The critical endpoint at large N_c

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Overview

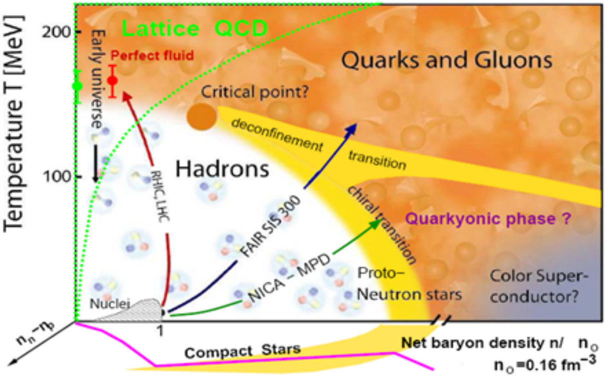
1. Introduction
   Elements of large N_c

2. The PLeLSM model
   Parameterization
   Earlier results at $T \neq 0$, $\mu_B = 0$ at $N_c = 3$

3. N_c scaling in the PLeLSM
   Condensates and masses
   Polyakov-loops at large N_c
   Field equations

4. Results in the UAE approximation
   $T = 0$, $\mu_q \neq 0$
   $T \neq 0$, $\mu_q = 0$
   Pressure
   Phase boundary

Envisaged phase diagram of QCD



Important details of the phase diagram is still unknown (mainly at large baryon density)

Properties of the phase diagram especially at finite baryon densities/baryochemical potential can be well investigated with the help of effective field theories of QCD ⟶ e.g. details of the phase boundary like existence and location of the CEP, in medium dependence of meson masses, or properties of compact stars etc.

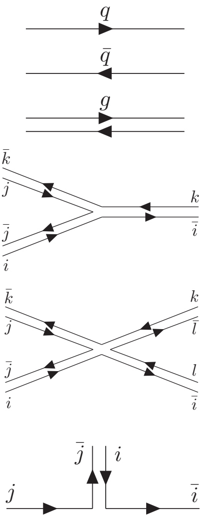
Basics of Large N_c I.

G. 't Hooft. (1974), Nucl. Phys. B 72:461

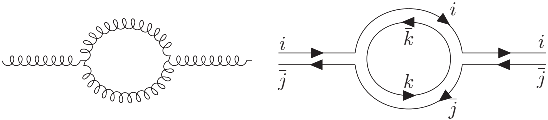
G. 't Hooft. (1974), Nucl. Phys. B 75:461–470

E. Witten. (1979), Nucl. Phys. B 160:57–115

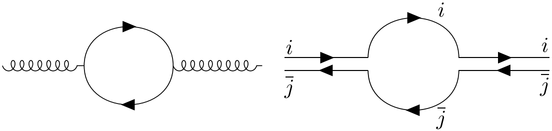
- No expansion parameter in QCD if $m_{u/d/s} \approx 0 \longrightarrow$ not so obvious expansion parameter: N_c
- $SU(3) \longrightarrow SU(N_c)$
- double line notation based on color structure of gluons: $A_j^{\mu;\,i} \sim q^i \bar{q}_j$
- 3-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} \partial^\mu A_i^{\nu;\,k}$
- 4-coupling: $A^i_{\mu;\,j} A^j_{\nu;\,k} A_l^{\mu;\,k} A_i^{\nu;\,l}$
- quark - gluon vertex: $\bar{q}_i \gamma^\mu q^j A^i_{\mu;\,j}$



Basics of Large N_c II.



N_c combinatorial factor due to closed color loop $\implies g \sim \frac{1}{\sqrt{N_c}}$



Quark loops are $1/N_c$ suppressed.

Leading diagrams are palanar diagrams with minimum number of quark loops

Investigation of N-point functions of quark bilinears ($J = \bar{q}q$, $\bar{q}\gamma^\mu q$) leads to the large N_c properties of mesons

Properties of mesons and baryons for Large N_c

- mesons are free, stable, and non-interacting
- mesons are pure $q\bar{q}$ states for large N_c
- meson masses $\sim N_c^0$
- meson decay amplitudes $\sim 1/\sqrt{N_c}$
- for one meson creation: $< 0|J|m >\sim \sqrt{N_c}$
- k meson vertex $\sim N_c^{1-k/2}$. Specifically, the three- and four-meson vertices are $\sim 1/\sqrt{N_c}$ and $\sim 1/N_c$, respectively
- baryon masses $\sim N_c$. Consequently constituent quark masses $\sim N_c^0$

Lagrangian of the PLeLSM model

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$\begin{aligned}\mathcal{L} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ & + c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\ & + \bar{\Psi}\left(i\gamma^\mu D_\mu - g_F(S - i\gamma_5 P)\right)\Psi - g_V\bar{\Psi}\left(\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\Psi,\end{aligned}$$

$$\begin{aligned}\Phi &= S + iP \equiv \sum_{a=0}^{8}(S_a\lambda_a + iP_a\lambda_a) \\ D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\ L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \left\{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\right\}, \\ R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \left\{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\right\}, \\ D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.\end{aligned}$$

