Observable Consequences of Partial Thermalization in Relativistic Nuclear Collisions

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Motivation

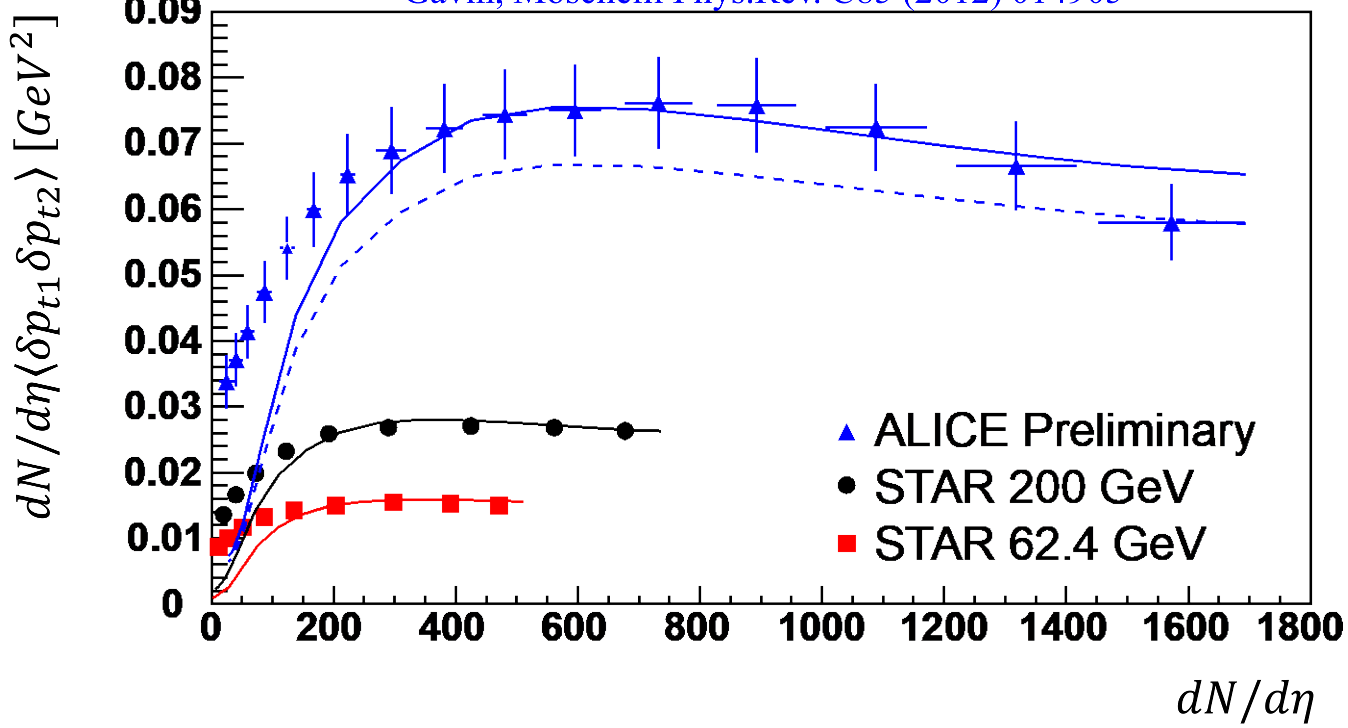
What are the observable consequences of incomplete thermalization of the medium created in nuclear collisions?

Net correlations of transverse momentum fluctuations

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$$

Gavin, Moschelli Phys.Rev. C85 (2012) 014905



STAR, Phys.Rev.C 72 (2005) 044902
ALICE preliminary, since published in Eur. Phys. J. C 74 (2014) 3077

Motivation

What are the observable consequences of incomplete thermalization of the medium created in nuclear collisions?

Net correlations of transverse momentum fluctuations

$$\langle \delta p_{T1}\delta p_{T2}\rangle = \frac{\int\int [\mathbf{corr.\,fn.}]\delta p_{T1}\delta p_{T2} d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2}{\langle N(N-1)\rangle}$$

$$\delta p_{Ti} = p_{Ti} - \langle p_T\rangle$$

$$d\boldsymbol{\omega} = d^3\boldsymbol{x} d^3\boldsymbol{p}$$

$d^3\boldsymbol{x} = \boldsymbol{p}^{\mu}\boldsymbol{d\sigma_{\mu}}/\mathbf{E}$ Cooper-Frye freeze-out

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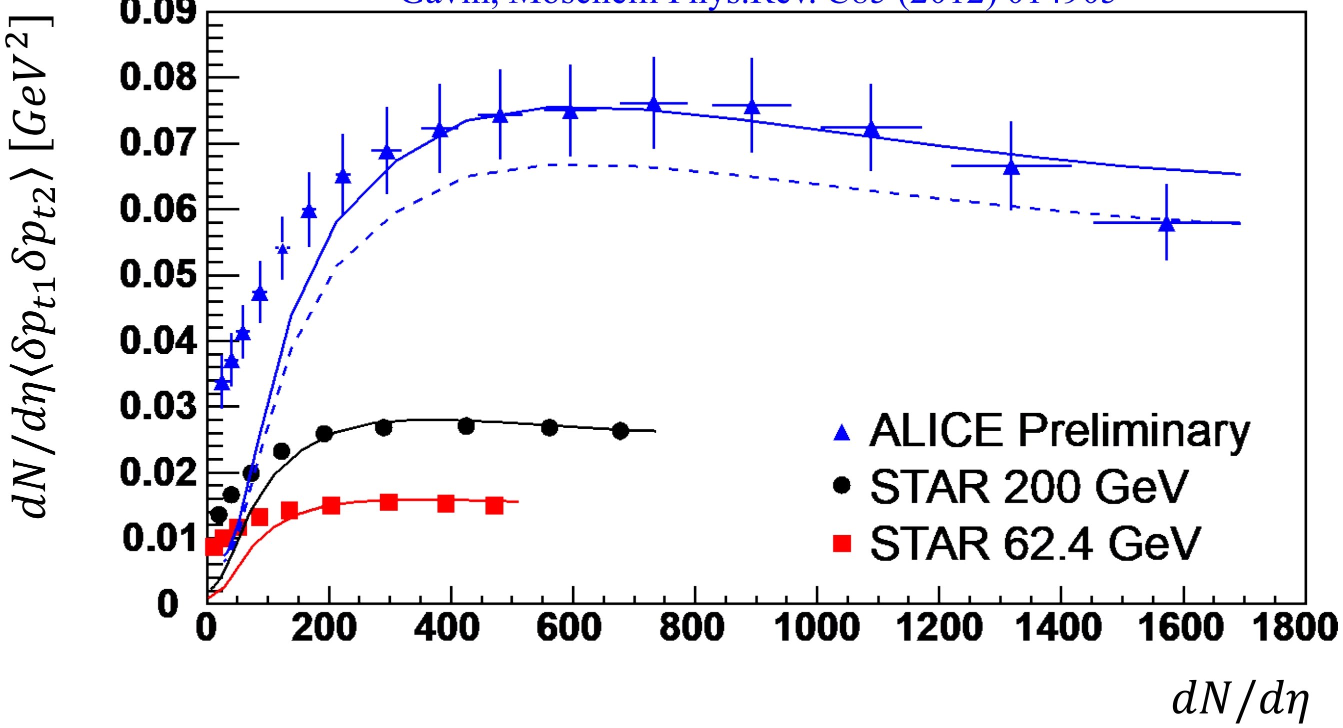

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Boltzmann Equation Relaxation Time Approximation

Boltzmann Eq.
Relaxation Time Approx.

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}\cdot\boldsymbol{\nabla}\right) f(\mathbf{p},\mathbf{x},t) = -\nu[\, f(\mathbf{p},\mathbf{x},t) - f^{eq}(\mathbf{p},\mathbf{x},t)\,]$$

Relaxation time ν^{-1}
Drift velocity $\mathbf{v_p} = \mathbf{p}/E$
Temperature T
Velocity $\boldsymbol{u}$
Chemical Potential μ

Local equilibrium distribution

$$f^{eq} = \exp\{-\gamma(E - \mathbf{p}\cdot\boldsymbol{u} - \mu)/T\}$$

Conservation laws require we choose T, $\boldsymbol{u}$, μ so that f^{eq} gives the same energy, momentum, and particle density as f. Use eigenfunctions φ_i with zero eigenvalue corresponding to the conserved quantities to define a projection operator.

$$P = \sum_{i=1}^{5} c(\mathbf{x},t)\varphi_i$$

$$Pf = f^{eq}$$

Linearized Boltzmann difference equation:

$$f(t+\Delta t) - f(t) = -\nu(1-P)f(t)\Delta t$$

Gavin, Moschelli, Zin Phys. Rev. C95 (2017) 6, 064901

Single Particle Equation

Difference equation $$f(t+\Delta t) - f(t) = -\nu(1-P)f(t)\Delta t$$

Gives $$\frac{d}{dt}\langle f\rangle = -\nu(1-P)\langle f\rangle$$

Solve using the method of characteristics to find

$$f(\mathbf{p},\mathbf{x},t) = \underset{\substack{\uparrow \\ \text{initial conditions}}}{f_0}\big(\mathbf{p},\mathbf{x}-\mathbf{v_p}t\big)S(t,t_0) + \underset{\substack{\uparrow \\ \text{equilibrium}}}{f^{eq}}\big(\mathbf{p},\mathbf{x}-\mathbf{v_p}t\big)\big(1-\underset{\substack{\uparrow \\ \text{survival probability}}}{S(t,t_0)}\big)$$

S is the probability particles escape the collision volume without suffering any collisions

$$S(t,t_0) = e^{-\int_{t_0}^{t}\nu(t')dt'}$$

As thermalization proceeds $S \to 0$.

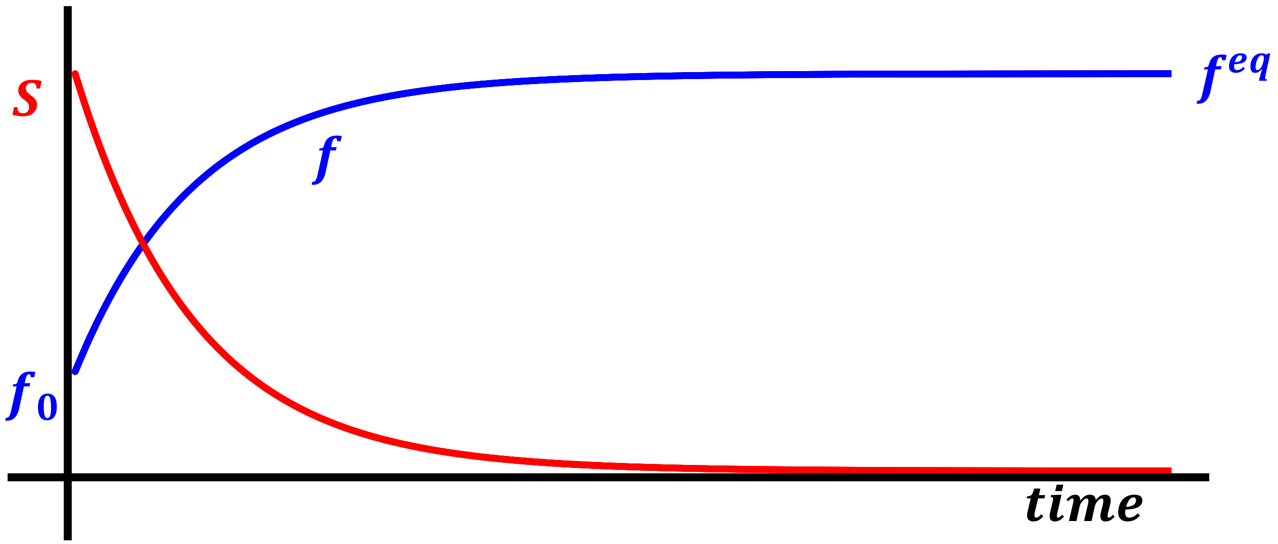
Survival Probability

$$f(\mathbf{p},\mathbf{x},t) = f_0(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)S(t,t_0) + f^{eq}(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)\big(1-S(t,t_0)\big)$$

initial conditions

equilibrium

survival probability



survival probability

$$S(t,t_0) = e^{-\int_{t_0}^{t}\nu(t')dt'}$$

$S = 1 \rightarrow$ no interaction
$S = 0 \rightarrow$ local equilibrium

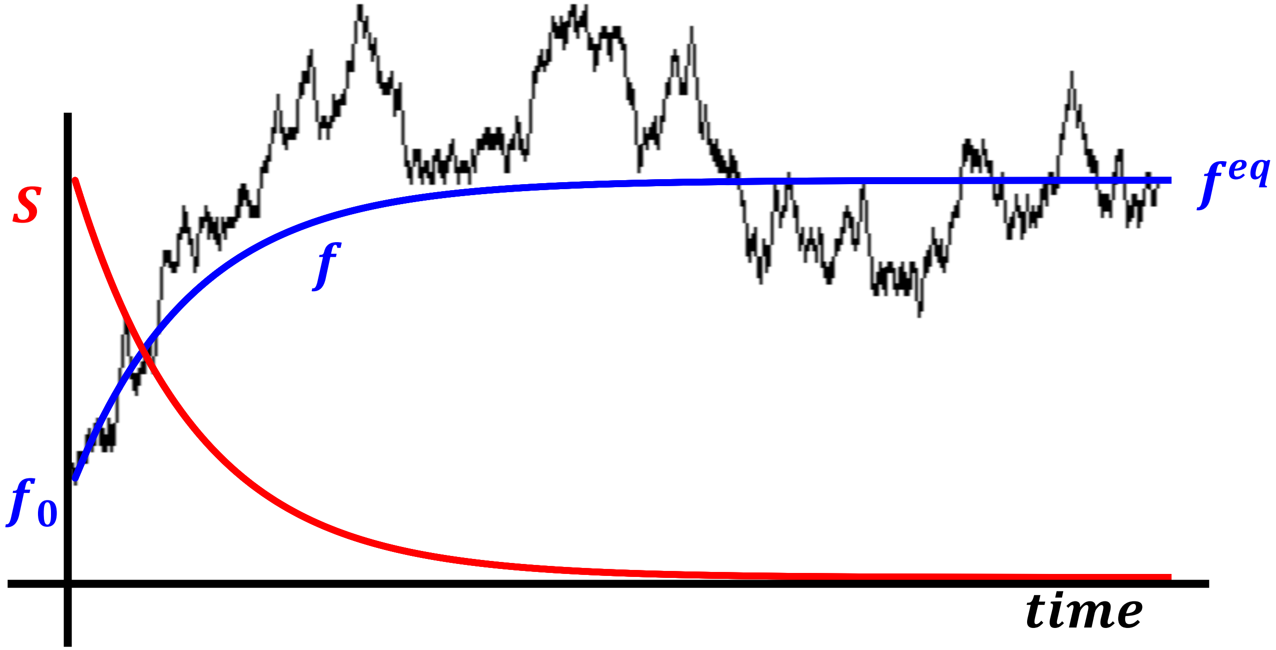
Survival Probability

$$f(\mathbf{p},\mathbf{x},t) = f_0(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)S(t,t_0) + f^{eq}(\mathbf{p},\mathbf{x}-\mathbf{v_p}t)(1-S(t,t_0))$$

initial conditions

equilibrium

survival probability



survival probability

$$S(t,t_0) = e^{-\int_{t_0}^{t}\nu(t')dt'}$$

$S = 1$ → no interaction
$S = 0$ → local equilibrium

Including Noise

$$\frac{d}{dt}\langle f\rangle = -\nu(1-P)\langle f\rangle + \boldsymbol{noise}$$

Linearized Boltzmann-Langevin Equation:

$$f(t+\Delta t) - f(t) = -\nu(1-P)f(t)\Delta t + \Delta W(t)$$

Stochastic term $\Delta W(t)$ represents a random change to f at each time step.

