Estimating transport coefficients of strongly interacting matter with an extended NJL model

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XQCD 2023, 27/07/2023, Coimbra, Portugal

Outline

Introduction

QCD: the Theory of **Strong Interactions**

- Very successfull pQCD at high energy
- Non-perturbative low energy regime requires the use of other tools for instance:
  - lQCD
  - AdS/QCD
  - Dyson-Schwinger
  - FRG
  - Chiral pertubation theory
  - Effective models
    - **Dynamical**/**Explicit** **Chiral Symmetry Breaking** plays a big role in low energy phenomenology

The model

$$\mathcal{L}_{\text{eff}} = \overline{\psi}\,(\imath\gamma^{\mu}\partial_{\mu} - \hat{m})\,\psi$$

The model

$$\mathcal{L}_{\text{eff}} = \overline{\psi}\left(\imath\gamma^{\mu}\partial_{\mu} - \hat{m}\right)\psi + \mathcal{L}_{NJL}$$

- **Nambu–Jona-Lasinio** (4 q)

$$\mathcal{L}_{NJL} = \frac{G}{2}\left(\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}\imath\gamma^5\lambda_a\psi\right)^2\right)$$

The model

$$\mathcal{L}_{\text{eff}} = \overline{\psi}\left(\imath\gamma^{\mu}\partial_{\mu} - \hat{m}\right)\psi + \mathcal{L}_{NJL} + \mathcal{L}_{H}$$

- $\mathcal{L}_{NJL} = \frac{G}{2}\left(\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}\imath\gamma^5\lambda_a\psi\right)^2\right)$
- **'t Hooft determinant** (6 q)

$$\mathcal{L}_H = 8\kappa\left(\det\left[\overline{\psi}P_L\psi\right] + \det\left[\overline{\psi}P_R\psi\right]\right)$$

The model

$$\mathcal{L}_{\text{eff}} = \overline{\psi}\left(\imath\gamma^{\mu}\partial_{\mu} - \hat{m}\right)\psi + \mathcal{L}_{NJL} + \mathcal{L}_{H} + \mathcal{L}_{8q}$$

- $\mathcal{L}_{NJL} = \frac{G}{2}\left(\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}\imath\gamma^5\lambda_a\psi\right)^2\right)$
- $\mathcal{L}_{H} = 8\kappa\left(\det\left[\overline{\psi}P_L\psi\right] + \det\left[\overline{\psi}P_R\psi\right]\right)$
- **Eight quark interaction term**

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$$

$$\mathcal{L}_{8q}^{(1)} = 16g_1\left[\left(\overline{\psi}_i P_R\psi_m\right)\left(\overline{\psi}_m P_L\psi_i\right)\right]^2$$

$$\mathcal{L}_{8q}^{(2)} = 16g_2\left(\overline{\psi}_i P_R\psi_m\right)\left(\overline{\psi}_m P_L\psi_j\right)\left(\overline{\psi}_j P_R\psi_K\right)\left(\overline{\psi}_K P_L\psi_i\right)$$

The model

$$\mathcal{L}_{\text{eff}} = \overline{\psi}\left(\imath\gamma^{\mu}\partial_{\mu} - \hat{m}\right)\psi + \mathcal{L}_{NJL} + \mathcal{L}_{H} + \mathcal{L}_{8q}$$

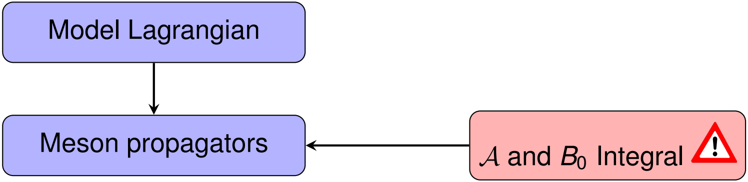
- $\mathcal{L}_{NJL} = \frac{G}{2}\left(\left(\overline{\psi}\lambda_a\psi\right)^2 + \left(\overline{\psi}\imath\gamma^5\lambda_a\psi\right)^2\right)$
- $\mathcal{L}_{H} = 8\kappa\left(\det\left[\overline{\psi}P_L\psi\right] + \det\left[\overline{\psi}P_R\psi\right]\right)$
- $\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$
  $\mathcal{L}_{8q}^{(1)} = 16g_1\left[\left(\overline{q}_iP_Rq_m\right)\left(\overline{q}_mP_Lq_i\right)\right]^2$
  $\mathcal{L}_{8q}^{(2)} = 16g_2\left(\overline{q}_iP_Rq_m\right)\left(\overline{q}_mP_Lq_j\right)\left(\overline{q}_jP_Rq_K\right)\left(\overline{q}_KP_Lq_i\right)$

