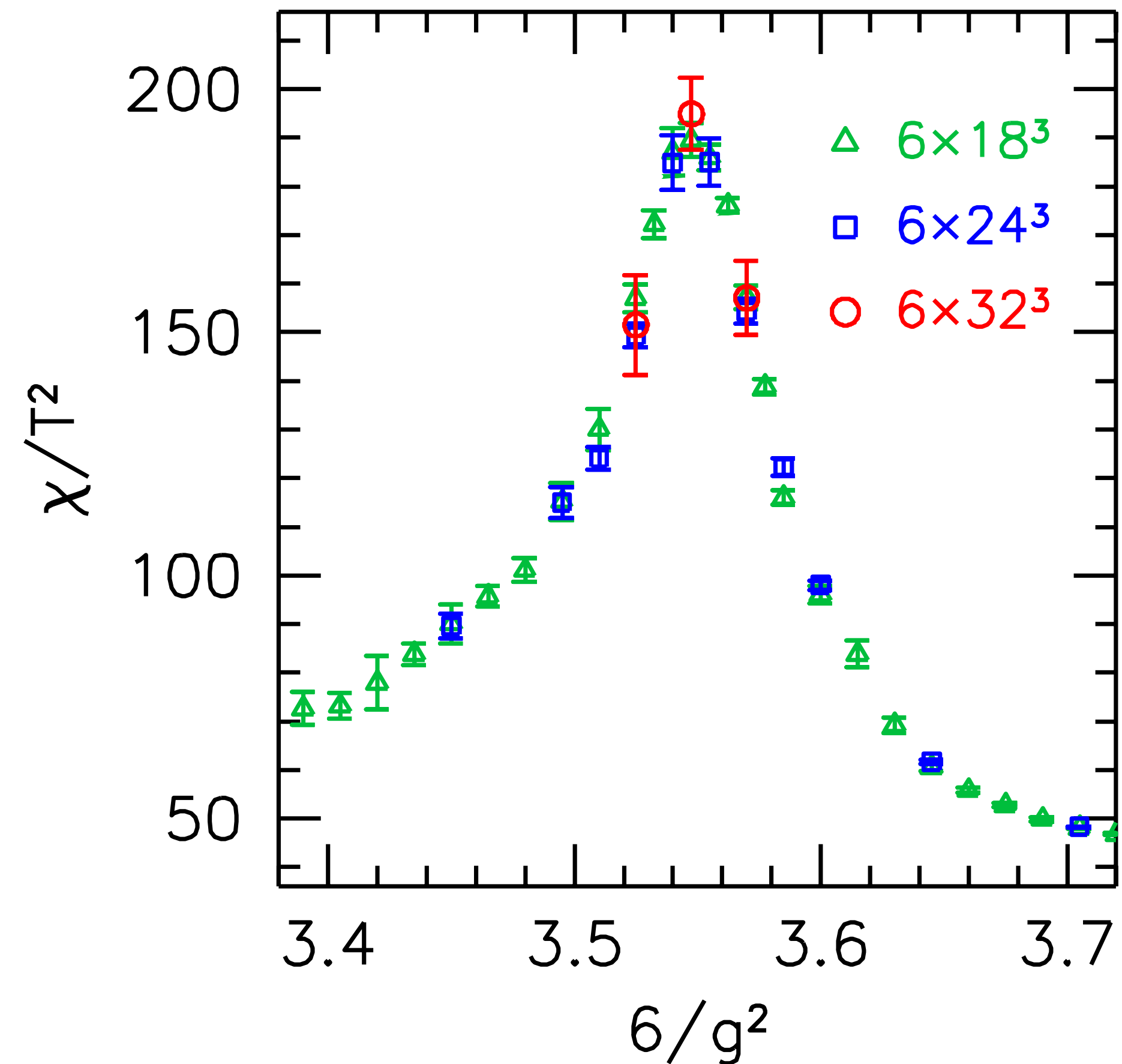
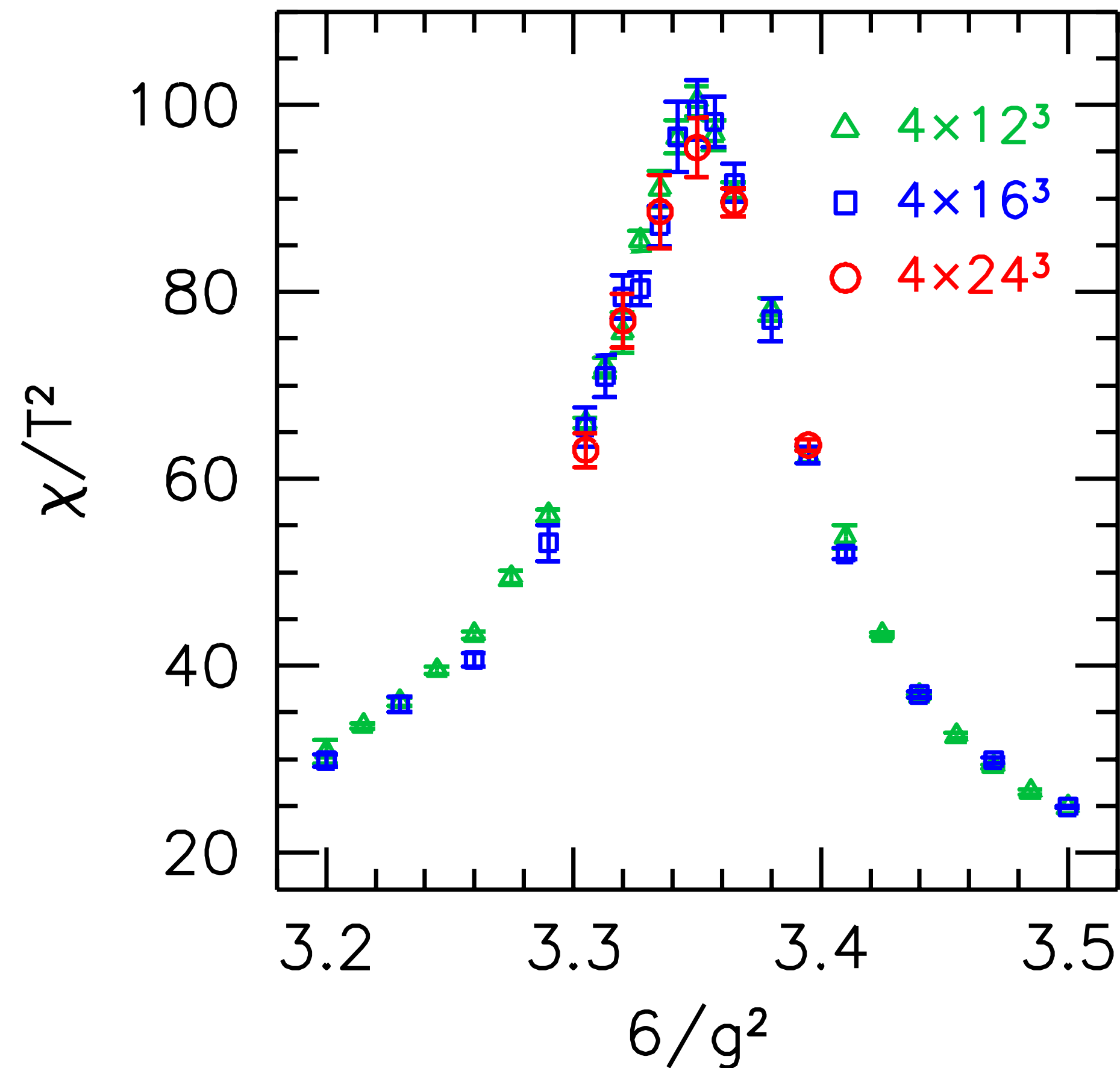
in collaboration with **K. Szabo, Z. Fodor**



