Transient effects of charge diffusion on EM fields in heavy-ion collision

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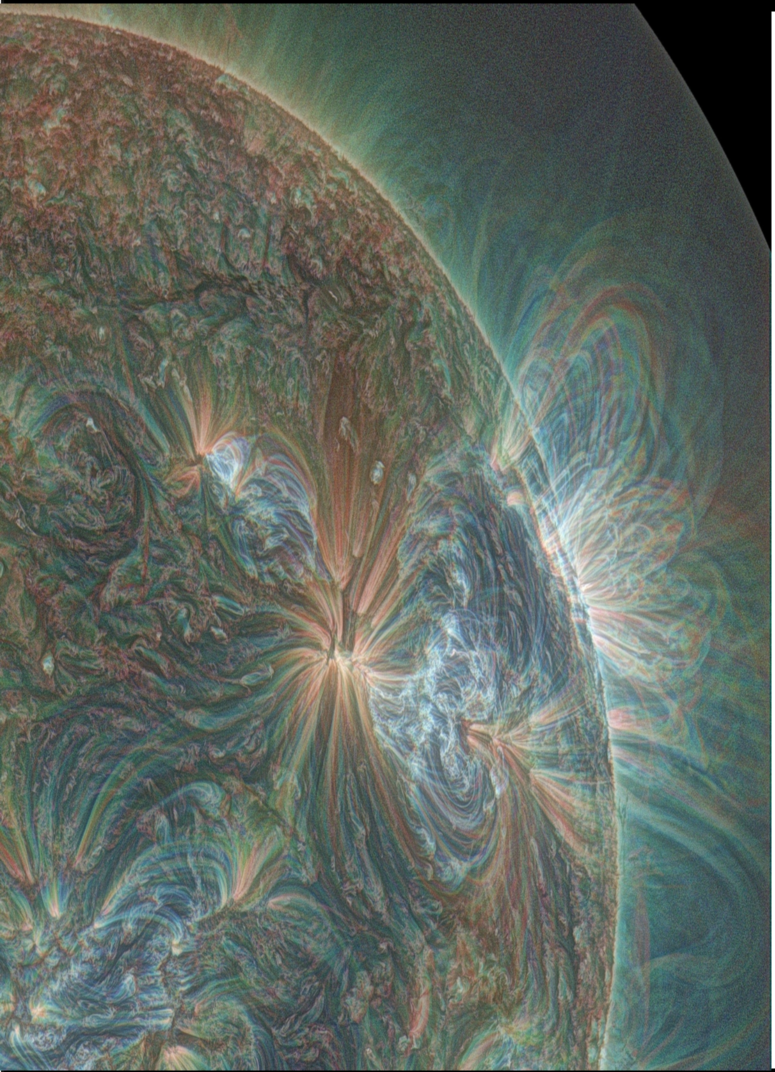


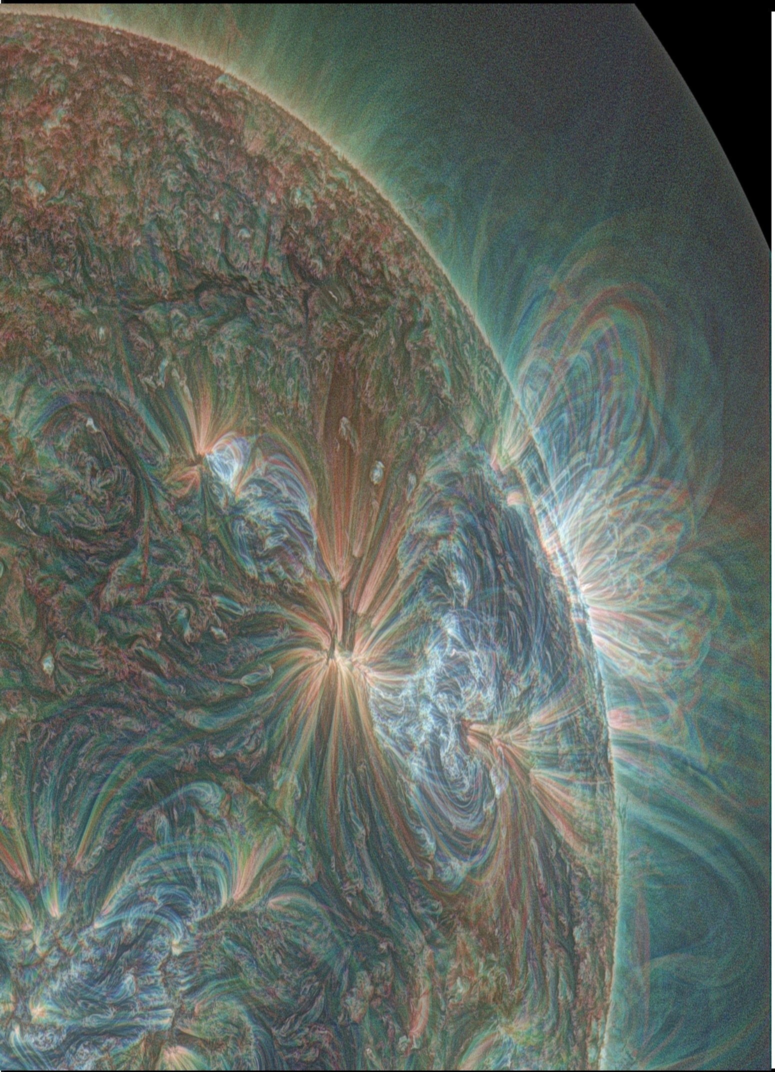
INTRODUCTION

- Magnetohydrodyanmics (MHD) describes the physics of electomagnetically charged plasmas [1].