OZI violation in $\mathcal{L}_H$ and $\mathcal{L}_{8q}^{(1)}$.

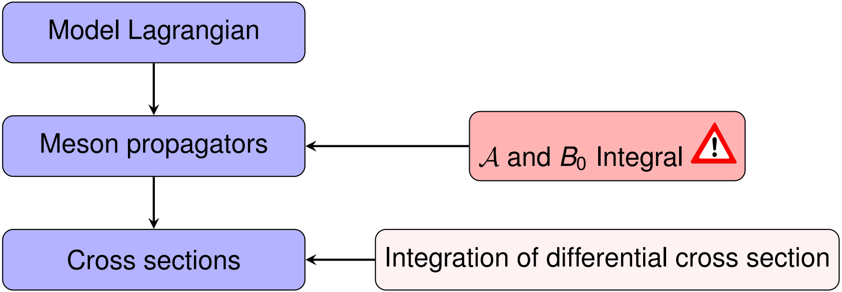
Strategy overlook

Model Lagrangian

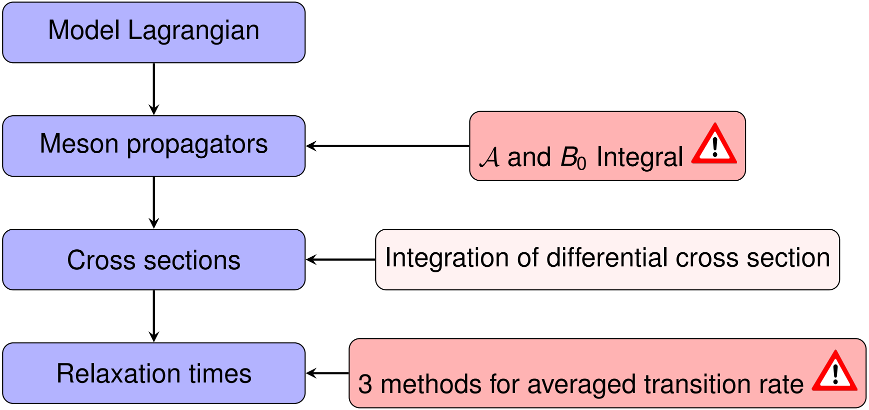
Strategy overlook



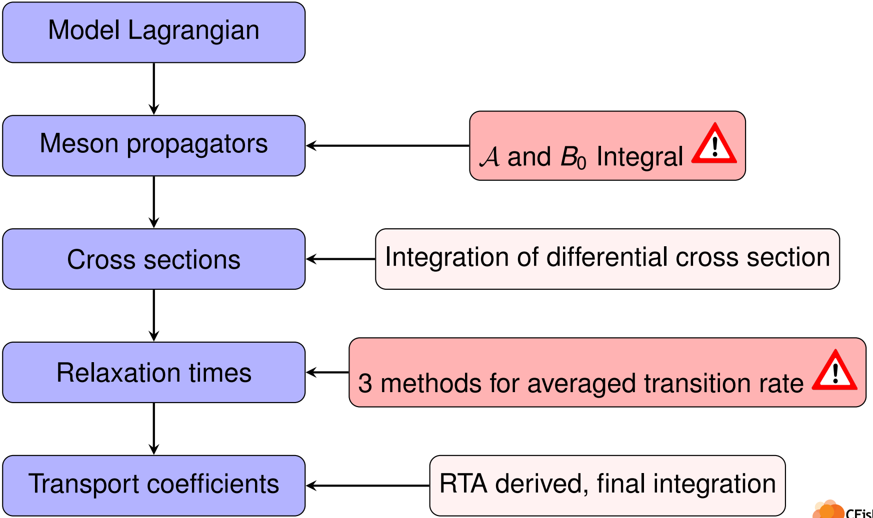
Strategy overlook



Strategy overlook



Strategy overlook



Meson propagators

Meson propagators can be obtained from the effective action and involve the scalar/pseudoscalar **meson projectors** (S and P) and the **quark polarization function** (Π^S and Π^P)

