Universal properties of ideal hydrodynamic evolution near the QCD critical point

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XQCD 2023, Coimbra, July 28

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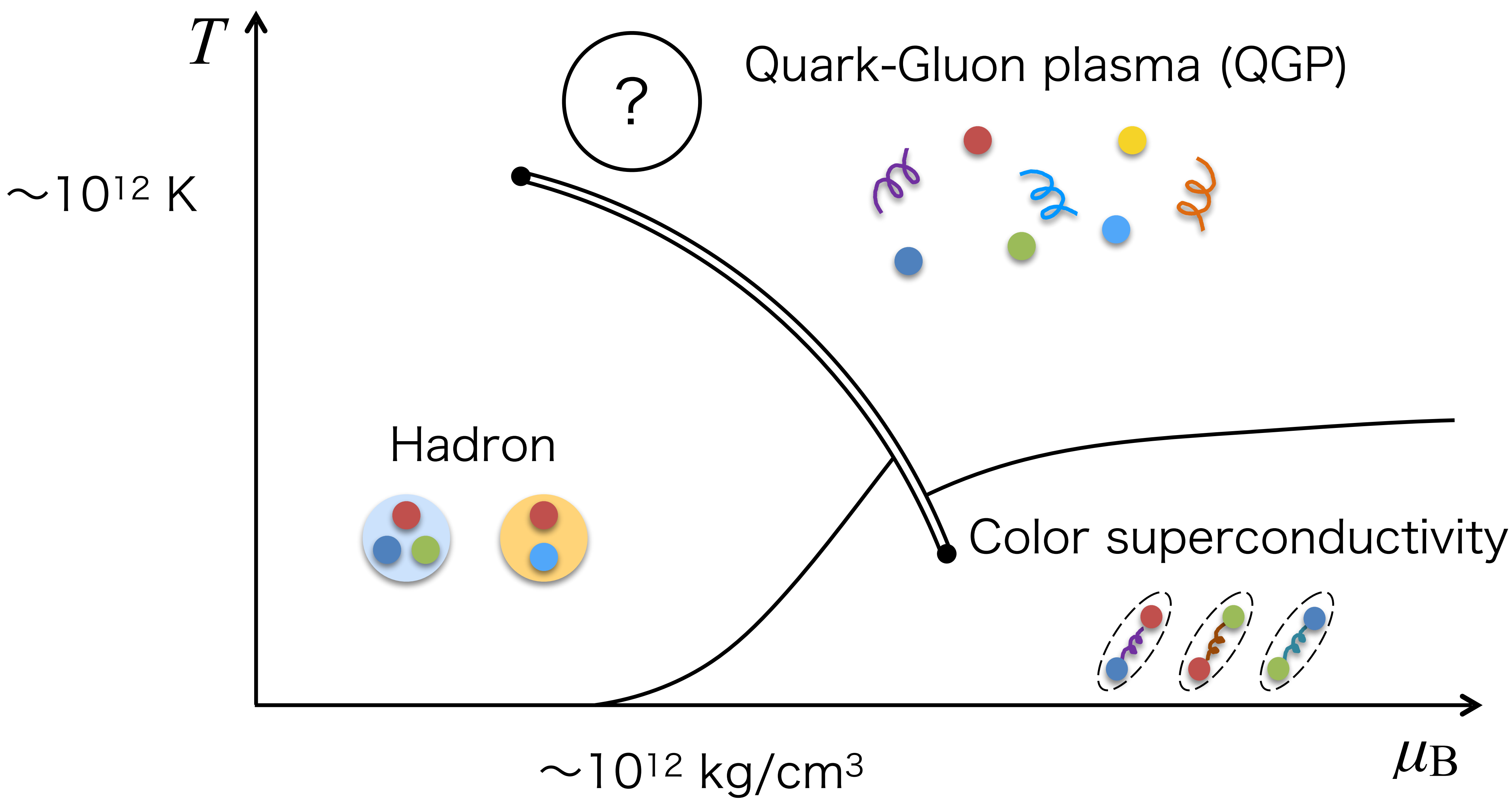
QCD phase diagram



Motivation

- Ideal hydrodynamics: 0th-order approximation in heavy-ion collisions
- Entropy and baryon number conservation:

$$s\tau = s_{\mathrm{ini}}\tau_{\mathrm{ini}}\,, \quad n\tau = n_{\mathrm{ini}}\tau_{\mathrm{ini}} \longrightarrow \hat{s} \equiv \frac{s}{n} = \frac{s_{\mathrm{ini}}}{n_{\mathrm{ini}}}$$

- Time evolution = $\hat{s}$ contours on the (μ, T) plane
- The universality of critical phenomena