- Electric fields are screened in a conducting plasma, due to the presence of electrically charged particles. Dynamics is dominated by magnetic fields called ideal MHD.

- Applications: Astrophysics including stars and the interstellar medium, solar physics, heavy-ion collisions etc.

[1] H. Alfvén, Nature 150, 405 (1942)



WHY MHD IS IMPORTANT ?

- Cosmology: suppose that a particle (say a proton) moves at the earth's solar distance R with the earth's orbital velocity v. The gravitational and EM forces are

$$\mathbf{F}_G = -G\frac{Mm\mathbf{R}}{R^3} \qquad \mathbf{F}_{EM} = e(\mathbf{v}/c) \times \mathbf{B}$$

Ratio of the forces ($\mathbf{B} = 10^{-4}\ G$): $\mathbf{F}_{EM}/\mathbf{F}_G \approx 10^7$

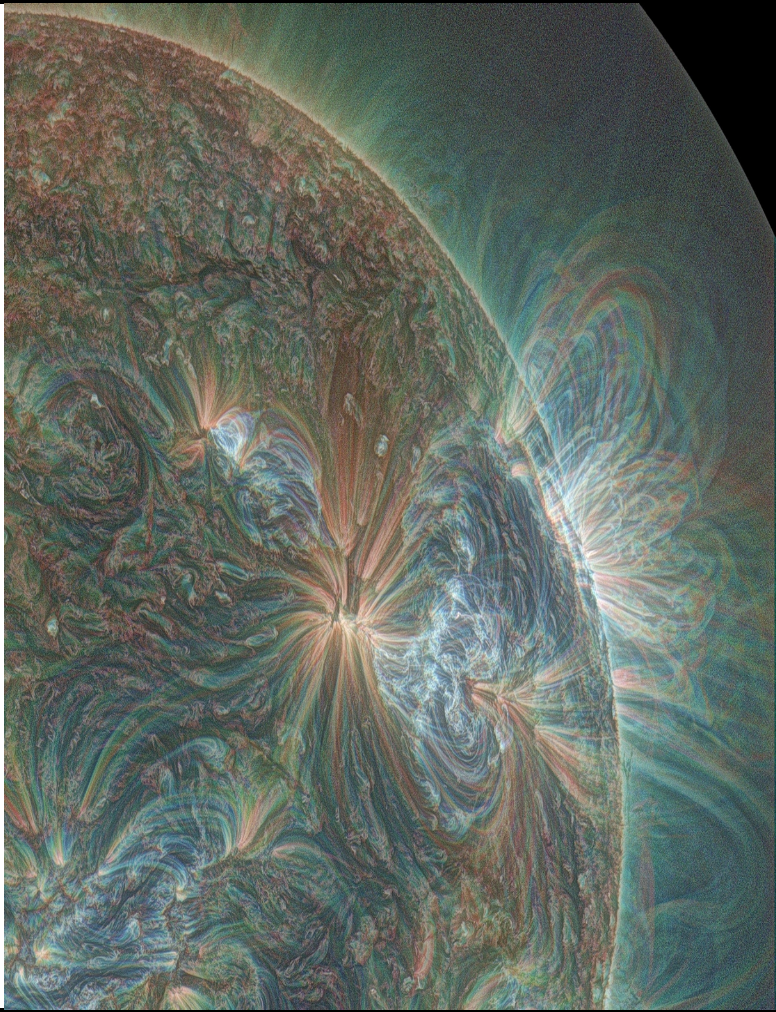
Interplanetary magnetic field

- Heavy ion collision:

$$eB \sim m_\pi^2 \sim 10^{18}\ G$$

RELATIVISTIC MAGNETO-HYDRODYNAMICS

Formulation



HYDRODYNAMICS AND ELECTROMAGNETISM

- Magnetohydrodyanmics (MHD) can be formulated as a charged fluid coupled to dynamical electromagnetic fields.

