Nontrivial topology in QCD, the Vacuum Energy and Large scale magnetic field of the Universe

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1. Preliminary: Energy in QCD

- We want to argue that there is a novel type of energy in strongly coupled QCD. This energy has "<u>non-dispersive</u>" nature, and can not be expressed in terms of conventional scattering amplitudes.

- It explicitly contradicts to the "folk theorem" that the S-matrix contains all the information about all physical observables.

- All these novel effects are due to the nontrivial topological sectors in the gauge systems and tunnelling transitions between them.

- The effect is non-local in nature, and can not be expressed in terms of local curvature in gradient expansion. It is expressed in terms of a non-local characteristics of the system -**the holonomy**.

2. Topological susceptibility

- A convenient way to explain the <u>nature</u> of new type of vacuum energy is to study the topological susceptibility (*it is the key element in the resolution of the so-called U(1) problem in QCD, Witten, Veneziano, 1979*).

$$\chi_{YM} = \int d^4x \, \langle q(x), q(0) \rangle \neq 0 \qquad \frac{\partial^2 E_{\rm vac}(\theta)}{\partial \theta^2} = \chi_{YM}$$

To avoid confusion: This is the <u>Wick's</u> T-product, not <u>Dyson's</u>

- χ_{YM} does not vanish, though $q(x) \sim \partial_\mu K^\mu(x)$. It has ``wrong sign", see below. It can not be related to any physical propagating degrees of freedom. Furthermore, it has a pole in momentum space

$$\lim_{k \to 0} \int d^4x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4}$$

- There is a <u>massless</u> pole (Veneziano ghost), but there are <u>no</u> any <u>physical massless</u> states in the system.

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0|q|n\rangle\langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

- conventional physical degrees of freedom always contribute with sign (-) while one needs sign (+) to satisfy WI and resolve the U(1) problem

$$\chi_{non-dispersive} = \int d^4x \, \langle q(x), q(0)\rangle = \frac{1}{N^2}|E_{vac}| > 0$$

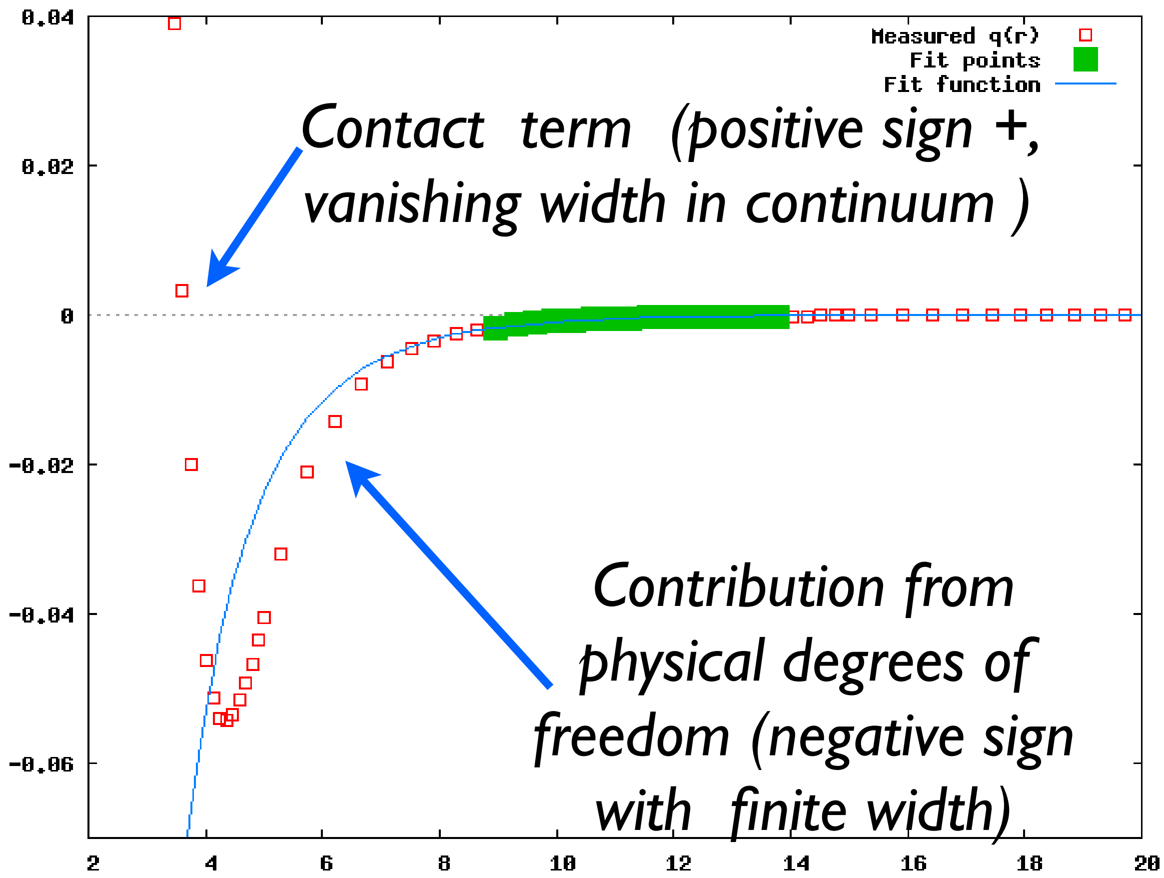
- Conventional terms (related to propagating degrees of freedom) always produce $\exp\left(-\Lambda_{QCD} L\right)$ behaviour at large distances.