+ Polyakov loop potential (for $T > 0$)

Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$

$a_1 \to a_1(1230), K_1 \to K_1(1270)$
$f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar ($\sim \bar{q}_i q_j$) and pseudoscalar ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star +} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star 0} \\ K_0^{\star -} & \bar{K}_0^{\star 0} & \sigma_S \end{pmatrix} \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

multiple possible assignments
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \to \pi(138), K \to K(495)$
mixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$
fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

In case of compact stars, also nonzero vector condensates

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2 ,$$

$(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N) \longrightarrow$ from the model, $Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$

multiparametric minimalization $\longrightarrow$ MINUIT

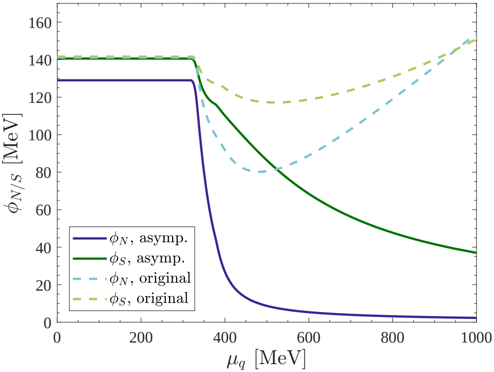
- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Curvature masses $\rightarrow$ 16 physical quantities: $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Importance of parameterization

- a naive parameterization → chiral symmetry would be broken at high densities
- investigating the asymptotic behavior we get an additional constraint for the parameters:

$$3h_1 + 2h_2 + 2h_3 < 0$$

- Phys.Rev.D 105 (2022) 10, 103014



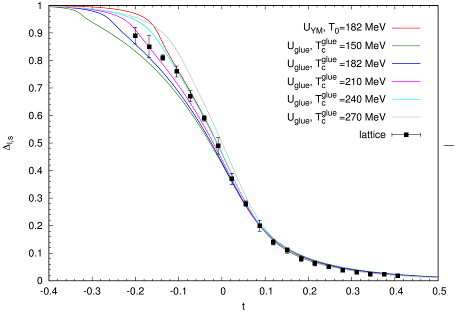
Features of our approximation

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – u, d, s constituent quarks ($m_u = m_d$)
  – (Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\mathrm{YM}}$ or $\mathcal{U}_{\log}^{\mathrm{glue}}$)
- no mesonic fluctuations, only fermionic ones
  $\mathcal{Z}=e^{-\beta V \Omega(T,\mu_q)} = \int\limits_{\mathrm{PBC}} \prod_a \mathcal{D}\xi_a \int\limits_{\mathrm{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x \left(\mathcal{L}+\mu_q \sum_f q_f^\dagger q_f\right)\right]$ approximated as $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M\rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$, $\tilde{\mu}_q = \mu_q - iG_4$

  $e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int\limits_{\mathrm{APBC}} \prod_{f,g} \mathcal{D}q_g \mathcal{D}q_f^\dagger \exp\left\{\int_0^\beta d\tau \int_x q_f^\dagger \left[\left(i\gamma_0 \vec{\gamma}\cdot\vec{\nabla} - \frac{\partial}{\partial\tau} + \tilde{\mu}_q\right)\delta_{fg} - \gamma_0 \mathcal{M}_{fg}|_{\xi_a=0}\right] q_g\right\}$
- tree-level (axial)vector masses
- fermionic thermal fluctuations included in the (pseudo)scalar curvature masses
- 2 (or 4) coupled T/μ_B-dependent field equations for the condensates $\phi_N, \phi_S, (\Phi, \bar{\Phi})$ at $N_c = 3$
- Polyakov-loops and fermionic vacuum fluctuations

t-dependence of the condensates compared to lattice results

The subtracted chiral condensate



– subtracted chiral condensate:

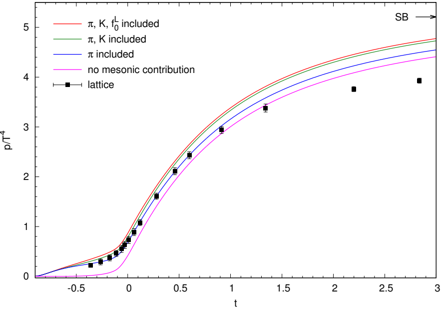
$$\Delta_{l,s} = \frac{\left(\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S\right)\Big|_T}{\left(\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S\right)\Big|_{T=0}}$$

– $U_{\rm log}^{\rm glue}$ with $T_c^{\rm glue} \in (210, 240)$ MeV gives good agreement with the lattice result of

Borsányi *et al.*, JHEP 1009, 073 (2010)