- The dynamical equations of MHD are the energy-momentum and charge conservaton

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_f = 0 \ ,$$

coupled to Maxwell's equations

$$\partial_\mu F^{\mu\nu} = J^\nu \ , \quad \epsilon^{\mu\nu\alpha\beta}\partial_\mu F_{\alpha\beta} = 0$$

- The dynamical fields of MHD are

$$u^\mu (u^\mu u_\mu = 1) \ , \ T \ , \ \mu \ , \ \mathcal{E}^\mu \equiv F^{\mu\nu} u_\nu \ , \ \mathcal{B}^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} u_\nu$$

MHD CONSTITUTIVE RELATIONS

- The plasma is characterised by its constituive relations

$$T^{\mu\nu}[u^\mu, T, \mu, \mathcal{E}^\mu, \mathcal{B}^\mu] \,, \quad J^\mu[u^\mu, T, \mu, \mathcal{E}^\mu, \mathcal{B}^\mu]$$

- For non-polarizable, non-magnetizable fluids, the energy-momentum tensor reads

$$T^{\mu\nu} = T^{\mu\nu}_f + T^{\mu\nu}_{em} \,.$$

$$T^{\mu\nu}_f \equiv w u^\mu u^\nu - P g^{\mu\nu} \,,$$
$$T^{\mu\nu}_{em} = -F^{\mu\lambda}F^\nu_{\lambda} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta}F_{\alpha\beta}$$

$$w \equiv \varepsilon + P \,, dP = s dT + q n_f d\mu \,, \varepsilon = sT + q n_f \mu - P$$

- Similarly, the fluid charge current reads

$$J^\mu_f \equiv q \left(n_f u^\mu + V^\mu_f \right) \,.$$

GRADIENT EXPANISION & MHD

- It is well known that **ordinary** hydrodynamics is an effective theory valid in the low-frequency, large-wavelength limit.

- Expansion parameter of this theory is the Knudsen number

$$\mathrm{Kn} \equiv \frac{\lambda}{L}$$

λ → Microscopic

L → Macroscopic

- Microscopic details of the system can be safely integrated out if Kn<<1, and hence the system can be defined by few macroscopic variables.

- At zeroth order in expansion the system is described by **ideal hydrodynamics**

- At first order in expansion the system is described by **Navier-Stokes hydrodynamics.**

$$\Pi = \lambda_\Pi \theta \; , V_f^\mu = \lambda_n \nabla^\mu \alpha \; , \pi^{\mu\nu} = \lambda_\pi \sigma^{\mu\nu}$$

Standard MHD appears as a natural extension of Navier-Stokes theory for conducting fluids

Navier-Stokes hydro violates causality

Consider the classic diffusuion model in 1+1 dimension which starts from the continuity equation

$$\frac{\partial n_f(t,x)}{\partial t} = -\nabla \cdot \mathbf{V}_f(t,x)$$

and use the NS form of diffusion equation : $\mathbf{V}_f(t,x) = -\mathcal{D}\nabla n_f(t,x)$

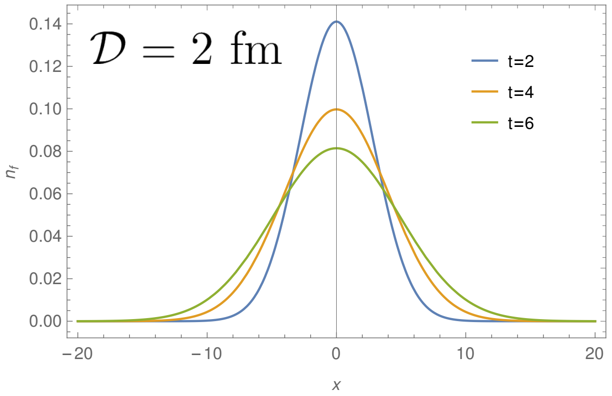
Diffusion coefficient

to get : $$\frac{\partial n_f(t,x)}{\partial t} = \mathcal{D}\nabla^2 n_f$$

which has the solution

$$p(t,x|t_0,x_0) = \left(\frac{1}{4\pi\mathcal{D}(t-t_0)}\right)^{1/2} \exp\left(-\frac{(x-x_0)^2}{4\mathcal{D}(t-t_0)}\right)$$

for an initial condition which is: $n_f(t_0,x) = \delta(x-x_0)$



Restoring causality

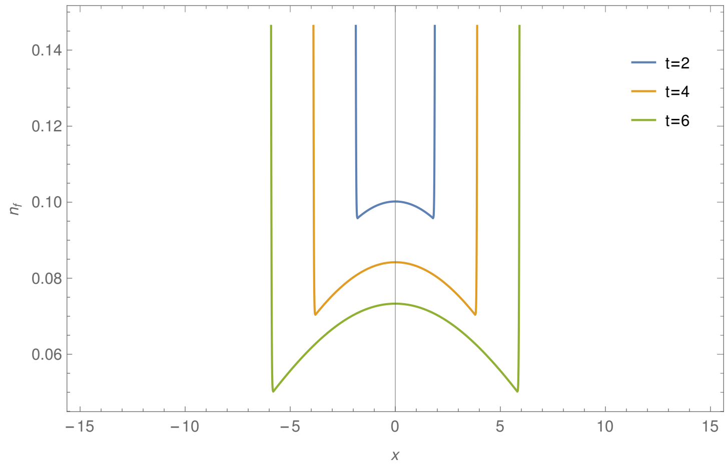
Phenomenologically this can be achived by adding a causal time lag to the diffusion equation, i.e.,

$$\tau_V \frac{\partial \mathbf{V}_f(t,x)}{\partial t} + \mathbf{V}_f(t,x) = -\mathcal{D}\nabla n(t,x)$$

τ_V → relaxation time

with the equation of charge diffusion as: $$\tau_V \frac{\partial^2 n_f(t,x)}{\partial t^2} + \frac{\partial n_f(t,x)}{\partial t} = \mathcal{D}\nabla^2 n_f$$

called the “Telegrapher’s equation” [2].



