Massive thermal loop integrals
& bulk viscosity in quark matter

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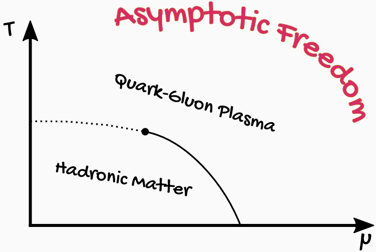
27/7/23, XQCD @ University of Coimbra, Portugal

setup

Perturbative thermal QCD

At sufficiently high chemical potentials μ_q and/or temperatures T, QCD is perturbative due to asymptotic freedom:
$\rightarrow$ Systematic small-g_s expansion through loop calculations



Particularly useful at large μ_q, since lattice is unavailable

Requires adding suitable thermal EFTs:
$\rightarrow$ HTL for density-effects, EQCD for temperature-effects

Neutron stars as probes of dense QCD

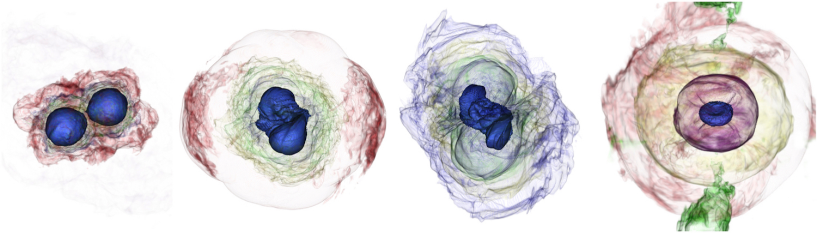
Even NS cores are sparser than the limit of pQCD applicability...
... But pQCD can be used to constrain NS properties[1], in particular the equation of state which gives the mass–radius relation of NSs

— u, d, s active, assume charge neutrality and β equilibrium
$2n_u - n_d - n_s = T^2\mu_e - \mu_e^3/\pi^2$, $\mu_s = \mu_d, \mu_u = \mu_d - \mu_e$ intact —

[1] Unashamed ad for 2307.08734: Brand-new paper where we compute $T = 0$ EOS to $O(g^6 \ln g)$ after years of work—please go look at Kaapo's poster

Neutron star mergers

Observing neutron star mergers has opened the avenue for more than just the EoS: $\rightarrow$ Final part of the talk



mass effects

Tyler Gorda & Saga Säppi,

Phys.Rev.D 105 (2022) 11, arXiv 2112.11472

Quarks have masses

In neutron stars, u, d, s active: u, d are very light (when, say, $\mu_B > 2.6\text{GeV}$); c, b, t are completely decoupled; but s is just right to have a mass that could be relevant...

... Except the effects on the pressure are tiny, so $m_q = 0 \forall q$ is still a common assumption.

Motivation and past efforts

Accuracy is getting better, and esp. transport is often much more sensitive to masses than EoS $\rightarrow$ reason to understand mass effects, especially in collisions where thermal effects are also vital

Massive pressure is known to $O(g_s^4)$ at $T = 0$, and to $O(g_s^2)$ at $T > 0$ …
… but only through cumbersome expressions involving numerical integrals which are inconvenient and (relatively) expensive to evaluate

— In contrast, massless loop integrals are for the most part analytic in terms of standard special functions and their derivatives —

$\rightarrow$ Call for a simpler and faster method

Soft masses

Two trivial observations made with Tyler Gorda during XQCD 2019:

1. The strange quark mass is soft, $m_s = O(g_s \mu_B)$, in neutron star core densities and beyond[2]

— this is a numerical relation, not anything deeper than that —

2. Massive thermal loop integrals can be consistently expanded for a small mass even in dimreg:

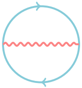
$$\sum\!\!\!\!\!\!\!\int_Q \frac{1}{Q^2 + m^2} = \sum\!\!\!\!\!\!\!\int_Q \frac{1}{Q^2} - m^2 \sum\!\!\!\!\!\!\!\int_Q \frac{1}{Q^4} + \dots$$