$$\left(D_S^{-1}\right)_{ab} = \frac{1}{2}\left(S^{-1}\right)_{ab} - \Pi^S_{ab}, \qquad \left(D_P^{-1}\right)_{ab} = \frac{1}{2}\left(P^{-1}\right)_{ab} - \Pi^P_{ab}$$

$$\Pi^S_{ij}\left[M_i, M_j, k_0, |\vec{k}|\right] = -\frac{N_c}{8\pi^2}\left(\mathcal{A}\left[M_i, |\vec{k}|\right] + \mathcal{A}\left[M_j, |\vec{k}|\right] + \left((M_i + M_j)^2 - k_0^2 + \vec{k}^2\right) B_0\left[M_i, M_j, \mathrm{Re}\left[k_0\right], |\vec{k}|\right]\right)$$

$$\Pi^P_{ij}\left[M_i, M_j, k_0, |\vec{k}|\right] = -\frac{N_c}{8\pi^2}\left(\mathcal{A}\left[M_i, |\vec{k}|\right] + \mathcal{A}\left[M_j, |\vec{k}|\right] + \left((M_i - M_j)^2 - k_0^2 + \vec{k}^2\right) B_0\left[M_i, M_j, \mathrm{Re}\left[k_0\right], |\vec{k}|\right]\right)$$

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Meson propagators

The relevant integrals are given, in Minkowski space-time, by:

$$\mathcal{A}\left[M_i, |\vec{k}|\right] = -16\pi^2 i \int_{\text{Reg}[k]} \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p_\mu p^\mu - M_i^2}$$

$$B_0\left[M_i, M_j, k_0, |\vec{k}|\right] = -16\pi^2 i \int_{\text{Reg}[k]} \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p_\mu p^\mu - M_i^2} \frac{1}{(p-k)_\mu (p-k)^\mu - M_j^2}$$

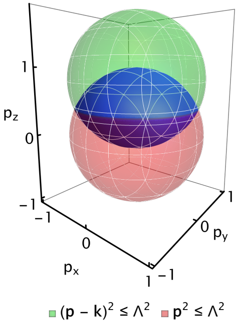
In order to ensure interchangeability of the quarks when evaluating a process, for instance, $u\,s \to u\,s \Leftrightarrow s\,u \to s\,u$, one should have

$$B_0\left[M_i, M_j, T, \mu_i, \mu_j, k_0, |\vec{k}|\right] \Leftrightarrow B_0\left[M_j, M_i, T, -\mu_j, -\mu_i, -k_0, |\vec{k}|\right] \quad (1)$$

Integration region

We shall consider the following integration region **both** for the B_0 and the $\mathcal{A}$ integrals in the polarization function

- Note that the regularization of the polarization is now external momentum dependent
- Relevant for the evaluation of cross sections



Relaxation times

$$n_i\left[T,\mu_i\right]=\begin{cases} 2N_c\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}f_{\mathrm{F}}\left[E_i-\mu_i,T\right] & i\in\{u,d,s\}\\ 2N_c\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}f_{\mathrm{F}}\left[E_i+\mu_i,T\right] & i\in\left\{\bar{u},\bar{d},\bar{s}\right\}\end{cases},$$

