Kinetic/Chemical Equilibration of Heavy DM Particles in Expanding Universe via Langevin equation simulation

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Preamble

- dark matter should exist
- LHC hasn't found new heavy mass particle $\rightarrow$ WIMP?
- lighter dark matter particles?
- more detailed consideration on heavy dark matter particles is needed

Number Density of DM Particles?

- what's the number density of DM particles?

- $\Omega_{\rm dm} \simeq 0.224$ (with $\Omega_{\rm total} = 1$) and $\rho_{\rm dm} \simeq 0.3\ \mathrm{GeV/cm^3}$ (WMAP), $\Omega_{\rm dm} \simeq 0.268$ (Planck)

- energy = number density $\times$ particle mass (if non-relativistic)

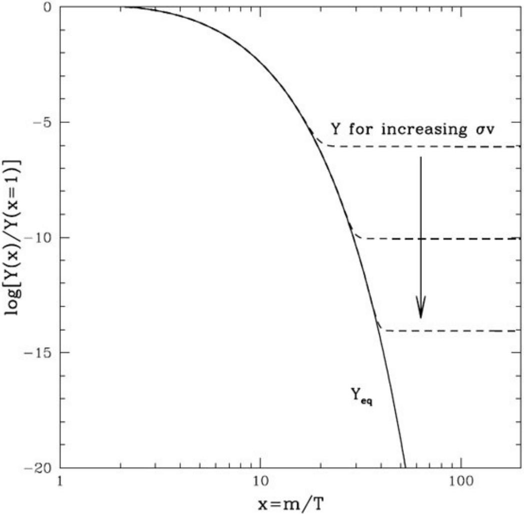
- relic density: Lee-Weinberg equation (B.W. Lee and S. Weinberg, PRL39 (1977) 165)

$$(\partial_t + 3H)Y = -\langle\sigma_{\rm eff} v\rangle(Y^2 - Y_{eq}^2)$$

- DM particles disappear through pair annihilation into SM particles

Number Density of DM Particles?

$$(\partial_t + 3H)Y = -\langle \sigma_{\rm eff} v \rangle (Y^2 - Y_{eq}^2)$$



Number Density of DM Particles?

in a nutshell

- initially, non-relativistic DM particles are in thermal equilibrium with the heat bath of thermal SM particles
- interaction rate starts to fall behind the expansion rate of the universe
- DM particle number density begins to “freeze out” and remains covariantly preserved

Number Density of DM Particles?

- change of the heavy DM particle number density is “chemical equilibration” process

- relic density or abundance of DM particle is determined by chemical equilibration

- momentum distribution affects cross-section

- cross-section affects chemical equilibration

Number Density of DM Particles?

overall, understanding relic density needs knowledge on

- general relativity for expanding universe
- quantum field theory for particle interactions
- non-equilibrium statistical physics for kinetic/chemical equilibration

Number Density of DM Particles?

kinetic equilibration of heavy DM at $\sim \mathcal{O}(1)$ GeV ?

- kinetic equilibration of heavy DM particles in the calculation of number density is usually assumed (T. Binder et al, "Early kinetic decoupling of dark matter: when the standard way of calculating the thermal relic density fails", Phs. Rev. D96 (2017) 115010 [1706.07433])

- which is questioned recently
(cf.K. Ala-Mattinen and K. Kainulainen, JCAP 09 (2020) 040 [1912.02870], T. Binder et al, Eur. Phys. J. C81 (2021) 577 [2103.01944])

that is, non-equilibrium statistical physics in expanding universe needs careful treatments

Challenges

- Boltzmann equation doesn't include thermal virtual corrections because its building blocks are real process $\rightarrow$ not consistent, modifies annihilation channels
(cf. M. Laine, JHEP 01 (2023) 157 [2211.06008])

- kinetic equilibration proceeds through elastic collisions with thermal bath particles (Standard Model particles).