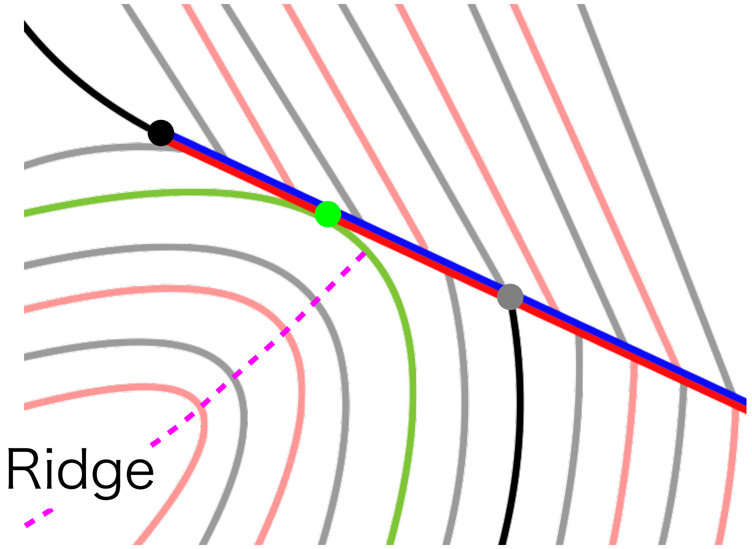
$\longrightarrow$ EOS of QCD ≃ 3D Ising mode

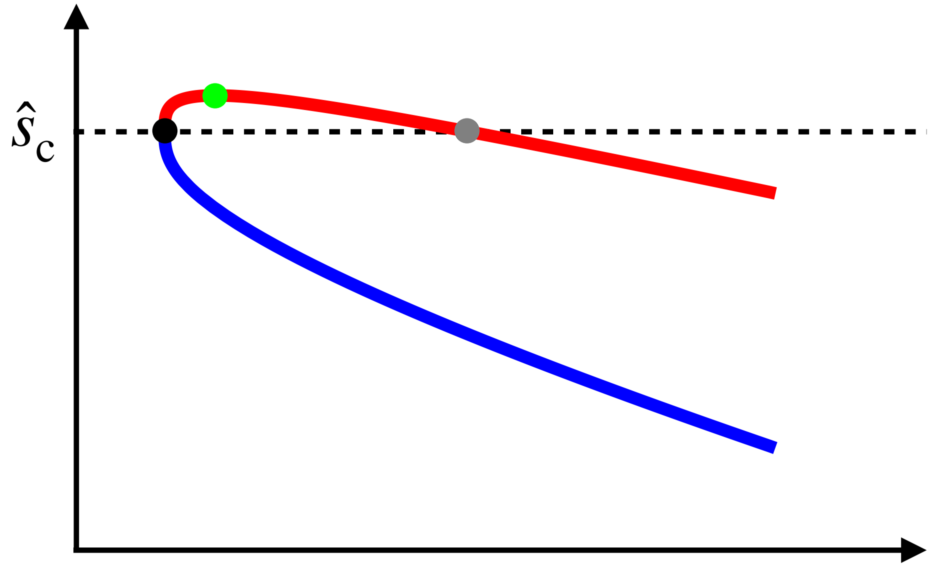
$\longrightarrow$ Universal properties on the $\hat{s}$ contour

Ridge and the hill-shape

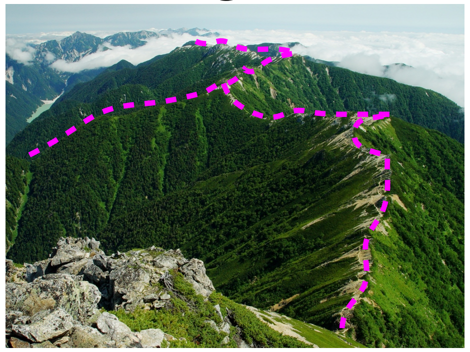
M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

$\hat{s} = \hat{s}(\mu, T)$ contour





Ridgeline



Wikipedia

Inevitable non-monotonic hill shape:

- Critical degeneracy: $\hat{s} \simeq \pm (T - T_\mathrm{c})^\beta$
- Third law of thermodynamics: $\hat{s}(T=0) = 0$

Outline

Mapping $(h, r) \leftrightarrow (\mu, T)$

- Pressure $P(h, r)$ with relevant parameters (h, r)

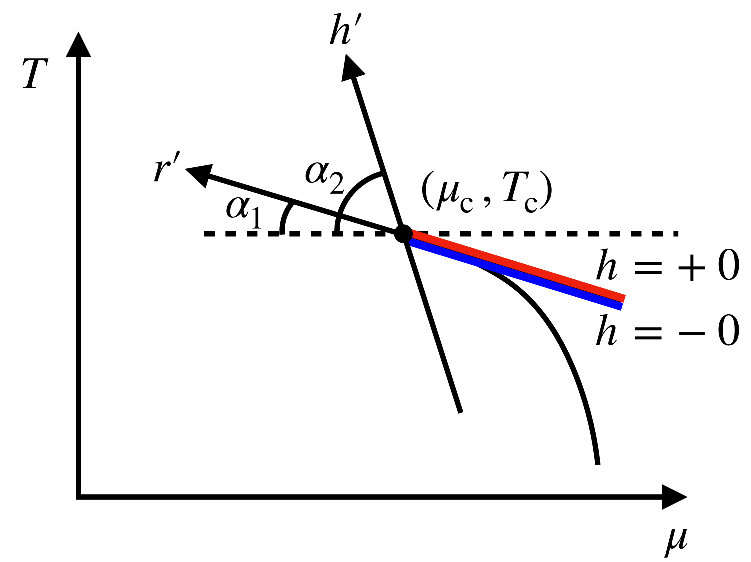
- Near the QCD critical point:

  C. Nonaka, M. Asakawa (2005), Parotto et al. (2020)

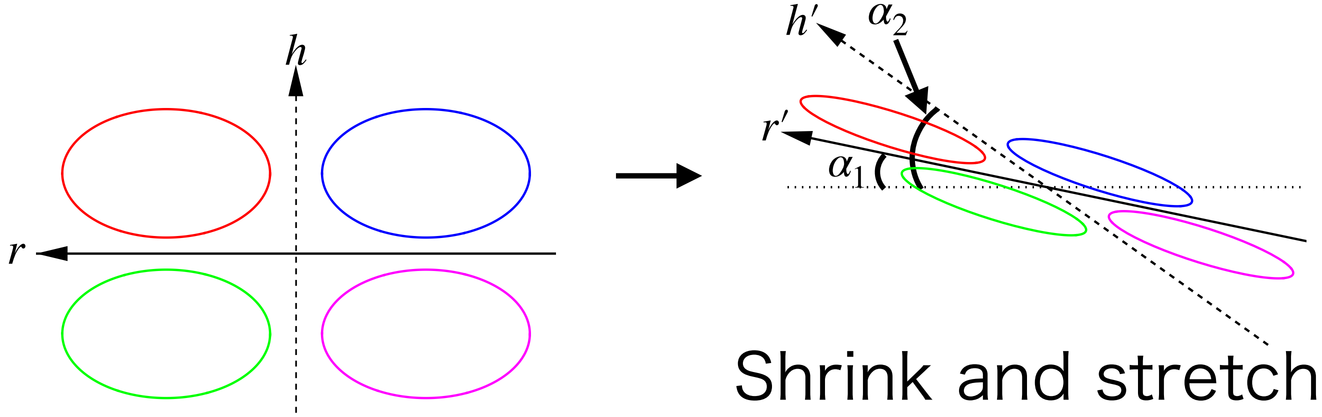
$$\frac{\mu - \mu_c}{T_c} = -w(r\rho\cos\alpha_1 + h\cos\alpha_2)$$

$$\frac{T - T_c}{T_c} = w(r\rho\sin\alpha_1 + h\sin\alpha_2)$$

$$\longrightarrow \quad h(\mu, T), r(\mu, T)$$



- α_2 : mapping angle



Shrink and stretch

Entropy/baryon near the critical point

- Thermodynamic quantities:

$$s \equiv \partial_T p + s_0 = m\partial_T h + \sigma\partial_T r + s_0 \quad (\because \text{Chain rule})$$

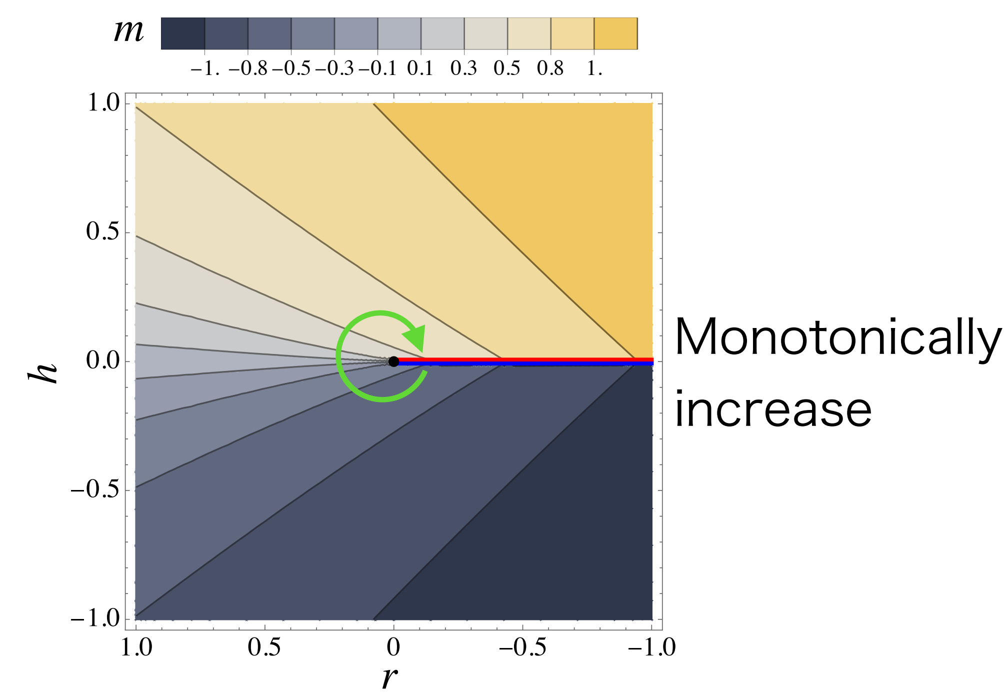
$$n \equiv \partial_\mu p + n_0 = m\partial_\mu h + \sigma\partial_T r + n_0$$