- Witten simply postulated this term, while Veneziano assumed the unphysical field, the so-called the "Veneziano ghost" to saturate "wrong" sign in χ.

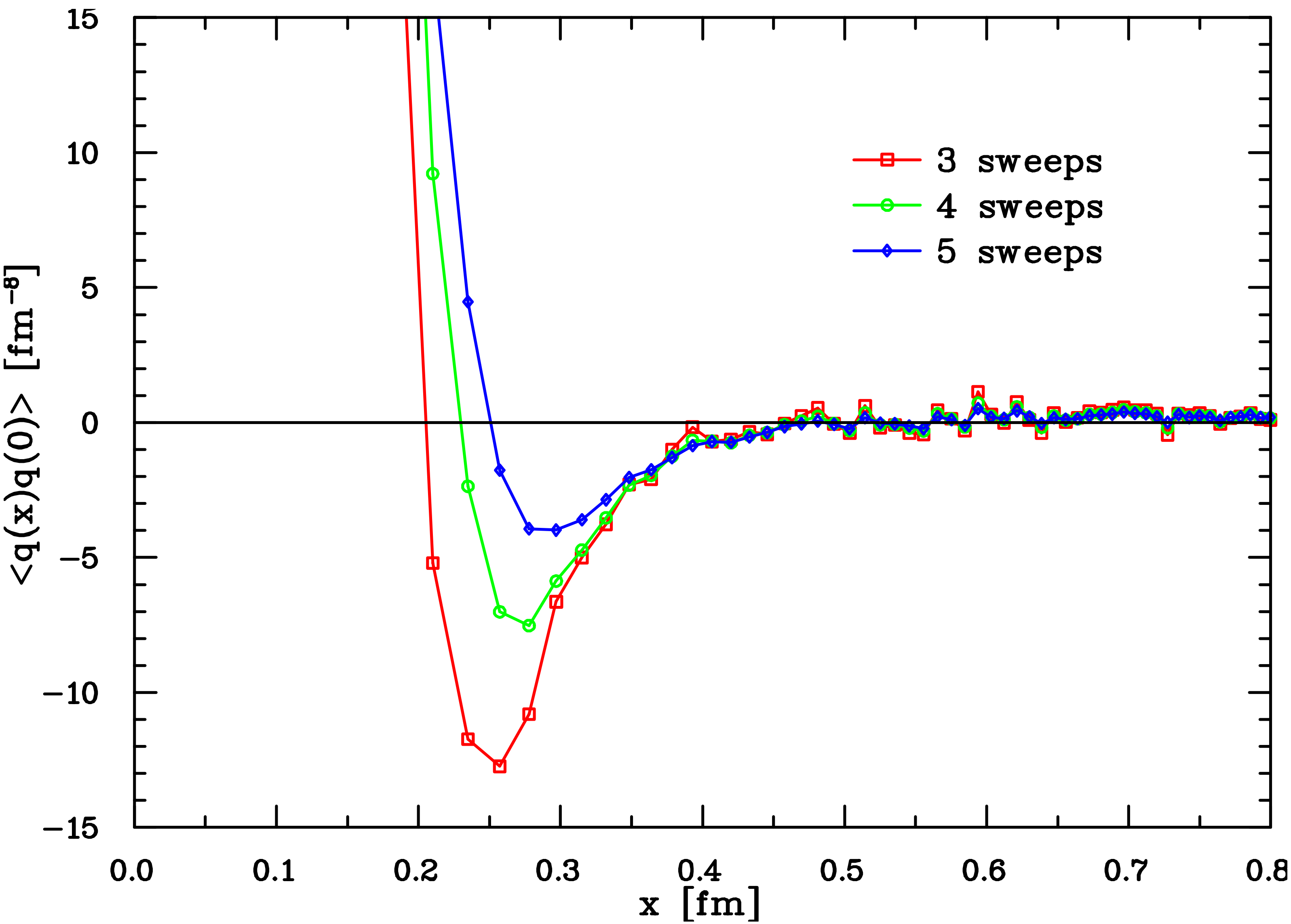
- In some models this contact non-dispersive term with "wrong" sign (+) can be explicitly computed. It is originated from the tunnelling effects between the degenerate topological sectors of the theory.