XQCD 2023

Thermal QCD phase transition

$N_f = 2 + 1$ stout smeared staggered fermions, $m_\pi = m_\pi^{(\text{phys})}$, $N_t = 4, 6$



[Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, 2006]

Chiral fermions on the lattice

Way to avoid Nielsen-Ninomiya «no-go» theorem

- Continuum chiral symmetry: $\gamma_5 D + D \gamma_5 = 0$
- Ginsparg-Wilson relation: $\gamma_5 D + D \gamma_5 = 2a D \gamma_5 D$
- Overlap fermions: $a D_{\text{ov}} = \frac{1}{2} \left(1 + \gamma_5 \text{sign}(\gamma_5 D_w(-m_w)) \right)$
- **Very expensive numerically**: require multiple tricks

[H.Neuberger, 1998]

Action details

- Symanzik improved gauge action
 - Fermion sector: 2 steps of HEX smeared gauge fields
 - $N_f = 2 + 1$ flavours of overlap quarks, **physical** masses
 - 2 flavours of Wilson fermions with mass $-m_W$
 - Two boson fields with mass $m_B a = 0.54$
 - Statistics: $O(2000 - 9000)$
- $a \rightarrow 0$: irrelevant
 - Keep **$Q = \text{const}$** ($Q = 0$)
 - Make calculations faster
- [H. Fukaya et al., 2006]

Implementing odd number of flavours

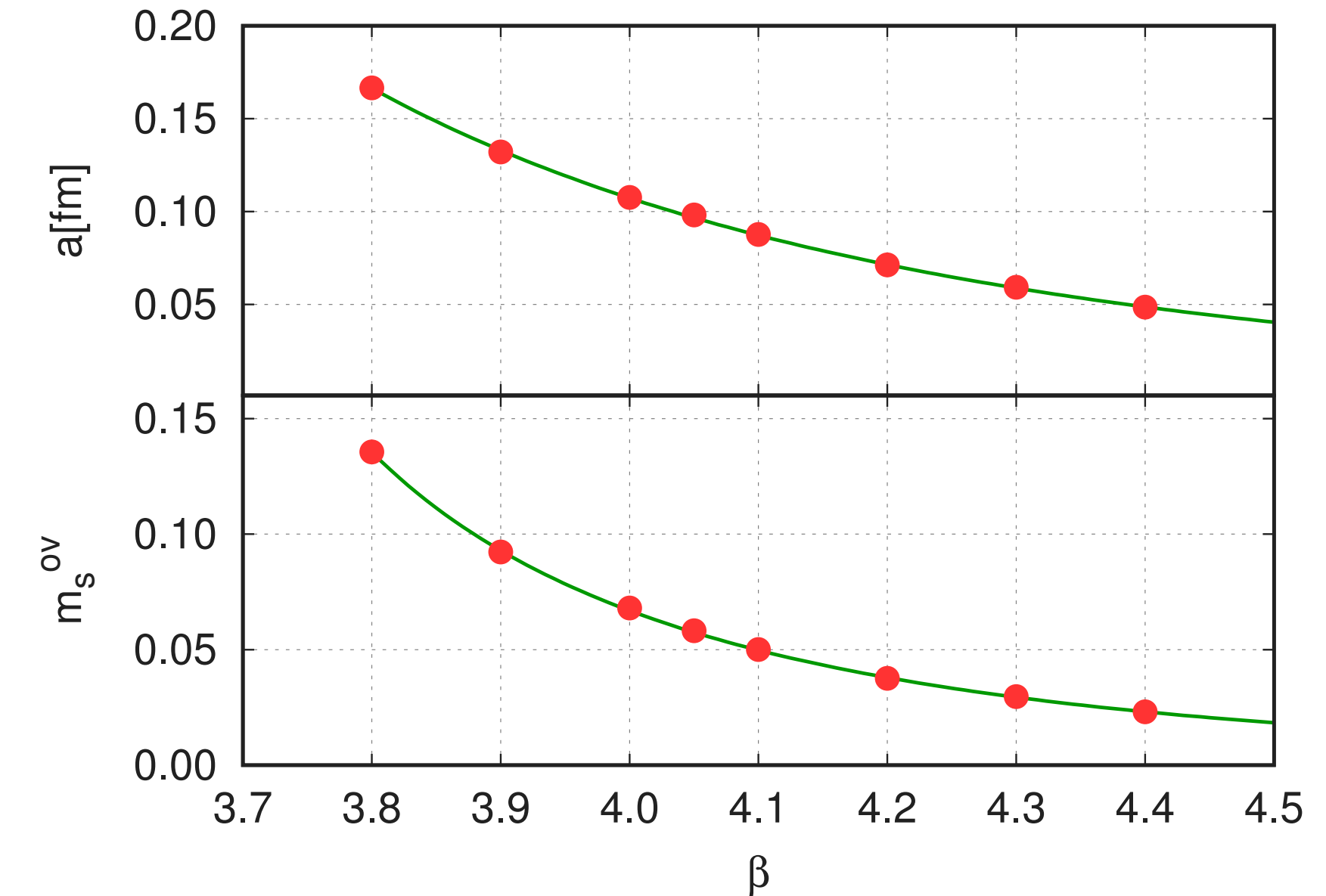
Exploiting $Q = \text{const}$

- Monte Carlo: determinant of a **hermitian** operator $H^2 = D_{\text{ov}} D_{\text{ov}}^\dagger$: $N_f = 2$
- To simulate $N_f = 1$ (strange quark): need to take the **square root**
- Chirality projectors: $P_\pm = \frac{1 \pm \gamma_5}{2}$, $H_\pm^2 = P_\pm H^2 P_\pm$
- Fixed topology $Q = \text{const}$:
 $\det H^2 \sim \det H_+^2 \det H_-^2 \sim (\det H_+^2)^2 \sim (\det H_-^2)^2$
- Take $\det H_+^2$ or $\det H_-^2$