Average of noise $\langle \Delta W\rangle = 0$

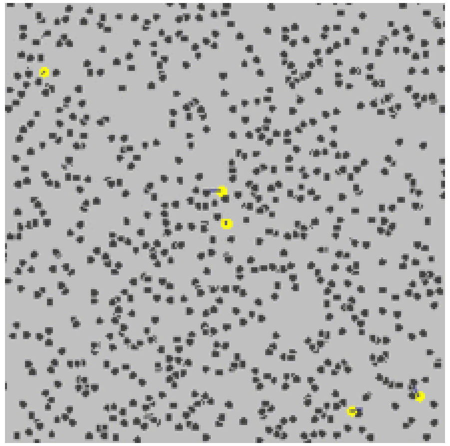
Noise doesn’t effect the single particle distribution

Brownian Motion

$$\underbrace{\mathrm{v}(t+\Delta t)-\mathrm{v}(t)}_{\text{change in velocity}} = -\gamma \mathrm{v}(t)\Delta t + \Delta W(t)$$

friction

collisions

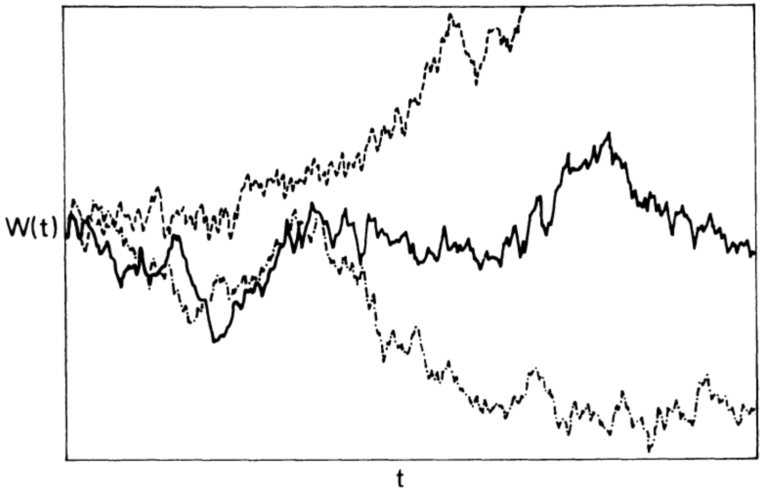


Average of noise $\langle \Delta W \rangle = 0$

Variance of noise $\langle \Delta W^2 \rangle = \Gamma \Delta t$

“strength” of the noise

Using the fluctuation dissipation theorem one can find $\Gamma = 2\gamma/m$



Gardiner, Handbook of Stochastic Methods 3ed

Two-Particle Equation

Two-particle correlations

$$G_{12} = \langle f_1 f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle - \langle f_1 \rangle \delta(1-2)$$

$$\delta(1-2) = \delta(\mathbf{x}_1 - \mathbf{x}_2)\delta(\mathbf{p}_1 - \mathbf{p}_2)$$

Using the Itô product rule:

$$\Delta\langle f_1 f_2 \rangle = \underbrace{\langle f_2 \Delta f_1 \rangle + \langle f_1 \Delta f_2 \rangle}_{-\nu(2 - P_1 - P_2)\langle f_1 f_2 \rangle} + \underbrace{\langle \Delta f_1 \Delta f_2 \rangle}_{\Gamma \Delta t}$$

$$\Gamma = \nu P_1 P_2 \big(\langle f_1 \rangle - f_1^{eq}\big)\delta(1-2)$$

Using the Itô product rule and the Boltzmann-Langevin Eq.

$$\left(\frac{d}{dt} + \nu(2 - P_1 - P_2)\right) G_{12} = \nu P_1 P_2 \big(\langle f_1 \rangle - f_1^{eq}\big)\delta(1-2)$$

Using the method of characteristics again

$$\boldsymbol{G_{12} = G_{12}^{eq} + A_{12} S + B_{12} S^2}$$

The initial phase space distribution determines the coefficients A_{12} and B_{12} as functions of the momenta and initial positions

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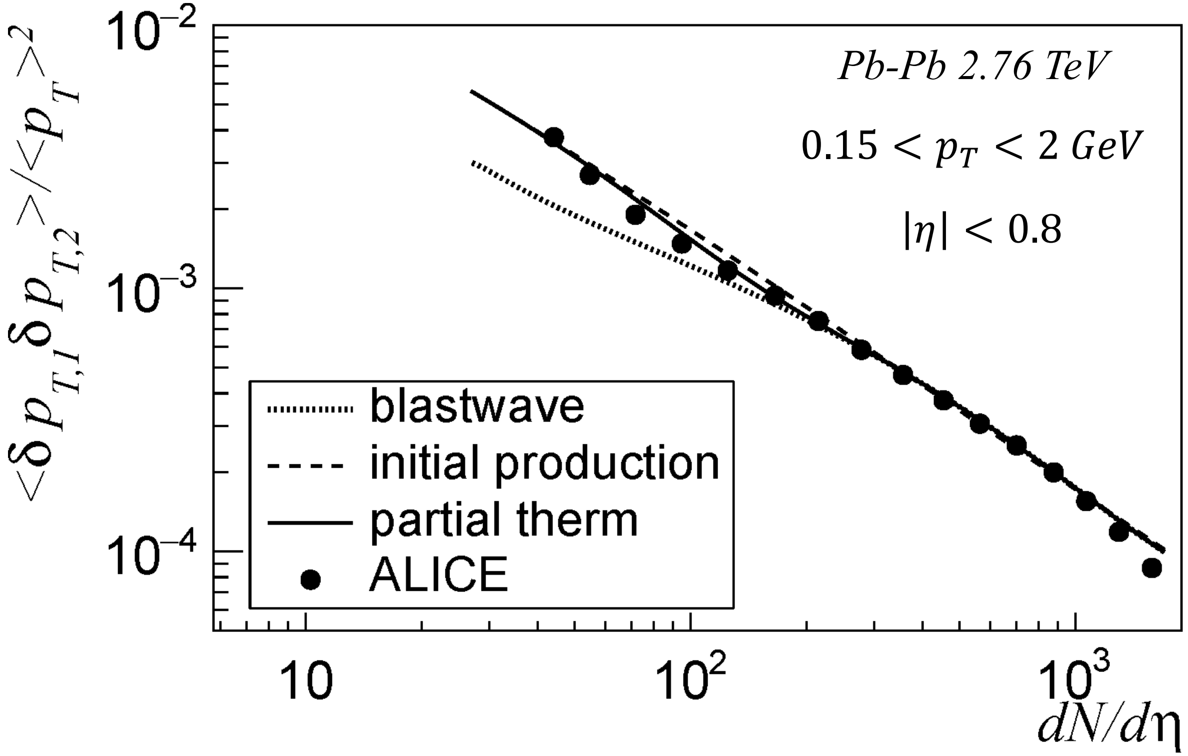
Partial Thermalization

$$G_{12} = G_{12}^{eq} + A_{12}\boldsymbol{S} + B_{12}\boldsymbol{S}^2$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\int\int G_{12}\, \delta p_{T1} \delta p_{T2}\, d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2}{\langle N(N-1) \rangle}$$

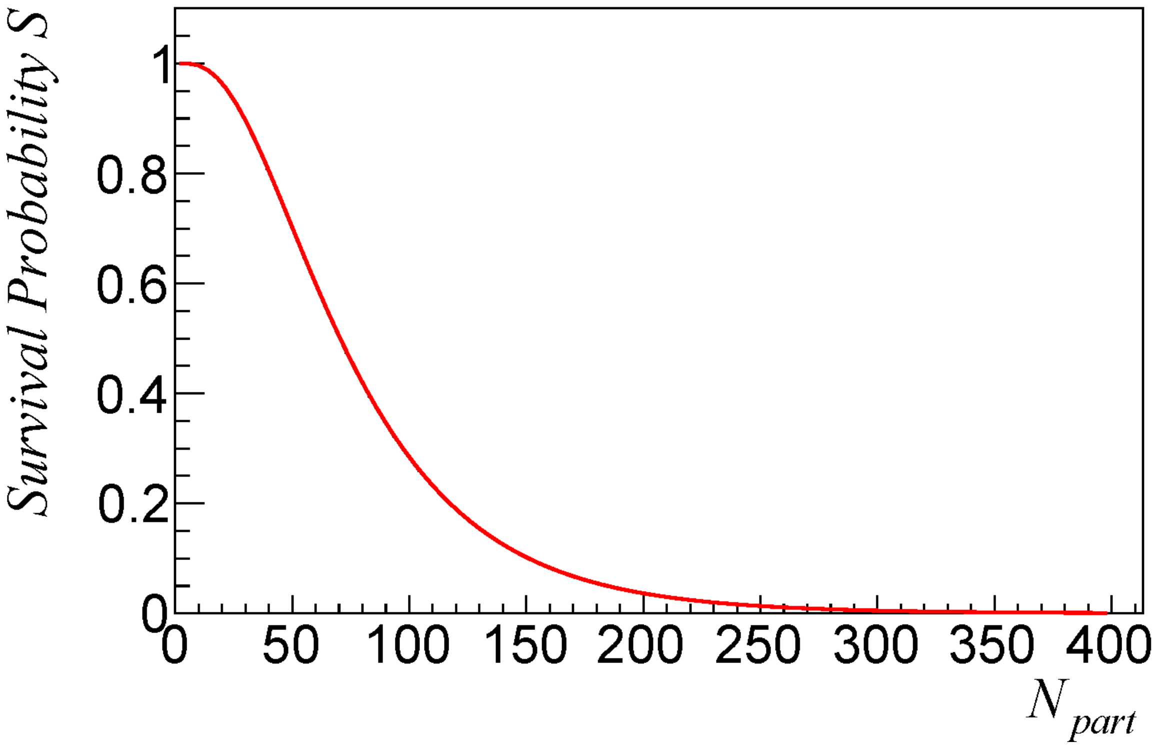
$$\delta p_{Ti} = p_{Ti} - \langle p_T \rangle$$

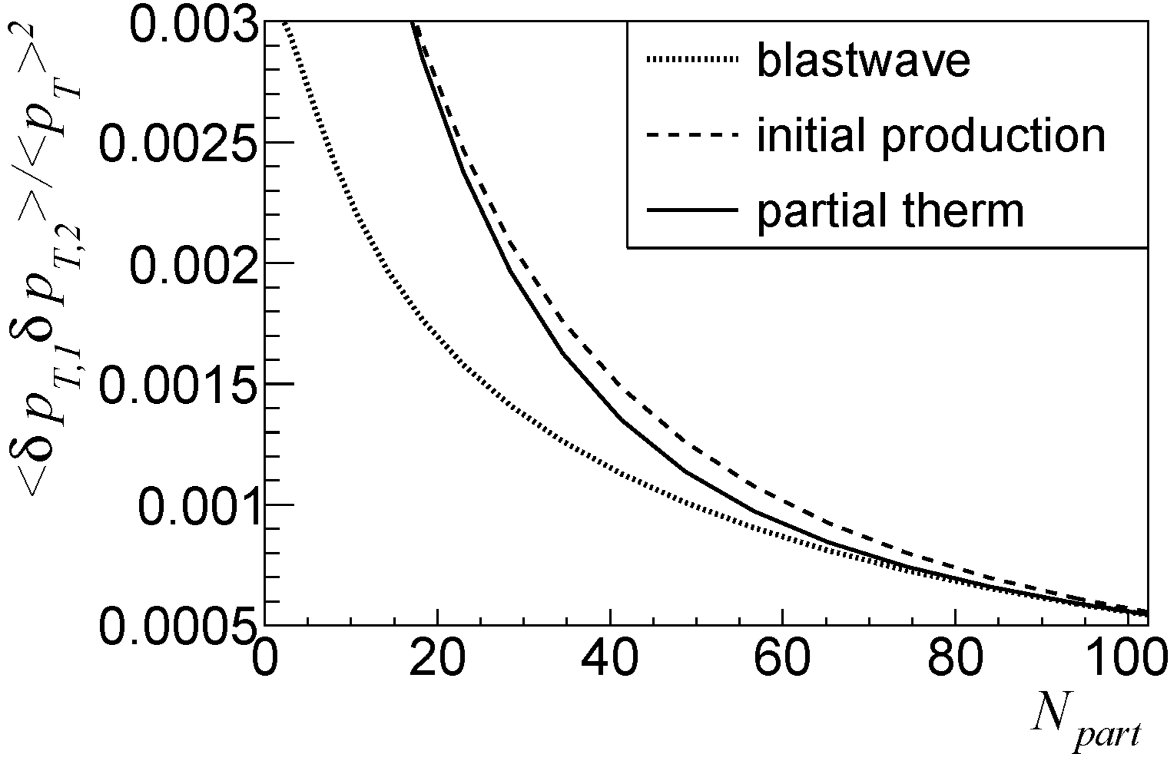
$$d\boldsymbol{\omega} = d^3\boldsymbol{x} d^3\boldsymbol{p}$$



Partial thermalization $\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 \boldsymbol{S}^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - \boldsymbol{S}^2)$

Partial Thermalization





$$\langle\delta p_{T1}\delta p_{T2}\rangle = \langle\delta p_{T1}\delta p_{T2}\rangle_0 S^2 + \langle\delta p_{T1}\delta p_{T2}\rangle_{eq}(1 - S^2)$$

Need a method for constraining S

Use Multiple Observables to Constrain Theory

Multiplicity Fluctuations

$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

Transverse Momentum Correlations

$$\mathcal{C} = \frac{\left\langle \sum_i \sum_{i\neq j} p_{Ti} p_{Tj} \right\rangle - \langle P_T\rangle^2}{\langle N\rangle^2}$$

Net Correlations of Transverse Momentum Fluctuations

$$\langle \delta p_{T1} \delta p_{T2}\rangle = \frac{\left\langle \sum_i \sum_{i\neq j} \delta p_{Ti} \delta p_{Tj} \right\rangle}{\langle N(N-1)\rangle}$$

Multiplicity-Momentum Correlations

$$\mathcal{D} = \frac{\left\langle \sum_i \sum_{i\neq j} \delta p_{Ti} \right\rangle}{\langle N\rangle^2} = \frac{Cov(P_T, N) - \langle p_T\rangle Var(N)}{\langle N\rangle^2}$$

Complementary Observables

$$\boldsymbol{(1+\mathcal{R})\langle \delta p_{T1} \delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle \mathcal{D} + \langle p_T\rangle^2 \mathcal{R} = 0}$$

Cody, Gavin, Koch, Kocherovsky, Mazloum, Moschelli, *Phys.Rev.C*107 (2023) 1, 014909

Use Multiple Observables to Constrain Theory

Complementary Observables	$\boldsymbol{(1+\mathcal{R})\langle \delta p_{T1}\delta p_{T2}\rangle - \mathcal{C} + 2\langle p_T\rangle\mathcal{D} + \langle p_T\rangle^2\mathcal{R} = 0}$
Multiplicity Fluctuations	$\mathcal{R}$ has no dependence on survival probability S
Transverse Momentum Correlations	$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1-S^2) + 2\langle p_T\rangle\big(\mathcal{D}_0 - \mathcal{D}_{eq}\big)S(1-S)$
Net Correlations of Transverse Momentum Fluctuations	$\langle \delta p_{T1}\delta p_{T2}\rangle = \langle \delta p_{T1}\delta p_{T2}\rangle_0 S^2 + \langle \delta p_{T1}\delta p_{T2}\rangle_{eq}(1-S^2)$
Multiplicity-Momentum Correlations	$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1-S)$