- These contributions can not be described in terms of conventional degrees of freedom (wrong sign);
- They are inherently non-local in nature as they are related to the tunnelling processes which are formulated in terms of the non-local large gauge transformation operator and holonomy;
- These terms may exhibit the long range features even through QCD has a gap (similar to the CM topological insulators);
- The effects have been explained in terms χ_{YM} . However, the θ -dependent portion of energy $E_{\rm vac}(\theta)$ (relevant for the cosmological applications) has all these unusual features due to the relation

$$\chi_{YM} = \frac{\partial^2 E_{\rm vac}(\theta)}{\partial^2 \theta}\Big|_{\theta=0}$$



The topological susceptibility $\chi(r)$ as a function of r. Wrong sign for χ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r=0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.



The topological susceptibility as a function of r (in physical units "fm"). Plot is adapted from *Misha Ilgenfritz* et al, PRD 2008 and shows sensitivity to smearing sweeps.

3. Definition of the Gravitating Energy

- We assume that the relevant (gravitating) energy which enters the Friedman's equation is the difference $\Delta E = (E_{\rm FLRW}(H) - E_{\rm Mink})$ similar to computations of the Casimir energy, when the difference ΔE is observed. This assumption was, in fact, originally formulated by Zeldovich in 1967.

- We can not (by technical reasons) to perform the computations in FLRW background. However, we can proceed with computations in a toy model formulated on hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$ when role of $H \sim 10^{-33}\ eV$ plays parameter $\kappa \to 0$

- We want to argue that a nontrivial holonomy generates a linear correction $\Delta E(\kappa) \sim \kappa$ in contrast with conventional expectation $\Delta E \sim R\,[\mathbb{H}^3_\kappa] \sim \kappa^2$

- Technically, we want to see a linear (rather than very small quadratic κ^2) correction in the ratio

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq 1 + \mathcal{O}\left(\frac{\kappa}{\Lambda_{QCD}}\right).$$

- If the same pattern persists in real FLRW Universe one could estimate

$$\Delta E_{\rm vac} \sim \kappa \sim L^{-1} \sim \Lambda^4_{QCD}\left(\frac{1}{\Lambda_{QCD} L}\right) \sim (10^{-3}{\rm eV})^4$$

- In other words, we interpret the observed Dark Energy as a modification of the QCD vacuum $\sim \kappa$ energy due to a nontrivial topology (not expressible in terms of local curvature $R\,[\mathbb{H}^3_\kappa] \sim \kappa^2$)

- It has the same non-dispersive nature (can not be expressed in terms of propagating DOF), it is non-local in nature (not expressible in terms of the local curvature), and it has a positive sign.