- SM particles such as quarks and gluons can behave non-perturbatively at $\mathcal{O}(1)$ GeV

Kinetic equilibration of heavy quarks in QGP

• kinetic equilibration of heavy quarks (charm and bottom quarks) in expanding Quark-Gluon Plasma (G.D. Moore and D. Teaney, Phys. Rev. C71 (2005) 064904 hep-ph/0412346)

• Langevin equation approach to kinetic equilibration

• separation of the mode: heavy quark (slow mode), light quarks and gluons (fast mode)

• QGP is expanding and light degrees of freedom can interact strongly among themselves

Our scheme

- kinetic equilibration of dark matter particles proceeds through elastic scattering with SM particles

- corresponding scattering rate is much smaller than the typical plasma interaction rates (DM particles are weakly coupled with SM particles)

- but plasma particles can interact strongly

Langevin equation for the kinetic equilibration of heavy DM particles
in the thermal bath of SM particles

Procedure

- chooese a model for heavy DM particle
- calculate shear viscosity and momentum diffusion constant for the model
- numerically solve the Langevin equation with these parameters
- get momentum distribution of heavy DM particles
- calculate chemical equiibration of heavy DM particles

In short

- question of kinetic/chemical equilibration of (heavy) DM particles in expanding universe

- similarity to equilibration of heavy quarks in expanding quark-gluon plasma (G.D. Moore and D. Teaney, hep-ph/0412346)

- allows possibility of plasma particles as a strongly-coupled system

- separation of the scale, “fast mode” and “slow mode” $\rightarrow$ Brownian motion

- “expanding universe”: Hubble rate

- chemical equilibrium (number density) vs. kinetic equilibrium (momentum distribution)

Langevin equation

- Langevin equation in an expanding universe

$$\frac{dp^i}{dt} = -(\eta(t) + H(t))p^i + f^i(t)$$

with $H(t) = \frac{da/dt}{a}$ (a is the scale factor) and

$$< f^i(t_1)f^j(t_2) >= \zeta\delta^{ij}\delta(t_1 - t_2)$$

- using dimensionless variables

$$x = \log\left(\frac{T_{\max}}{T}\right), \quad \hat{p}^i = \frac{p^i}{s^{1/3}}, \quad \hat{\eta} = \frac{\eta}{3c_s^2 H}, \quad \hat{\zeta} = \frac{\zeta}{3c_s^2 H}$$

we get

$$\frac{d\hat{p}^i(x)}{dx} = -\hat{\eta}(x)\hat{p}^i(x) + \hat{f}^i(x)$$

with $< \hat{f}^i(x_1)\hat{f}^j(x_2) >= \hat{\zeta}\delta^{ij}\delta(x_1 - x_2)$

Discretized Langevin equation

- Itoh calculus:

$$\hat{p}^i_{n+1} = \hat{p}^i_n - \hat{\eta}_n \hat{p}^i_n dx + \hat{f}^i_n \sqrt{dx}, \qquad \langle \hat{f}^i_n \, \hat{f}^j_m \rangle = \hat{\zeta}_n \delta^{ij} \delta_{mn}$$

- a slight complication from normal Langevin equation in that the coefficients, the friction and the momentum diffusion coefficient are time-dependent

Model

- scalar singlet model for heavy DM particle (φ)

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial^{\mu}\varphi\partial_{\mu}\varphi - \left[\frac{1}{2}\left(m_{\varphi 0}^2 + \kappa\phi^{\dagger}\phi\right)\varphi^2 + \frac{1}{4}\lambda_{\phi}\varphi^4\right]$$

- $m_{\varphi}^2 \simeq m_{\varphi 0}^2 + \kappa v^2/2 = 60$ GeV ($v \simeq 246$ GeV), couples to SM Higgs

- start from e.g., $T = 5$ GeV equilibrium distribution

Model

- the momentum diffiusion coefficient, ζ is given as

$$\zeta = \frac{\frac{1}{3}\sum_{i=1,\cdots 3}\int d^3x\{\mathcal{F}^i(\vec{x}),\mathcal{F}^i(\vec{0})\}}{\chi}$$

where

$$\{\mathcal{F}^i(\vec{x}),\mathcal{F}^i(\vec{0})\} = \lim_{\omega\to 0^+}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\frac{1}{2}\Big(\mathcal{F}^i(\vec{x})\mathcal{F}^i(\vec{0})+\mathcal{F}^i(\vec{0})\mathcal{F}^i(\vec{x})\Big)\rangle$$