Lattice details, scale setting

Scale setting from simulations with large m_π

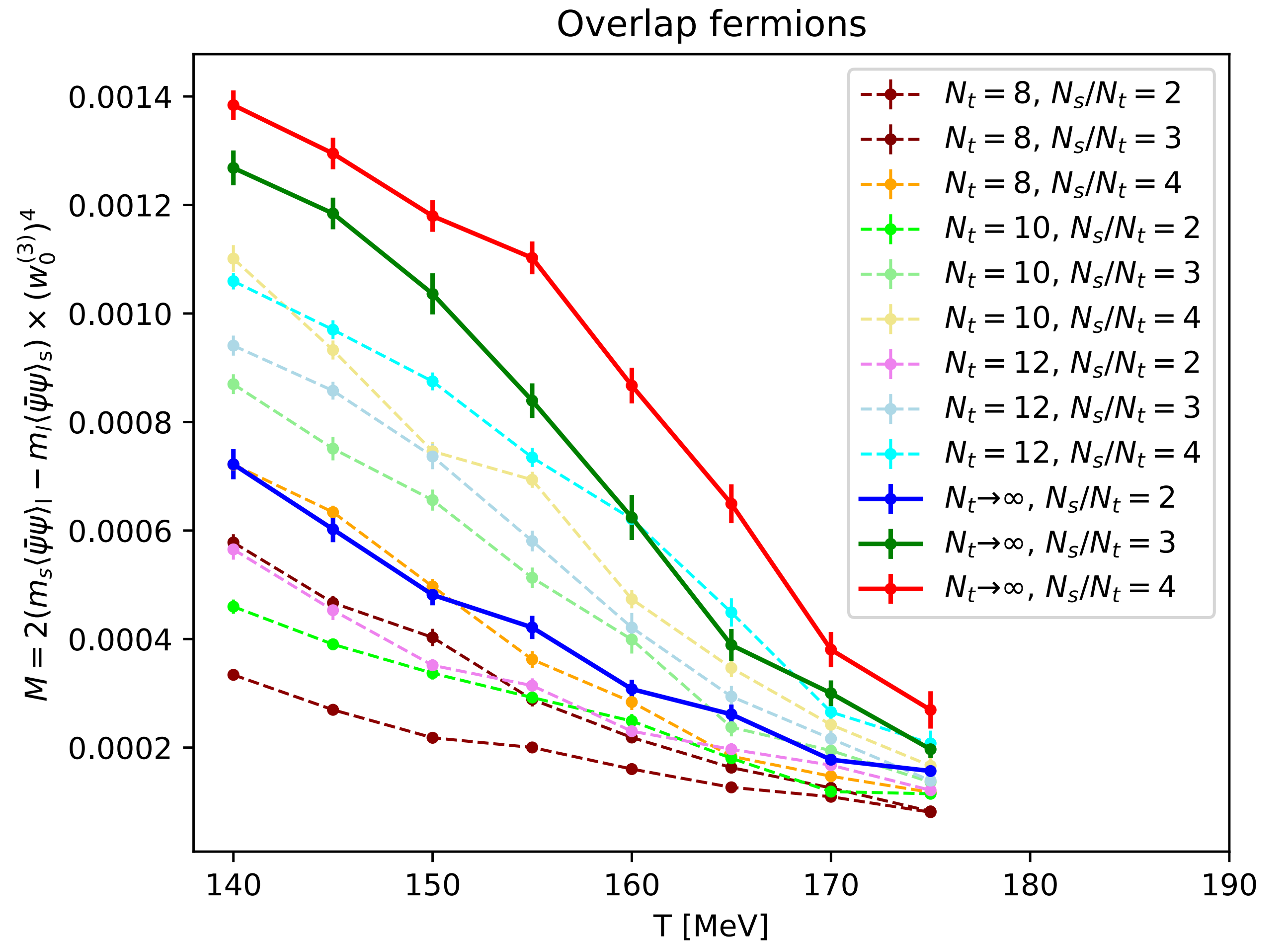
- Simulations are done along the LCP
- Scale setting: require $T = 0$ simulations
- $N_f = 3$ staggered simulations, $T = 0$, $w_0^{(3)} = 0.153(1)$ fm, $m_\pi^{(3)} = 712(5)$ MeV
- $N_f = 3$ overlap simulations, $T = 0$, at each β tune m_s^{ov} to have $m_\pi w_0 \equiv m_\pi^{(3)} w_0^{(3)}$
- $N_f = 2 + 1$ overlap simulations, $T \neq 0$: $m_s = m_s^{\text{ov}}$, $m_{ud} = R m_s^{\text{ov}}$, $a = w_0^{(3)} / w_0^{\text{ov}}$
- Physical point: $m_{ud} = m_{ud}^{(\text{phys})}$, $m_s = m_s^{(\text{phys})}$



[Sz. Borsanyi et al., 2016]