$$\bar{\sigma}_{ab\to X}\left[T,\boldsymbol{\mu_Q}\right]=\frac{1}{n_a\left[T,\mu_a\right]n_b\left[T,\mu_b\right]}\int\frac{\mathrm{d}^3\vec{p}_a}{(2\pi)^3}\int\frac{\mathrm{d}^3\vec{p}_b}{(2\pi)^3}$$
$$\times\left(2N_c f_{\mathrm{F}}^a\left[E_a\mp\mu_a,T\right]\right)\left(2N_c f_{\mathrm{F}}^b\left[E_b\mp\mu_b,T\right]\right)v_{\mathrm{rel}}\left[M_a,\vec{p}_a,M_b,\vec{p}_b\right]$$
$$\times\,\sigma_{ab\to X}\left[T,\boldsymbol{\mu_Q},s\right]$$

$$\tau_q^{-1}\left[T,\mu\right]=\sum_f\sum_X n_f\left[T,\mu\right]\bar{\sigma}_{qf\to X}\left[T,\mu\right]$$

We compared three different methods for the evaluation of the relaxation times

- Method I: direct evaluation of the integral above
- Method II: numerical approximation following Rehberg *et al*, Nucl. Phys. A608, 356–388
- Method II: numerical approximation following Zhuang *et al*,Phys. Rev. D 51, 3728

Transport coefficients

We will use the following **relaxation time approximation** derived expressions

- **Shear viscosity** quantifies the resistance of a fluid to shear flow

$$\eta[T,\boldsymbol{\mu_Q}] = \frac{2N_c}{15T}\sum_{i=q,\bar{q}}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{\vec{p}^4}{E_i^2}\,\tau_i\, f_{\mathrm{F}}^i\left[E_i\mp\mu_i,T\right]\left(1-f_{\mathrm{F}}^i\left[E_i\mp\mu_i,T\right]\right)$$

- **Bulk viscosity** quantifies the resistance of a fluid to volumetric expansion

$$\zeta[T,\boldsymbol{\mu_Q}] = \frac{2N_c}{T}\sum_{i=q,\bar{q}}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{E_i^2}\,\tau_i\, f_{\mathrm{F}}^i\left[E_i\mp\mu_i,T\right]\left(1-f_{\mathrm{F}}^i\left[E_i\mp\mu_i,T\right]\right)$$
$$\times\left(\left(\frac{1}{3}-c_s^2\Big|_\rho\right)\vec{p}^2 - c_s^2\Big|_\rho\left(M_i^2 - TM_i\frac{\mathrm{d}M_i}{\mathrm{d}T} - \mu_B M_i\frac{\mathrm{d}M_i}{\mathrm{d}\mu_B}\right) + \frac{\partial p}{\partial\rho}\Big|_\epsilon\left(M_i\frac{\mathrm{d}M_i}{\mathrm{d}\mu_B} - E_i b_i\right)\right)^2$$

Remember that $\tau_i,\ M_i,\ E_i\ldots$ are all T and $\boldsymbol{\mu_Q}\equiv\{\mu_u,\mu_d,\mu_s\}$ dependent

Parametrizations

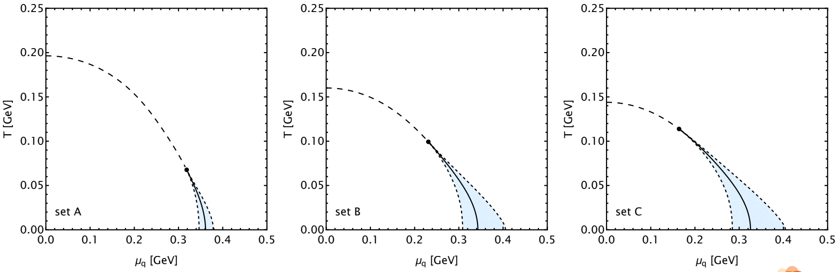
Within the NJLH8q there is a family of parametrizations which leaves the vacuum meson properties almost unchanged (raising g_1 and lowering G, M_σ decreases)

The chosen illustrative parametrization sets were:

- Set A: no 8q interactions, a “standard” set
- Set B: with 8q interactions (weak OZI-violating 8q)
- Set C: with 8q interactions (strong OZI-violating 8q)