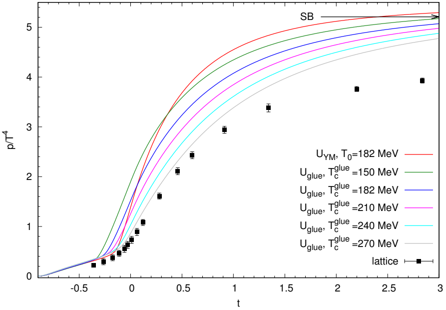
here we use the reduced temperature: $t = (T - T_c)/T_c$

Normalized pressure and the effects of meson contributions



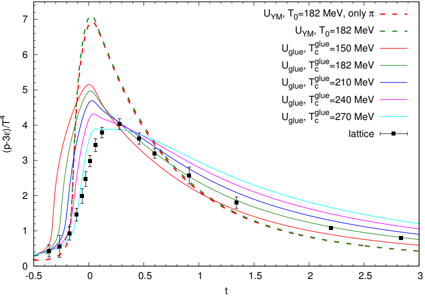
– we used $U_{\rm glue}$ with $T_c^{\rm glue} = 270$ MeV
– pions dominate the pressure at small T
– contribution of the kaons is important
– at high T the pressure overshoots the lattice data of Borsányi *et al.*, JHEP 1011, 077 (2010)

– overshooting increases with decreasing $T_c^{\rm glue}$



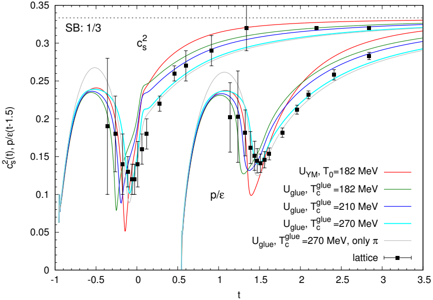
Scaled interaction measure, speed of sound and p/ϵ

Scaled interaction measure



By properly setting the T_c^{glue} parameter $\longrightarrow$ good agreement with lattice

Speed of sound and p/ϵ



N_c scaling of the Lagrange parameters

The parameters are: $m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S,\ \Phi_N, \Phi_S, g_F,\ h_{N/S}$

- $m_0^2,\ m_1^2,\ \delta_s \sim N_c^0$, because terms of tree level meson masses
- $g_1,\ g_2 \sim \frac{1}{\sqrt{N_c}}$, three couplings
- $\lambda_2,\ h_2,\ h_3 \sim \frac{1}{N_c}$, four couplings
- $\lambda_1,\ h_1 \sim \frac{1}{N_c^2}$, four couplings with different trace structure
- $c_1 \sim \frac{1}{N_c^{3/2}}$ $U_A(1)$ anomaly term has extra $1/N_c$ suppression
- $h_{N/S} \sim \sqrt{N_c}$, Goldstone-theorem: $m_\pi^2 \Phi_N = Z_\pi^2 h_N$ + PCAC: $\Phi_N = Z_\pi f_\pi$
- $g_F \sim \frac{1}{\sqrt{N_c}}$, $m_{u/d} = g_F \Phi_N$

We expect $\Phi_{N/S} \sim \sqrt{N_c}$, since $\Phi_N = Z_\pi f_\pi,\ f_\pi \sim \sqrt{N_c}$, but have to check!

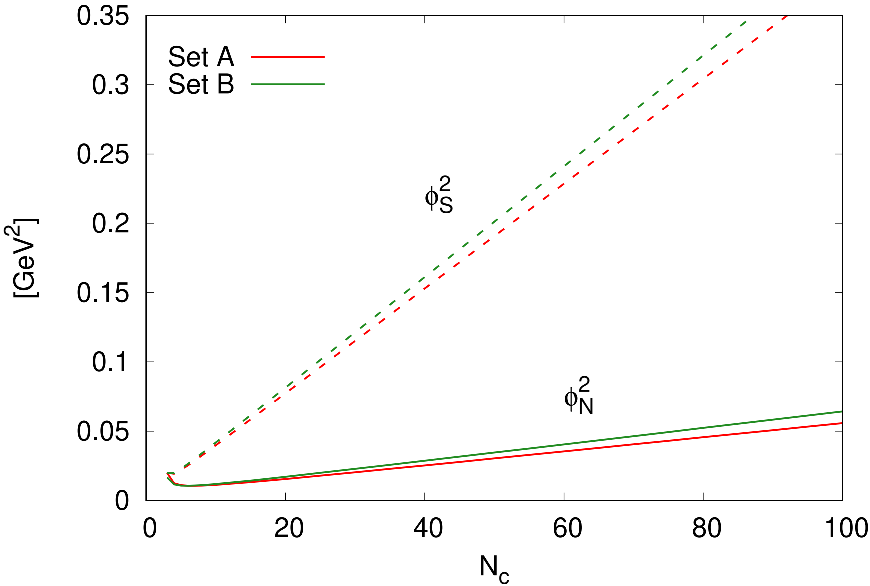
practically: $g_1 \longrightarrow g_1 \sqrt{\frac{3}{N_c}},\ h_{N/S} \longrightarrow h_{N/S} \sqrt{\frac{N_c}{3}} \ldots$ *etc.*