— at $T = 0, \mu = 0$, every expanded integral vanishes as scalefree —

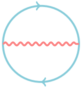
[2] To be more precise, m_q/m_D where m_D is the Debye mass is bounded by unity

Soft-mass expansion

We get an expansion scheme for soft masses, e.g. $(d = 3 - 2\varepsilon)$:



$$
\begin{aligned}
= -\, g^2 d_{\mathrm{A}} \Bigg\{ & \SumInt_{\{\widetilde{P},\widetilde{Q}\}} \frac{2m^2}{(P^2+m^2)(Q^2+m^2)(P-Q)^2} \\
& - (1-\epsilon) \left[\SumInt_{P} \frac{1}{(P^2+m^2)} \right]^2 + 2(1-\epsilon) \SumInt_{P} \frac{1}{P^2} \SumInt_{Q} \frac{1}{(Q^2+m^2)} \Bigg\}
\end{aligned}
$$



$$
\begin{aligned}
= \Big|_{m=0} - 2g^2 m^2 d_{\mathrm{A}} \Bigg\{ & \SumInt_{\{\widetilde{P},\widetilde{Q}\}} \frac{1}{P^2 Q^2 (P-Q)^2} \\
& + (1-\epsilon) \SumInt_{\{\widetilde{P},\widetilde{Q}\}} \frac{1}{P^2 (Q^2)^2} - (1-\epsilon) \SumInt_{P,\{\widetilde{Q}\}} \frac{1}{P^2 (Q^2)^2} \Bigg\} + O(m^4)
\end{aligned}
$$

— Works as long as $\exists r \in \mathbb{Q} : m = O(g^r \mu)$ or $m = O(g^r T)$: feel free to use this to add mass effects to your favourite thermal systems —

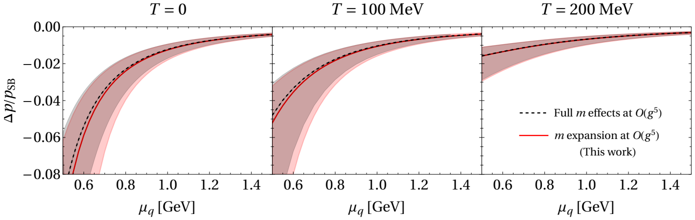
Massive Debye Mass

Fun example of quark mass effects: Change in (LO) Debye mass

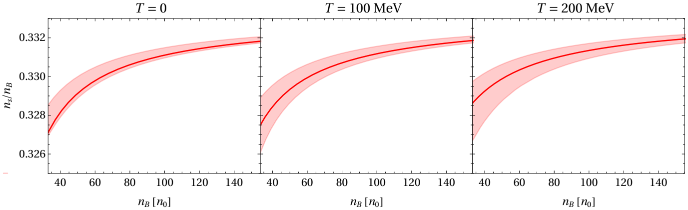
$$
\begin{aligned}
\Delta m_{\mathrm{E}}^2 &\simeq g^2 T^2 \frac{2(d-1)\Gamma\left(2-\frac{d}{2}\right)}{(d-2)\sqrt{\pi}} \\
&\times \sum_f \mathrm{Re}\left[\zeta\left(2-d; \frac{1}{2}+\mathrm{i}\frac{z_f}{2\pi}, \frac{y_f}{2\pi}\right) - \zeta\left(2-d, \frac{1}{2}+\mathrm{i}\frac{z_f}{2\pi}\right)\right] \\
&- g^2 T^2 \frac{2\Gamma\left(2-\frac{d}{2}\right)}{\sqrt{\pi}} \sum_f \left(\frac{y_f}{2\pi}\right)^2 \mathrm{Re}\,\zeta\left(4-d; \frac{1}{2}+\mathrm{i}\frac{z_f}{2\pi}, \frac{y_f}{2\pi}\right) \\
&= -\frac{g^2}{4\pi^2} \sum_f m_f^2 + O(g^2 m_f^4, d-3),
\end{aligned}
$$

where $y_f = \beta m_f$ and $z_f = \beta \mu_f$, and ζ is a generalised ζ function $\zeta(s; z, a) \equiv \sum_{n=0}^{\infty} [(n+z)^2 + a^2]^{-s/2}$