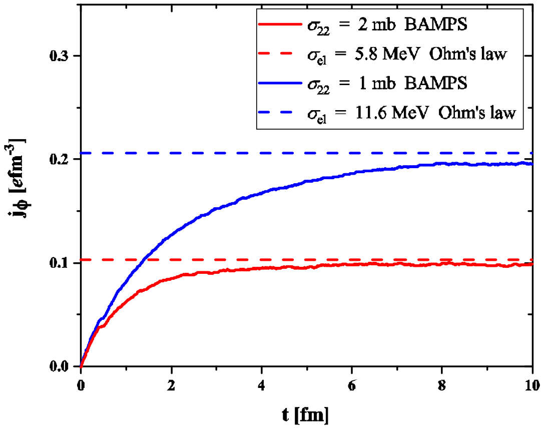
[2] Hunt, Bruce J. (2005). The Maxwellians, Ithaca, USA: Cornell University Press.

OHM'S LAW & CHARGE DIFFUSION CURRENT

- Ohm's law, in its simplest covariant Navier-Stokes-type form, reads [3]

$$qV_f^\mu = q\kappa\nabla^\mu\alpha + \sigma\mathcal{E}^\mu \ .$$

- The modification of the standard form of Ohm's law also required since the build-up of the corresponding charge diffusion current needs a finite time [4].
- Causal second-order evolution equations for dissipative quantities can be derived from a fundamental microscopic theory, e.g kinetic theory [5,6].



[3] C. Palenzuela, L. Lehner, O. Reula, and L. Rezzolla, Mon. Not. Roy. Astron. Soc. 394, 1727 (2009)
[4] Z. Wang, J. Zhao, C. Greiner, Z. Xu, and P. Zhuang, Phys. Rev. C 105, L041901 (2022)
[5] G. S. Denicol, E. Molnár, H. Niemi, and D. H. Rischke, Phys. Rev. D 99, 056017 (2019)
[6] A. K. Panda, A. Dash, R. Biswas, and V. Roy, Phys. Rev. D 104, 054004 (2021)

OHM'S LAW & CHARGE DIFFUSION CURRENT

- In its most simplest form, second-order equation for charge diffusion reads

$$\tau_V q \dot{V}_f^{\langle\mu\rangle} + q V_f^\mu = q\kappa \nabla^\mu \alpha + \sigma \mathcal{E}^\mu$$

- In the rest frame of the fluid, and assuming conductivity and relaxation time as constant, the above equation can be cast into the following form:

$$\ddot{V}_f^i + 2\,\omega_0\,\zeta_d\,\dot{V}_f^i + \omega_0^2 V_f^i = \frac{\omega_0^2}{q}\,\epsilon^{ijk}\partial_j B_k$$

which is the equation for a **damped, driven harmonic oscillator.**

$$\omega_0 \equiv \sqrt{\sigma/\tau_V}\,,\zeta_d \equiv 1/(2\sqrt{\sigma\tau_V})$$

APPLICATION TO HEAVY-ION COLLISIONS

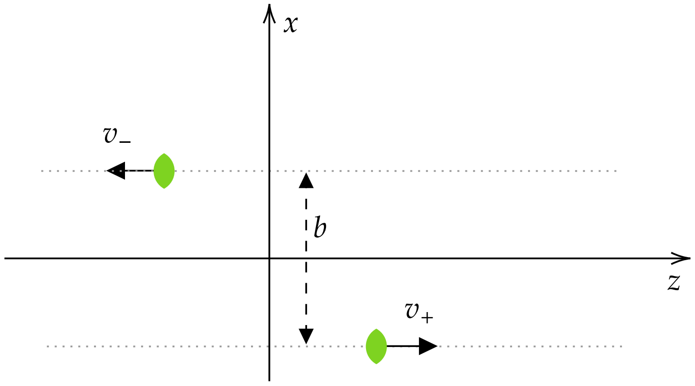
Simplified setup

COLLISION GEOMETRY & SETUP

- The electromagnetic four-potential in the Lorenz gauge is given as

$$A^{\mu}_{\pm} = \left(\frac{Z\alpha_{\mathrm{EM}}\gamma}{r_{\pm}}, 0, 0, v_{\pm}\frac{Z\alpha_{\mathrm{EM}}\gamma}{r_{\pm}} \right) ,$$

$$r_{\pm}(x,y,z,t) \equiv \sqrt{(x \pm b/2)^2 + y^2 + \gamma^2(z - v_{\pm}t)^2}$$