Multiplicity Fluctuations

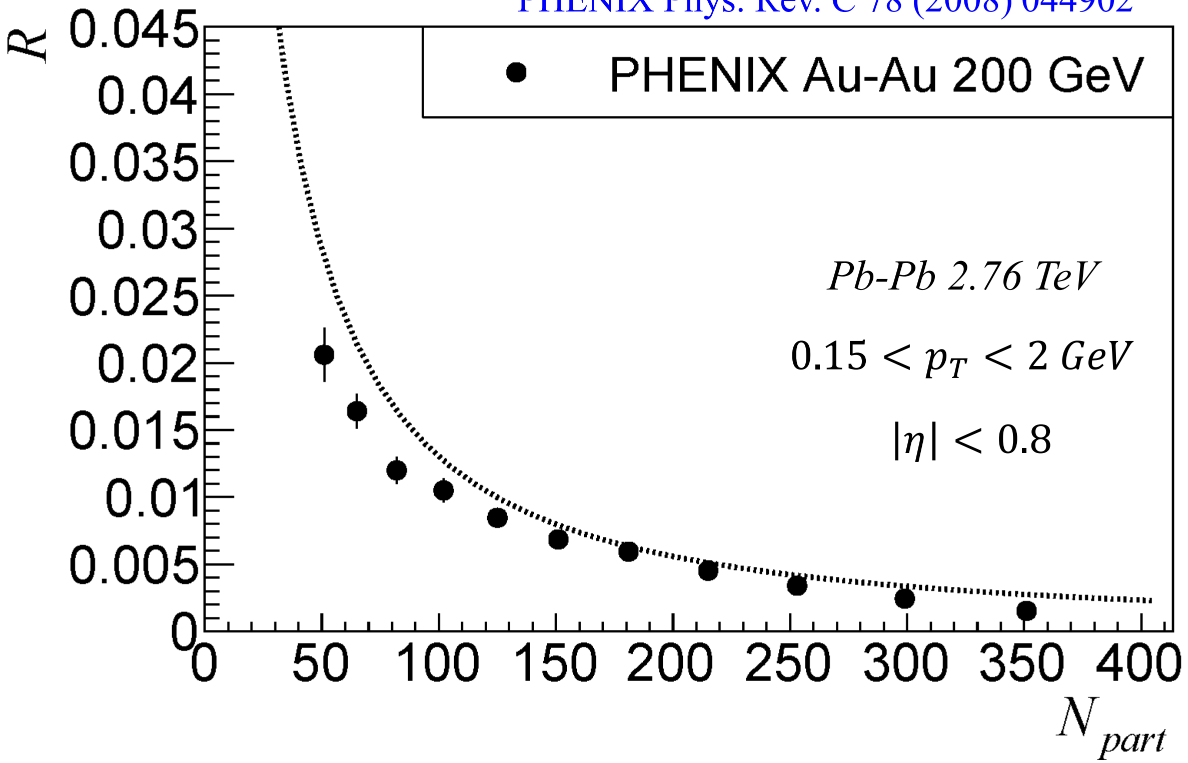
$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

$$\mathcal{R} = \mathcal{R}^{eq}$$

$$\mathcal{R} \propto \frac{const}{\langle N\rangle}$$

constant is adjusted for fit to $\langle \delta p_{T1} \delta p_{T2} \rangle$ data

PHENIX Phys. Rev. C 78 (2008) 044902



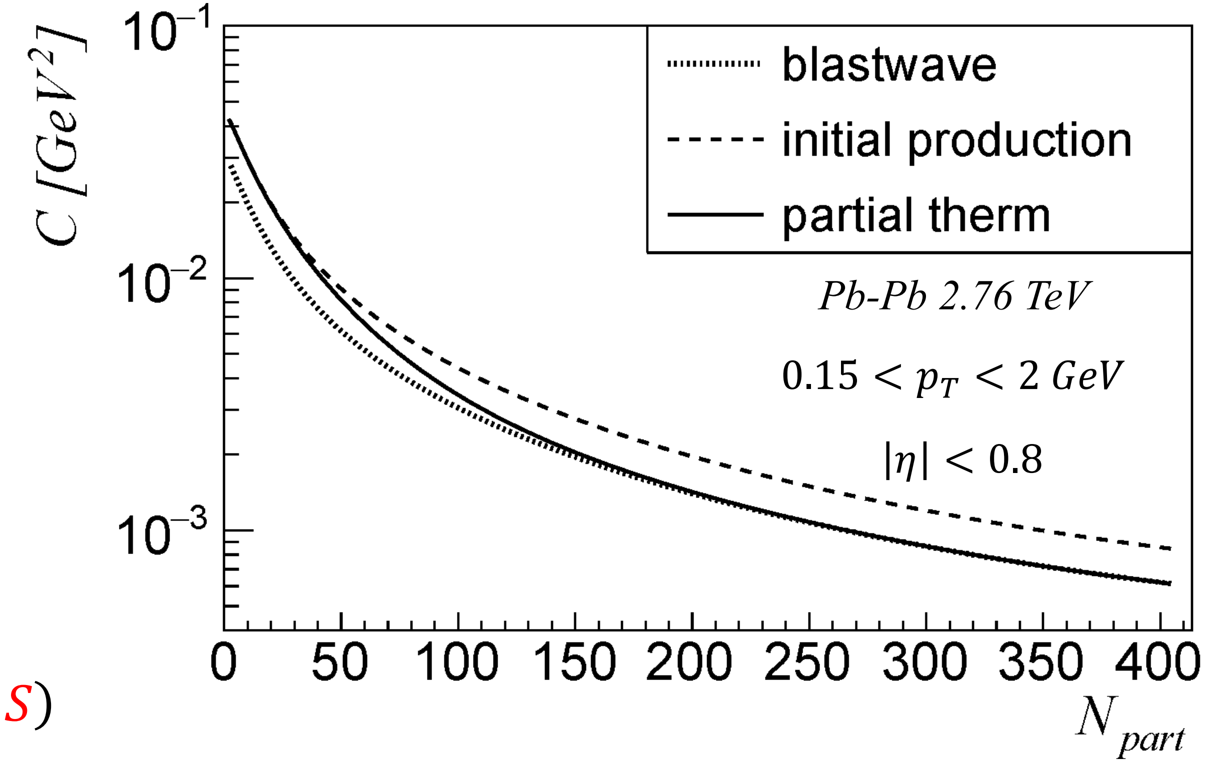
- Sets the scale for all two-particle correlations
- Measures number density “volume fluctuations”
- Related to isothermal compressibility in GCE

Transverse Momentum Correlations

$$\mathcal{C} = \frac{\langle \sum_i \sum_{i\neq j} p_{Ti} p_{Tj} \rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

Partial thermalization

$$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S (1 - S)$$



- Sensitive to both number density fluctuations and transverse momentum fluctuations
- Can estimate shear viscosity and shear relaxation time

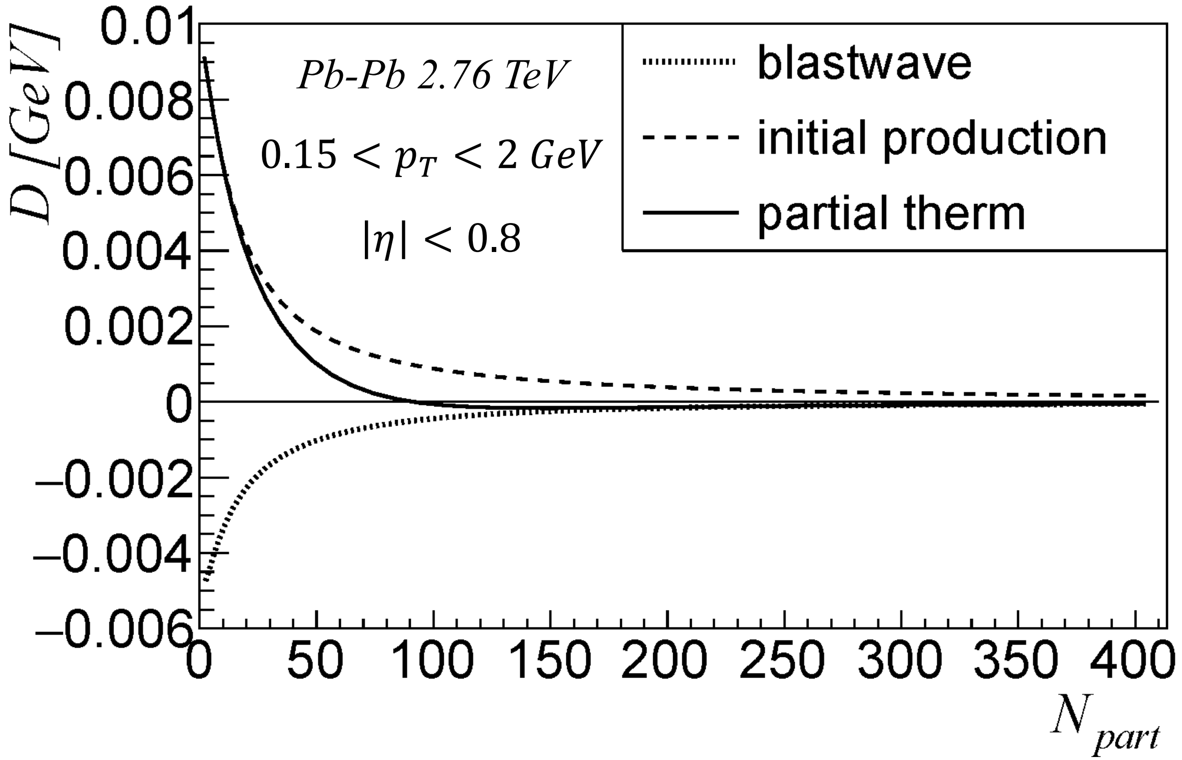
Phys Rev C94, 024921 (2016)
Nucl Phys A982, 311 (2019)

Multiplicity-Momentum Correlations

$$\mathcal{D} = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \rangle}{\langle N \rangle^2} = \frac{Cov(P_T, N) - \langle p_T \rangle Var(N)}{\langle N \rangle^2}$$

Partial thermalization

$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$$



- Removes multiplicity fluctuations
- We find $\mathcal{D} = 0$ in equilibrium in GCE
- Negative in blastwave
- Positive in PYTHIA

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Summary

Boltzman-Langevin evolution of correlations is sensitive to incomplete thermalization

- $\mathcal{R}$ has no dependence on survival probability S
- $\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$
- $\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq}(1 - S^2)$
- $\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T \rangle \left(\mathcal{D}_0 - \mathcal{D}_{eq}\right) S(1 - S)$
- Simultaneous comparison to multiple observables with different powers of S constrains extraction of S

Complementary observables $\boldsymbol{(1 + \mathcal{R})\langle \delta p_{T1} \delta p_{T2} \rangle - \mathcal{C} + 2\langle p_T \rangle \mathcal{D} + \langle p_T \rangle^2 \mathcal{R} = 0}$

- $\mathcal{R}$ Multiplicity Fluctuations
- $\mathcal{C}$ Transverse Momentum Correlations
- $\langle \delta p_{T1} \delta p_{T2} \rangle$ Correlations of Transverse Momentum Fluctuations
- $\mathcal{D}$ Multiplicity-Momentum Correlations
    - $\boldsymbol{\mathcal{D} = 0}$ **expected in equilibrium,** nonzero $\mathcal{D}$ can indicate incomplete thermalization
- All derived from the same parent correlation function

- Use for validation of measurement or calculation of observables
- Challenge theories and models to address all observables simultaneously
- Interpret one observable in terms of physics contributions of the others

Initial Production Estimate

$$\langle \delta p_{T1} \delta p_{T2} \rangle_0 = \frac{\mathcal{R}}{\mathcal{R}_{pp}} \left(\frac{\langle p_T \rangle}{\langle p_T \rangle_{pp}} \right)^2 \langle \delta p_{T1} \delta p_{T2} \rangle_{pp} \, \frac{(1 + \mathcal{R}_{pp})}{(1 + \mathcal{R})}$$

$$\mathcal{C}_0 = \frac{\mathcal{R}}{\mathcal{R}_{pp}} \left(\frac{\langle p_T \rangle}{\langle p_T \rangle_{pp}} \right)^2 \mathcal{C}_{pp}$$

$$\mathcal{D}_0 = \frac{\mathcal{R}}{\mathcal{R}_{pp}} \, \frac{\langle p_T \rangle}{\langle p_T \rangle_{pp}} \, \mathcal{D}_{pp}$$

Moments of the Distribution of Correlated Pairs

$$\mathcal{R} = \frac{1}{\langle N\rangle^2}\int\int G_{12}\ d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2$$

$$\mathcal{C} = \frac{1}{\langle N\rangle^2}\int\int G_{12}\ p_{T1}p_{T2}\ d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2$$

$$\langle \delta p_{T1}\delta p_{T2}\rangle = \frac{\int\int G_{12}\ \delta p_{T1}\delta p_{T2}\ d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2}{\langle N(N-1)\rangle}$$

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Moments of the Distribution of Correlated Pairs

$$\mathcal{R} = \frac{1}{\langle N\rangle^2}\int\int G_{12}\ d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2 \xrightarrow{\qquad\qquad} \mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

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Moments of the Distribution of Correlated Pairs

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$$\mathcal{C} = \frac{1}{\langle N\rangle^2}\int\int G_{12}\; p_{T1}p_{T2}\; d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2 \longrightarrow \mathcal{C} = \frac{\left\langle \sum_i \sum_{i\neq j} p_{Ti}p_{Tj}\right\rangle - \langle P_T\rangle^2}{\langle N\rangle^2}$$

$$\langle \delta p_{T1}\delta p_{T2}\rangle = \frac{\int\int G_{12}\; \delta p_{T1}\delta p_{T2}\; d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2}{\langle N(N-1)\rangle}$$

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Moments of the Distribution of Correlated Pairs

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Moments of the Distribution of Correlated Pairs

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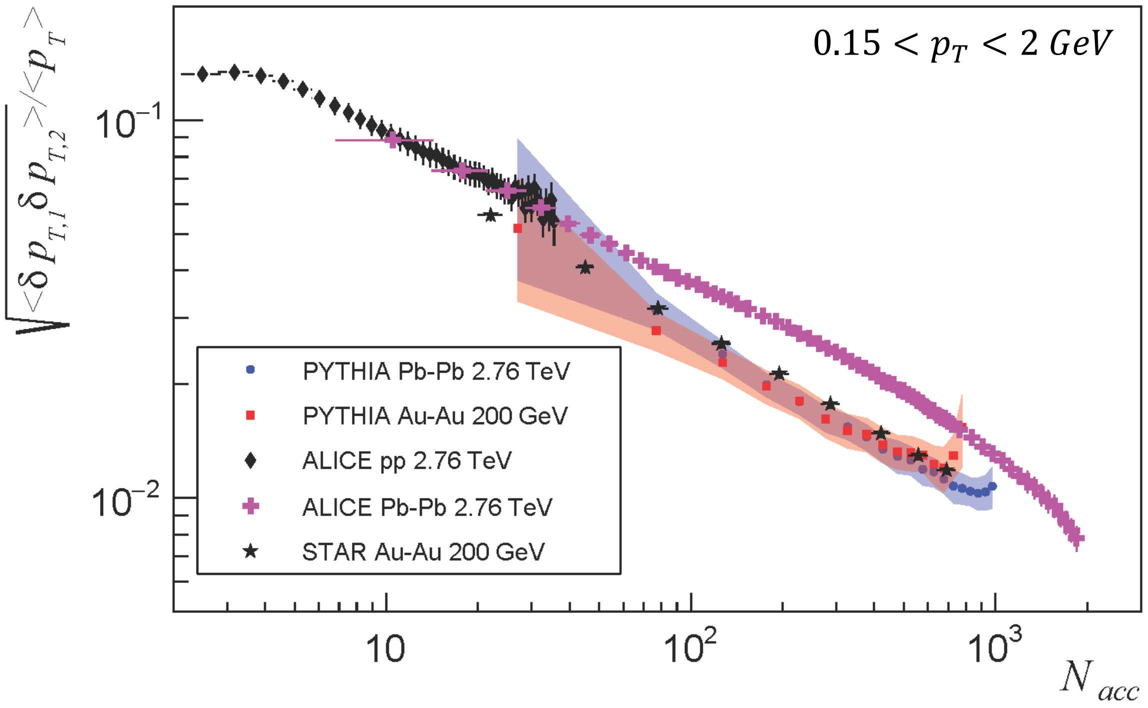
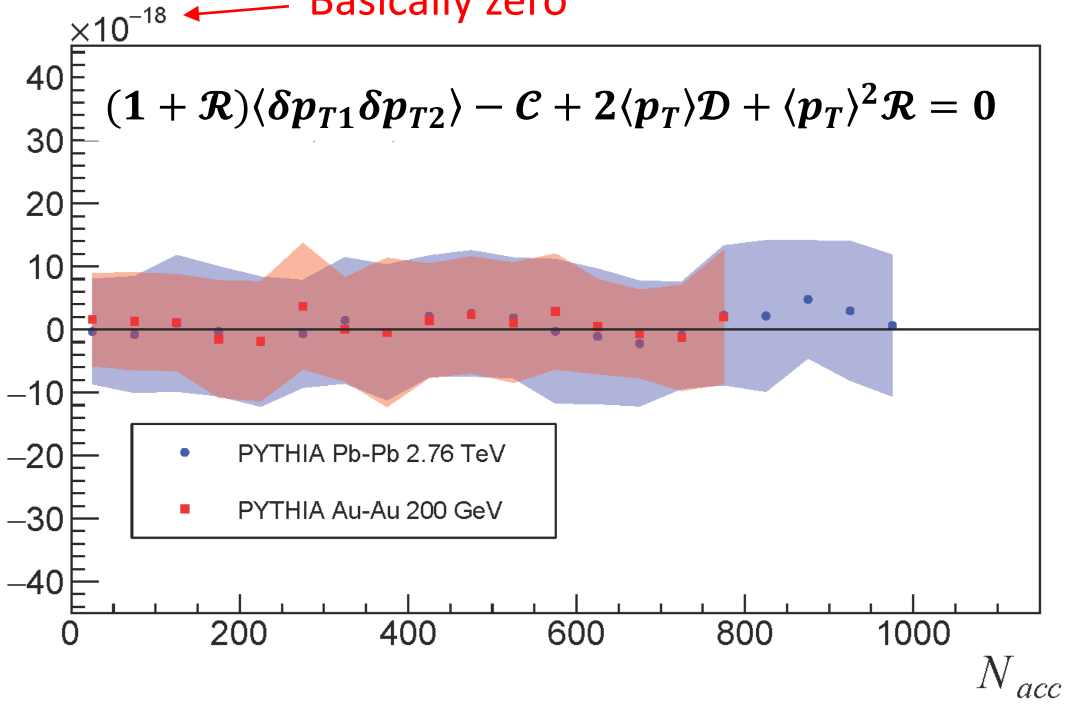
$$\mathcal{D} = \frac{1}{\langle N\rangle^2}\int\int G_{12}\; \delta p_{T1}\; d^3\boldsymbol{\omega}_1 \mathrm{d}^3\boldsymbol{\omega}_2 \quad\longrightarrow\quad \mathcal{D} = \frac{\left\langle \sum_i \sum_{i\neq j} \delta p_{Ti}\right\rangle}{\langle N\rangle^2} = \frac{Cov(P_T, N) - \langle p_T\rangle Var(N)}{\langle N\rangle^2}$$