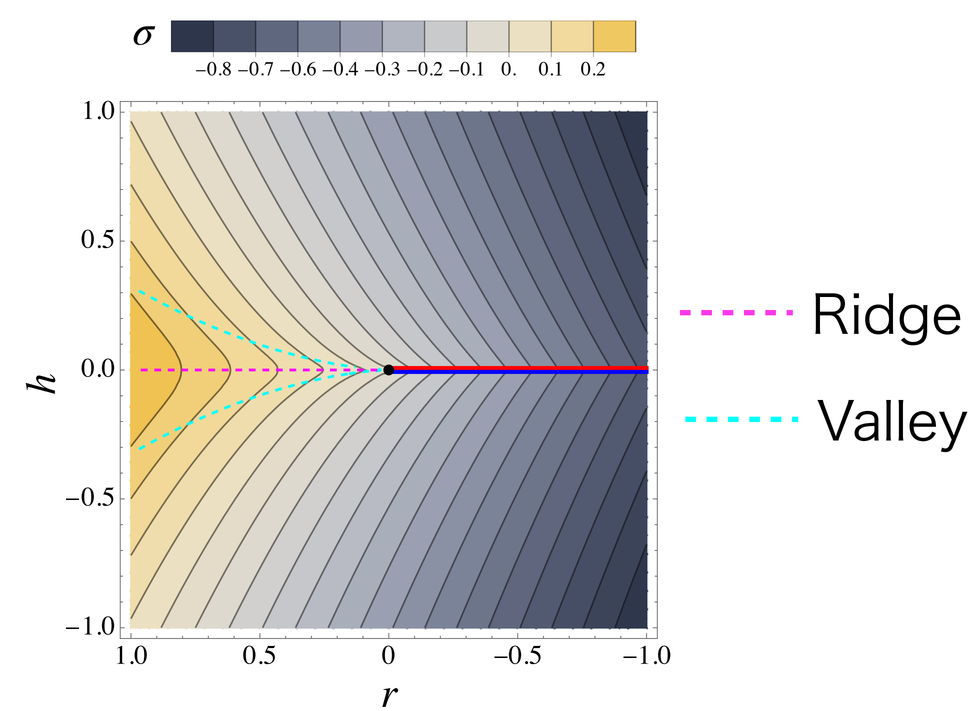
- Relevant parameter derivatives:
  - Order parameter: $m = \partial_h p$
  - Ising entropy: $\sigma = \partial_r p$

P. Schofield (1969) R. Guida and J. Zinn-Justin (1997)

- Entropy/baryon number:

$$\boxed{\hat{s} \simeq \hat{s}_0 + (\partial_m \hat{s})_0 m + (\partial_\sigma \hat{s})_0 \sigma}$$





Along the first-order line

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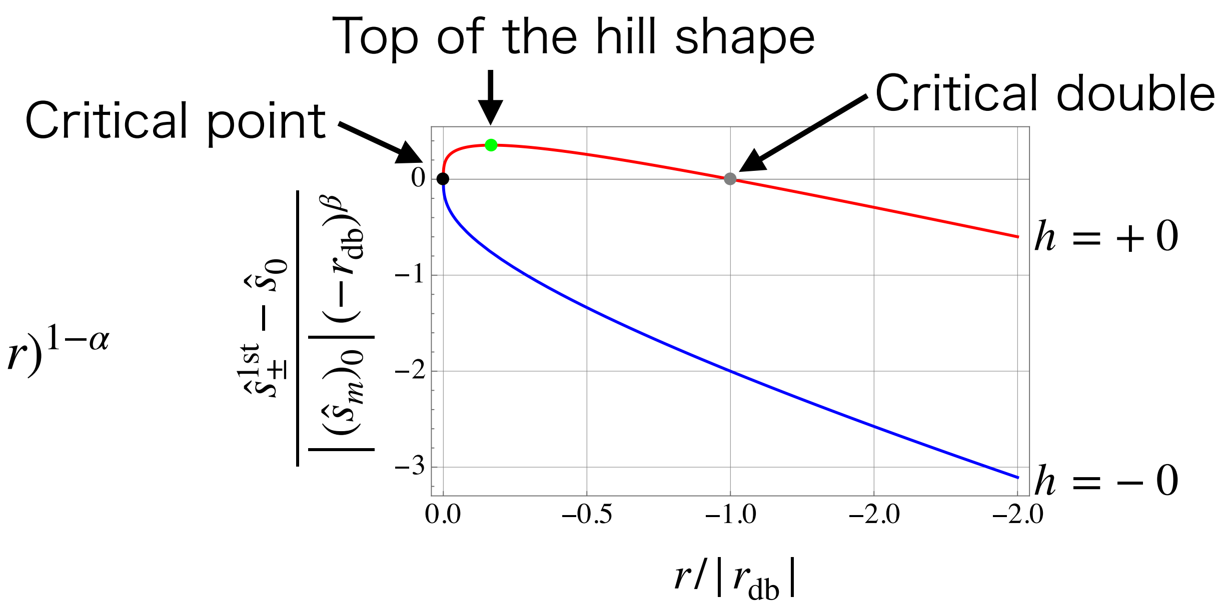
- Set $h = \pm 0\,,\ r \leq 0$