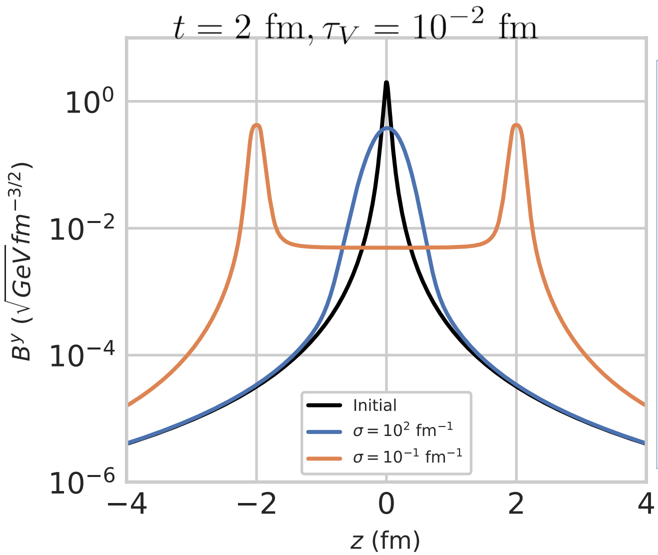
- We assume that the system is homogeneous in the transverse plane, hence consider only the electromagnetic field near $\mathbf{x}_{\perp} = 0$

$$B^{y}(0,0,z,t) = \frac{b}{2} Z\alpha_{\mathrm{EM}} \left(\frac{1}{r^3_{0,+}} + \frac{1}{r^3_{0,-}} \right) \sinh Y_{\mathrm{bm}} ,$$

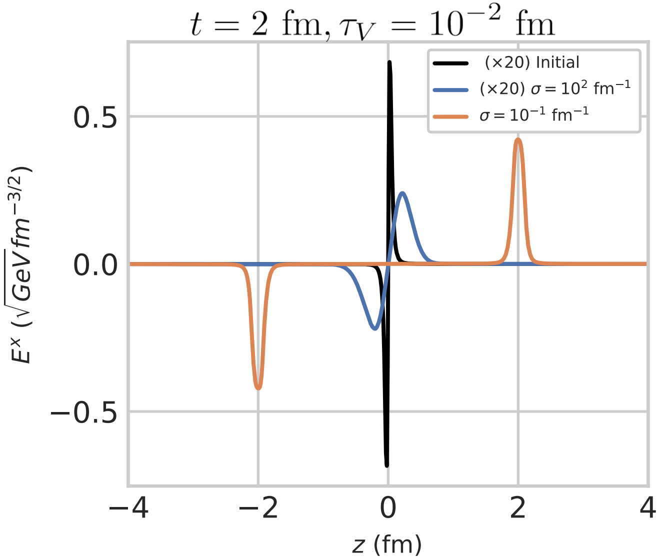
$$E^{x}(0,0,z,t) = \frac{b}{2} Z\alpha_{\mathrm{EM}} \left(\frac{1}{r^3_{0,+}} - \frac{1}{r^3_{0,-}} \right) \cosh Y_{\mathrm{bm}} ,$$