- Historical comments: many people from different fields had advocated (after *Zeldovich, 1967*) a similar idea on the RHS for the Friedman's equation

$$\Delta E(L) = [E(L) - E_{\rm Mink}]$$

- $$E(L) \equiv -(\beta V)^{-1} \ln \mathcal{Z}$$

- *James Bjorken (particle physics), 2001,*
  Ralf Schuetzhold (GR), PRL, 2002;
  Grisha Volovik (CM physics), 2008 +many more

- I personally adopted this idea in 2009, mostly due to the intense (and never ending) discussions with Grisha Volovik in the relation with his COSLAB (Cosmology in a Laboratory) activities.

4. Holonomy and the linear correction $\kappa \sim 1/\mathcal{T}$ in hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$

- Normally it is expected that all corrections due to the time-dependent (curved) background are proportional to the local curvature $R\,[\mathbb{H}^3_\kappa] \sim \kappa^2$
- We want to test these ideas in gauge theories with nontrivial holonomy. In this case corrections are not reduced to the local observables. The IR regularization plays key role in all computations.
- Specifically, we compute the ratio which explicitly shows the linear correction $\sim \kappa$

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq \left(1 - \frac{\nu\bar{\nu}}{2} \cdot \frac{\kappa}{\Lambda_{QCD}}\right). \quad E_{\rm vac} \equiv -\frac{1}{\beta V} \ln \mathcal{Z}$$

- The computations are based on KvBLL calorons with nontrivial holonomy (Kraan-van Baal-Lee-Lu)

$$\frac{1}{2}TrL = \frac{1}{2}Tr\mathcal{P}\exp\left(i\int_0^\beta dx_4 A_4(x_4, |\mathbf{x}| \to \infty)\right) = \cos(\pi\nu)$$

- Normally, nontrivial holonomy ($\nu \neq 0, 1$) generates zero contribution to the partition function in thermodynamical limit. However, the KvBLL configurations are known to generate IR -finite contribution to the free energy (in huge contrast with conventional instantons).

- The KvBLL configurations can be thought as a superposition of "N" different monopoles which carry the fractional topological charge $Q = \pm 1/N$

- Confinement can be understood as **percolation** of these fractionally charged monopoles which enter the partition function in sets of "N" .

- The crucial role in generating this result is zero-mode determinant. These modes are drastically different in hyperbolic and in Euclidean spaces.

- This difference in these two cases is determined by asymptotic behaviour (different cutoff: $v \longleftrightarrow \rho$)

$$A_4^M(r) = \left(v \coth(vr) - \frac{1}{r}\right) \frac{\tau^3}{2} \qquad \text{on} \quad \mathbb{R}^3$$

$$A_\chi^M(\rho) = \left((\nu+1)\coth\left[(\nu+1)\kappa\rho\right] - \coth\kappa\rho\right)\frac{\kappa\tau^3}{2} \quad \text{on} \quad \mathbb{H}^3_\kappa$$

- Eventually, this difference translates into the difference in fugacities (and <u>vacuum energies</u>) as claimed above

$$f^2 = \left[\frac{4\pi\beta\Lambda_{QCD}^4}{g^4}\right]^2 \cdot \langle \frac{[1+2\pi\nu\bar{\nu}\frac{r_{12}}{\beta}]}{(\Lambda_{QCD}\ r_{12})^{2/3}}[1+2\pi\nu\frac{r_{12}}{\beta}]^{\frac{8}{3}\nu-1}[1+2\pi\bar{\nu}\frac{r_{12}}{\beta}]^{\frac{8}{3}\bar{\nu}-1}\rangle$$