$$\hat{s}^{\rm 1st}_{\pm} = \hat{s}_0 \pm (\partial_m \hat{s})_0 (-r)^{\beta} + (\partial_\sigma \hat{s})_0 C (-r)^{1-\alpha}$$

Competition between $m \propto (-r)^{\beta}$ and $\sigma \propto (-r)^{1-\alpha}$

$$\big(\beta = 0.326\,,\ \alpha = 0.11\big)$$

$\longrightarrow$ non-monotonic behavior



- The critical double: $(-r_{\rm db})^{1-\alpha-\beta} = \left|\dfrac{(\partial_m \hat{s})_0}{C(\partial_\sigma \hat{s})_0}\right| \propto \left|\dfrac{\cos\alpha_1 - \hat{s}_0 \sin\alpha_1}{\cos\alpha_2 - \hat{s}_0 \sin\alpha_2}\right|$

- Solve $h(\mu, T) = 0\,,\ r(\mu, T) = r_{\rm db} \longrightarrow (\mu, T) = (\mu_{\rm db}\,, T_{\rm db})$

$$\boxed{\mu_{\rm db} - \mu_{\rm c} \sim \begin{cases} T_{\rm c} & (\alpha_2 \simeq 0^\circ) \\ \dfrac{T_{\rm c}}{\hat{s}_0^2} & (\alpha_2 \simeq 90^\circ) \end{cases}}$$

$$\big(0^\circ < \alpha_1 \ll 90^\circ\,,\ \hat{s}_0 \gg 1\,,\ \hat{s}_0 \alpha_1 \sim \mathcal{O}(1)\big)$$

Contours: parameter sets

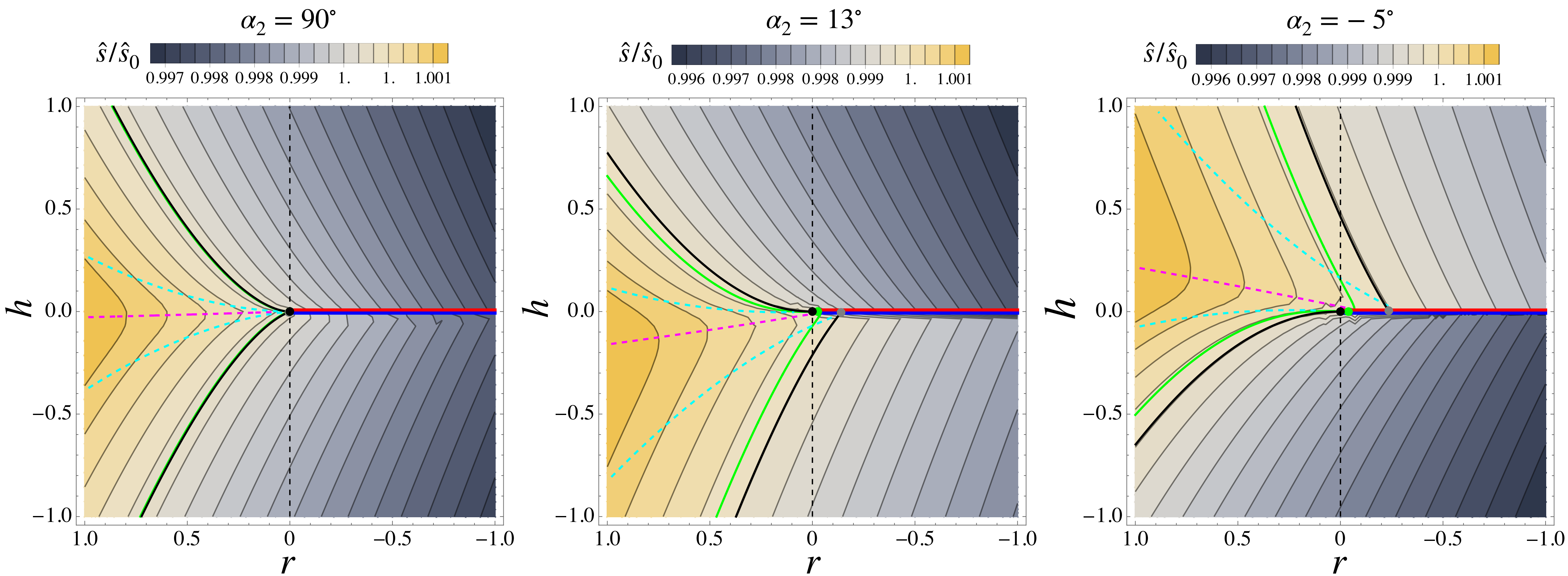
- Default: $(\mu_{\mathrm{c}}, T_{\mathrm{c}}, s_0, n_0, w, \rho) = (350\,\mathrm{MeV}\,,\ 143.2\,\mathrm{MeV}\,,\ 21\,,\ 1\,,\ 1\,,\ 2)$
  Parotto et al. (2020)
- Vary slope angles α_1 (the first-order line) and α_2 (shrink/stretch rate):
  - Crossover from lattice QCD $\longrightarrow$ $0 < \alpha_1 \ll 90^\circ$
  - $|\alpha_1 - \alpha_2|$ can be suppressed by m_{q} M. Pradeep and M. Stephanov (2019)
  - α_2 can be negative
- Hillside (μ, T) plane: $H_{\mathrm{side}}^{(T)} \equiv \mathrm{sgn}\,(\partial_m \hat{s})_0 (\partial_\sigma \hat{s})_0 \alpha_2 = -\,\mathrm{sgn}\left(\cot\alpha_1 - \hat{s}_0\right)\left(\cot\alpha_2 - \hat{s}_0\right) = \begin{cases} 1 & (T > T_{\mathrm{bd}}(\mu)) \\ -1 & (T < T_{\mathrm{bd}}(\mu)) \end{cases}$