$$Y_{\mathrm{bm}} = \mathrm{Artanh}\sqrt{1 - 4m_N^2/s_{NN}}$$

Au-Au collisions @ $\sqrt{s_{NN}} = 200$ GeV



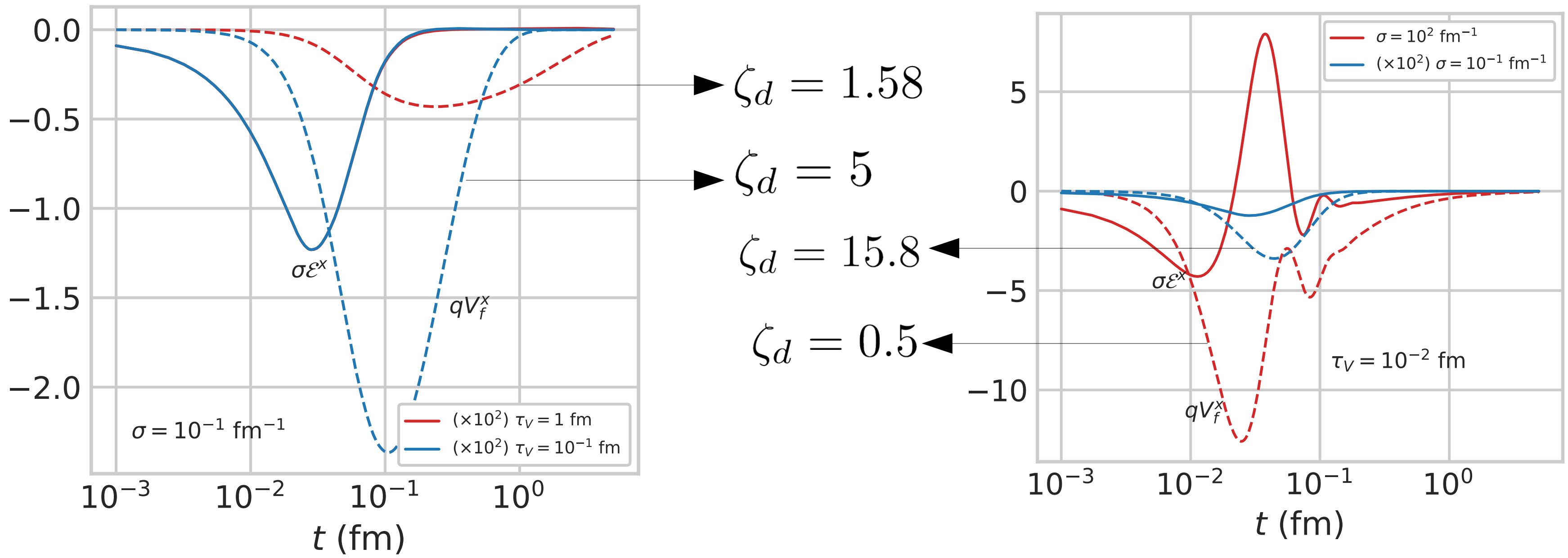
$$b = 10 \text{ fm},$$
$$t_0 = 10^{-3} \text{ fm},$$
$$P = 18.33 \text{ GeV/fm}^3,$$
$$\varepsilon = 3P.$$



- σ and τ_V are kept as free parameters.
- Closer to initial config.. for large σ (frozen flux theorem), diffusive tails for small σ.

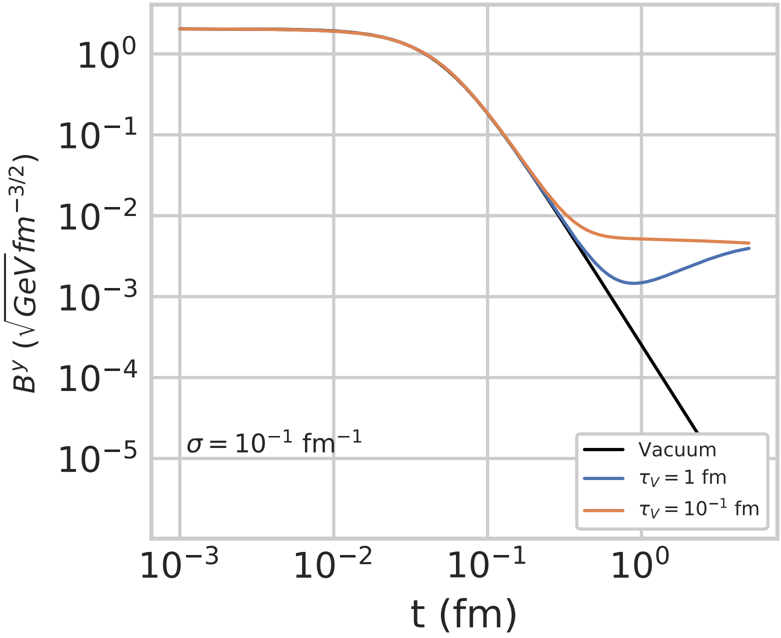
PRD 107, 056003 (2023), AD, M Shokri, L Rezzolla, D Rischke

Time evolution of the charge diffusion current



- Large τ_V takes longer to approach its Navier-Stokes value and the magnitude is also smaller.
- Oscillations in underdamped case.

Time evolution of the magnetic field



- Longer τ_V (the solid blue vs the solid orange line) means an incomplete response of the charge diffusion current and hence leads to faster decay of the magnetic field at early times.