Consistency Checks





Basically zero

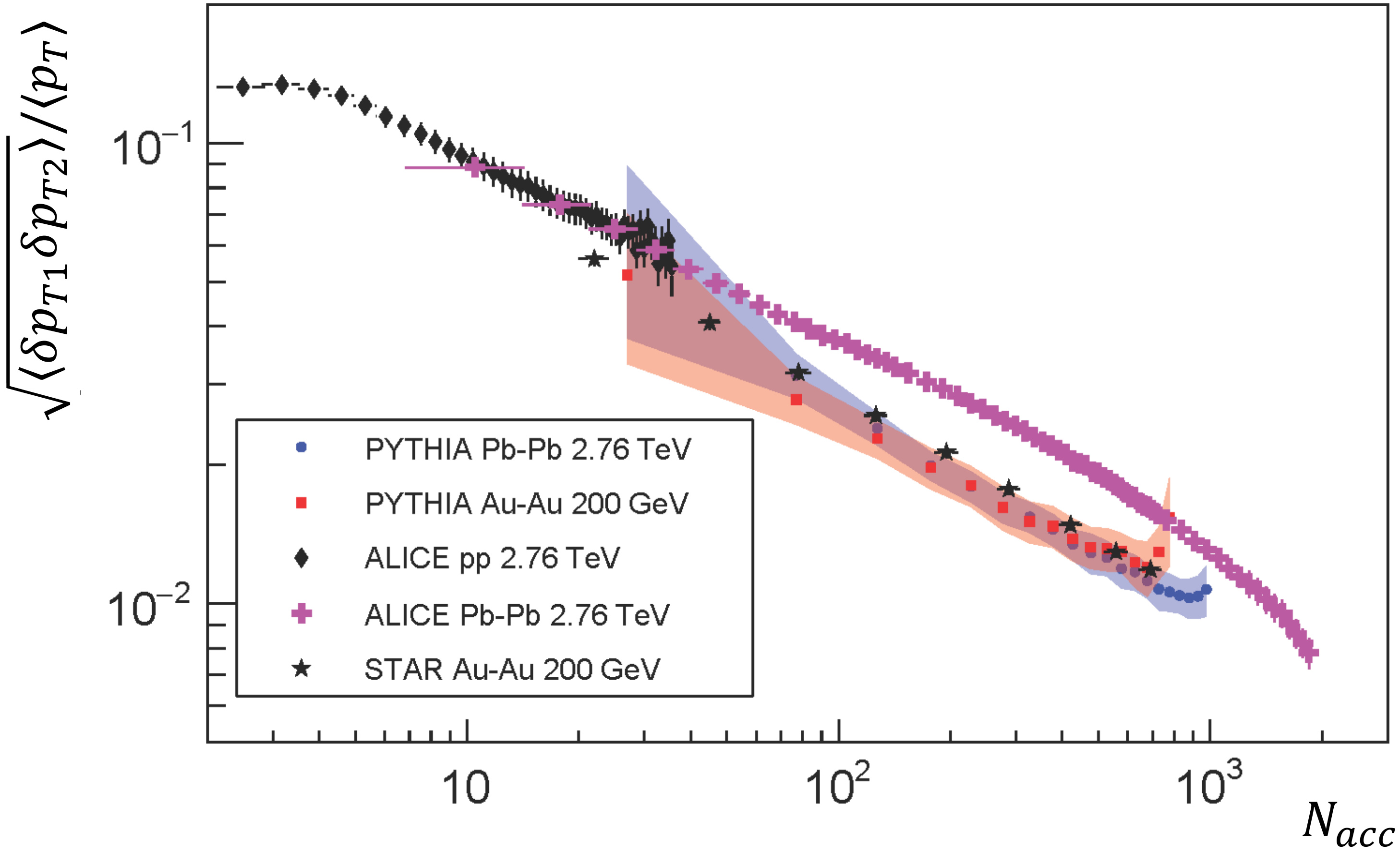
$$(1+\mathcal{R})\langle \delta p_{T1} \delta p_{T2} \rangle - \mathcal{C} + 2\langle p_T \rangle \mathcal{D} + \langle p_T \rangle^2 \mathcal{R} = 0$$

- Validates consistent calculation of observables using PYTHIA
- Theories or models that explain one observable should be able to explain all.
- Can interpret one observable in terms of the physics contributions of the others

Not zero!

- Example: $$\langle \delta p_{T1} \delta p_{t2} \rangle = \frac{\mathcal{C} - 2\langle p_T \rangle \mathcal{D} - \langle p_T \rangle^2 \mathcal{R}}{(1+\mathcal{R})}$$

Correlations of Transverse Momentum Fluctuations



ALICE: Eur. Phys. J C74, 3077 (2014)
STAR: Phys. Rev. C99, 044918 (2019)

Multiplicity – Momentum Correlations

$$\mathcal{D} = \frac{\left\langle \sum_i \sum_{i\neq j} \delta p_{Ti} \right\rangle}{\langle N \rangle^2} \quad \longrightarrow \quad \mathcal{D} = \frac{Cov(P_T, N) - \langle p_T \rangle Var(N)}{\langle N \rangle^2}$$

- Removes multiplicity fluctuations
- $\mathcal{D} = 0$ in thermal equilibrium (Grand Canonical Ensemble)
- PYTHIA: positive, nonzero $\mathcal{D}$
- Consistent with increase in $\langle p_T \rangle$ with multiplicity

PYTHIA pp	$\boldsymbol{\mathcal{D}}$ **[GeV]**	$\boldsymbol{\langle \delta p_{T1} \delta p_{T2} \rangle}$ **[GeV^2]**
200 GeV	$\mathbf{0.01685} \pm 9.32 \times 10^{-5}$	$\mathbf{0.00257} \pm 2.27 \times 10^{-5}$
2.76 TeV	$\mathbf{0.0348} \pm 1.68 \times 10^{-4}$	$\mathbf{0.00446} \pm 3.67 \times 10^{-5}$

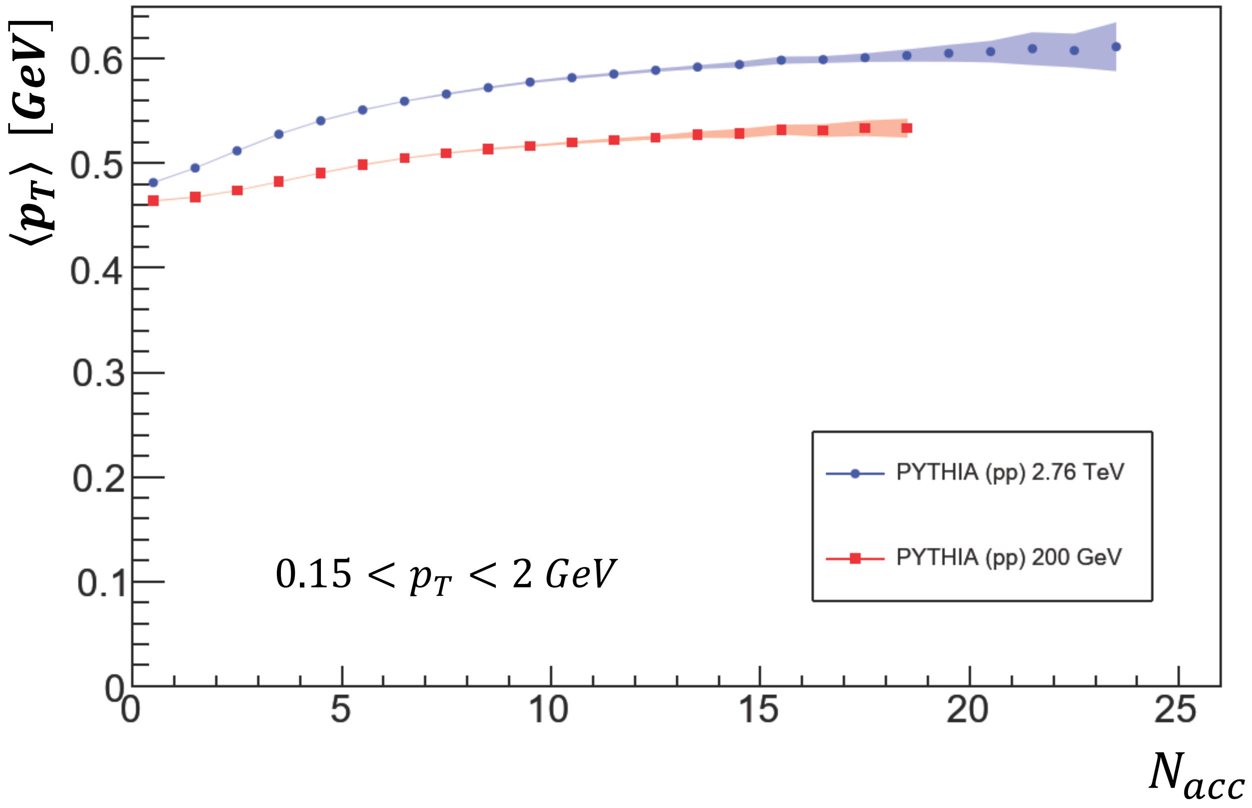
PYTHIA pp	$\boldsymbol{\langle N \rangle \mathcal{D} / \langle p_T \rangle}$	$\boldsymbol{\langle N \rangle (1 + \mathcal{R}) \langle \delta p_{T1} \delta p_{T2} \rangle / \langle p_T \rangle^2}$
200 GeV	$\mathbf{0.230} \pm 1.28 \times 10^{-3}$	$\mathbf{0.0919} \pm 8.53 \times 10^{-4}$
2.76 TeV	$\mathbf{0.549} \pm 2.71 \times 10^{-3}$	$\mathbf{0.191} \pm 1.64 \times 10^{-3}$

Can $\mathcal{D}$ signal thermalization?

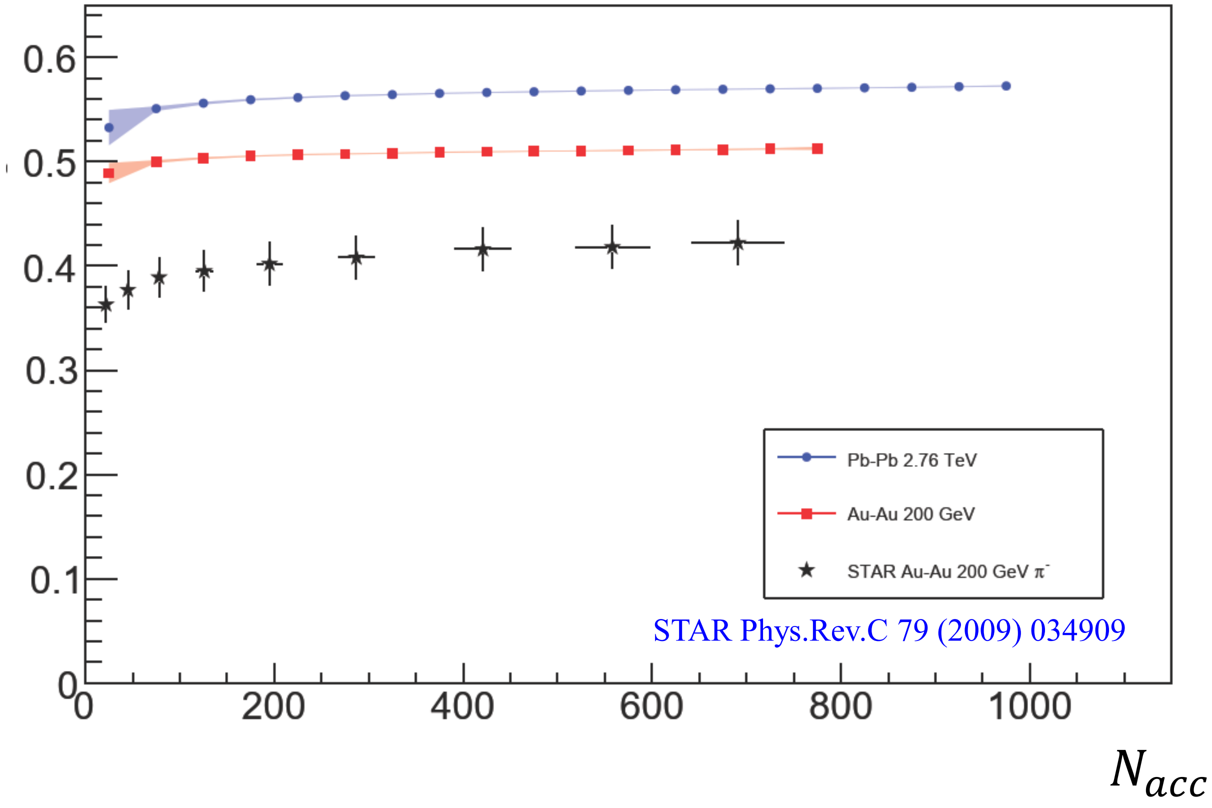
$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq} (1 - S)$$

Cody, Gavin, Koch, Kocherovsky, Mazloum, Moschelli *Phys.Rev.C*107 (2023) 1, 014909 arXiv: 2110.04884