- Though linear terms $\sim r_{12}/\beta$ can be modified as a result of interactions. However, they cannot be exactly cancelled. The effect is proportional to holonomy $\nu(1-\nu)$, and vanishes for conventional instantons with $\nu = 0, 1$

- With these assumptions the non-dispersive correction to the energy (at very small $\kappa \to 0$) is

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq \left(1 - \frac{\nu\bar{\nu}}{2} \cdot \frac{\kappa}{\Lambda_{QCD}}\right).$$

- For the same effect to persist in our FLRW Universe one should have a nontrivial part $\mathbb{S}^1_{\kappa^{-1}}$ (spatial or temporal) along which the holonomy to be computed. The $\mathbb{S}^1_{\kappa^{-1}}$ has Universe size $L = 2\pi\kappa^{-1}$

- No contradictions with observations as long as size is sufficient large $L \geq H^{-1}$, in which case the DE is

$$\Delta E_{\rm vac} \sim \kappa \sim L^{-1} \sim \Lambda^4_{QCD} \left(\frac{1}{\Lambda_{QCD} L}\right) \sim (10^{-3}{\rm eV})^4$$

- *Q: How a system with a gap could be ever sensitive to arbitrary large distances?*

- A1: The long range order in gapped QCD is similar to Aharonov -Casher effect. If one inserts an external charge into superconductor when electric field is screened $\exp(-r/\lambda)$ a neutral magnetic fluxon will be still sensitive to external charge at arbitrary large distances.

- A2: Long range order in the system emerges because the large gauge transformation operator and holonomy are non-local operators sensitive to far IR-physics, similar to "modular operator" in Aharonov -Casher effect.

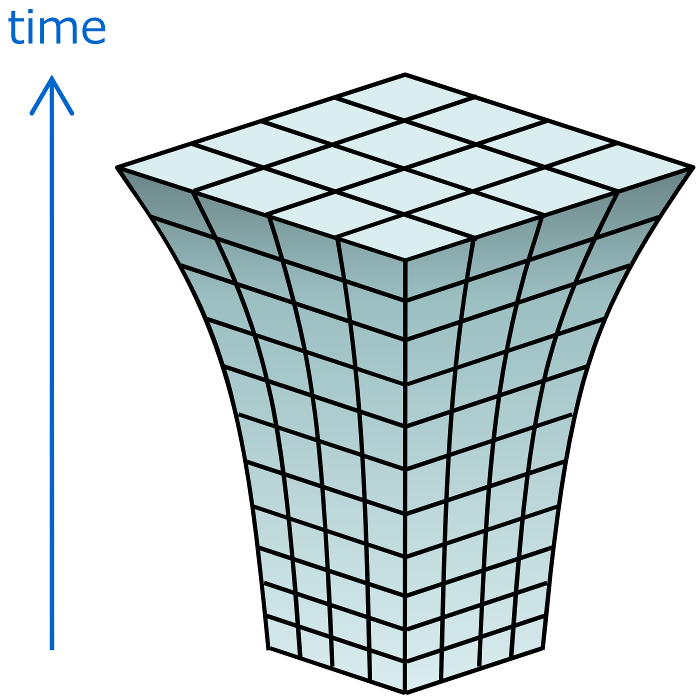
- Are there other hints on a linear dependence on cosmological scale $L \sim H^{-1}$ in a gapped system? (locality suggests quadratic behaviour as $R \sim H^2$)