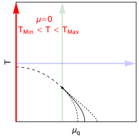
Phase diagram

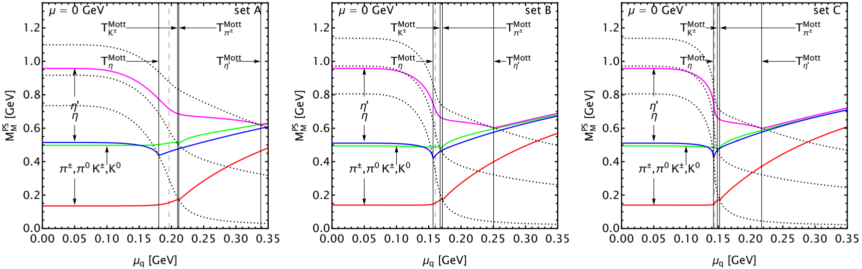
- Increasing the OZI violating eight quark interactions coupling, g_1 , (and lowering the NJL coupling) has a big impact in the thermodynamics of the model
  - Faster transition behavior for sets with higher g_1
  - Two CEPs are a possibility (which we will not consider here)



Meson properties: pseudoscalar masses

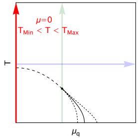
$\mu_q = 0,\ 0 < T < 0.35$ GeV

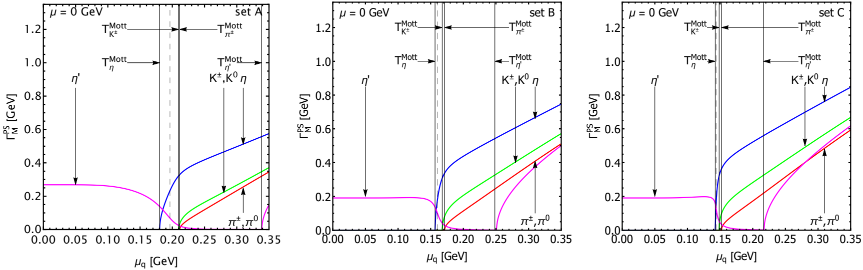




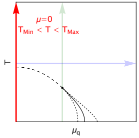
Meson properties: pseudoscalar widths

$\mu_q = 0,\ 0 < T < 0.35\ \mathrm{GeV}$

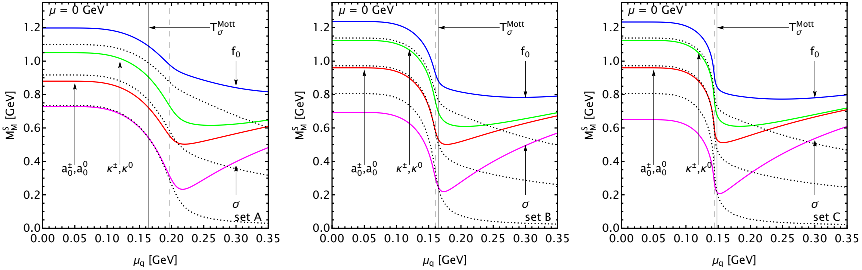




Meson properties: scalar masses

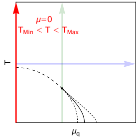


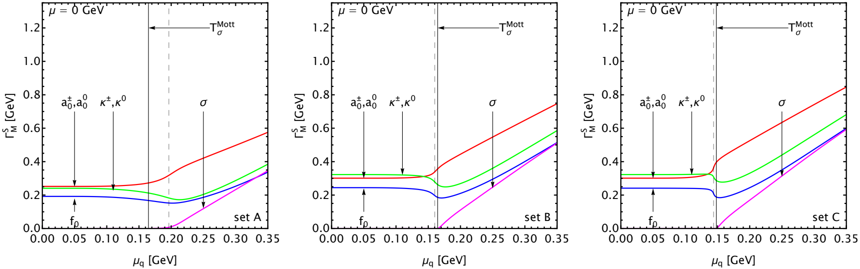
$\mu_q = 0,\ 0 < T < 0.35$ GeV



Meson properties: scalar widths

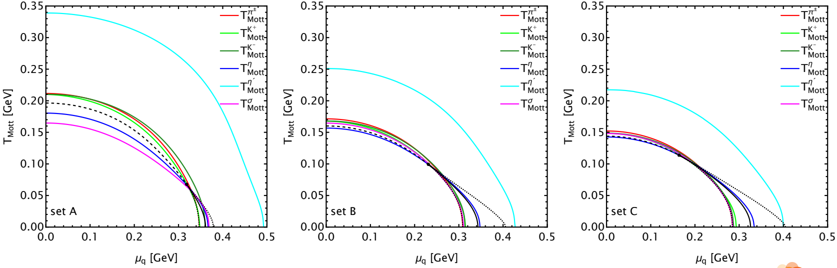
$\mu_q = 0$, $0 < T < 0.35$ GeV



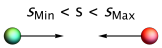


Mott temperatures/chemical potential