Parameter sets at $N_c = 3$

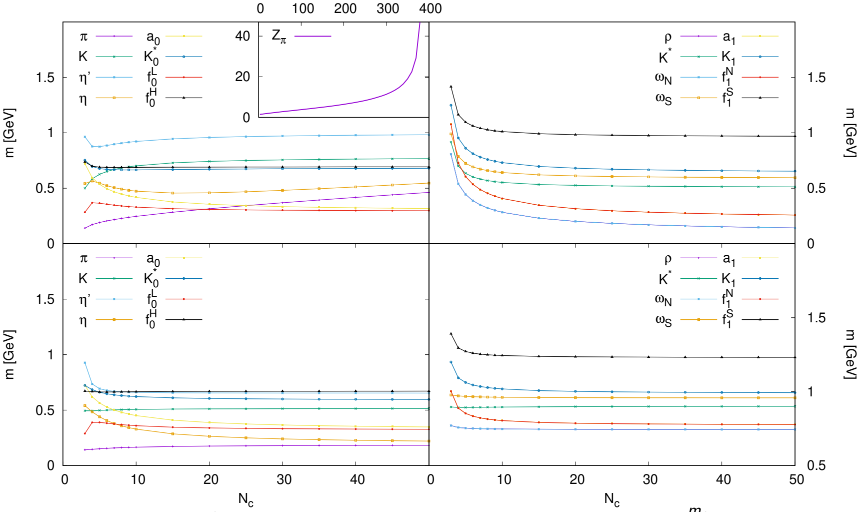
Parameter	Set A	Set B
ϕ_N [GeV]	0.1411	0.1290
ϕ_S [GeV]	0.1416	0.1406
m_0^2 [GeV^2]	2.3925_{E-4}	-1.2370_{E-2}
m_1^2 [GeV^2]	6.3298_{E-8}	0.5600
λ_1	−1.6738	−1.0096
λ_2	23.5078	25.7328
c_1 [GeV]	1.3086	1.4700
δ_S [GeV^2]	0.1133	0.2305
g_1	5.6156	5.3295
g_2	3.0467	−1.0579
h_1	37.4617	5.8467
h_2	4.2281	−12.3456
h_3	2.9839	3.5755
g_F	4.5708	4.9571
M_0 [GeV]	0.3511	0.3935

- Set A: $m_\sigma = 290$ MeV from Phys.Rev.D 93 (2016) 11, 114014
- Set B: similar m_σ mass and additional constraint, $3h_1 + 2h_2 + 2h_3 < 0$ from Phys.Rev.D 105 (2022) 10, 103014

N_c scaling of the $\phi_{N/S}$ condensates



N_c scaling of the tree level masses



Top figures using Set A, bottom figures using Set B;

$$Z_\pi = \frac{m_{a_1}}{\sqrt{m_{a_1}^2 - g_1^2\phi_N^2}}$$

for every meson we see: $m_{meson} \sim N_c^0$

Introduction to Polyakov-loops/Polyakov-loop variables

Definition of Polyakov-loop

$$L(\vec{x}) = \mathcal{P} \exp \left\{ i \int_0^\beta A_4 dt \right\}$$

Polyakov-loop variables:

$$\Phi(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c L(\vec{x}), \text{ and } \bar{\Phi}(\vec{x}) = \frac{1}{N_c} \mathrm{Tr}_c L(\vec{x})^\dagger, \text{(non center } (C_n) \text{ symmetric)}$$

If $\Delta F_{q/\bar{q}}$ is a change in the free energy, when an infinitely heavy quark (or antiquark) is added to the system, than

$$\langle \Phi(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_q(\vec{x})} \quad , \quad \langle \bar{\Phi}(\vec{x}) \rangle_\beta = e^{-\beta \Delta F_{\bar{q}}(\vec{x})}$$ *Phys. Rev. D* 24 (1981) 450

- C_n symm. phase $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta = 0 \rightarrow \Delta F_{q/\bar{q}} = \infty \rightarrow$ confinement
- C_n NON symm. phase $\rightarrow \langle \Phi(\vec{x}) \rangle_\beta \neq 0 \rightarrow \Delta F_{q/\bar{q}} < \infty \rightarrow$ deconfinement

Thus $\Phi(\vec{x})$ and $\bar{\Phi}(\vec{x})$ can be used as order parameters for confinement

Polyakov-loop variables at large N_c (I.)