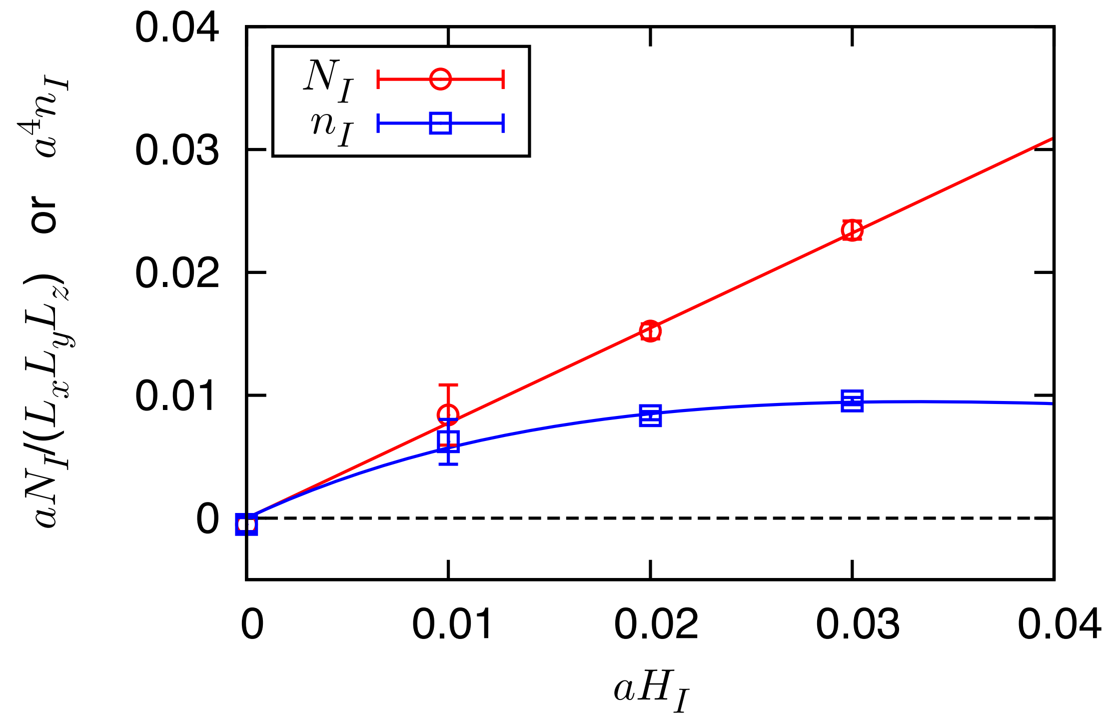
- 1. A number of analytical computations in some simplified models (e.g. deformed QCD).

- 2a. Lattice numerical simulations. In this case the computations of a real part of the energy -momentum tensor $Re\langle T_{\mu\nu}\rangle$ is a hard problem.

- 2b. However, the imaginary (absorptive) portion of the energy-momentum tensor $Im\langle T_{\mu\nu}\rangle$ due to particle production, can be computed, see plot below.

- 2c. Analyticity suggests that the dependence on H must be the same in $Re\langle T_{\mu\nu}\rangle$ and $Im\langle T_{\mu\nu}\rangle$





The plots from A. Yamamoto, arxiv 1405.6665.

1. The expansion in Euclidean space-time was parametrized by the "imaginary" Hubble constant when the lattice action is positively defined;
2. Red curve describes the particle production rate per unit volume per unit time in the background H_I ;
3. The linear dependence on H_I has been computed, $Im[\langle T_{\mu\nu}\rangle] \sim H_I$. It strongly supports our arguments.

5. Applications to the Dark Energy

- Lesson 1: There is a fundamentally new type of the vacuum energy which can not be expressed in terms of any local field.

- Lesson 2: It emerges as a result of tunnelling processes between degenerate topological sectors, and formulated in terms of the "non-dispersive" contact terms and nonlocal holonomy.

- Lesson 3: We identify this new type of energy with cosmological vacuum dark energy (DE).