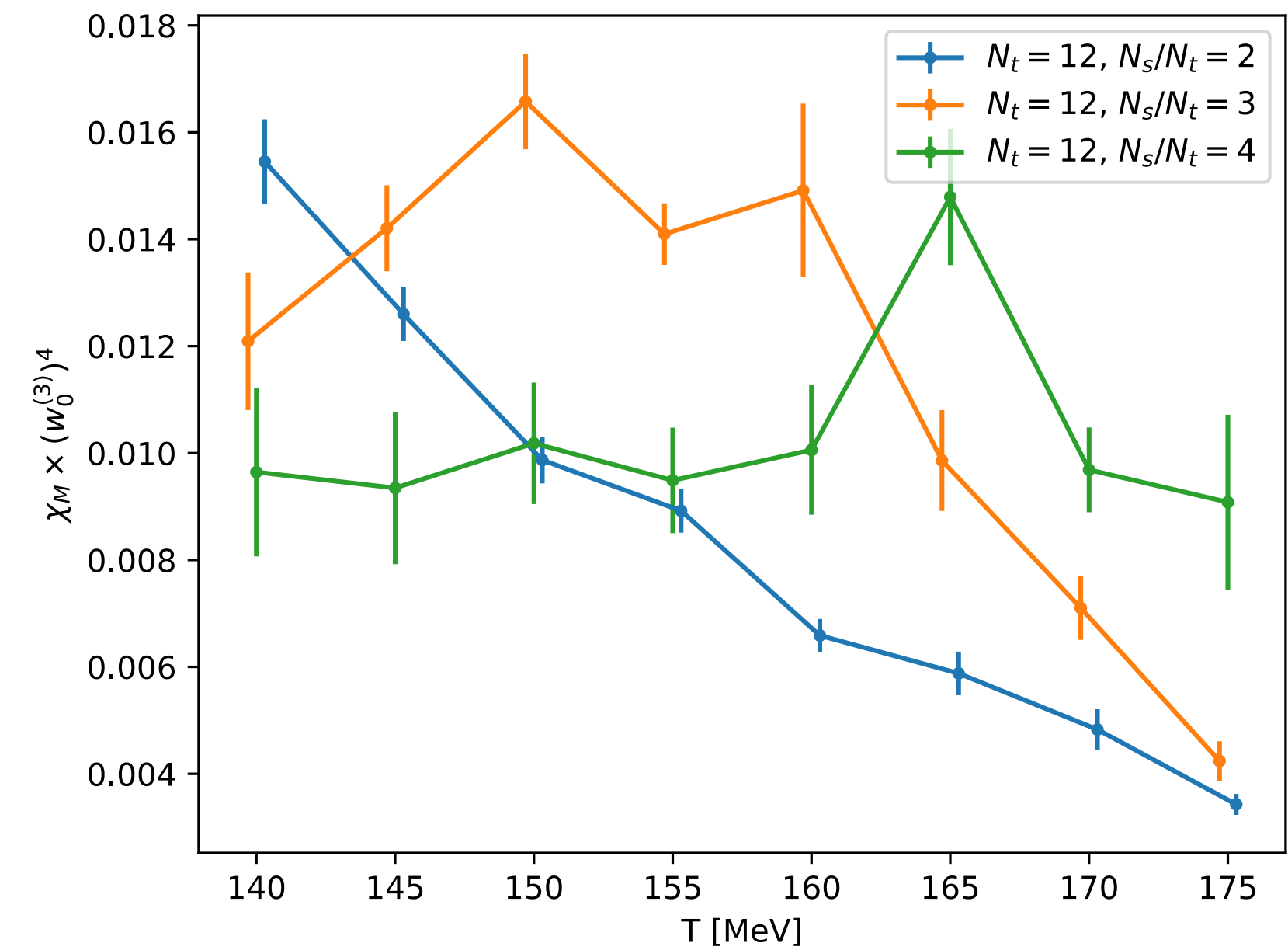
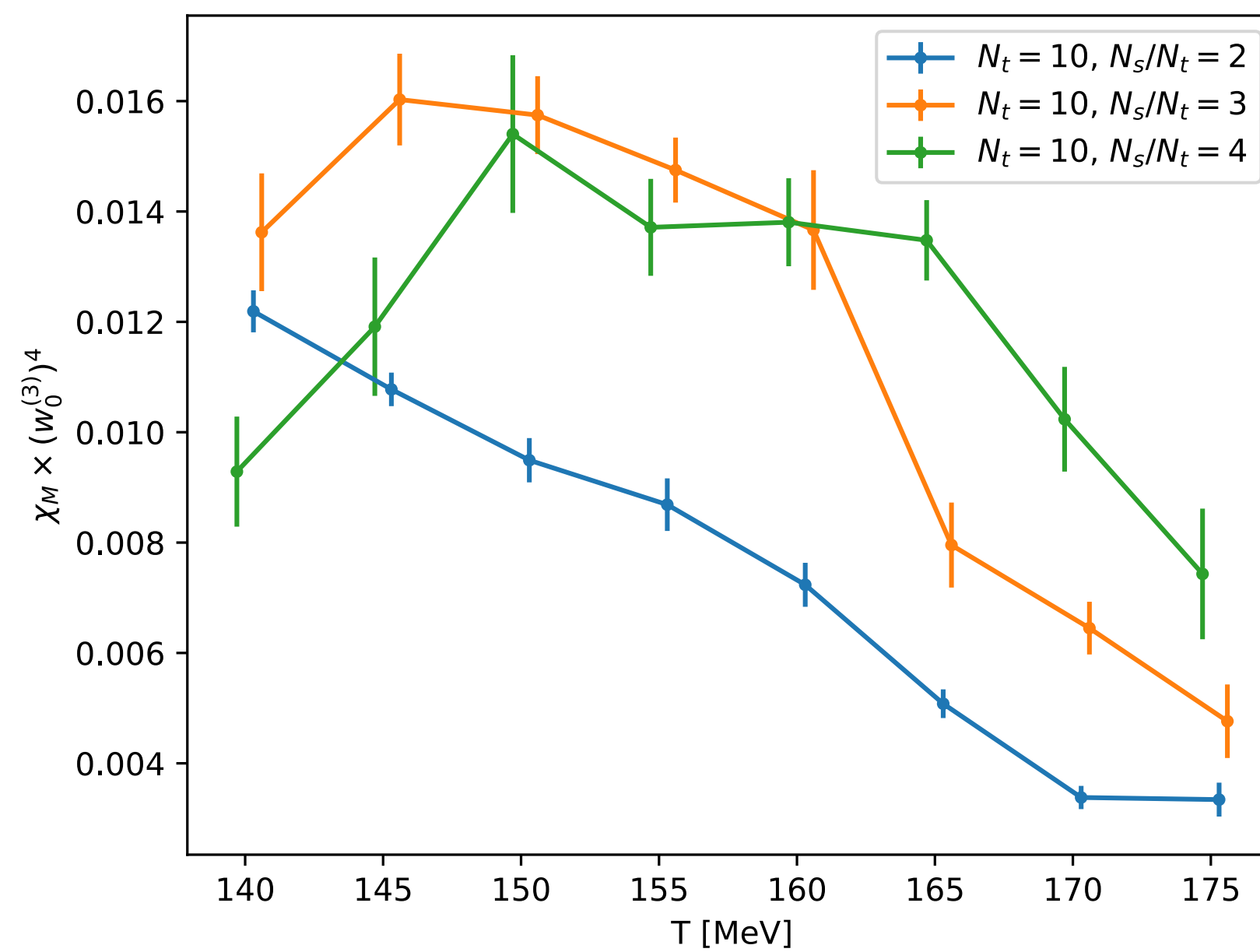
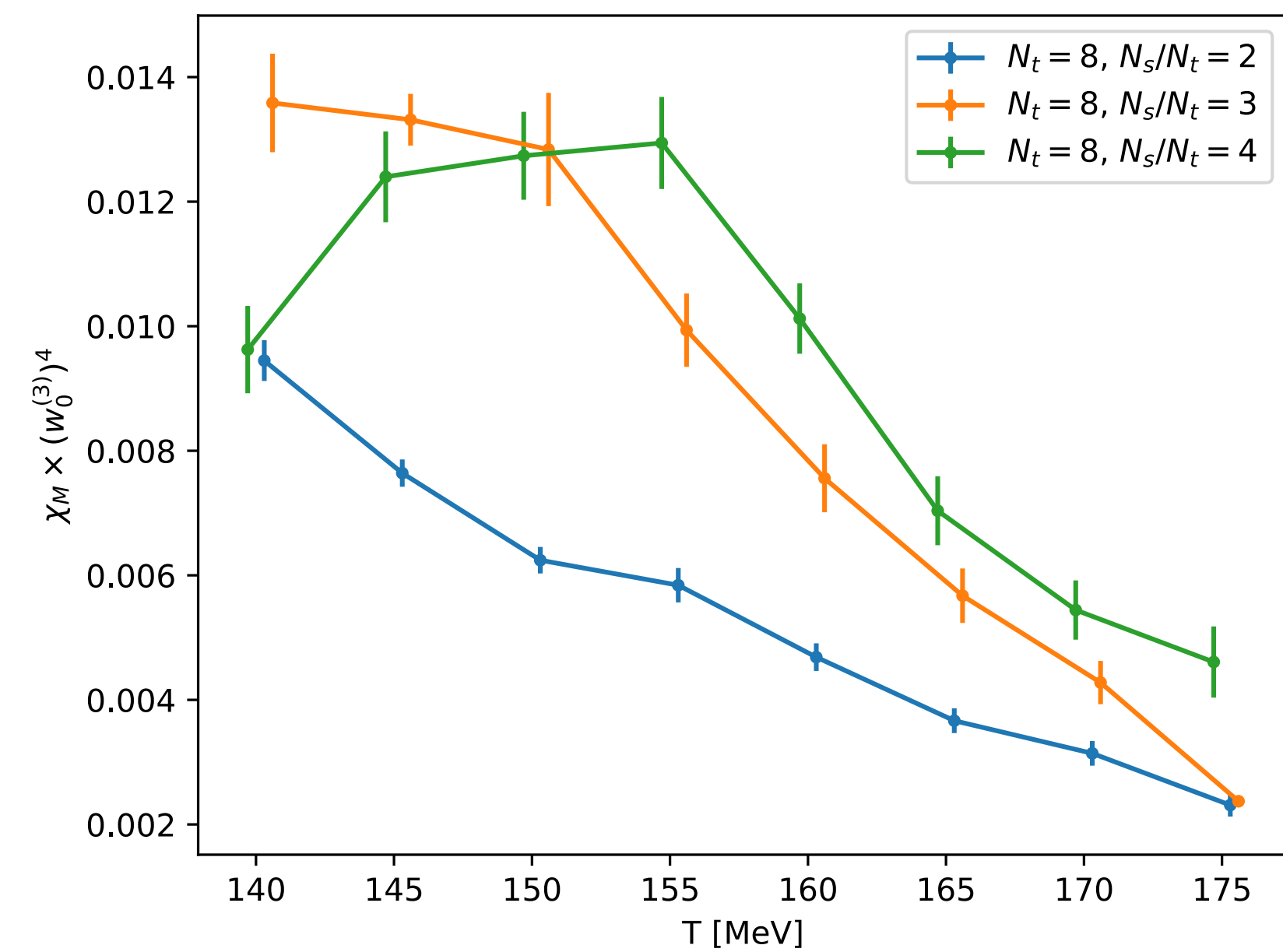
Chiral condensate