Polyakov gauge → A_4 is diagonal and time independent
→ further simplification: homogeneous gluon filed

$$L = e^{i\beta A_4} = \mathrm{diag}\left(e^{iq_1}, \dots, e^{iq_{N_c}}\right), \quad q_j \in \mathbb{R}, \quad \sum_j q_j = 0$$

$N_c - 1$ independent diagonal $SU(N_c)$ matrix → Φ and $\bar{\Phi}$ above are not enough

$$\Phi_n = \frac{1}{N_c}\mathrm{Tr}_c L^n, \quad \bar{\Phi}_n = \frac{1}{N_c}\mathrm{Tr}_c {L^\dagger}^n, \quad n \in \left(1, \dots, \lfloor \frac{N_c}{2} \rfloor\right),$$ Phys. Rev. D 86, 105017 (2012)

our approx. → quarks prop. on a const. gluon background → color dep. chem. pot.

$$\Omega^{(0)}_{\bar{q}q}(T,\mu_q) = \Omega^{(0)\mathrm{v}}_{\bar{q}q} + \Omega^{(0)\mathrm{T}}_{\bar{q}q}(T,\mu_q),$$

$$\Omega^{(0)\mathrm{T}}_{\bar{q}q}(T,\mu_q) = -2T\sum_f \int \frac{d^3p}{(2\pi)^3}\left[\ln g_f^+(p) + \ln g_f^-(p)\right]$$

$$\ln g_f^+(p) \equiv \mathrm{Tr}_c \ln\left[\mathbb{1} + L^\dagger e^{-\beta(E_f(p)-\mu_q)}\right] = \ln \mathrm{Det}_c\left[\mathbb{1} + L^\dagger e^{-\beta(E_f(p)-\mu_q)}\right]$$

$$\ln g_f^-(p) \equiv \mathrm{Tr}_c \ln\left[\mathbb{1} + L e^{-\beta(E_f(p)+\mu_q)}\right] = \ln \mathrm{Det}_c\left[\mathbb{1} + L e^{-\beta(E_f(p)+\mu_q)}\right]$$

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Polyakov-loop variables at large N_c (*II.*)

$$g_f^+ = 1 + e^{-N_c \beta E_f^+} + N_c \left[\bar{\Phi}_1 e^{-\beta E_f^+} + \Phi_1 e^{-(N_c-1)\beta E_f^+} \right] + [\text{terms with } 2 \text{ to } N_c - 2 \text{ phases}]$$

Only first line in case of $N_c = 3$. For $N_c > 3$ more and more Φ_ks appear:

$$\begin{aligned} g_f^+ =& 1 + e^{-N_c \beta E_f^+} \\ &+ N_c \left[\bar{\Phi} e^{-\beta E_f^+} + \Phi e^{-(N_c-1)\beta E_f^+} \right] \\ &+ \frac{1}{2} \left(N_c^2 \bar{\Phi}^2 - N_c \bar{\Phi}_2 \right) e^{-2\beta E_f^+} \\ &+ \frac{1}{2} \left(N_c^2 \Phi^2 - N_c \Phi_2 \right) e^{-(N_c-2)\beta E_f^+} \\ &+ [\text{terms with } 3 \text{ to } N_c - 3 \text{ phases}] \end{aligned}$$

All the Φ_ks are unknown → At N_c there would be $N_c + 1$ field equations → approximation: Uniform eigenvalue ansatz

Uniform eigenvalue ansatz

We express the q_j angles with the $N_c - 1$ group angles of the Cartan subgroup of $SU(N_c)$: Phys. Rev. D 103, 074026 (2021)

$$\vec{q} \equiv (q_1, \ldots, q_{N_c}) = \sum_{j=1}^{N_c-1} \gamma_j \vec{v}_j, \quad \{\vec{v}_j\}_{j=1}^{N_c-1} \text{ (set of basis vectors)}$$

Ansatz (UEA): $\gamma_1 \neq 0$, $\gamma_i = 0, i \neq 1$, a direction is fixed in the subalgebra,

$$\vec{v}_1 = \left(-1, -\left(1 - \frac{2}{N_c - 1}\right), \ldots, -\left(1 - (j-1)\frac{2}{N_c - 1}\right), \ldots, 1\right), \; j = 1, \ldots, N_c$$

$$L = \text{diag}\left(e^{-i\gamma}, e^{-i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{-i\left(1-2\frac{2}{N_c-1}\right)\gamma}, \ldots, (e^0), \ldots, e^{i\left(1-2\frac{2}{N_c-1}\right)\gamma}, e^{i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{i\gamma}\right)$$

Instead of $N_c - 1$ unknown variables, only one: $\gamma_1 \equiv \gamma$ and $\Phi_n = \bar{\Phi}_n \in \mathbb{R}$

$$\Phi_n = \frac{1}{N_c}\left(2\sum_{j=1}^{\lfloor \frac{N_c}{2} \rfloor} \cos\left[\left(1 - 2\frac{j-1}{N_c - 1}\right) n\gamma\right] + \alpha\right)$$