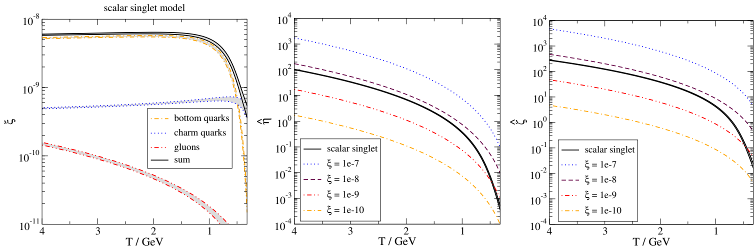
with

$$\mathcal{F}^i(\vec{x}) = m_\varphi\partial_0\mathcal{J}^i = -\frac{\kappa v}{2m_\varphi}\psi^*(\partial_i h)\psi, \quad \varphi \simeq \frac{1}{\sqrt{2m_\varphi}}\left(\psi e^{-im_\varphi t}+\psi^* e^{im_\varphi t}\right)$$

$$\chi = \int d^3x\langle\mathcal{J}^0(0^+,\vec{x})\mathcal{J}^0(0,\vec{0})\rangle, \quad \mathcal{J}^0 = \psi^*\psi$$

Langevin equation coefficients

- scalar singlet model: coupling (ξ), friction and momentum diffusion



Result

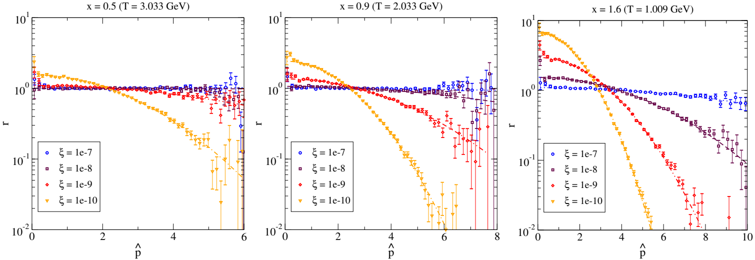
- $\Delta x = 10^{-6}$, $N = 10^5$ Langevin simulation
- usual check with systematics
- for an easy comparision,

$$r \equiv \frac{\mathcal{P}}{\mathcal{P}_{\mathrm{eq}}}$$

where $\mathcal{P}$ is the momentum distribution from Langevin simulation and $\mathcal{P}_{\mathrm{eq}}$ is the equilibrium (gaussian) momentum distribution

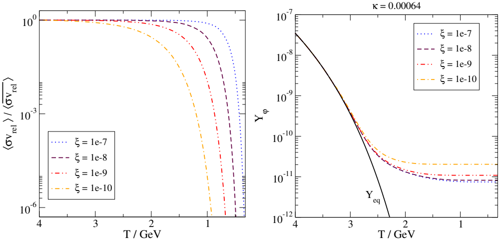
Result

- result: momentum distribution



Result

- result: the source term in the Lee-Weinberg equation and the resulting number distribution



where Y_φ is from

$$\partial_x Y_\varphi \simeq -\frac{s}{3c_s^2 H}\left[\langle\sigma v_{\rm rel}\rangle Y_\varphi^2 - \langle\sigma\bar{v}_{\rm rel}\rangle \bar{Y}_\varphi^2\right]$$

Conclusion and Discussion

- Langevin equation allows us to investigate non-perturbatively the kinetic/chemical equilibration process in expanding universe (similar to kinetic equilibration of heavy quark in QGP) if there is a mode separation

- Feasibility is shown for a scalar singlet model

- For this model system, the kinetic equilibrium is achieved with a red-tilted spectrum

- Annihilation cross-section is reduced

- For the “correct abundance”, somewhat stronger coupling ($< 20\%$) than the perturbative expectation is needed