- $M = 2 (m_s \langle \bar{\psi} \psi \rangle_l - m_l \langle \bar{\psi} \psi \rangle_s)$
- Large cutoff effect and FV effects
- $T_{pc} \approx 163(6) \text{ MeV}$



Chiral susceptibility

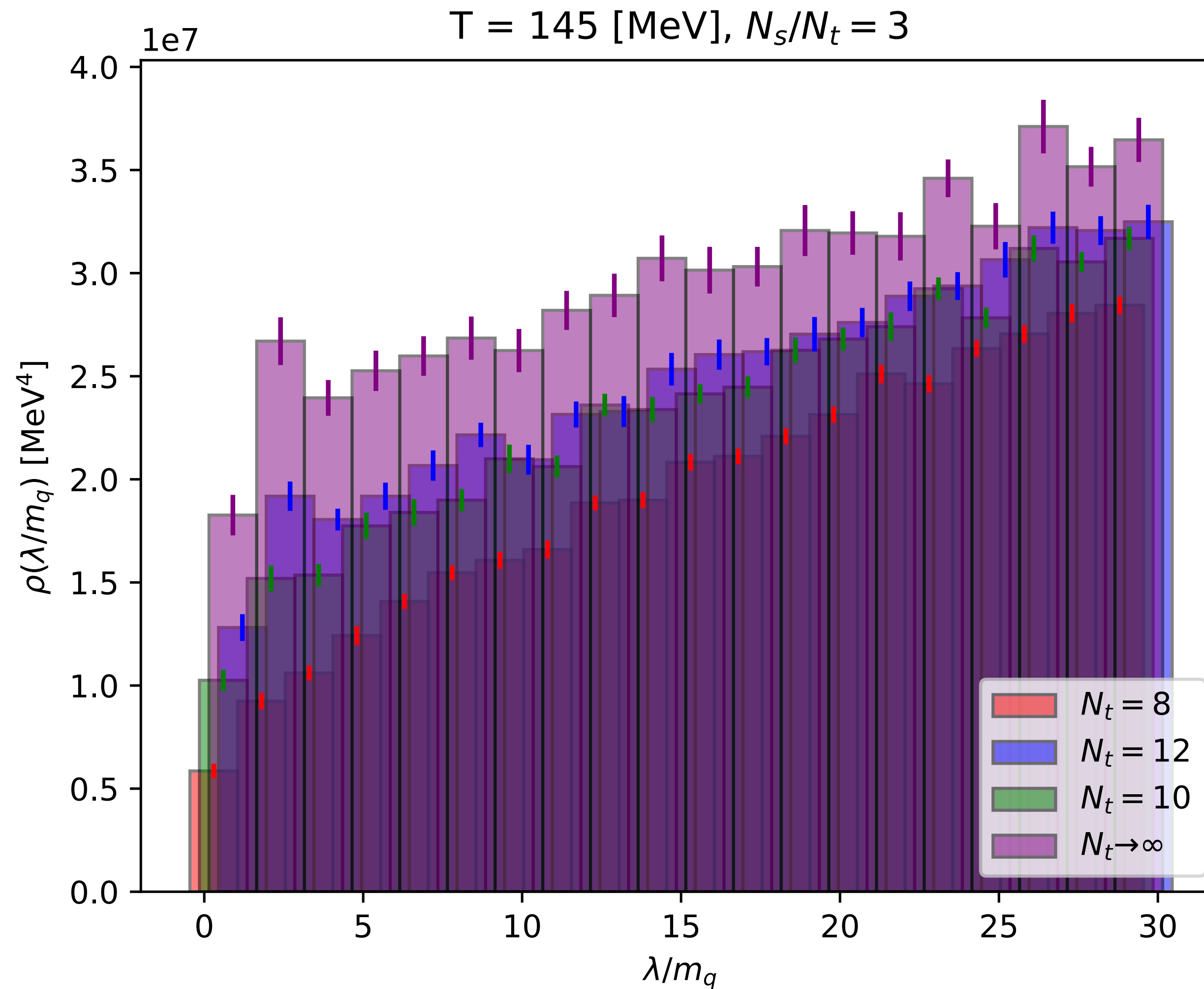
$$\chi_M = m \partial_m M$$



$T_{pc} \sim 160$ MeV

Eigenvalue spectrum

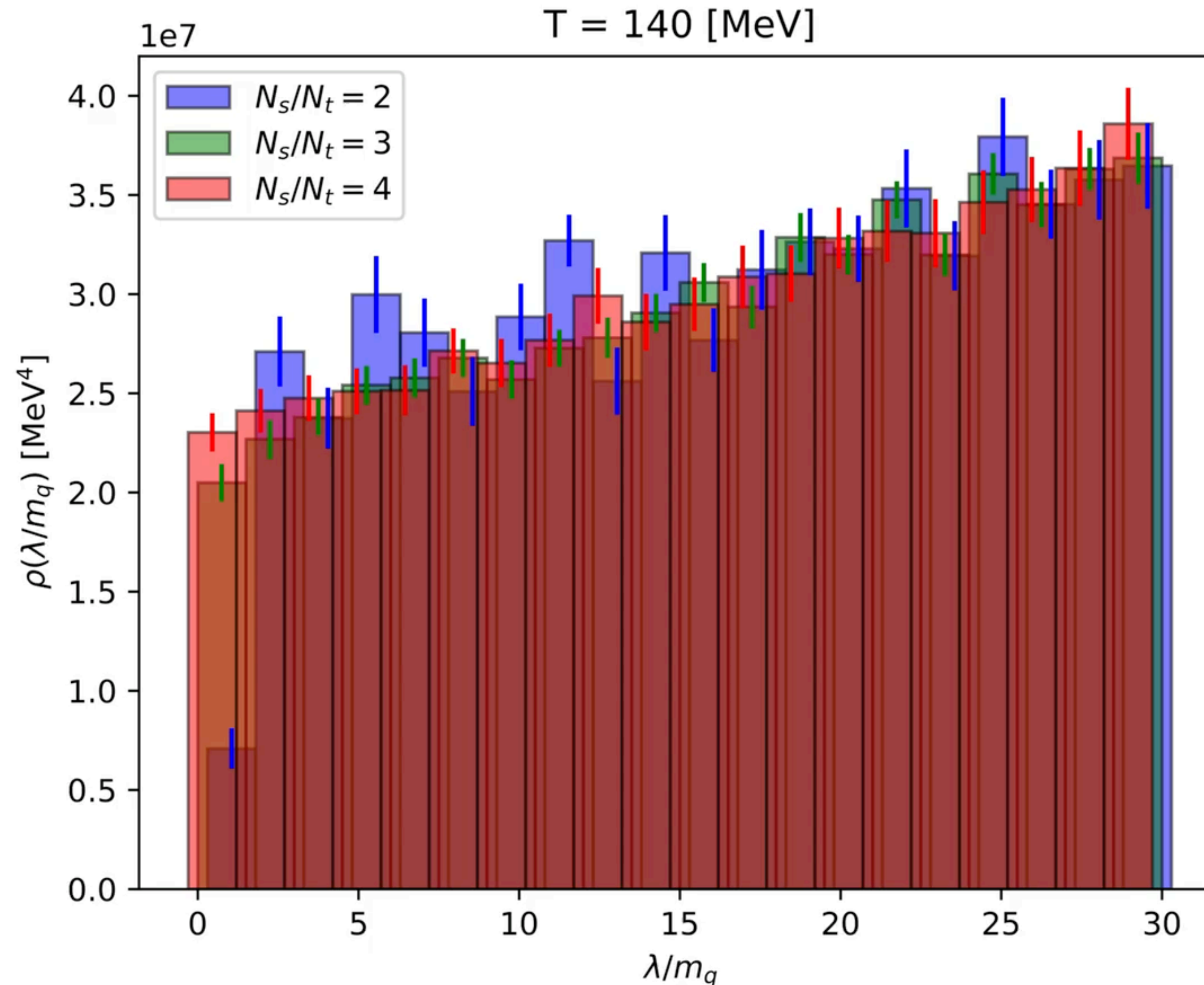
$$D_{\text{ov}}^\dagger(m=0)D_{\text{ov}}(m=0)|e_i\rangle = \lambda_i^2|e_i\rangle$$