Polyakov-loop potential at large N_c

$$U_{\text{Pol}} = U_{\text{conf}} + U_{\text{glue}},$$ Phys. Rev. D 103, 074026 (2021)

$$U_{\text{conf}} = -\frac{b}{2} T \ln H = -\frac{b}{2} T \ln g'_A, \quad g'_A = \text{Det}' \left(\mathbb{1}_A - L_A e^{-\beta E_A(p)} \right)$$

$$U_{\text{glue}} = n_{\text{glue}} T \int \frac{d^3p}{(2\pi)^3} \ln g_A, \qquad g_A = \text{Det} \left(\mathbb{1}_A - L_A e^{-\beta E_A(p)} \right)$$

$$L_A = \text{diag}(e^{iQ_1}, \ldots, e^{iQ_{N_c^2-1}}),$$

$$\vec{Q} = (\underbrace{0, \ldots, 0}_{N_c-1}, \underbrace{q_1 - q_2, \ldots, (q_j - q_k)|_{j\neq k}, \ldots q_{N_c-1} - q_{N_c}}_{N_c(N_c-1)}).$$

L_A Polyakov-loop op. in the adjoint repr.; Q_j adjoint angles
$b = (0.1745 \text{ GeV})^3$, $n_{\text{glue}} = 2$, and $m_A = 0.756$ GeV

Field equation in the UEA

$$0 = \frac{dU_{\mathrm{Pol}}}{d\gamma} - 2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(h_f^+(p) + h_f^-(p)\right),$$

$$0 = m_0^2 \phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^3 + \lambda_1 \phi_N \phi_S^2 - \frac{c}{\sqrt{2}}\phi_N\phi_S - h_{0N} + \frac{g_F}{2}\sum_{l=u,d}\langle \bar{q}_l q_l \rangle_T,$$

$$0 = m_0^2 \phi_S + (\lambda_1 + \lambda_2)\phi_S^3 + \lambda_1 \phi_N^2 \phi_S - \frac{\sqrt{2}c}{4}\phi_N^2 - h_{0S} + \frac{g_F}{\sqrt{2}}\langle \bar{q}_s q_s \rangle_T,$$

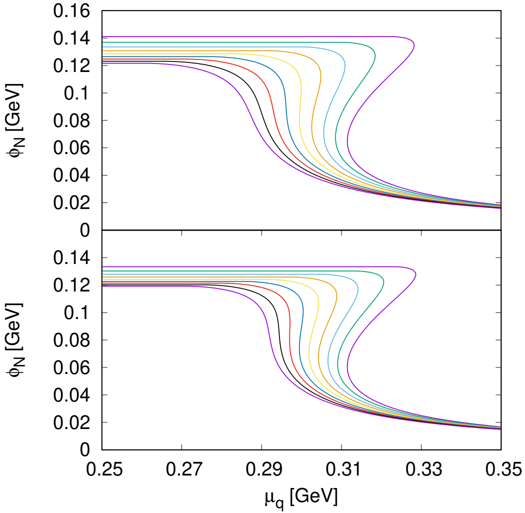
with

$$h_f^\pm = \frac{1}{g_f^\pm}\frac{\partial g_f^\pm}{\partial \gamma},$$

$$\langle \bar{q}_f q_f \rangle_T = -4N_c m_f \left[\frac{m_f^2}{16\pi^2}\left(\frac{1}{2} + \ln\frac{m_f^2}{M_0^2}\right) + \mathcal{T}_f^{\mathrm{matt}}\right]$$

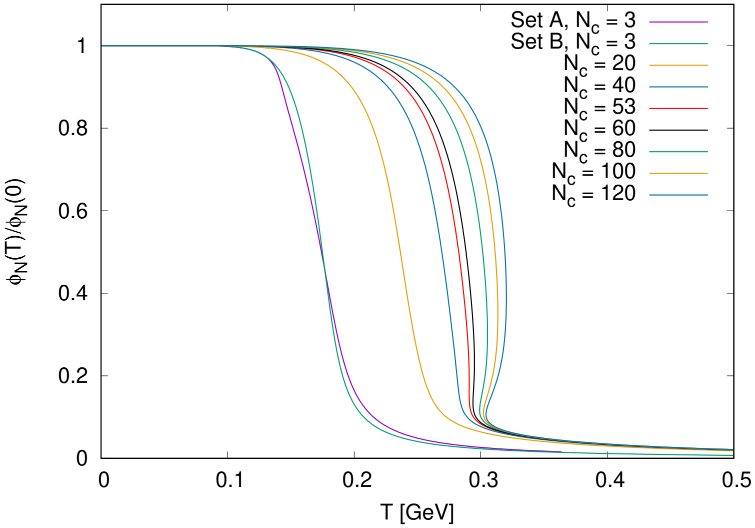
$$\mathcal{T}_f^{\mathrm{matt}} = \frac{T}{N_c}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2m_f}\left(\frac{1}{g_f^+}\frac{\partial g_f^+}{\partial m_f} + \frac{1}{g_f^-}\frac{\partial g_f^-}{\partial m_f}\right)$$