Multiplicity – Momentum Correlations



PYTHIA proton-proton



Angantyr nucleus-nucleus

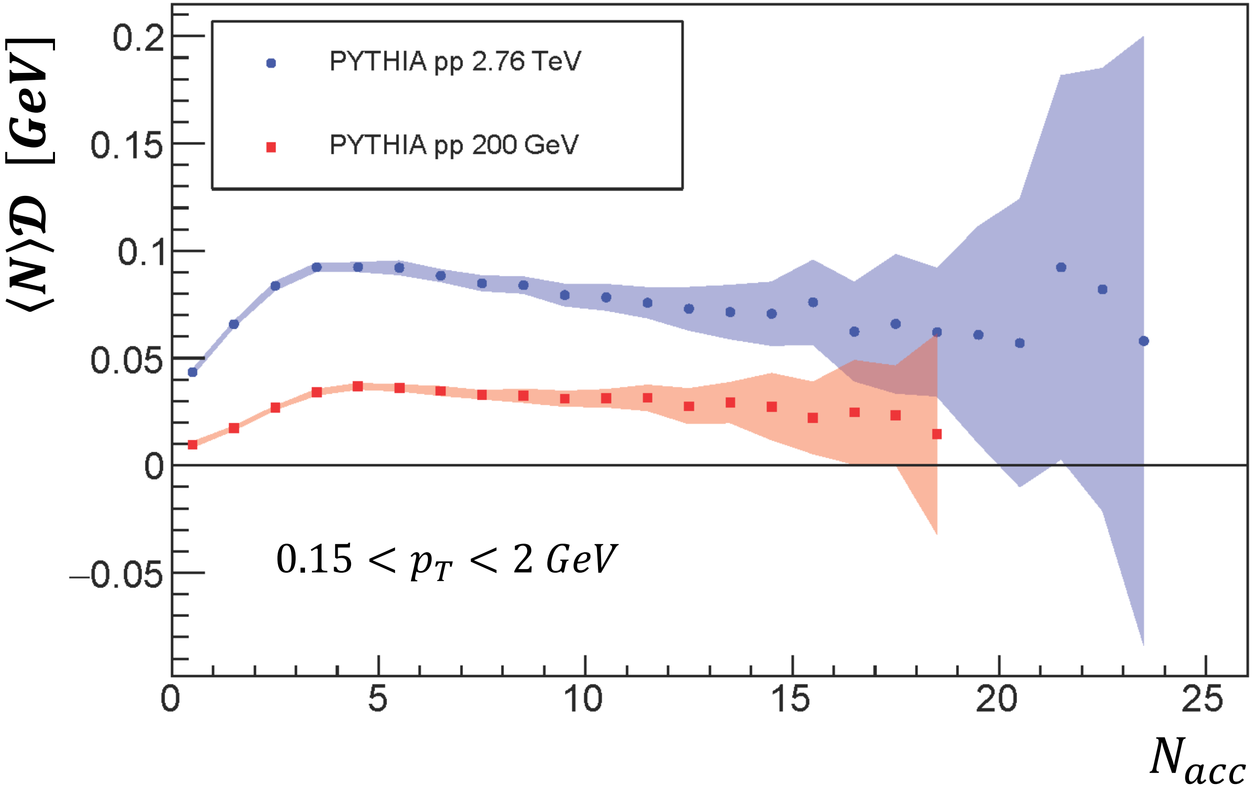
STAR Phys.Rev.C 79 (2009) 034909

- The average transverse momentum per particle increases with increasing multiplicity. This is a positive correlation between event P_T and multiplicity.

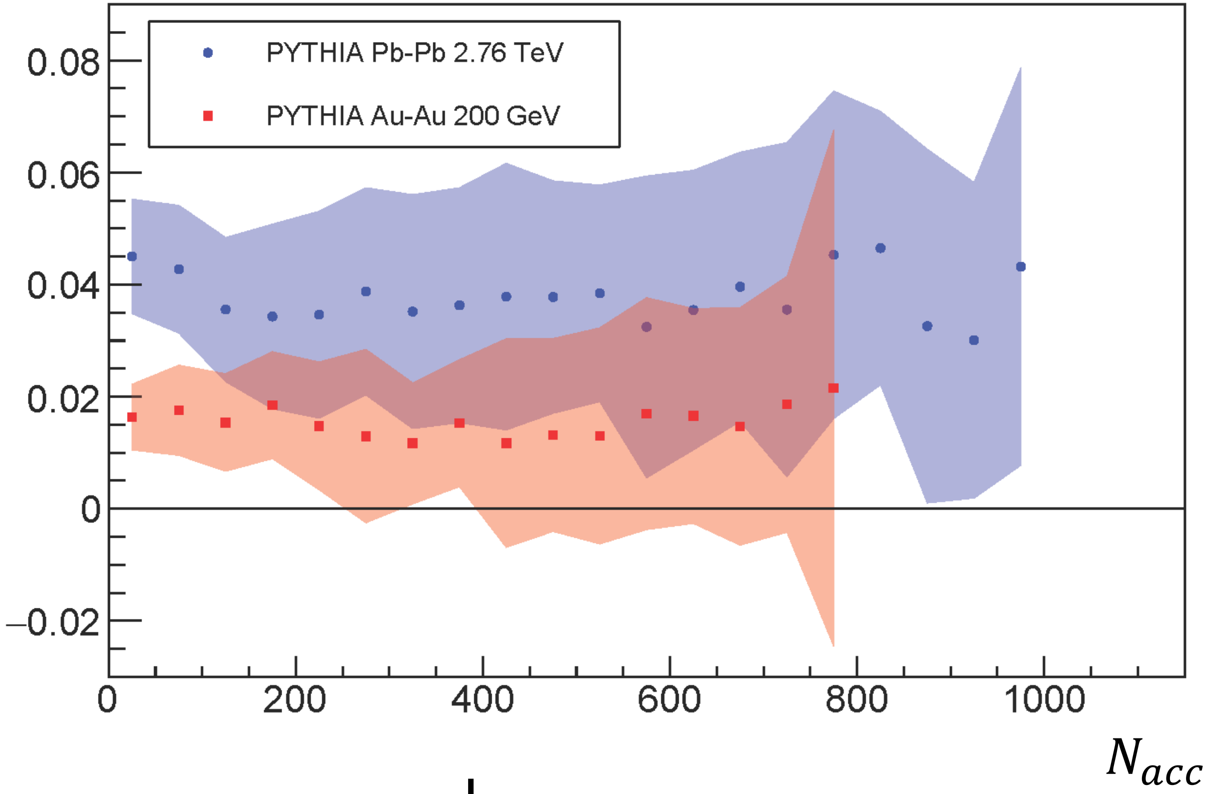
PYTHIA Comput.Phys.Commun. 191 (2015) 159-177, arXiv:1410.3012
Angantyr 10 (2018) 134, arXiv:1806.10820JHEP