  Mapping

α_1	3.85° $(\cot\alpha_1 < \hat{s}_0)$			1° $(\cot\alpha_1 > \hat{s}_0)$		
α_2	90°	13°	-5°	90°	13°	-13°

Contours (r, h) plane

$\alpha_1 = 3.85^\circ \quad (\cot \alpha_1 < \hat{s}_0)$

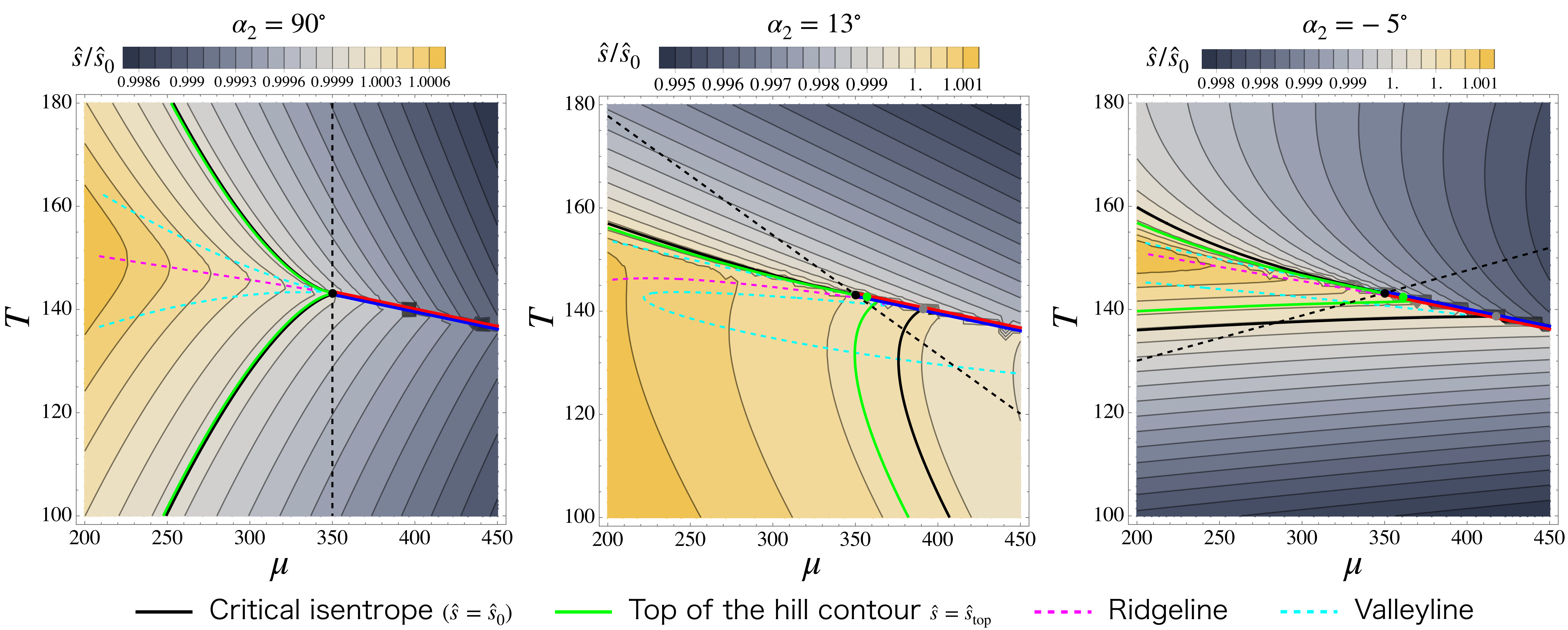


Critical isentrope $(\hat{s} = \hat{s}_0)$ — Top of the hill contour $\hat{s} = \hat{s}_{\rm top}$ — Ridgeline — Valleyline