- The obtained relevant parameters are amazingly close to the observed DE values:

$$\mathcal{T}^{-1} \sim H \sim \frac{\Lambda^3_{QCD}}{M^2_{PL}} \sim 10^{-33} eV, \quad \rho_{\rm DE} \sim H \Lambda^3_{QCD} \sim (10^{-3} eV)^4, \quad \mathcal{T} \sim H^{-1} \sim \frac{M^2_{PL}}{\Lambda^3_{QCD}} \sim 10\,\mathrm{Gyr},$$

- It explicitly shows that the vacuum QCD contribution to the Hubble constant and DE is expressed in terms of the $\Lambda_{\rm QCD}$

- This energy will be eventually transferred to the Maxwell EM fields (so-called helical instability) on the time scale of $\alpha^{-2} H^{-1} \sim \alpha^{-2} 10^{10}$ years

- Result: the large scale magnetic field (with correlation length of entire visible Universe) will be generated.

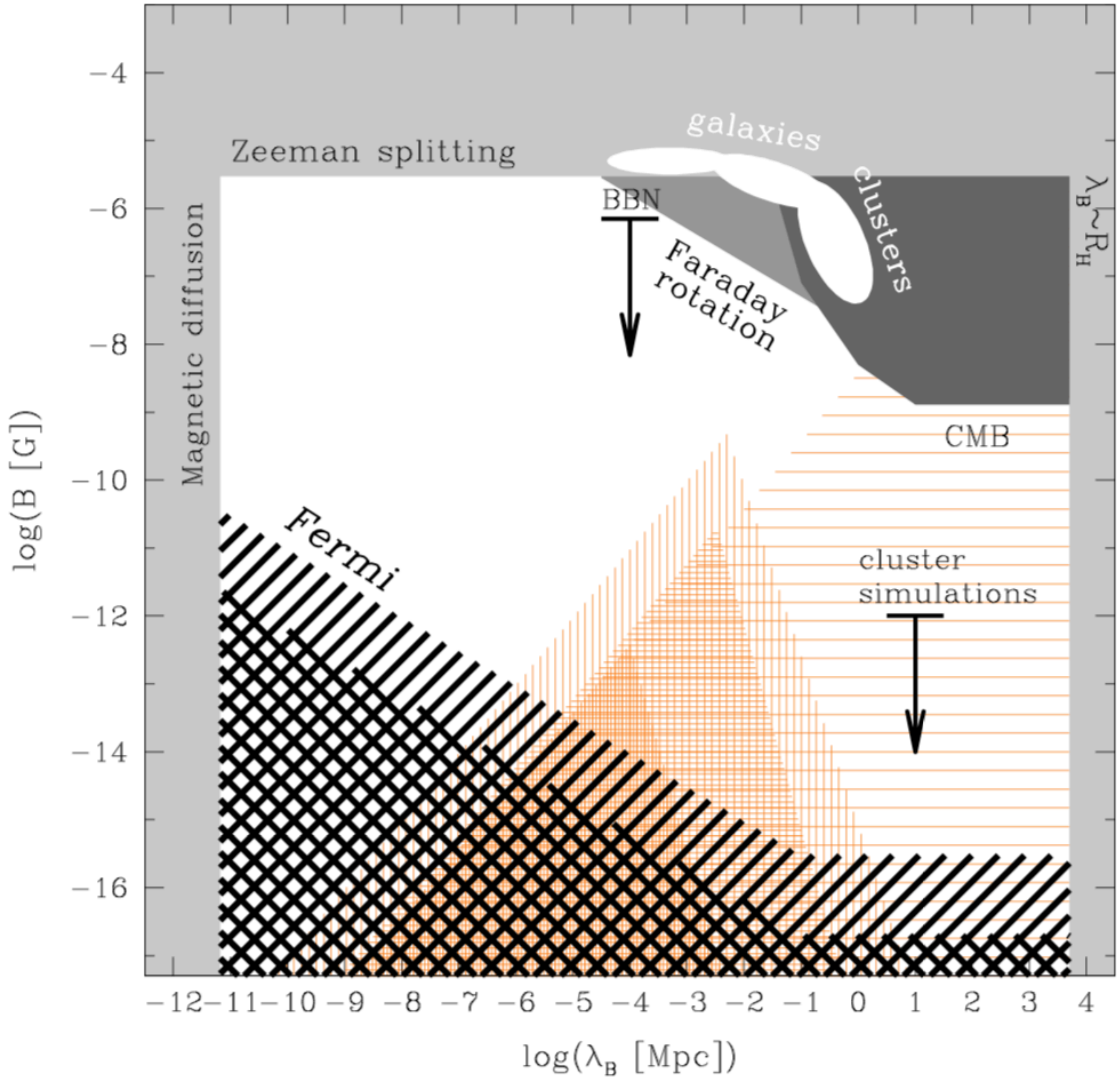
6. Large Scale Magnetic field

- There are many ideas how the observed cosmological magnetic field is generated including primordial mechanisms

- The B field correlated on enormous (Gpc) scale must exist. What is the origin of such correlation?

- We advocate unorthodox mechanism which is dramatically different from all previous approaches: the B- field is generated with enormous coherence scale from DE

- No need for any amplification mechanisms as it is characterized by the largest possible scale at the moment of formation



Constraints on the B field [From Neronov & Vovk]. The B-field correlated on Gpc scales must exist: $10^{-15}G \lesssim B \lesssim 10^{-9}G$

- The starting point is the conventional Effective Lagrangian in terms of the auxiliary field

$$\mathcal{L}_{b\gamma\gamma}(x) = \frac{\alpha}{4\pi} N \frac{\sum_i Q_i^2}{N_f} \left[\theta + b(x,H)\right] \cdot F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$$

- It generates well known extra term with $\mu_5 \sim H_0$

$$\vec{\nabla} \times \vec{B} = \sigma \vec{E} + \frac{\alpha}{2\pi} N \frac{\sum_i Q_i^2}{N_f} \cdot \left(\mu_5 \vec{B}\right), \quad \mu_5 \equiv \langle \dot{b}(x,H) \rangle$$