Multiplicity – Momentum Correlations

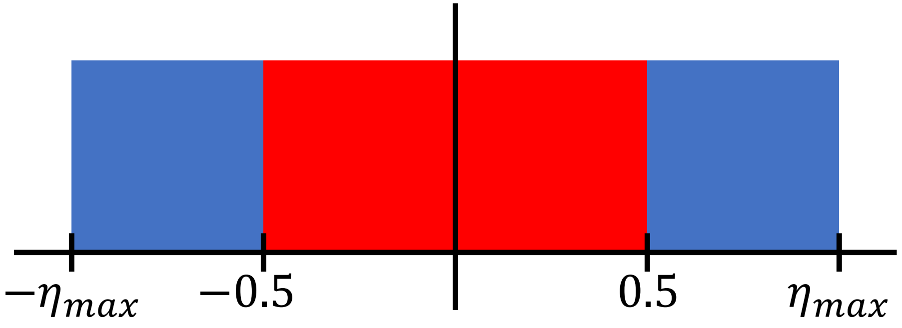
PYTHIA proton-proton



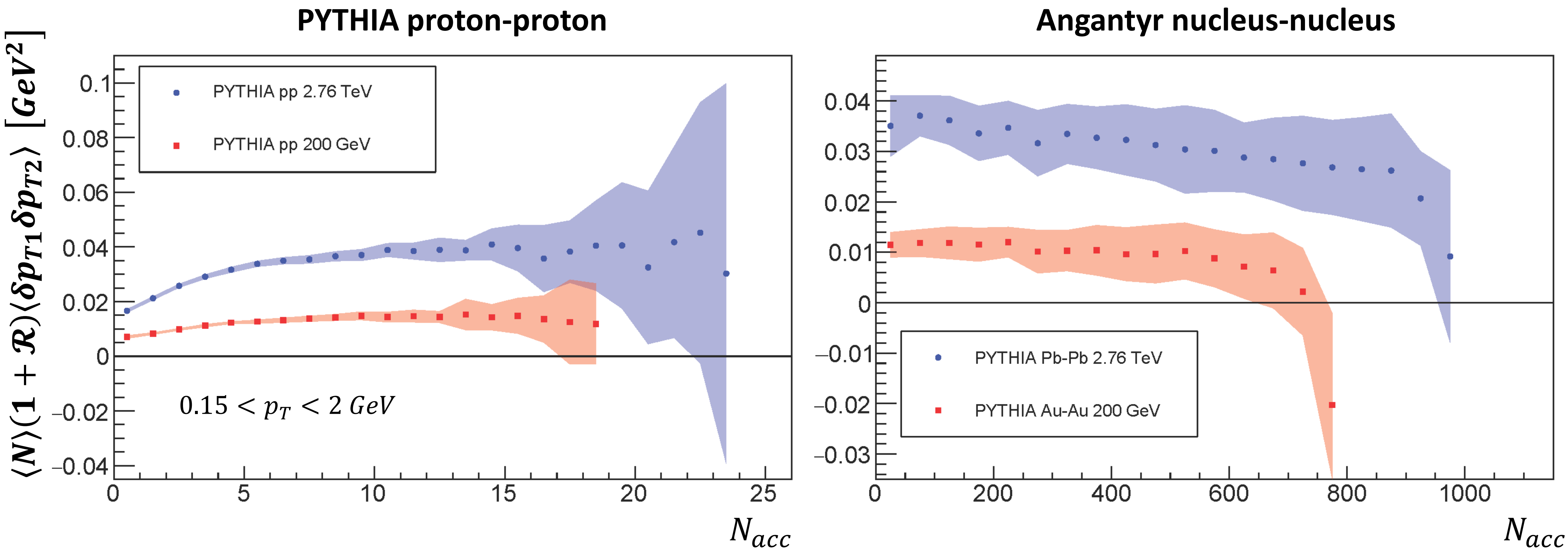
Angantyr nucleus-nucleus



- Correlated pairs $|\eta| < 0.5$
- N_{acc} particles from remaining acceptance

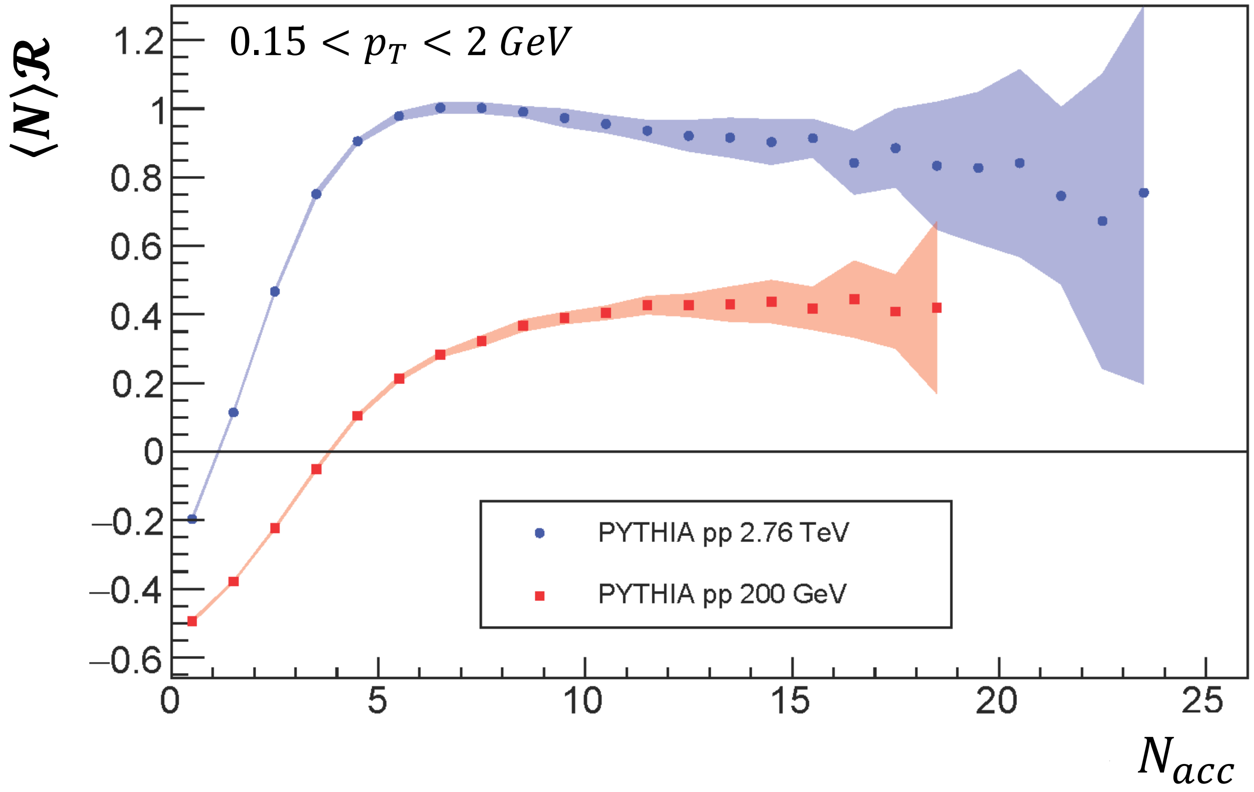


Correlations of Transverse Momentum Fluctuations

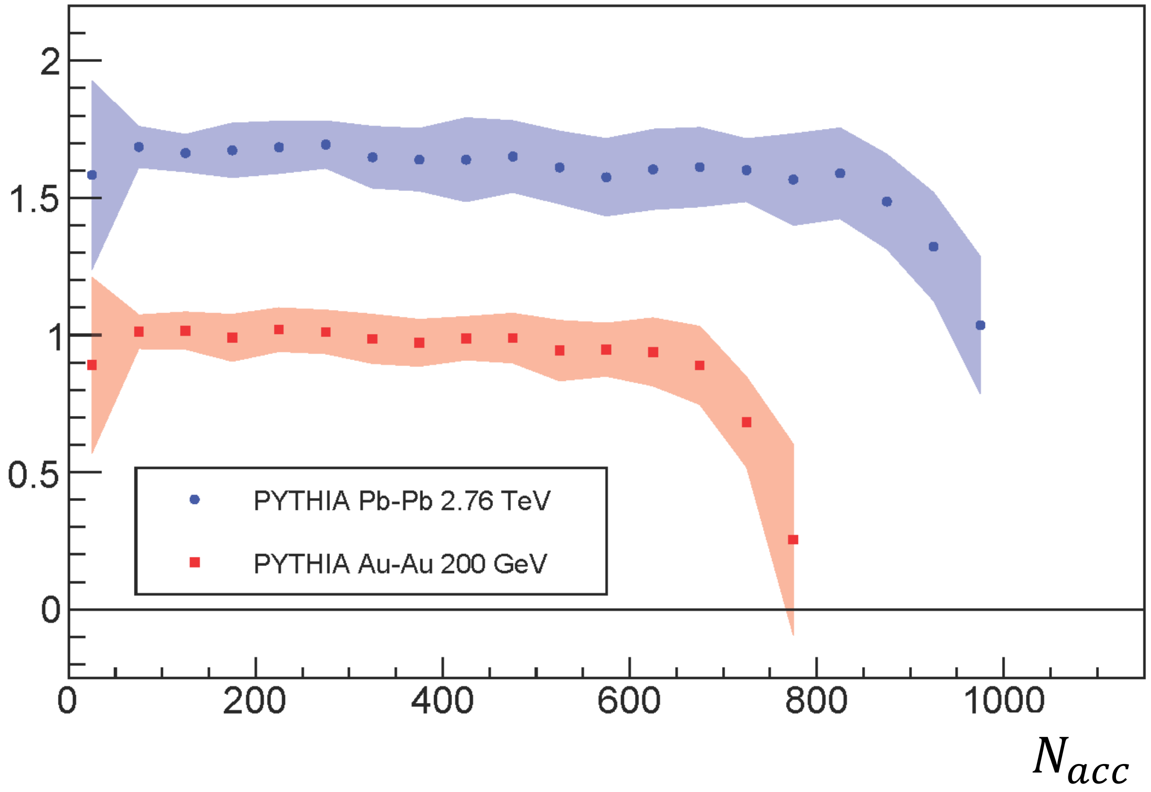


Multiplicity Fluctuations

PYTHIA proton-proton



Angantyr nucleus-nucleus

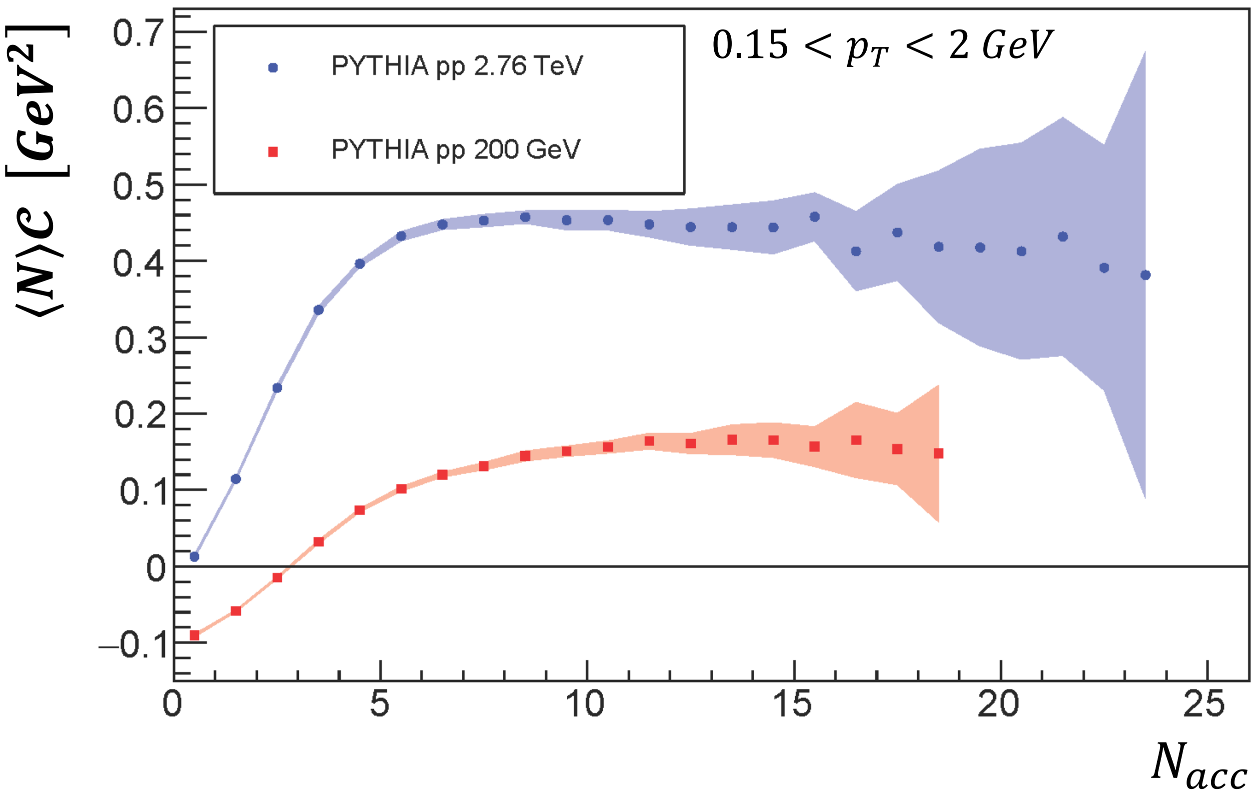


- $\mathcal{R}$ measures correlations, sets an overall scale
- $\mathcal{R} = 0$ for independent particle production
- $\mathcal{R} \propto 1/\langle N \rangle$ for independent source and NBD distributions
- Large contribution from “volume fluctuations”

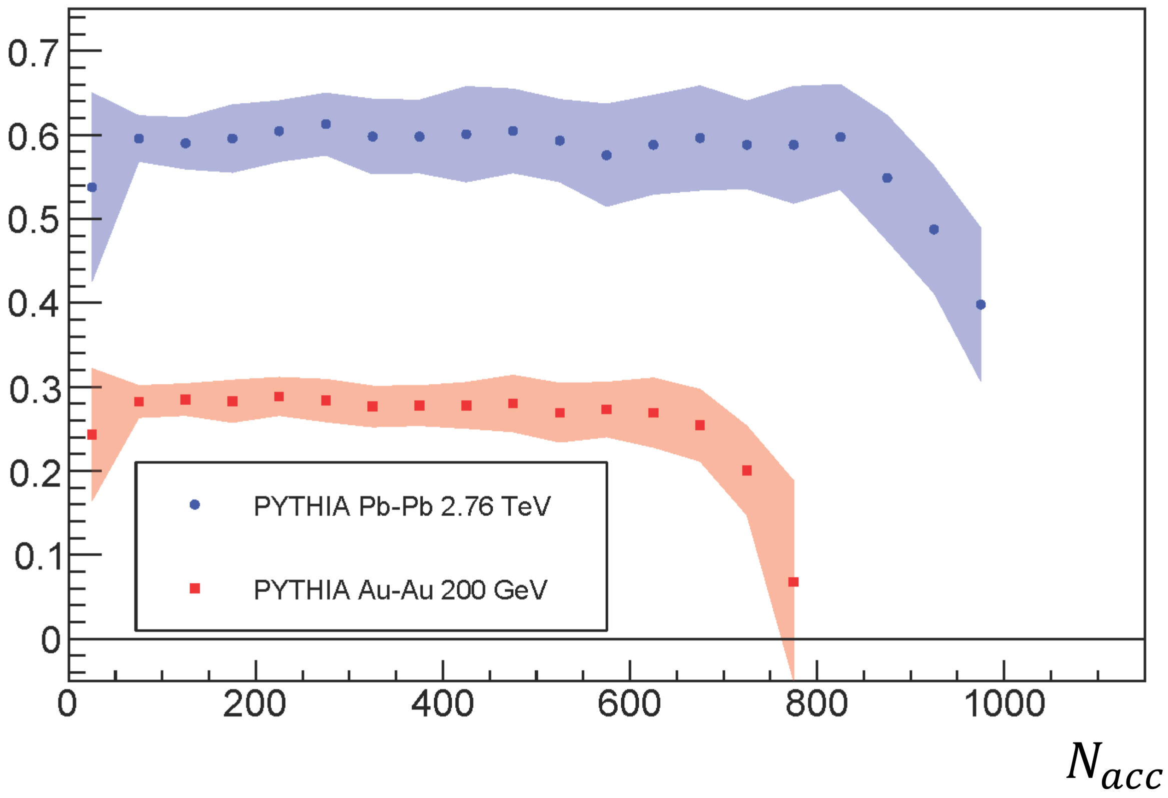
No dependence on Survival probability S

Transverse Momentum Correlations

PYTHIA proton-proton



Angantyr nucleus-nucleus



- $\mathcal{C}$ is the momentum weighted version of $\mathcal{R}$
- Sensitive to both number density fluctuations and transverse momentum fluctuations
- Can estimate shear viscosity and shear relaxation time

$$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T\rangle\left(\mathcal{D}_0 - \mathcal{D}_{eq}\right)S(1 - S)$$

Phys Rev C94, 024921 (2016) arXiv:1606.02692
Nucl Phys A982, 311 (2019) arXiv:1807.06532

Multiplicity Fluctuations

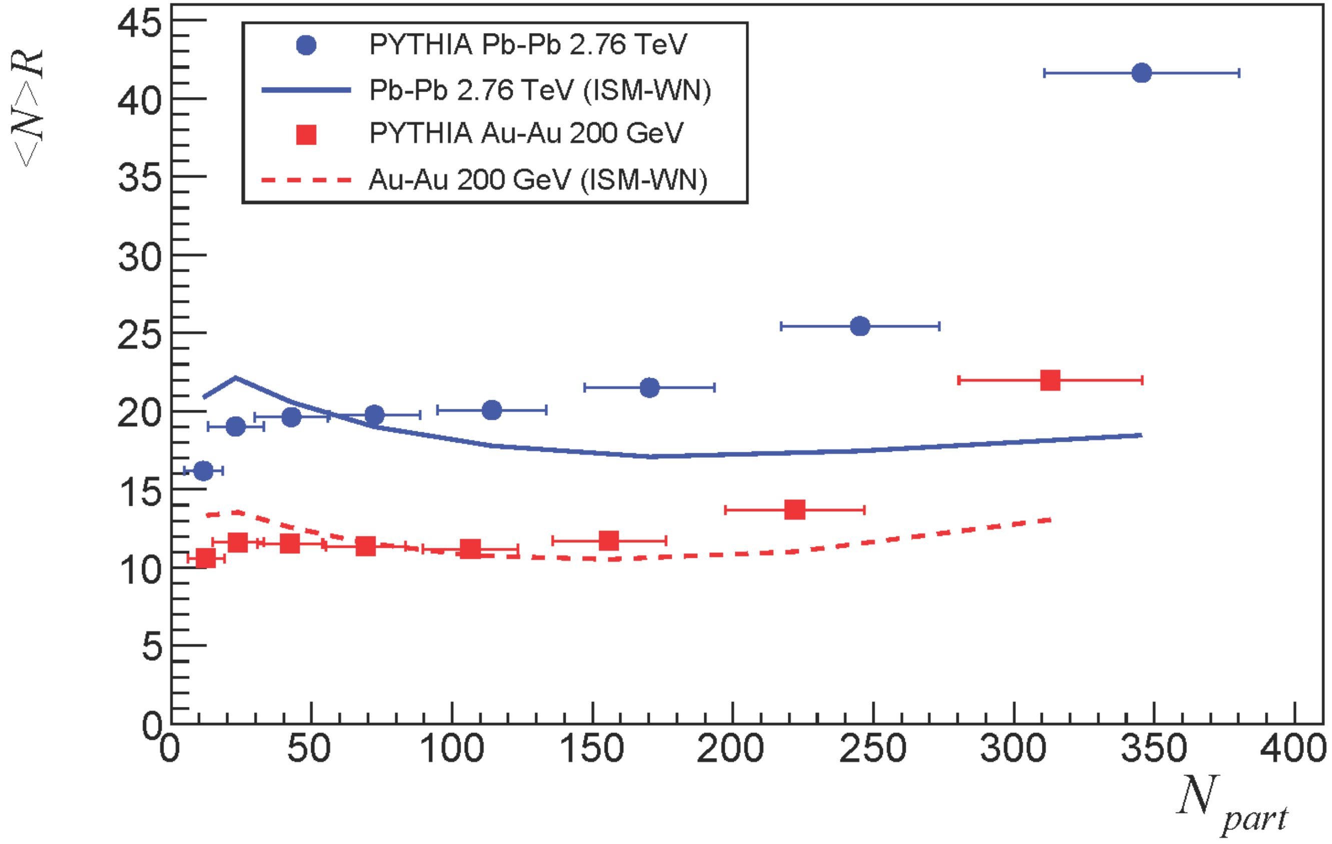
$$\mathcal{R} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{Var(N) - \langle N\rangle}{\langle N\rangle^2}$$

No dependence on Survival probability S

Independent Source Model

$$\mathcal{R} = \frac{2\mathcal{R}_{pp}}{\langle N_{part}\rangle} + \frac{\langle N_{part}^2\rangle - \langle N_{part}\rangle^2}{\langle N_{part}\rangle^2}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



Transverse Momentum Correlations

$$\mathcal{C} = \frac{\langle \sum_i \sum_{i\neq j} p_{Ti} p_{Tj} \rangle - \langle P_T \rangle^2}{\langle N \rangle^2}$$

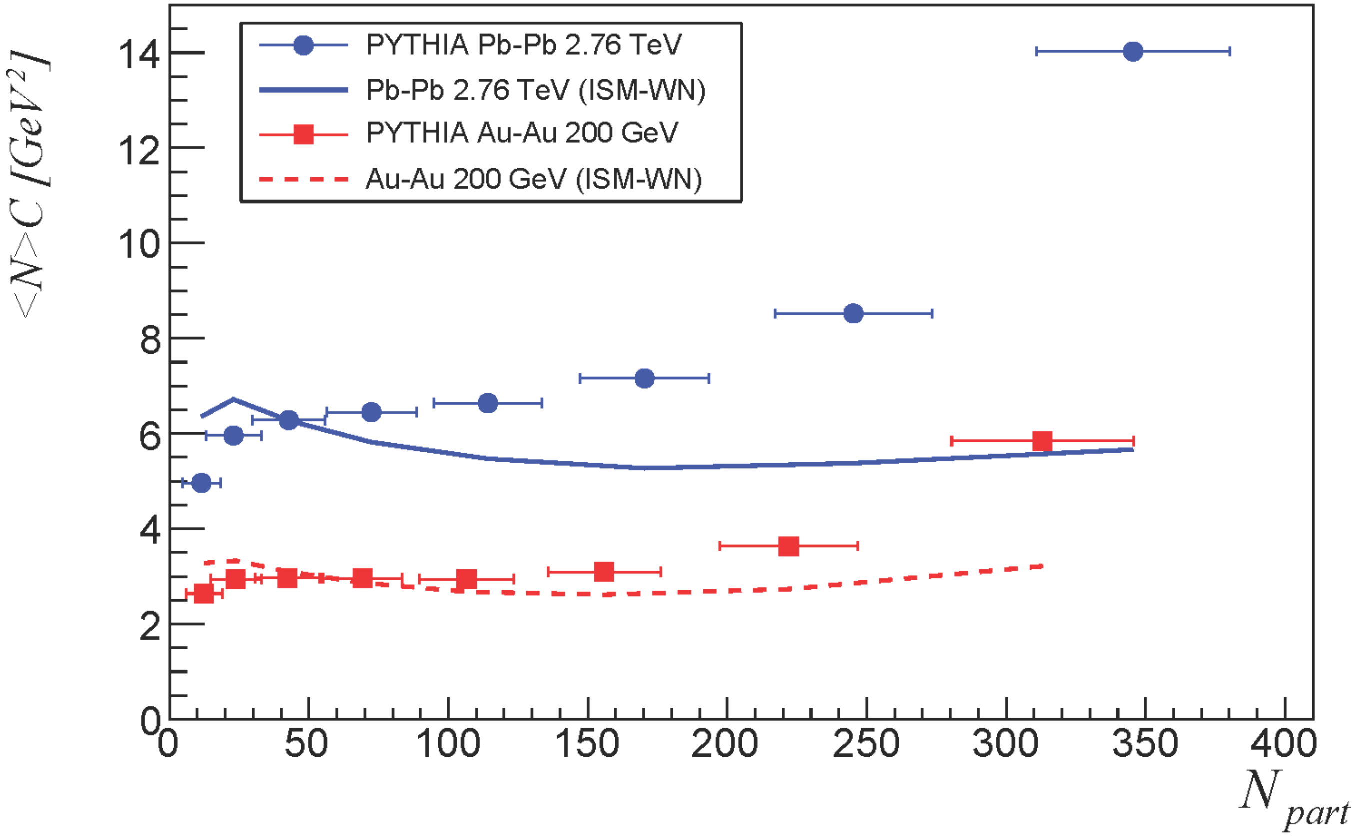
$$\mathcal{C} = \mathcal{C}_0 S^2 + \mathcal{C}_{eq}(1 - S^2) + 2\langle p_T \rangle (\mathcal{D}_0 - \mathcal{D}_{eq}) S(1 - S)$$

Independent Source Model

$$\mathcal{C} = \frac{2\mathcal{C}_{pp}}{\langle \mathrm{N}_{part} \rangle} + \left(\frac{\langle N_{part}^2 \rangle - \langle N_{part} \rangle^2}{\langle N_{part} \rangle^2} \right) \langle p_t \rangle^2$$