QCD mass effects: strangeness fraction and pressure



normalised pressure-Δ —bands from RG variation



strangeness fraction —1/3 if $m_s = 0$

— The expansion works—Next up, bulk viscosity! —

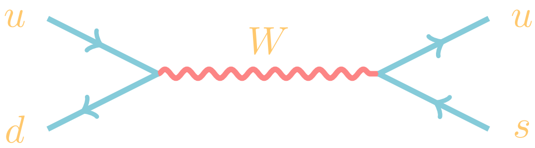
bulk viscosity

Jesús Cruz Rojas, Tyler Gorda, Carlos Hoyos, Niko Jokela, Matti Järvinen, Aleksi Kurkela, Risto Paatelainen, Saga Säppi, and Aleksi Vuorinen: arXiv soon?

Bulk viscosity in neutron stars

Bulk viscosity ζ tells us something about the deformability of plasma

In neutron stars, bulk of the bulk viscosity is associated with the rate λ of the weak process $u + d \to u + s$



— cf. hot plasma, where bulk viscosities can be very different! —

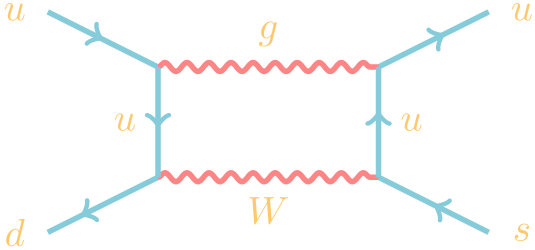
Pocket formula: $$\zeta \propto \lambda \frac{A_1^2}{\omega^2 + C_1^2},$$

with ω a neutron star constant, A_1, C_1 are functionals of pressure...
... and this is where pQCD methodology comes in!

Assumptions and applying the soft-mass expansion

Without the soft-mass expansion, we need numerical derivatives of barely-convergent messy numerical integrals
With it, we simply have derivatives of ζs and can easily evaluate A_1, C_1 for a wide range of densities and temperatures to $O(g_s^5)$

A_1, C_1 computed at three-loop order $[O(g^5)]$ — state of-the-art for QCD when T, μ are both finite — but we are still using leading-order λ: Big assumption but corrections via eg. box diagrams are complicated and left out for now

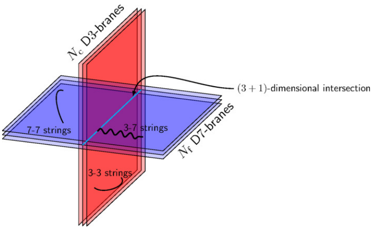


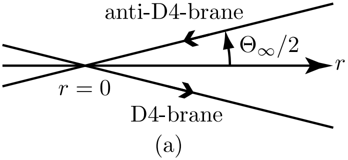
Holography and getting to low densities

Can't go low enough with pQCD $\rightarrow$ need something nonperturbative

Holographic sort-of-like QCD models can access NS core densities

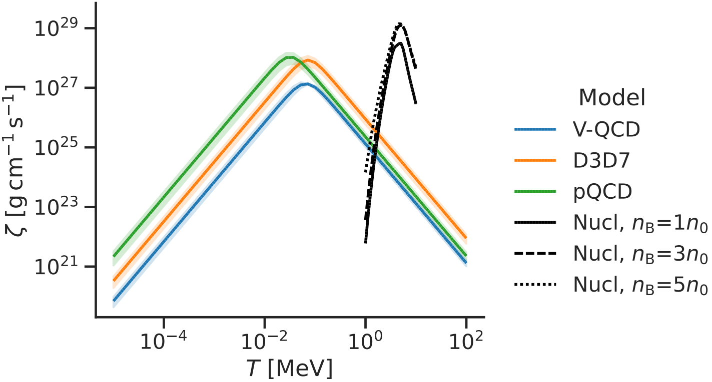
They aren't QCD, but we can compare the two approaches





Specifically, we use D3–D7 (top-down) and VQCD (bottom-up) which are matched to QCD results to fix eg. masses (which will be unrealistically large, but that's why we have pQCD to compare with!)