- Mixing of m and σ $\longrightarrow$ The ridge(valley) lines of σ moved and curved

Contours (μ, T) plane

$\alpha_1 = 3.85^\circ \quad (\cot\alpha_1 < \hat{s}_0)$

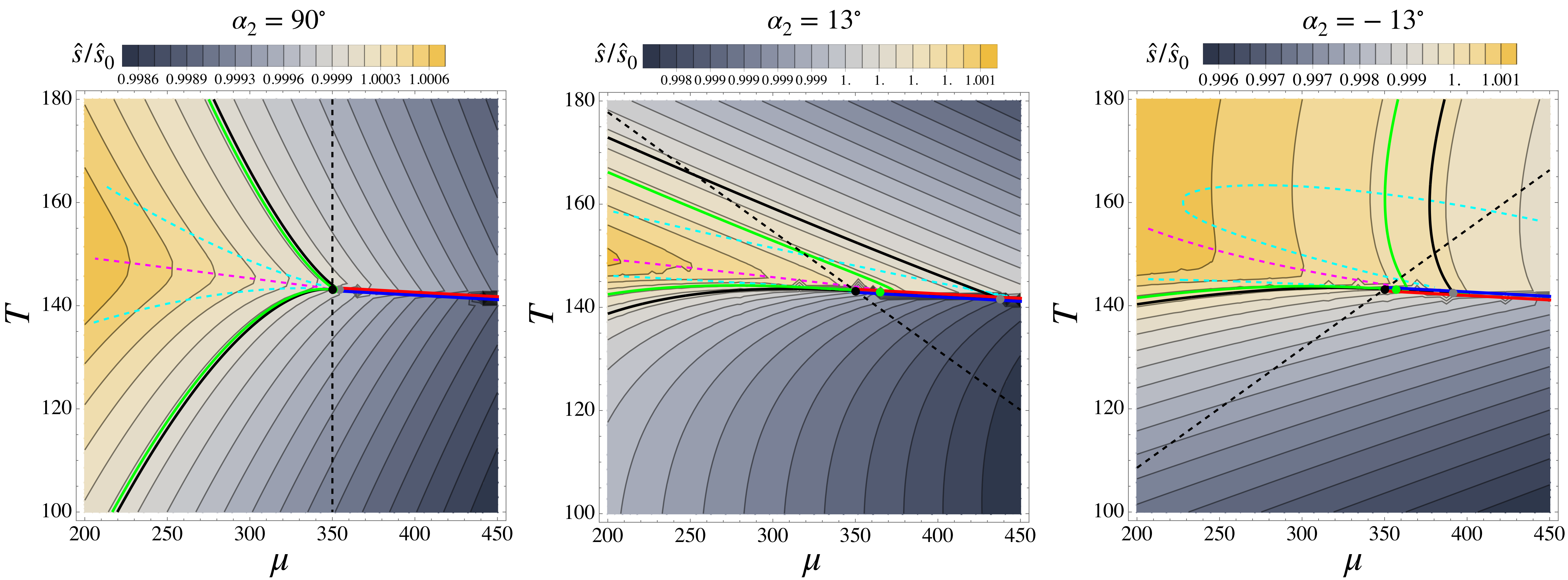


- Ridgeline and valley line are robust under the shrink and stretch of $\hat{s}(r, h)$

- $H^{(T)}_{\mathrm{side}} = -1$

Contours (μ, T) plane

$\alpha_1 = 1^\circ \quad (\cot\alpha_1 > \hat{s}_0)$



Critical isentrope $(\hat{s} = \hat{s}_0)$ — Top of the hill contour $\hat{s} = \hat{s}_{\rm top}$ — Ridgeline — Valleyline

- $H^{(T)}_{\rm side} = 1$ for narrow range: $0 < \alpha_1 < \cot^{-1}\hat{s}_0 \approx 2.72631^\circ$ or $0 < \alpha_2 < \alpha_1 \ll 90^\circ$