- Correspond to the appearance of an imaginary part in the pole of the meson propagator (at vanishing external momentum)
  - The η' starts as a resonance (finite decay width) at vanishing T and μ_q but with increasing temperature at vanishing chemical potential the width vanishes and then reappears



Cross sections: impact of the regularization procedure

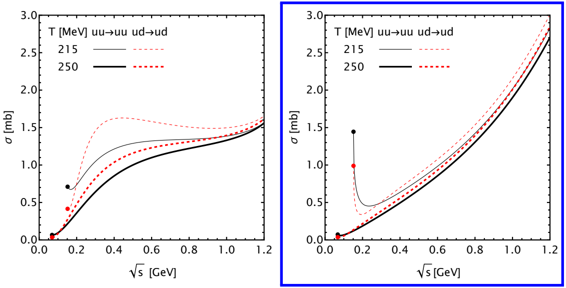


Parameter **set A** (no 8q)

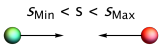
$uu \to uu$, $ud \to ud$

$\mu = 0$, $T = 215$ MeV, $T = 250$ MeV

New regularization on the **right-hand** side



Cross sections: impact of the regularization procedure

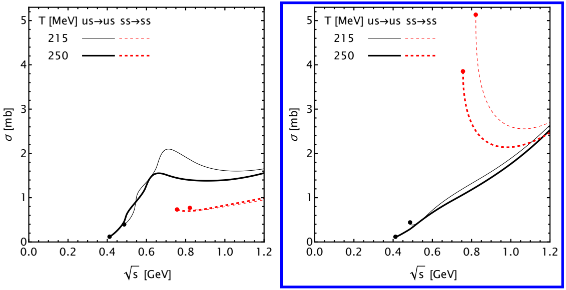


Parameter **set A** (no 8q)

$us \to us$, $ss \to ss$

$\mu = 0$, $T = 215$ MeV, $T = 250$ MeV

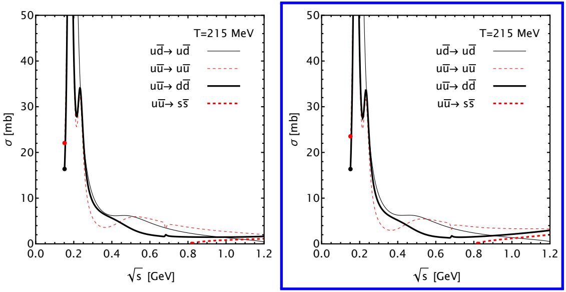
New regularization on the **right-hand** side



Cross sections: impact of the regularization procedure

$s_{Min} < s < s_{Max}$

Parameter **set A** (no 8q)
$u\bar{d} \to u\bar{d}$, $u\bar{u} \to u\bar{u}/d\bar{d}/s\bar{s}$
$\mu = 0$, $T = 215$ MeV
New regularization on the **right-hand** side