μ_q dependence of the ϕ_N condensates at diff. N_cs



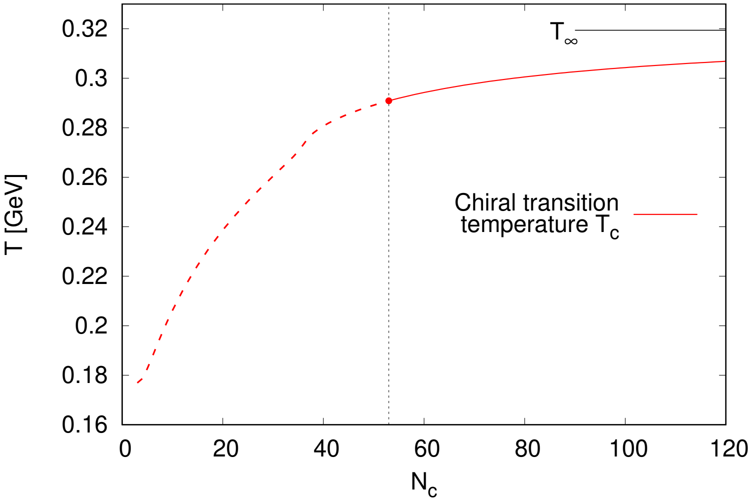
- Upper fig.: set A
- Lower fig.: set B
- Crossover already for $N_c = 3.45$

T dependence of the ϕ_N condensates at diff. N_cs



$N_c = 3$: $T_c = 178.6$ MeV and $T_c = 176.9$ MeV, for set A and set B

Saturation of the pseudocritical temp. T_c



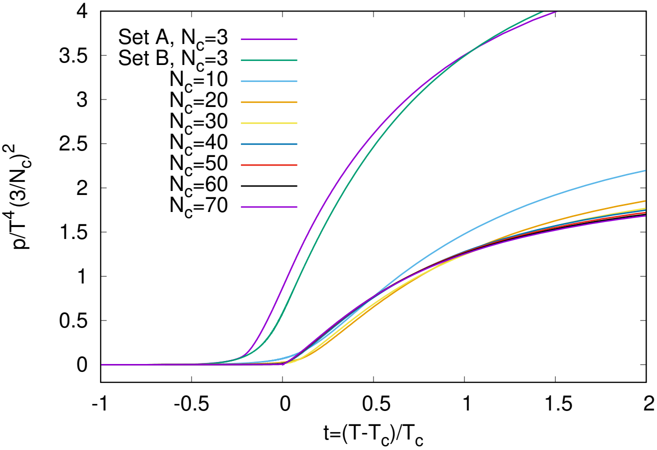
fit:

$$T_c(N_c) = \alpha/(N_c + \beta) + T_\infty;\ T_\infty = 0.3192\ \mathrm{MeV}$$

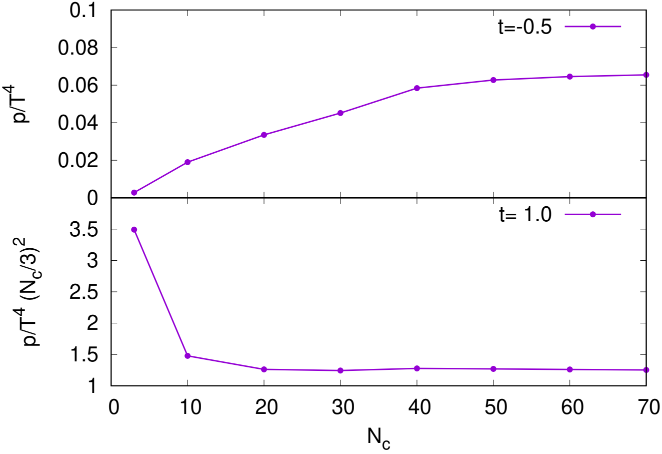
Pressure for different N_c values at $\mu_q = 0$

$$p(T,\mu_q) = -\left(\Omega(T,\mu_q,\phi_{N/S}(T,\mu_q),\gamma(T,\mu_q)) - \Omega(T,\mu_q,\phi_{N/S}(0,0),\gamma(0,0))\right)$$

