Maximum entropy freezeout of fluctuations

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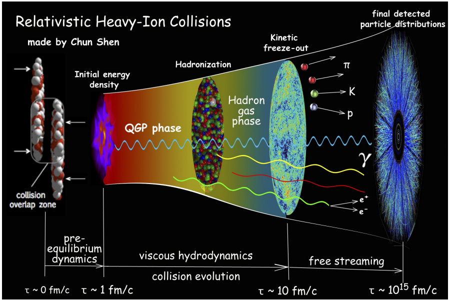
with Maneesha Pradeep: 2211.09142

Heavy-ion collisions and freezeout



- Hydrodynamics describes evolution of HIC fireball.
- However, experiments do not measure hydrodynamic variables, but particle multiplicities.
- Freezeout is an essential step connecting theory with experiment.

Standard procedure (Cooper-Frye)

- In each hydro cell at freezeout local $T(x)$, $\mu(x)$, $u(x)$

  is translated into

  local phase space distribution $f_A(x) = e^{\alpha(x)q_A+\beta(x)u(x)\cdot p_A}$.

  $\alpha = \mu/T$ and $\beta = 1/T$,
  A – set of particle quantum numbers
  such as charge (q_A), 4-momentum (p_A).

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- f_A gives us single-particle observables.

  For fluctuations we need (at least) $\langle\delta f_A \delta f_B\rangle$.

Fluctuations

- Hydrodynamics is intrinsically stochastic: dissipation means there are fluctuations.
- Fluctuations, non-gaussian, in particular, are essential for mapping QCD phase diagram and locating QCD critical point.
- Hydrodynamic evolution of fluctuations
  — a lot of recent progress — a subject for a different talk.
- This talk — freezeout of fluctuations.

Earlier work, problems and questions

- "Fluctuating Cooper-Frye:" *Kapusta-Muller-MS 2011*

$$\delta f_A = \left(\delta\alpha\frac{\partial}{\partial\alpha} + \delta\beta\frac{\partial}{\partial\beta} + \delta u\frac{\partial}{\partial u}\right) f_A(\alpha,\beta,u)$$

Then, multiplicity fluctuation correlator:

$$\langle\delta f_A \delta f_B\rangle = \underbrace{\langle\delta\alpha\delta\alpha\rangle}_{\text{from hydro}} \left(\frac{\partial}{\partial\alpha} f_A\right)\left(\frac{\partial}{\partial\alpha} f_B\right) + \ldots \qquad (*)$$

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- Problem:
  consider ideal gas, no correlations, i.e. $\langle f_A f_B\rangle = \delta_{AB} f_A$

  but there are fluctuations of $\delta\alpha$, $\delta\beta$, etc. even in ideal gas $\Rightarrow$ equation $(*)$ produces incorrect result: spurious correlations.

Source of the problem and a solution

- Pairs of correlated particles erroneously include “pairs” made of the same particle counted twice.
- A solution *Li-Springer-MS ’13, Plumberg-Kapusta ’20*
  for charge fluctuations – subtract the contribution of ideal gas to $\langle\delta n\delta n\rangle$ in hydrodynamics and apply equation $(*)$ only to the remainder:

$$\langle\delta n\delta n\rangle \equiv \langle\delta n\delta n\rangle_{\rm ideal} + \Delta\langle\delta n\delta n\rangle$$

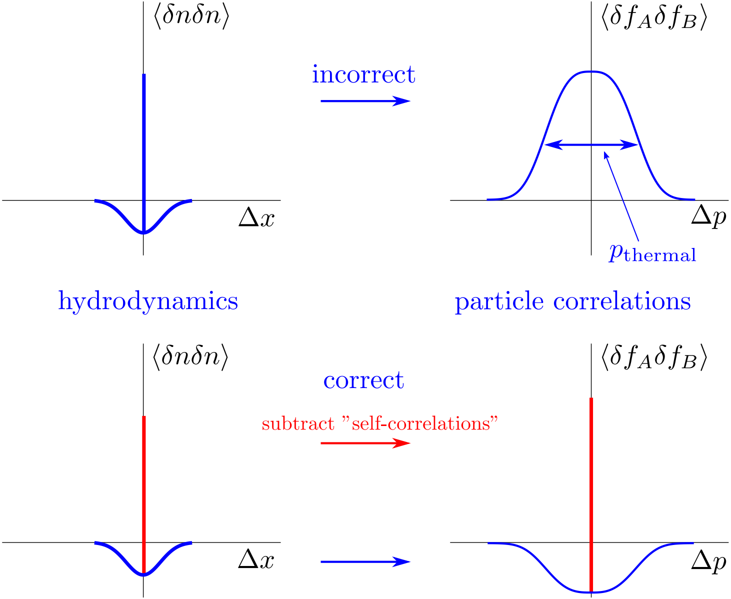
$$\langle\delta f_A\delta f_B\rangle = \delta_{AB}f_A + \underbrace{\Delta\langle\delta n\delta n\rangle\left(\frac{\partial}{\partial n}f_A\right)\left(\frac{\partial}{\partial n}f_B\right)}_{\text{balance function}}$$

- Similarly, for critical contribution to fluctuations, $\langle\delta\sigma\delta\sigma\rangle_{\rm critical}\sim\xi^2$ translates into deviation from baseline:

$$\langle\delta f_A\delta f_B\rangle = \underbrace{\delta_{AB}f_A}_{\text{baseline}} + \underbrace{\mathcal{O}(\xi^2)}_{\text{critical contribution}}$$

MS-Rajagopal-Shuryak 1999

Thermal smearing and “self-correlations”



incorrect

hydrodynamics

particle correlations

correct

subtract ”self-correlations”

Open questions

How to deal with

- Temperature, velocity fluctuations?
- Non-critical fluctuations?
- Non-gaussian fluctuations?