Comparing methods for the bulk viscosity



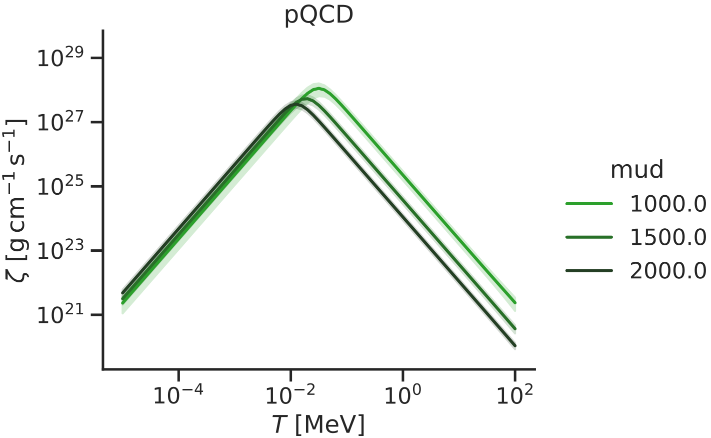
Bulk viscosity at $\mu_B = 450$ (VQCD) 620 (D3–D7) 1000 MeV (pQCD)

Pretty cool: Qualitative agreement with different approaches, robust estimate in the deconfined phase distinct from nuclear models

— Sorry for the units, I was outvoted —

pQCD progression

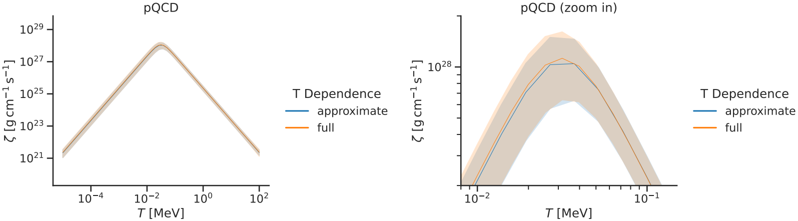
Nice and clear progression as μ is decreased



— Shaded bands show RG scale variation —

T-dependence

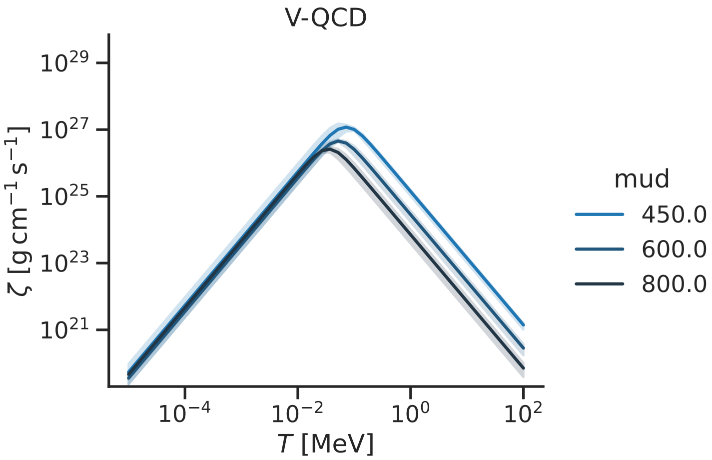
T can just be fixed to some small (but nonzero) value without much effect when computing A_1, C_1



Important to confirm explicitly with pQCD, done in holographic models for computational efficiency

V-QCD progression

V-QCD has the same qualitative behaviour as μ is varied



— should be more realistic than D3–D7 (cf. first plot) —

Conclusions

- Expanding thermal loop integrals gives a simplified way of taking into account the effects of a strange quark mass in neutron stars
- Applicable whenever masses scale with a thermal scale; maybe useful elsewhere?
- Strange quark mass effects give bulk viscosity estimates
- Comparing with holography gives a robust idea for how the viscosity behaves in cool quark matter