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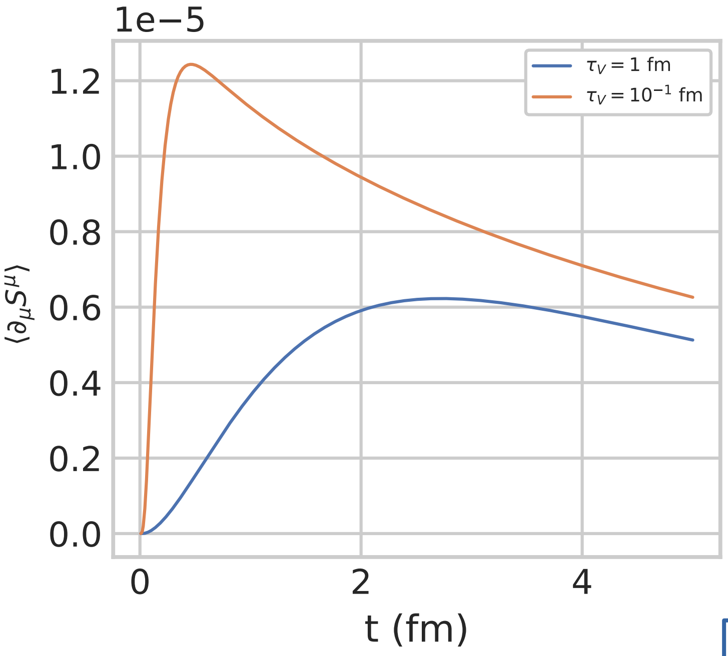
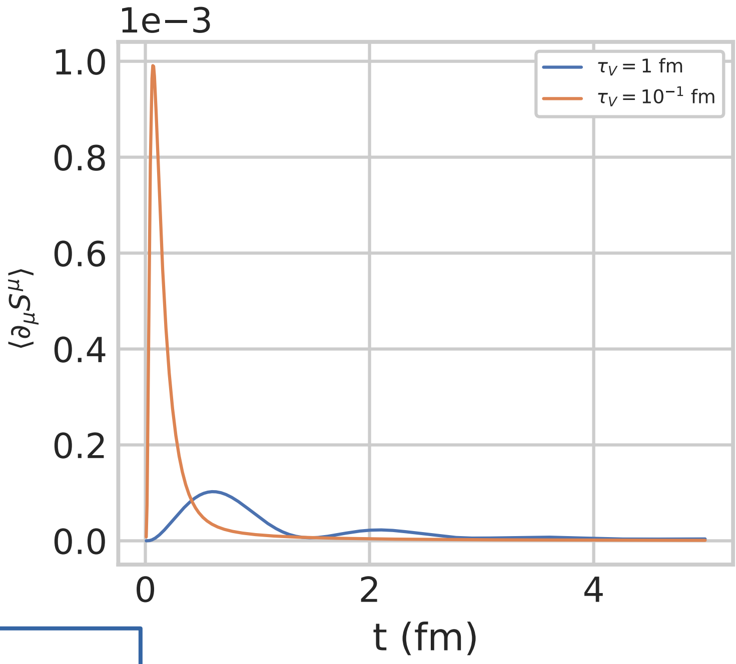
Entropy production



$\sigma = 10^{-1}\ \mathrm{fm}^{-1}$



$\sigma = 10\ \mathrm{fm}^{-1}$

$$\partial_\mu S^\mu = -\frac{q^2}{\sigma T} V_f^\mu V_{f,\mu}$$

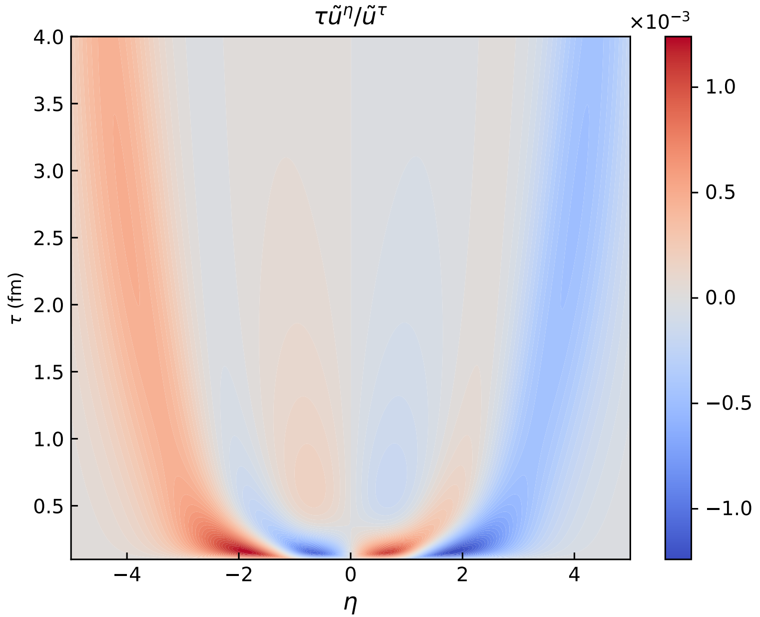
APPLICATION TO HEAVY-ION COLLISIONS

Initially expanding case

Setup

- System initially expanding according to Bjorken flow v_z=z/t
- We will use Milne coordirnates: $\tilde{x}^\mu = (\tau, x, y, \eta)$
- Energy density is constant in rapidity and fluid velocity is $\tilde{u}^\mu = (1,0,0,0)$
- Electromagnetic fields transform as: $\tilde{F}^{\mu\nu}(\tau_0, \mathbf{x}_\perp, \eta) = \frac{\partial \tilde{x}^\mu}{\partial x^\rho}\frac{\partial \tilde{x}^\nu}{\partial x^\sigma} F^{\rho\sigma}(\tau_0 \cosh\eta, \mathbf{x}_\perp, \tau_0 \sinh\eta)$
- **Goal**: to study the backreaction of EM fields on the fluid