Maximum entropy freezeout: *Pradeep-MS 2211.09142*

Revisit one-point/single-particle observables

- Locally matching conserved quantities before/after freezeout:

  $n(x) = \sum_A q_A f_A(x)$ and $\epsilon(x) u^\mu(x) = \sum_A p^\mu_A f_A(x)$.

  Problem: these equations for f_A have infinitely many solutions.

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- Which solution maximizes Bolzmann entropy?

  $S_0 = -\sum_A f_A \log f_A$

  Answer: $f_A = e^{\alpha_A q_A + \beta u \cdot p_A}$ — Cooper-Frye.

  - Matching also dissipative viscous stress and diffusive current gives $f_A = e^{\alpha_A q_A + \beta u \cdot p_A} + \underbrace{\Delta f_A}_{\text{non-equilibrium correction}}$. *(Everett-Chattopadhyay-Heinz 2021)*

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- Fluctuations?

Pradeep-MS 2022

Maximum entropy freezeout of fluctuations

- We want to match fluctuations of $\{n, \epsilon, u^\mu\} \equiv \Psi_a$, to fluctuations of f_A so that $\Psi_a = \sum_A P_a^A f_A$ event-by-event

  i.e., $G_{AB} \equiv \langle \delta f_A \delta f_B \rangle$ must obey $(P_a^A = \{q_A, p_A, \dots\})$

$$\underbrace{\langle \delta\Psi_a \delta\Psi_b \rangle}_{H_{ab}} = \sum_{AB} P_a^A P_b^B \underbrace{\langle \delta f_A \delta f_B \rangle}_{G_{AB}}$$

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- Entropy? Is a functional of fluctuations, i.e., of G_{AB}, G_{ABC}, etc. E.g,

$$S_2 = S_0 + \underbrace{\frac{1}{2}\mathrm{Tr}\left[\log GC + GC + 1\right]}_{\text{relative entropy, } G = -C^{-1} \equiv \bar{G}} \text{, where } C_{AB} = \frac{\delta^2 S}{\delta f_A \delta f_B}.$$

Maximum entropy solution

- Relative entropy is maximized (subject to constraints) by

  $$G^{-1}_{AB} = \bar{G}^{-1}_{AB} + (H^{-1} - \bar{H}^{-1})_{ab} P^a_A P^b_B$$

  - Also for non-gaussian correlators *(Pradeep-MS 2022)*.
  - Note: when $H = \bar{H} \to G_{AB} = \bar{G}_{AB} = f_A \delta_{AB}$.

- Linearizing in $\Delta H \equiv H - \bar{H}$ we obtain the desired generalization of earlier results:

  $$G = \underbrace{\bar{G}}_{\text{baseline}} + \underbrace{(\bar{H}^{-1} P \bar{G})^T \Delta H (\bar{H}^{-1} P \bar{G})}_{\text{correlations}}$$

Non-gaussian correlators ($n \geq 3$ particles)

Linearied equations are simple and intuitive:

$$G_{AB} = \bar{G}_{AB} + \Delta G_{AB}, \quad H_{ab} = \bar{H}_{ab} + \Delta H_{ab},$$

$$G_{ABC} = \Big[\underbrace{\bar{G}_{ABC}}_{A\bullet\!\bullet^{B}_{C}} + \underbrace{3\Delta G_{AD}\delta_{DBC}}_{A\bullet\!\sim\!\sim\!\sim\!\bullet^{B}_{C}} + \underbrace{\widehat{\Delta} G_{ABC}}_{\text{irreducible correlation}} \Big]_{\overline{ABC}} \leftarrow \text{permutation average}$$

$$H_{abc} = \Big[\bar{H}_{abc} + 3\Delta H_{ad}\delta_{dbc} + \widehat{\Delta} H_{abc} \Big]_{\overline{abc}}$$

Maximum entropy method gives:

$$\Delta G_{AB} = \Delta H_{ab} (\bar{H}^{-1} P \bar{G})^{a}_{A} (\bar{H}^{-1} P \bar{G})^{b}_{B}$$

$$\hat{\Delta} G_{ABC} = \hat{\Delta} H_{abc} (\bar{H}^{-1} P \bar{G})^{a}_{A} (\bar{H}^{-1} P \bar{G})^{b}_{B} (\bar{H}^{-1} P \bar{G})^{c}_{C}$$

Critical fluctuations

- The contribution of critical fluctuations matches the simple model often used in the literature *(MS 2011)*:

  $$\delta f_A^{\text{critical}} = \delta\sigma \left(\frac{\partial}{\partial\sigma} f_A \right)$$

  where critical field σ couples to mass so that $\delta m_A = g_A \delta\sigma$.

  Thus $$\langle \delta f_A \delta f_B \rangle = \underbrace{\delta_{AB} f_A}_{\text{Poisson baseline}} + \underbrace{\langle \delta\sigma\delta\sigma \rangle \left(\frac{\partial}{\partial\sigma} f_A \right) \left(\frac{\partial}{\partial\sigma} f_B \right)}_{\text{critical contribution} \sim g_A g_B}$$

- Now, within maximum entropy approach, we can determine the couplings g_A of the critical mode from the equation of state.

Concluding summary

- Maximum entropy approach for single-particle observables $=$ traditional Cooper-Frye freezeout.
- Maximum entropy approach solves the problem of freezing out of hydrodynamic fluctuations.
- The method is very general and works for gaussian and non-gaussian, for critical and non-critical fluctuations.
- Agrees with existing methods where such are available.
- Allows determination of critical field coupling parameters crucial to predicting the magnitude of CP signatures in terms of the EOS parameters.