Classification of contours

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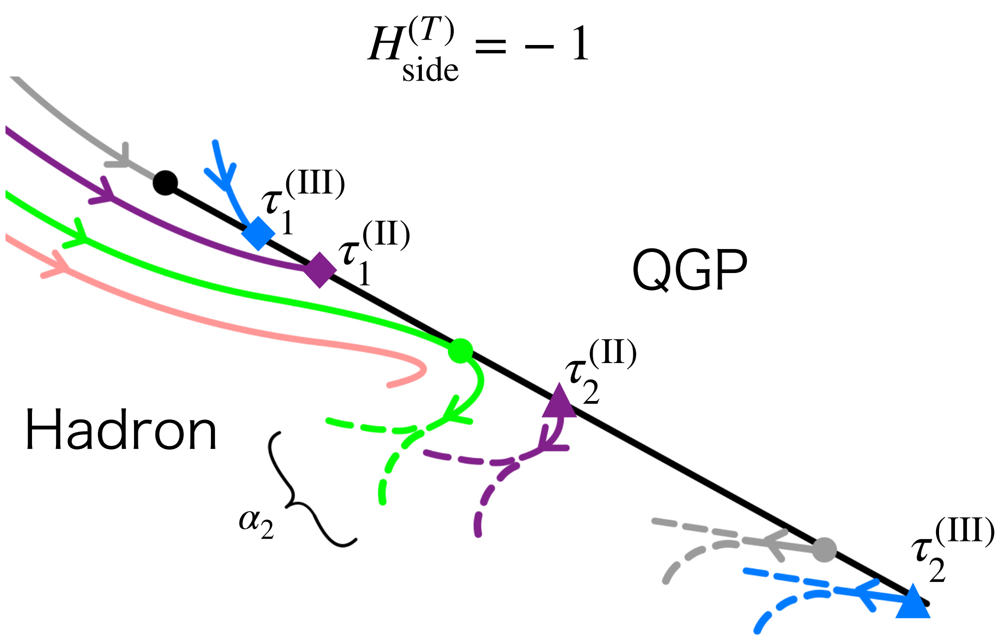

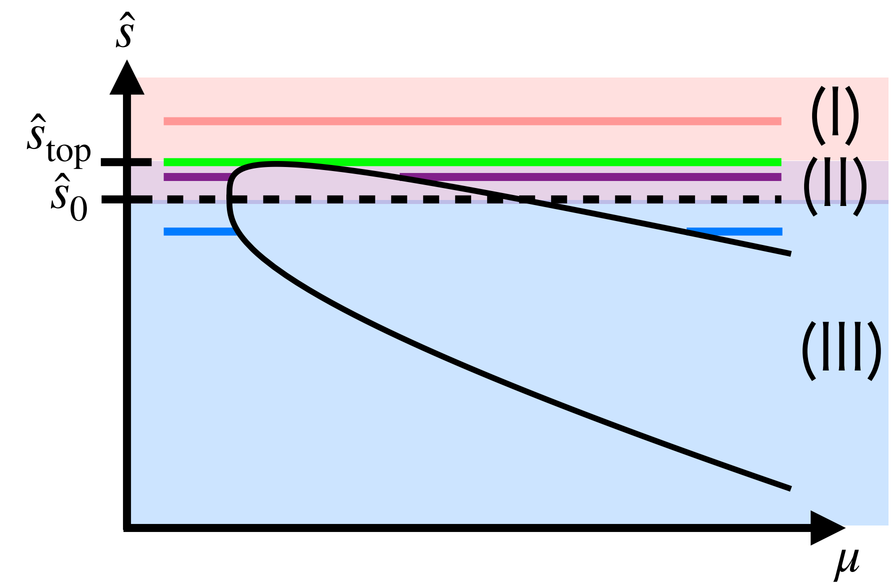

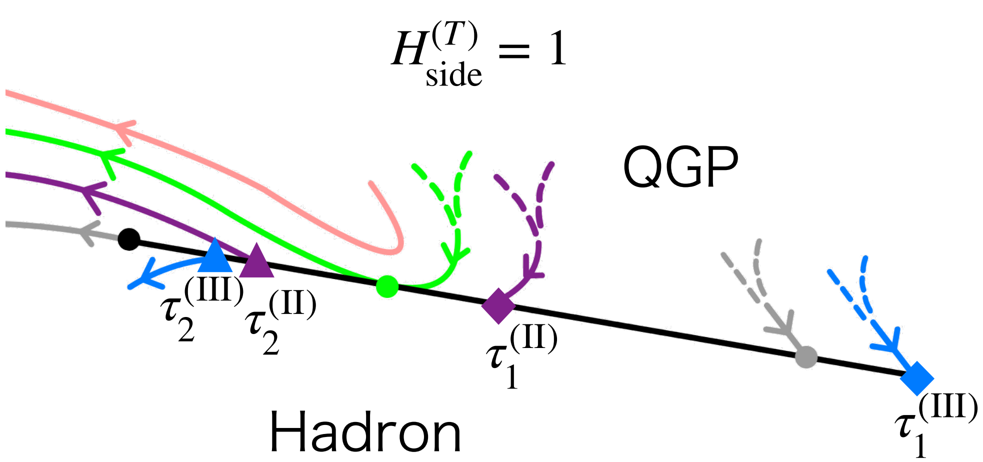

	$H^{(T)}_{\rm side} = -1$	$H^{(T)}_{\rm side} = 1$
(I)	Crossover	
(II)	Hadron → Hadron	QGP → QGP
(III)	QGP → Hadron	

Summary

M. Pradeep, M. Stephanov, NS, and H. Yee (in preparation)

- Universal properties of s/n contours = Ideal hydro. trajectory
- Ridge structure and the hill shape along the first-order boundary