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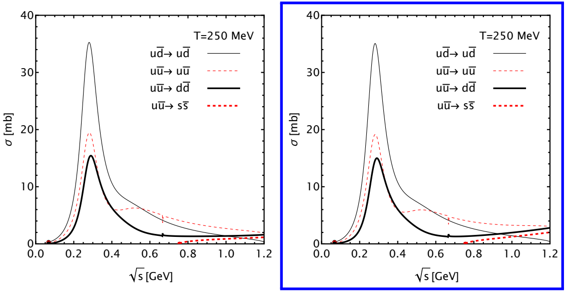
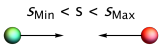
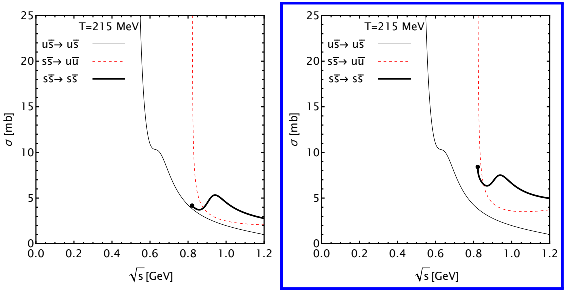


Cross sections: impact of the regularization procedure



Parameter **set A** (no 8q)
$u\bar{s} \to u\bar{s}$, $s\bar{s} \to u\bar{u}/s\bar{s}$
$\mu = 0$, $T = 215$ MeV
New regularization on the **right-hand** side



Cross sections: impact of the regularization procedure

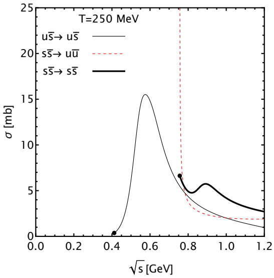
$s_{Min} < s < s_{Max}$

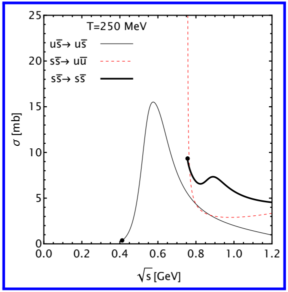
Parameter **set A** (no 8q)

$u\bar{s} \to u\bar{s}$, $s\bar{s} \to u\bar{u}/s\bar{s}$

$\mu = 0$, $T = 250$ MeV

New regularization on the **right-hand** side





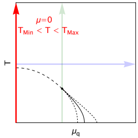
Cross sections: impact of the regularization procedure

- In $q_i\, q_j \rightarrow q_i\, q_j$ (and $\overline{q}_i\, \overline{q}_j \rightarrow \overline{q}_i\, \overline{q}_j$) we see a **striking difference**
- In $q\, \overline{q} \rightarrow q\, \overline{q}$ we see a **small difference at higher center of mass energy**

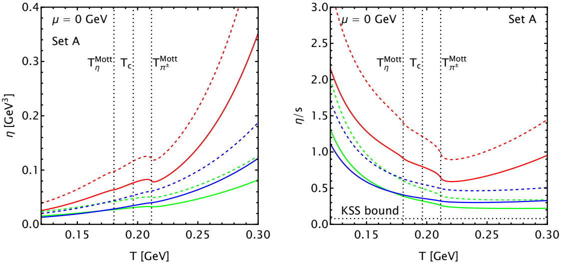
The reason for this behavior can be easily understood in terms of Mandelstam variables and the intervening channels:

- the spacial component of the exchanged momentum is 0 in the s channel is zero (and the new regularization coincides with the traditional result)
- the difference grows for larger spacial component of the exchanged momentum

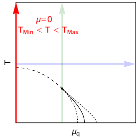
Transport coefficients: shear viscosity



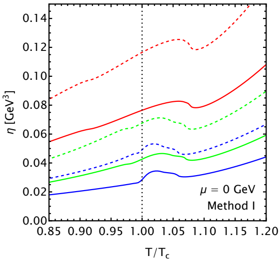
Parameter **set A** (no 8q)
$\mu_q = 0,\ 0.12\ \mathrm{GeV} < T < 0.3\ \mathrm{GeV}$
Method I, **method II** and **method III** comparison

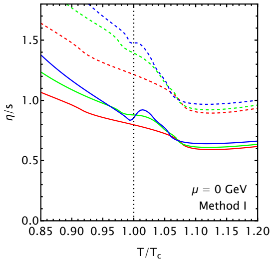


Transport coefficients: shear viscosity

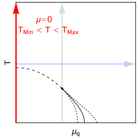


Parameter **set A**, **set B** and **set C** comparison
$\mu_q = 0$, $0.83\ T_c < T < 1.53\ T_c$
Method I