Eigenvalue spectrum

$\rho(\lambda \rightarrow 0)$ disappears at $T \sim 160 - 165$ MeV

$$D_{\text{ov}}^\dagger(m=0)D_{\text{ov}}(m=0)|e_i\rangle = \lambda_i^2|e_i\rangle$$



Renormalised condensate from Dirac spectrum

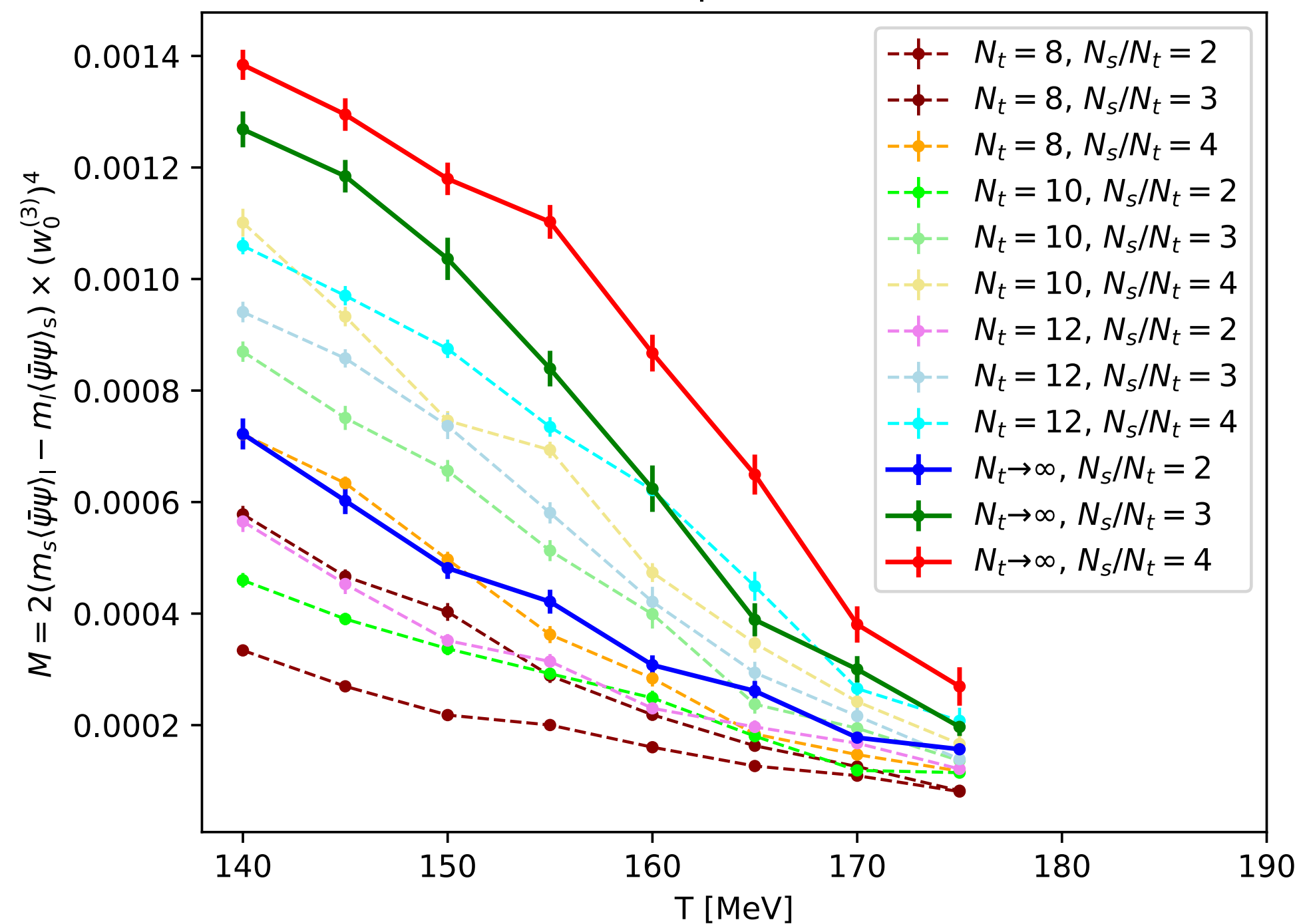
$$D_{\text{ov}}^\dagger(m=0)D_{\text{ov}}(m=0)|e_i\rangle = \lambda_i^2|e_i\rangle$$

$$\bullet \langle \bar{\psi}\psi \rangle_{\text{ren}} = \int_0^{m_s} d\lambda \rho(\lambda) \frac{m_l}{\lambda^2 + m_l^2}$$