$0.15 < p_T < 2\ GeV$

$|\eta| < 1$



Correlations of Transverse Momentum Fluctuations

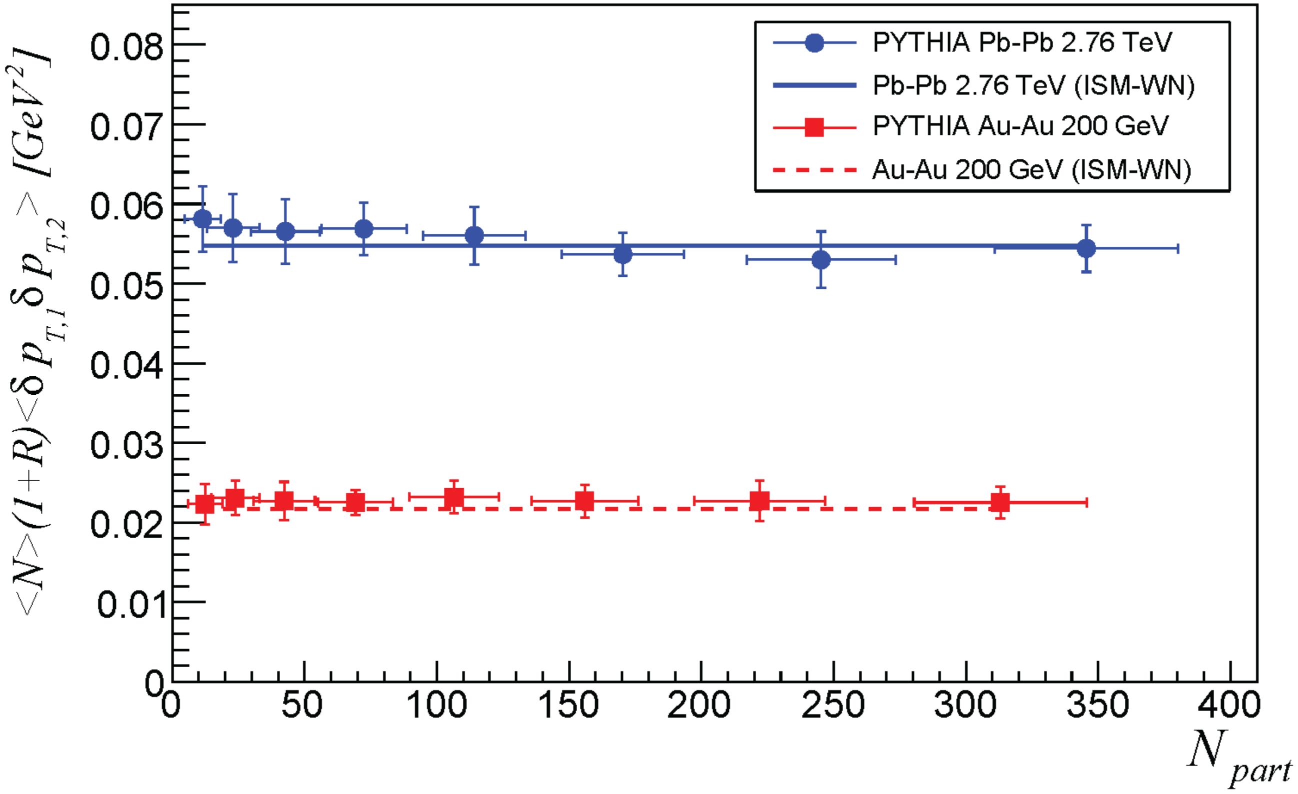
$$\langle \delta p_{T1} \delta p_{T2} \rangle = \frac{\langle \sum_i \sum_{i \neq j} \delta p_{Ti} \delta p_{Tj} \rangle}{\langle N(N-1) \rangle}$$

$$\langle \delta p_{T1} \delta p_{T2} \rangle = \langle \delta p_{T1} \delta p_{T2} \rangle_0 S^2 + \langle \delta p_{T1} \delta p_{T2} \rangle_{eq} (1 - S^2)$$

Independent Source Model

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{\langle N_{part} \rangle} \frac{(1 + \mathcal{R}_{pp})}{(1 + \mathcal{R})}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$



Lines – independent source model
Points - PYTHIA/Angantyr

PYTHIA Comput.Phys.Commun. 191 (2015) 159-177, arXiv:1410.3012
Angantyr 10 (2018) 134, arXiv:1806.10820JHEP

Correlations of Transverse Momentum Fluctuations

$$\langle \delta p_{T1}\delta p_{T2}\rangle = \frac{\langle \sum_i \sum_{i\neq j} \delta p_{Ti}\delta p_{Tj}\rangle}{\langle N(N-1)\rangle}$$

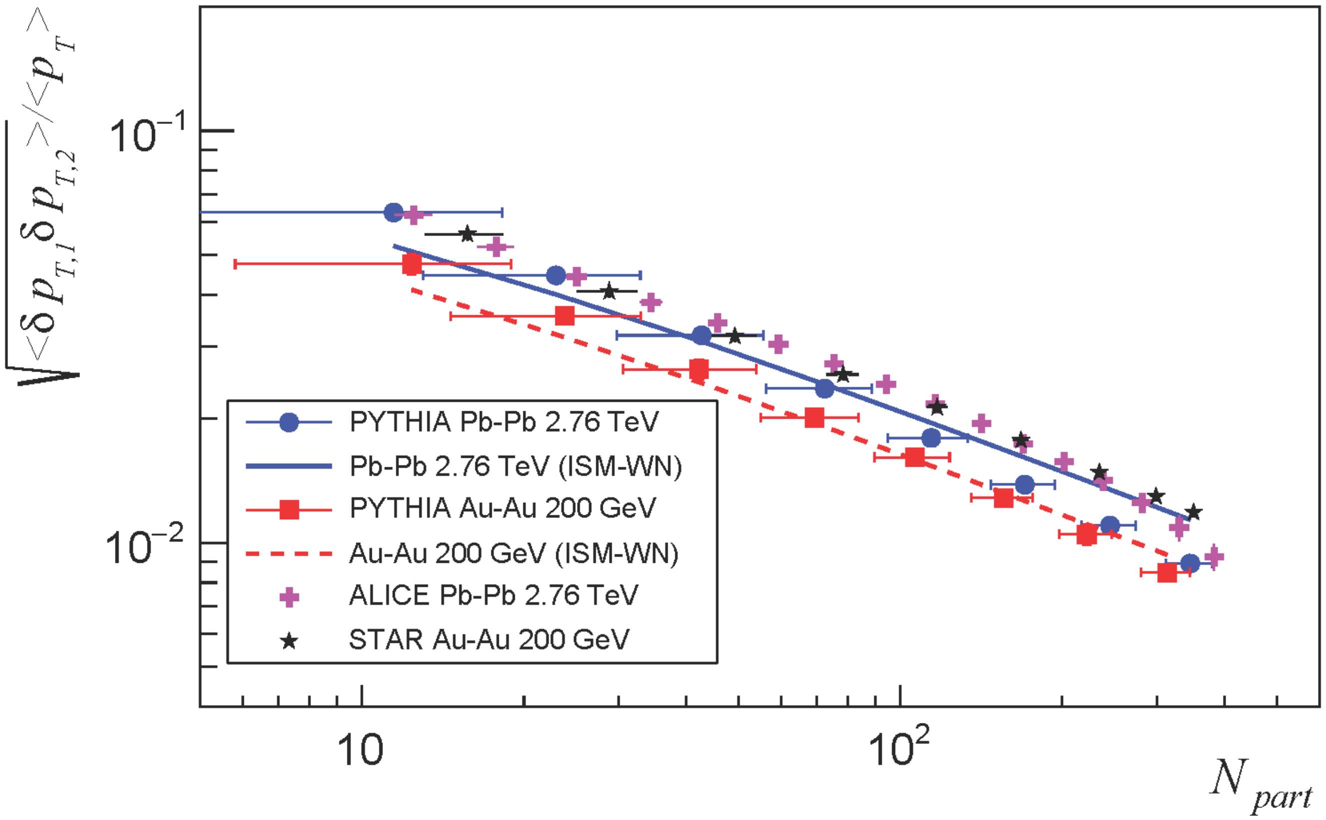
$$\langle \delta p_{T1}\delta p_{T2}\rangle = \langle \delta p_{T1}\delta p_{T2}\rangle_0 S^2 + \langle \delta p_{T1}\delta p_{T2}\rangle_{eq}(1-S^2)$$

Independent Source Model

$$\langle \delta p_{t1}\delta p_{t2}\rangle = \frac{2\langle \delta p_{t1}\delta p_{t2}\rangle_{pp}}{\langle N_{part}\rangle}\frac{(1+\mathcal{R}_{pp})}{(1+\mathcal{R})}$$

$0.15 < p_T < 2\ GeV$

$|\eta| < 1$



ALICE Eur.Phys.J.C 74 (2014) 10, 3077

STAR Phys.Rev.C 99 (2019) 4, 044918

Multiplicity – Momentum Correlations

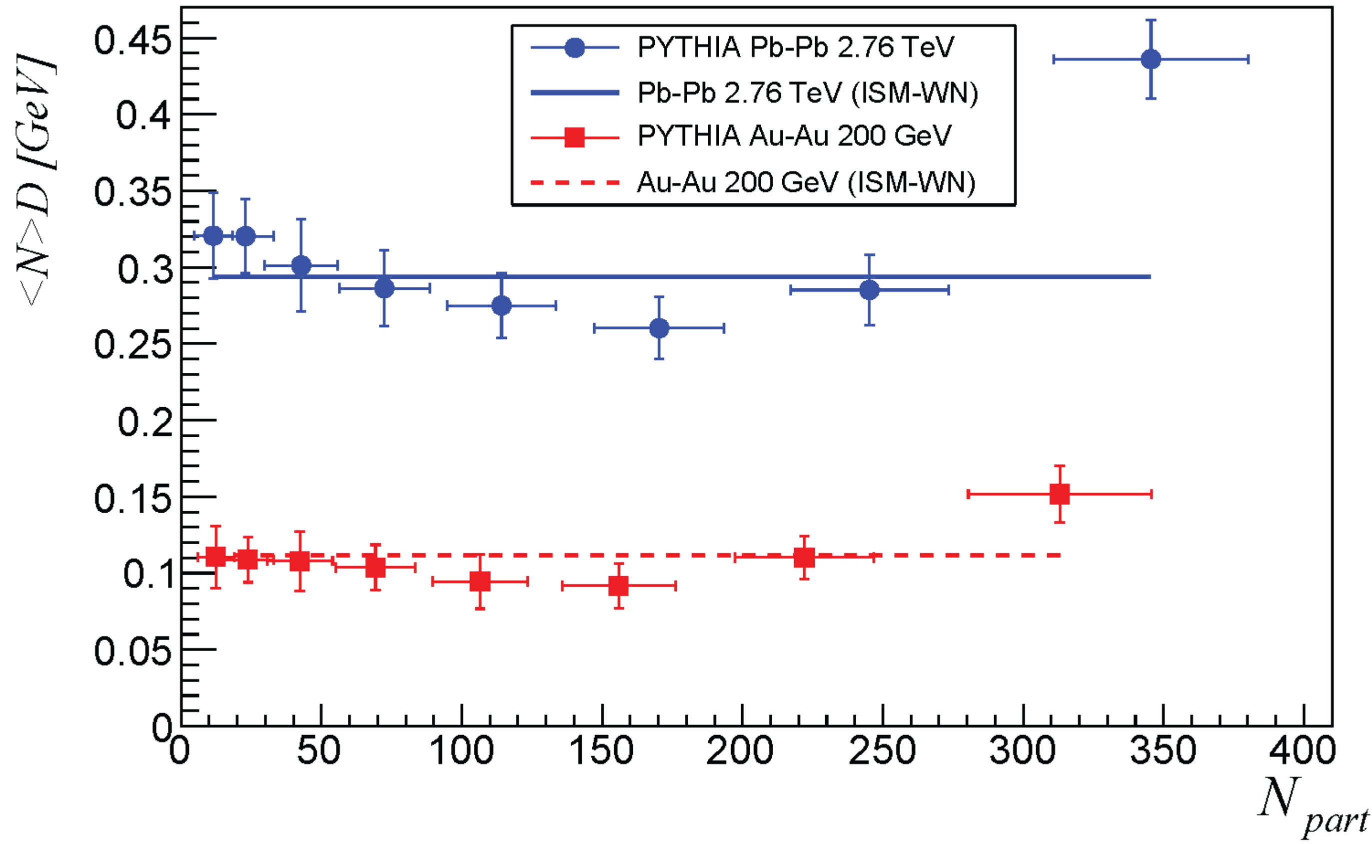
$$\mathcal{D} = \frac{Cov(P_T, N) - \langle p_T \rangle Var(N)}{\langle N \rangle^2}$$

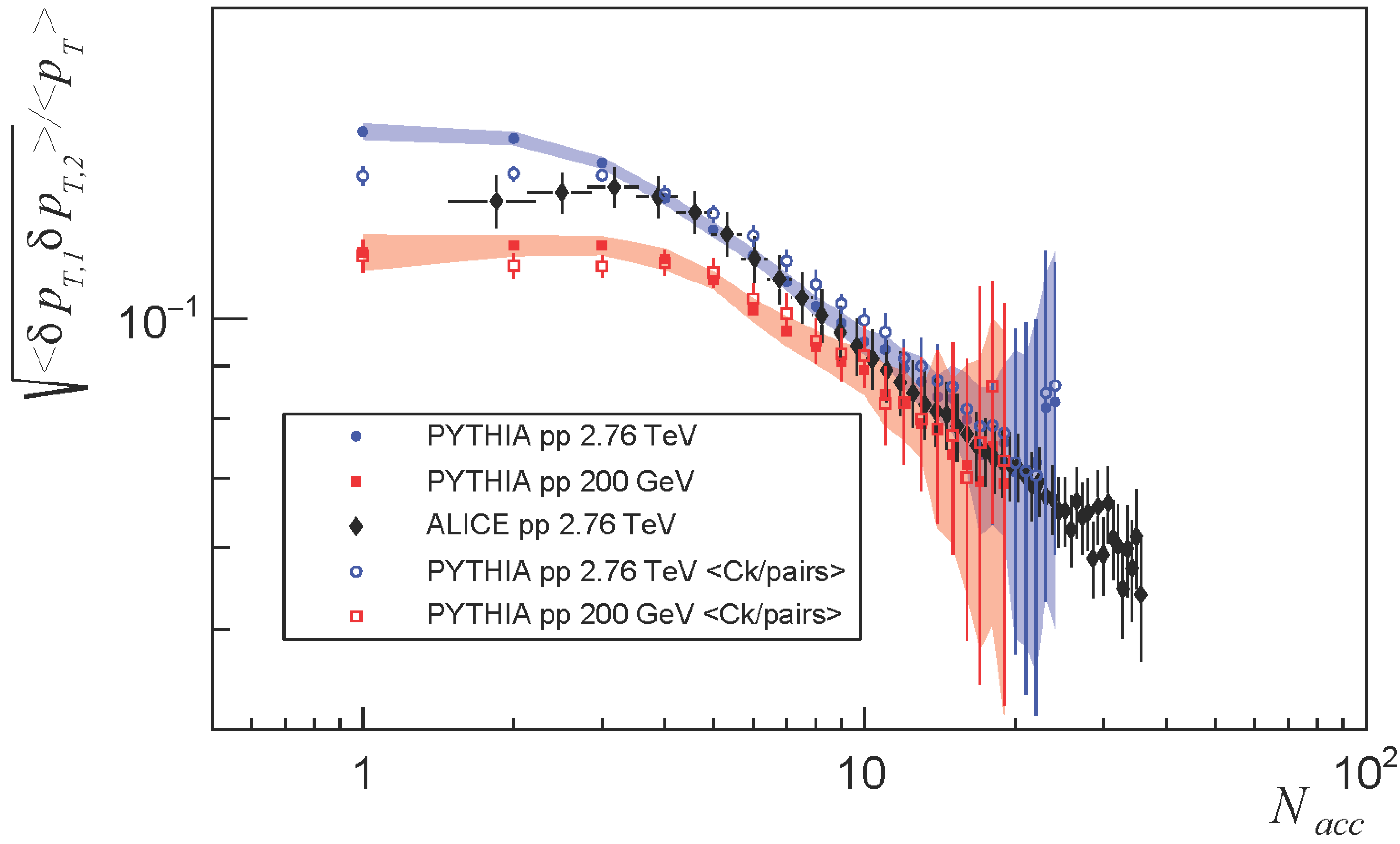
$$\mathcal{D} = \mathcal{D}_0 S + \mathcal{D}_{eq}(1 - S)$$