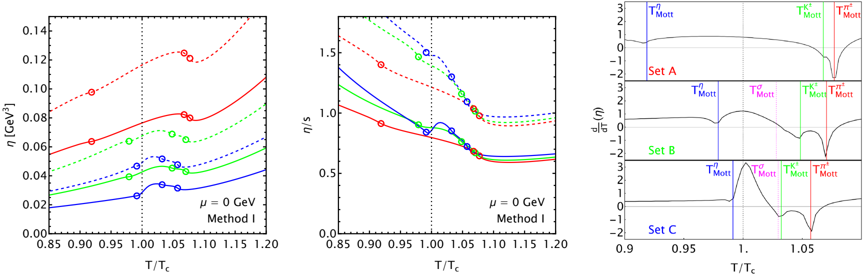
Transport coefficients: shear viscosity



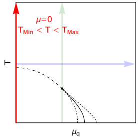
Parameter **set A**, **set B** and **set C** comparison

$\mu_q = 0$, $0.83\,T_c < T < 1.53\,T_c$

Method I



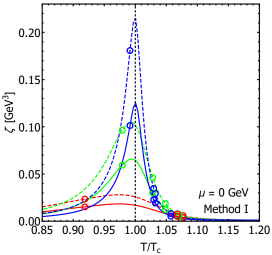
Transport coefficients: bulk viscosity

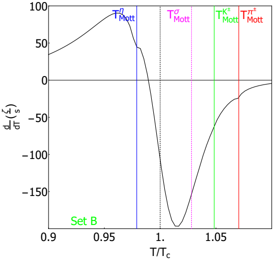


Parameter **set A**, **set B** and **set C** comparison

$\mu_q = 0$, $0.83\ T_c < T < 1.53\ T_c$

Method I





Transport coefficients and Mott temperatures connections

- η, shear viscosity
  - T^{η}_{Mott}, $T^{K^{\pm}}_{\text{Mott}}$ and $T^{\pi^{\pm}}_{\text{Mott}}$ correspond to minima of $\frac{\mathrm{d}\eta}{\mathrm{d}T}$ (**inflection points** of $\eta\,[T]$, $\frac{\mathrm{d}^2\eta}{\mathrm{d}T^2}=0$)
- ζ, bulk viscosity
  - T^{η}_{Mott}, T^{σ}_{Mott}, $T^{K^{\pm}}_{\text{Mott}}$, $T^{\pi^{\pm}}_{\text{Mott}}$ and $T^{\eta'}_{\text{Mott}}$ correspond to inflection points in $\frac{\mathrm{d}\eta}{\mathrm{d}T}$ (zeros of the third derivative, $\frac{\mathrm{d}^3\zeta}{\mathrm{d}T^3}=0$)

Conclusions

- The **novel regularization** procedure:
  - the external momentum dependent restriction of the integration region should be applied to **both** $\mathcal{A}$ and B_0 integrals when considering the quark polarization function
  - enables a **consistent** treatment of the $(anti)quark - (anti)quark$ cross sections
  - has a **significant impact** in some of the cross sections
- **Method I** is advised. Method II and II for the evaluation of the averaged transition rates appear to wash away some of the features
- Some signs of the influence of **meson properties** on **transport coefficients**
  - **Mott temperatures** show up as **special points** (inflexion point or inflexion point of the derivative) in the temperature dependency of **shear** and **bulk viscosity**

Additional information:

- "A new approach to the 3-momentum regularization of the in-medium one and two fermion line integrals with applications to cross sections in the Nambu–Jona-Lasinio model",
  Renan Pereira, J. Moreira, P. Costa, Constança Providência,
  in preparation
- "A new look into mesonic properties in the QCD phase diagram using an extended NJL model",
  J. Moreira, Renan Pereira, P. Costa, Constança Providência,
  in preparation
- "Transport coefficients of hot and dense quark matter from an extended Nambu–Jona-Lasinio model",
  J. Moreira, Renan Pereira, P. Costa, Constança Providência,
  in preparation
- I would like to thank the support of FCT through a research grant under project PTDC/FIS-NUC/29912/2017