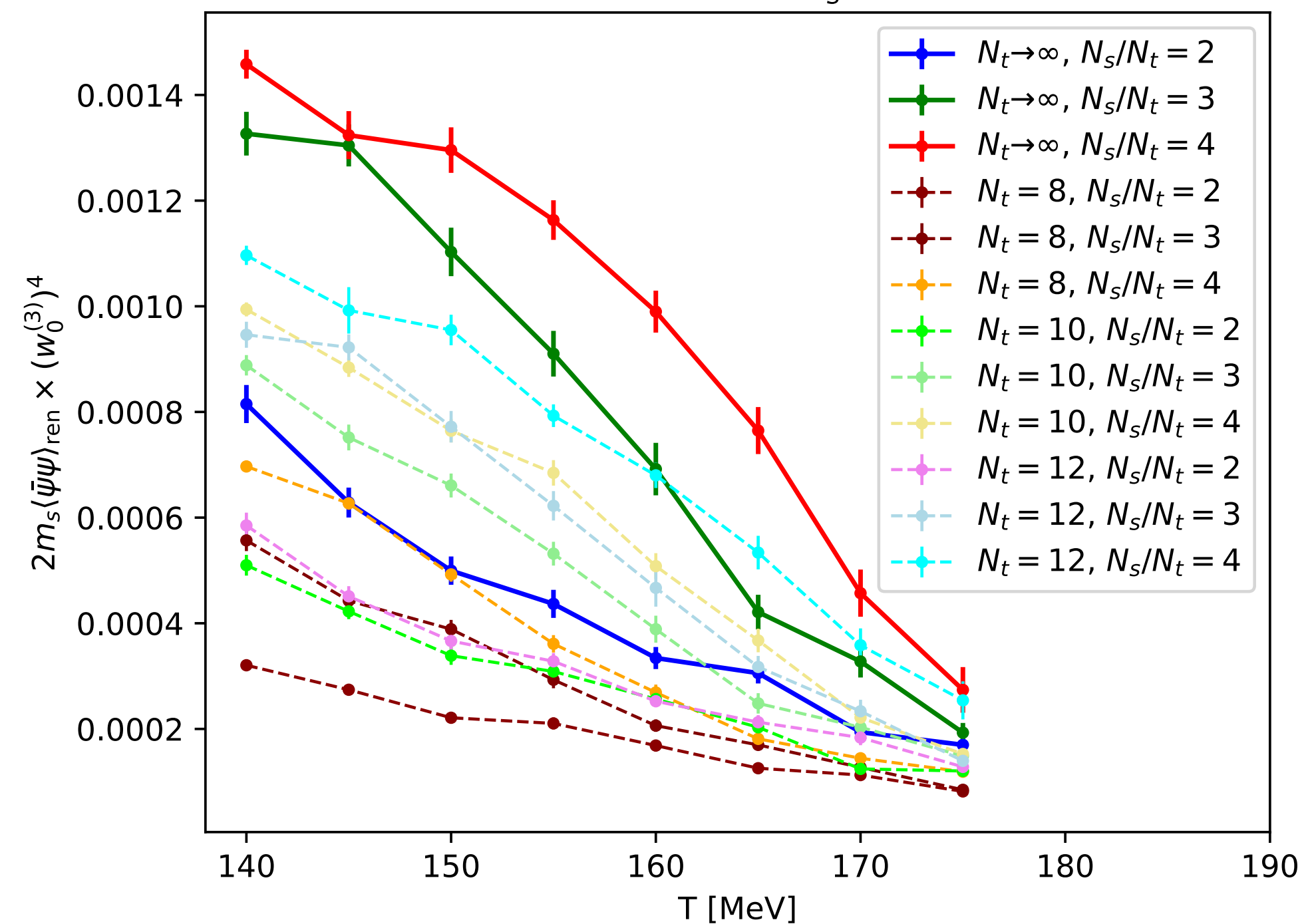
(continuum expression)