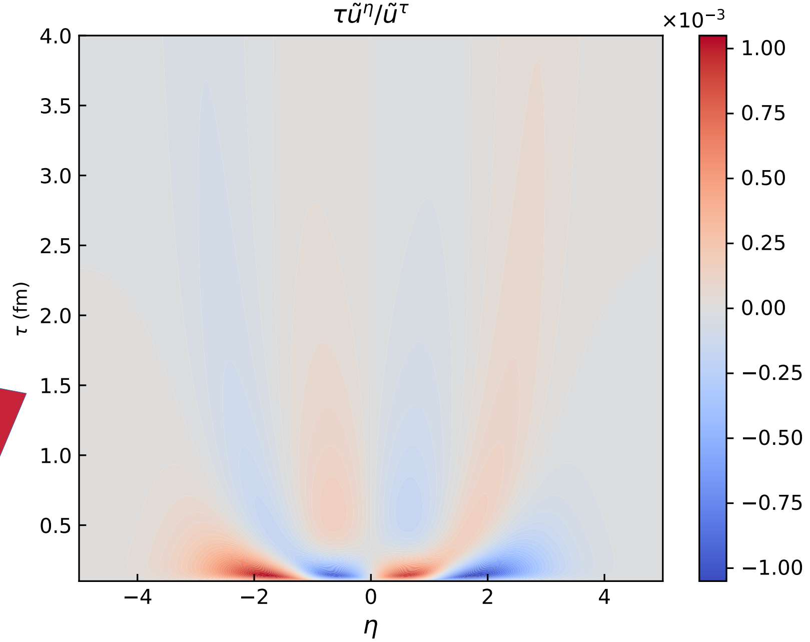
Why resistive MHD is needed?



$\sigma = 10^{-1}\ \mathrm{fm}^{-1}$

$$\partial_\mu T_f^{\mu\nu} = F^{\mu\lambda} J_{f,\lambda}\,.$$

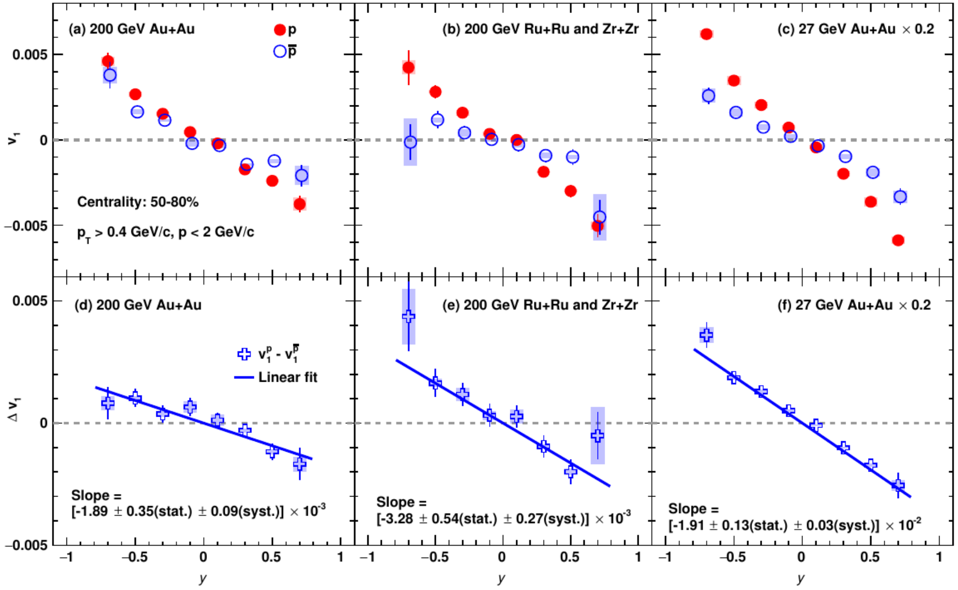
Joule heating negligible
Entropy prouction negligible !!



$\sigma = 10\ \mathrm{fm}^{-1}$

$$\partial_\mu T_f^{\mu\nu} \approx 0\,.$$

Experimental Developments



STAR Collaboration: arXiv:2304.03430

$$v_n(y) \equiv \frac{\int p_T dp_T d\phi \frac{dN}{dyp_T dp_T d\phi} \cos[n\phi]}{\int p_T dp_T d\phi \frac{dN}{dyp_T dp_T d\phi}}$$

- Ideal MHD not sufficient to describe charged directed flow splitting [1].
- Back reaction is important and in fact has the same order of magnitude as the splittiing.
- Development of 3+1D relativistic resistive MHD simulation underway.

[1] G. Inghirami et al., Eur.Phys.J.C 80 (2020) 3, 293

Take away...

- The standard Navier-Stokes form of Ohm’s law is acausal.
- Generalization of the law is analogous to an equation for driven damped harmonic oscillator.
- Longer relaxation time would lead to an incomplete generation of the charge diffusion current and hence faster decay of the magnetic field at early times.
- We have also studied the back-reaction of EM fields on the fluid and found that it would be important to describe charged particle directed flow.
- Development of 3+1D causal relativistic resistive MHD simulation underway.