- Similar equations have been studied before (e.g. dynamical axion field). The difference here is that the μ_5 does not satisfy any classical equation of motion as there is no canonical kinetic term for auxiliary field $b(x,H)$. The μ_5 is background field

- The $b(x,H)$ field was introduced as the Lagrange multiplier to account for tunnelling events

- It is known that the presence of the μ_5 term leads to the helical instability. In the present context it implies the generation of the magnetic field on the huge scales where μ_5 is correlated.

- The instability develops for large wavelengths:

$$B(t) = B_0 \exp(\gamma t), \quad k < \frac{\alpha}{\pi}\mu_5, \quad \mu_5 \sim H$$

- This effect leads to the generation of the magnetic field correlated on the enormous scales.

- The order of magnitude estimates suggest (present time)

$$B \sim 10^{-10} G$$

Concluding comments on Dark Energy & B field

- QCD vacuum energy is different for different background (Minkowski vs deSitter). This difference generates correct order of magnitude for the observed DE today

$$\rho_{\rm DE} \equiv \Delta E \equiv (E_{\rm deSitter} - E_{\rm Mink})$$

$$H \sim \frac{\Lambda^3_{\rm QCD}}{M^2_{\rm PL}} \sim 10^{-33}{\rm eV}, \qquad \rho_{\rm DE} \sim H\Lambda^3_{\rm QCD} \sim (10^{-3}{\rm eV})^4$$

- The vacuum energy will be eventually transferred to the magnetic energy in $\alpha^{-2}H^{-1}{\rm years}$

- The magnetic field at present time could be large: $B \sim 10^{-10}G$. It must be correlated on the scale of the entire visible Universe

Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study this new type of energy:

- When the Maxwell system is formulated on four-torus there will be an <u>extra contribution</u> to the Casimir pressure, not related to the physical propagating photons with two transverse polarizations (4-torus has nontrivial holonomy).

- This setting based on 4-torus topology should be contrasted with conventional setting when the Casimir energy is generated between two conducting plates (trivial holonomy).

- The Maxwell system on the 4-torus shows all signs (degeneracy, etc) which are normally attributed to the topologically ordered systems.