Independent Source Model

$$\mathcal{D} = \frac{2\mathcal{D}_{pp}}{\langle N_{part} \rangle}$$

$0.15 < p_T < 2\ GeV$
$|\eta| < 1$





$\sqrt{\langle\delta p_{T,1}\delta p_{T,2}\rangle}/\langle p_T\rangle$
10^{-1}
1
10
10^2
N_{acc}
● PYTHIA pp 2.76 TeV
■ PYTHIA pp 200 GeV
◆ ALICE pp 2.76 TeV
○ PYTHIA pp 2.76 TeV <Ck/pairs>
□ PYTHIA pp 200 GeV <Ck/pairs>

$\sqrt{s}$	200 GeV	$\pm$	2.76 TeV	$\pm$
$\langle N\rangle_{pp}$	6.635	3.65×10^{-3}	8.453	8.10×10^{-3}
$\langle p_t\rangle_{pp}$	0.4860	1.33×10^{-4}	0.5356	1.78×10^{-4}
$\mathcal{R}_{pp}$	0.2731	7.58×10^{-4}	0.453	1.02×10^{-3}
$\mathcal{C}_{pp}$	0.0842	2.20×10^{-4}	0.1738	4.84×10^{-4}
$\langle\delta p_{t1}\delta p_{t2}\rangle_{pp}$	0.00257	2.27×10^{-5}	0.00446	3.67×10^{-5}
$\mathcal{D}_{pp}$	0.01685	9.32×10^{-5}	0.0348	1.68×10^{-4}

Correlations and Fluctuations

Momentum Density of Particles $$\int \rho_1(\mathbf{p}_1)\, d^3\mathbf{p}_1 = \langle N \rangle$$

Pair Momentum Density of Particles $$\int \rho_2(\mathbf{p}_1, \mathbf{p}_2)\, d^3\mathbf{p}_1\, d^3\mathbf{p}_2 = \langle N(N-1) \rangle$$

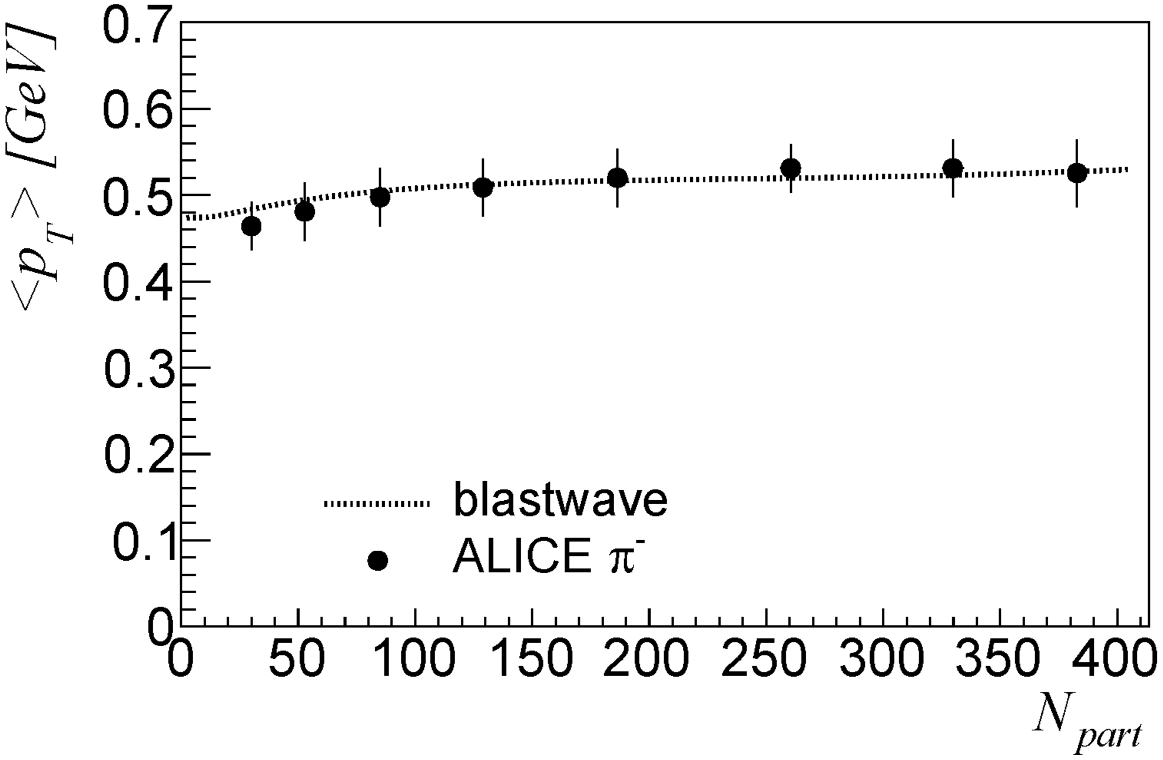
$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2) + r(\mathbf{p}_1, \mathbf{p}_2)$$

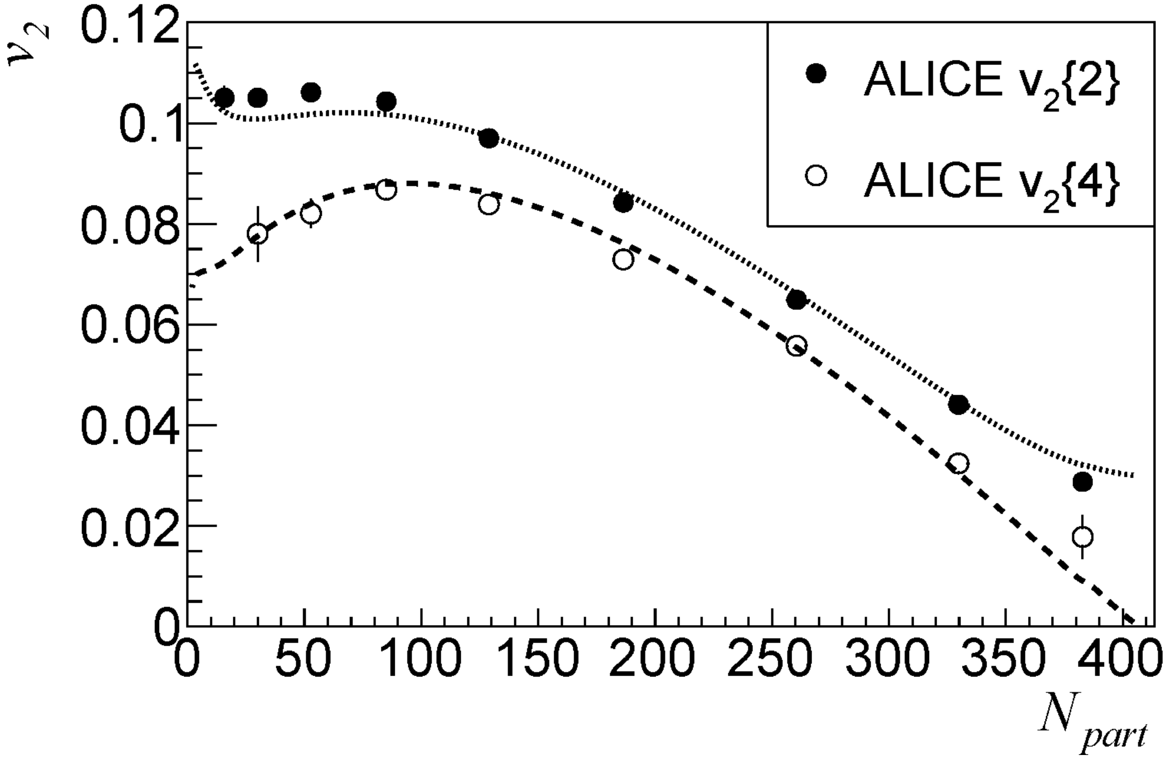
Correlated Pair Distribution $$r(\mathbf{p}_1, \mathbf{p}_2) = \rho_2(\mathbf{p}_1, \mathbf{p}_2) - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$$

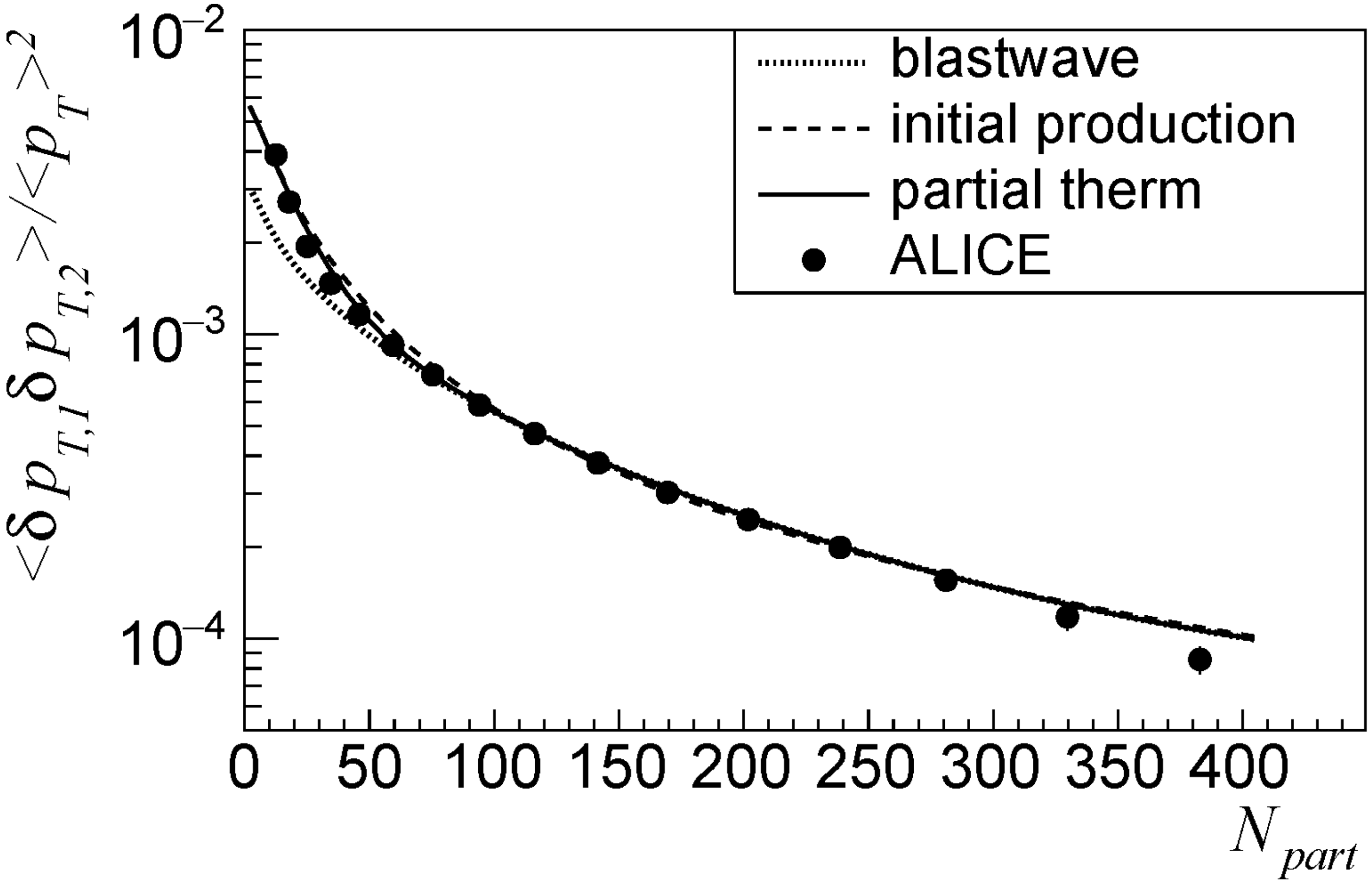
In the case of no correlations $\rho_2(\mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)$

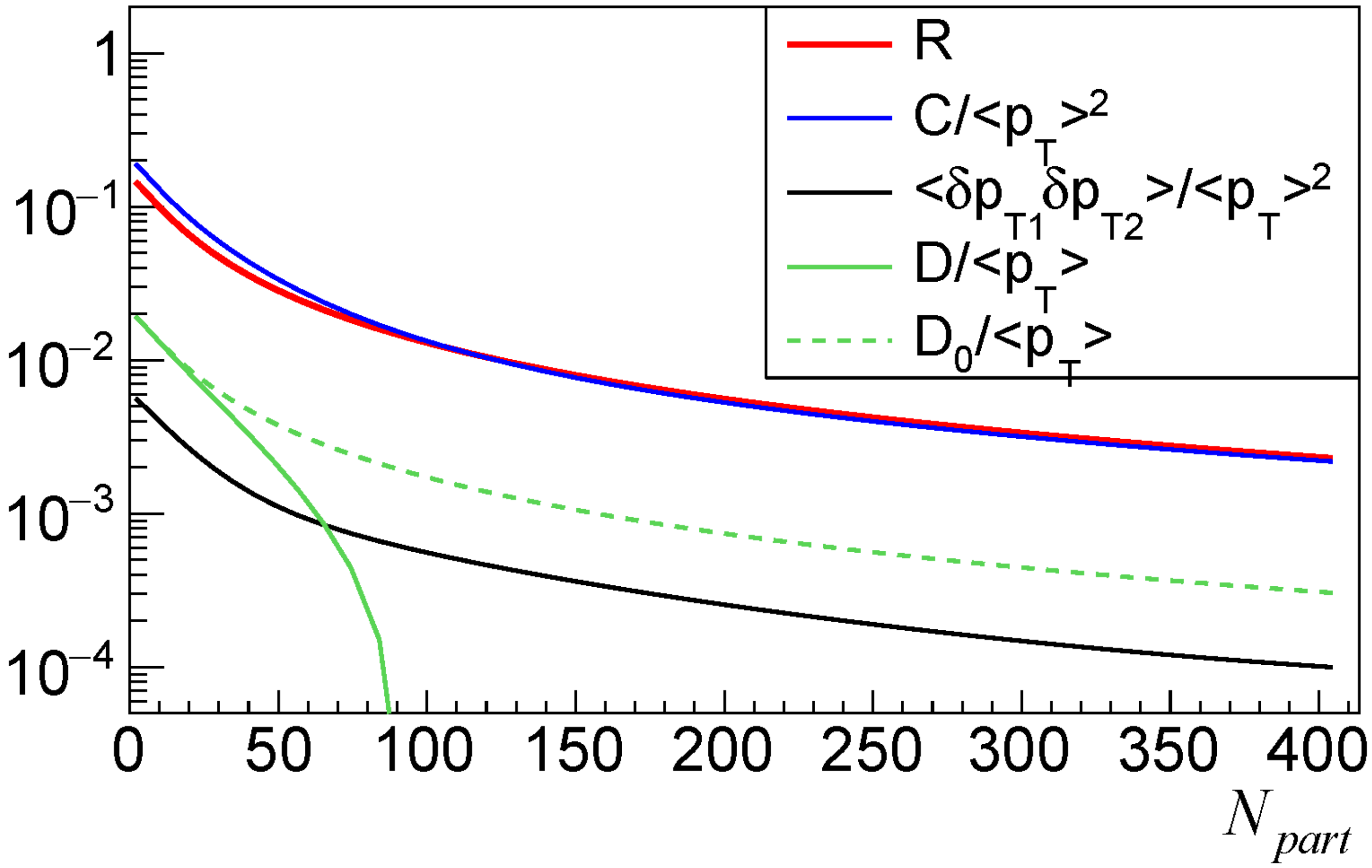
$$r(\mathbf{p}_1, \mathbf{p}_2) = 0$$

Constraining the Blastwave






10^{-2}
10^{-3}
10^{-4}
$<\delta p_{T,1}\delta p_{T,2}>/<p_T>^2$
0
50
100
150
200
250
300
350
400
N_{part}
blastwave
initial production
partial therm
ALICE



R
$C/\langle p_T\rangle^2$
$\langle\delta p_{T1}\delta p_{T2}\rangle/\langle p_T\rangle^2$
$D/\langle p_T\rangle$
$D_0/\langle p_T\rangle$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
50
100
150
200
250
300
350
400
N_{part}