- No logarithmic divergence

Overlap fermions

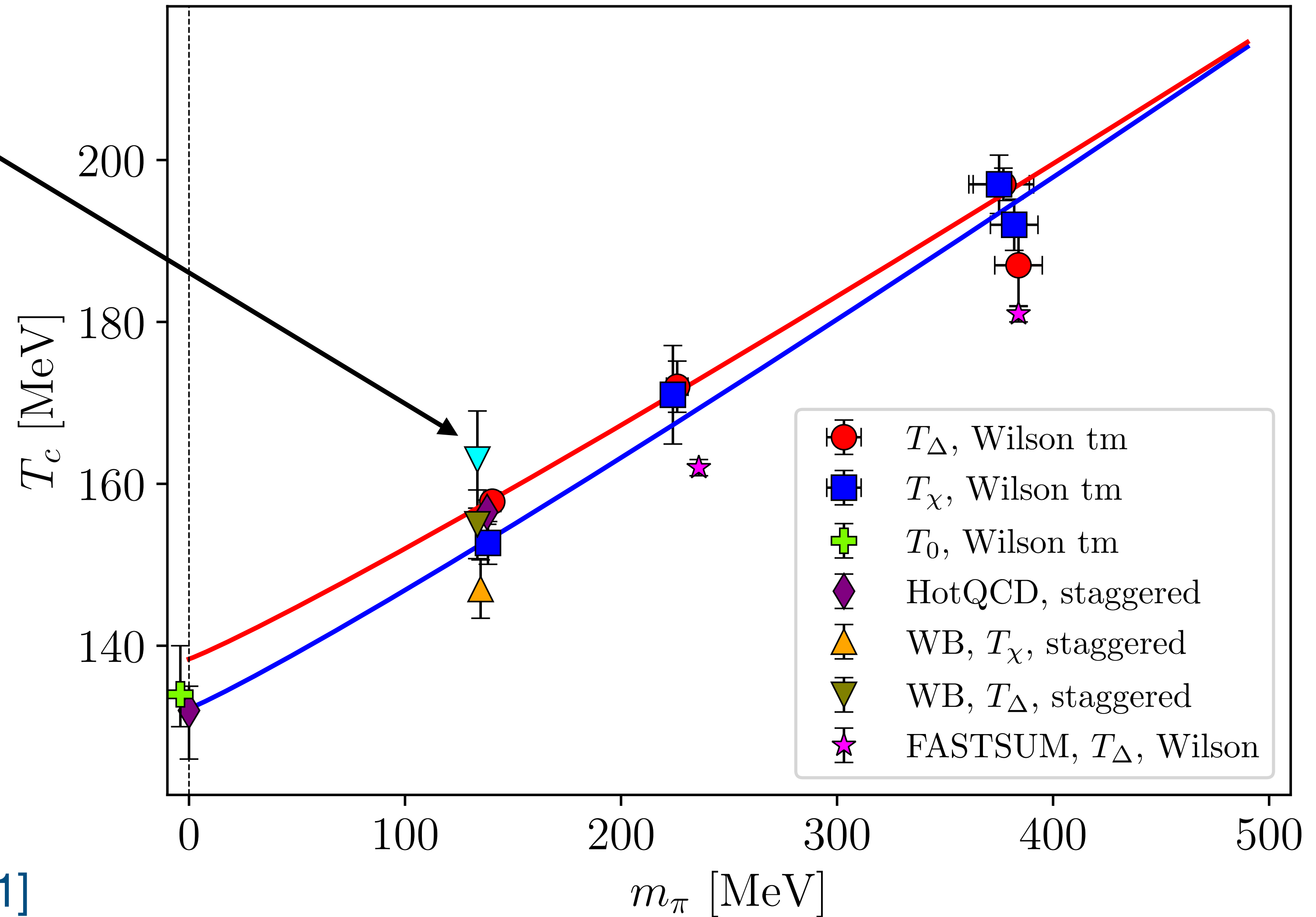


$\lambda_{DD^\dagger} \in [0, m_s^2]$



Critical temperature, other fermions

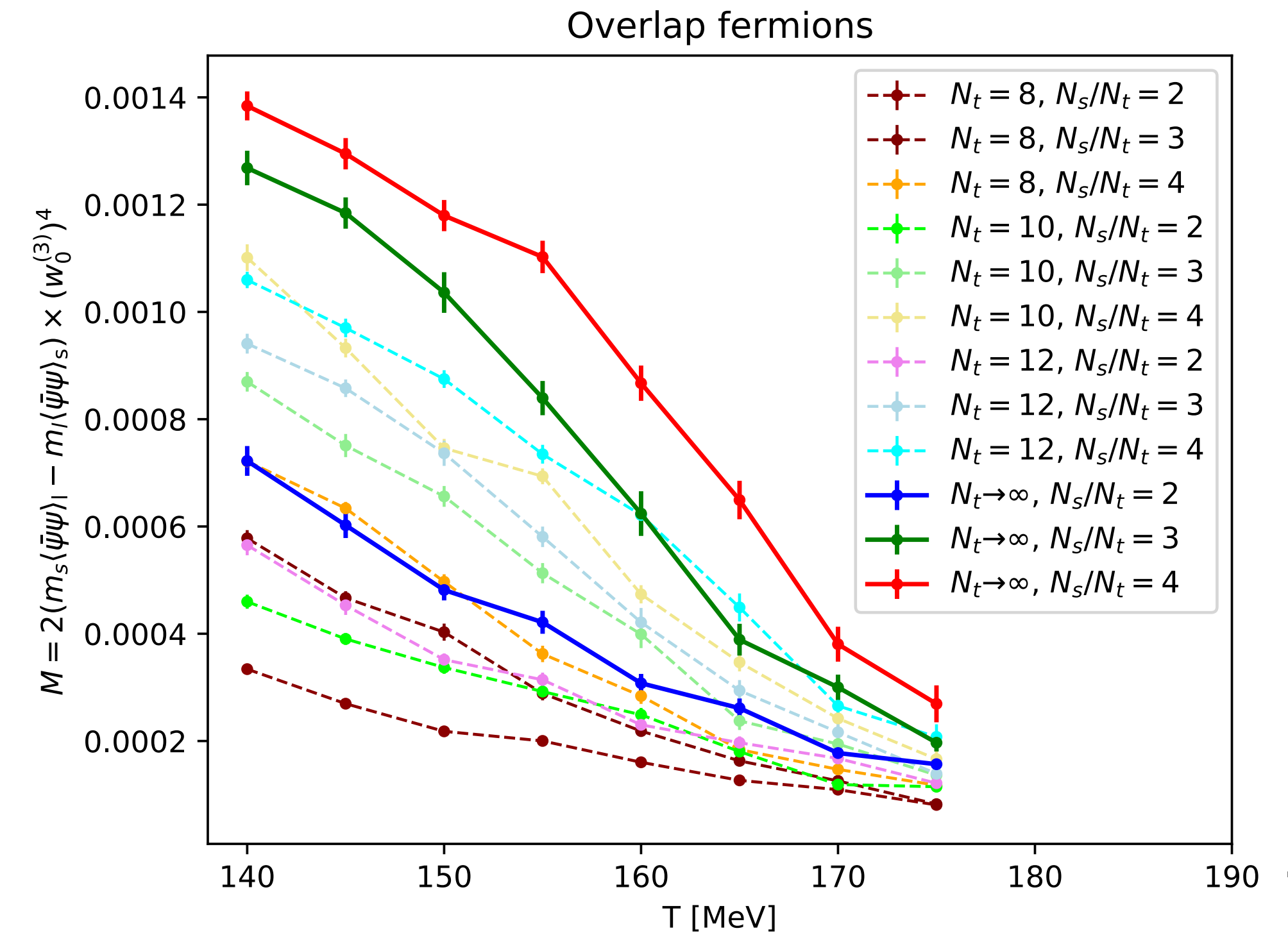
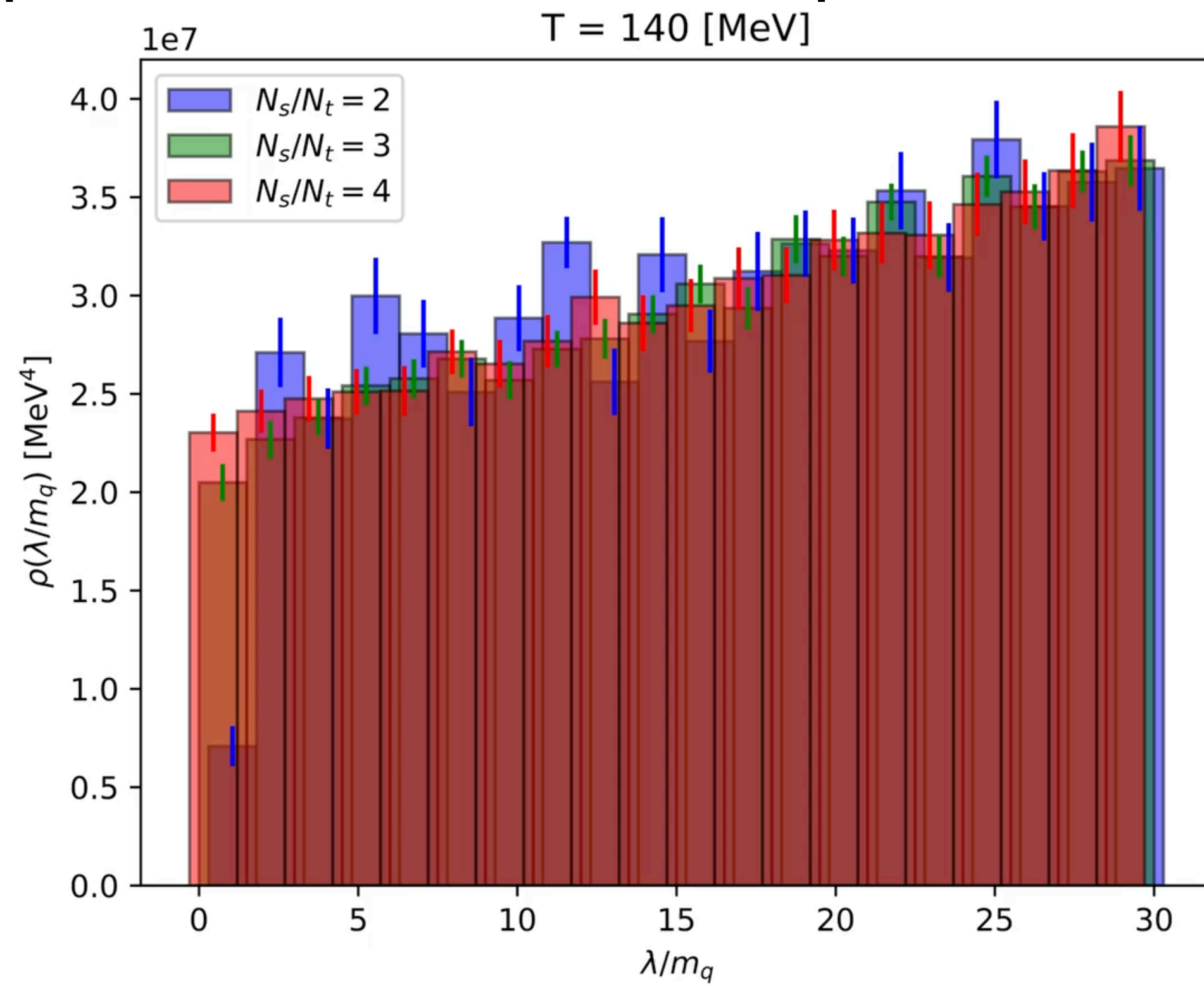
Overlap fermions



[TWEXT, 2021]

Summary

- Thermal QCD phase transition with overlap fermions
- $T_{pc} = 163(6)$ MeV
- Spectrum of the Dirac operator: consistent with the same picture



Summary

Thank you for your attention!

- Thermal QCD phase transition with overlap fermions
- $T_{pc} \approx 163(6)$ MeV
- Spectrum of the Dirac operator: consistent with the same picture