non-trivial subtraction, however field eq. doesn't change

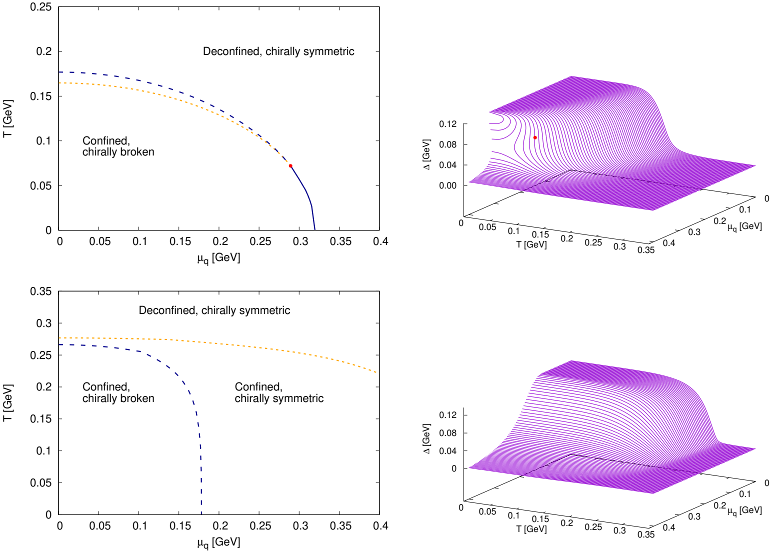


Scaling of the pressure at $\mu_q = 0$



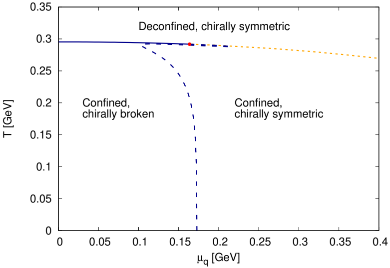
$t = -0.5$ (top: below ph. t. $\sim N_c^0$) and $t = 1$ (bottom: above ph. t. $\sim N_c^2$)

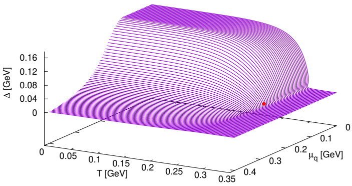
Phase boundary for different N_cs *I.*



top: $N_c = 3$, CEP exist, crossover for small T; *bottom* : $N_c = 33$ crossover everywhere

Phase boundary for different N_cs *II.*

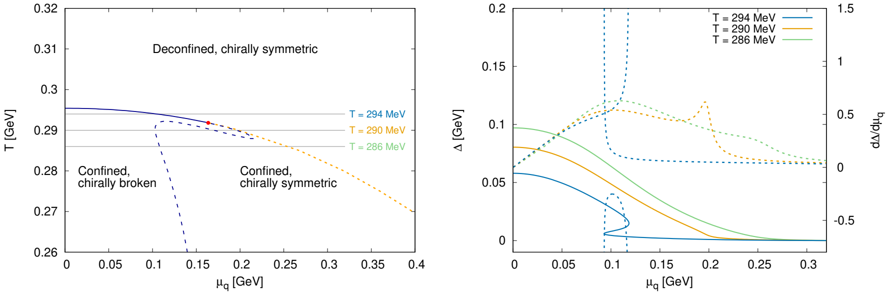




$N_c = 63$ CEP exist again, crossover for large T

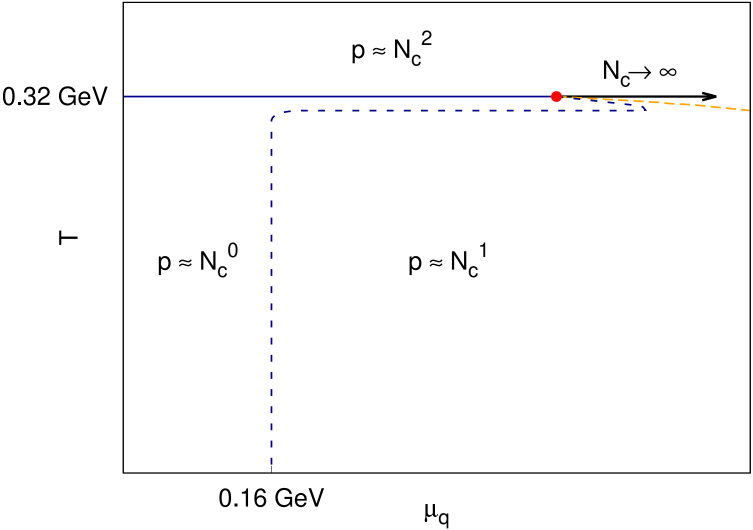
$$\Delta(T, \mu_q^{\mathrm{fix}}) = \frac{\left(\phi_N - \frac{h_N}{h_S}\phi_S\right)\big|_{T,\mu_q^{\mathrm{fix}}}}{\left(\phi_N - \frac{h_N}{h_S}\phi_S\right)\big|_{T=0,\mu_q^{\mathrm{fix}}}} \qquad (1)$$

A close look on the phase structure



$N_c = 63$: The subtracted condensate $\Delta(T_0, \mu_q)$ (solid lines) and its μ_q derivatives (dashed lines) calculated along the horizontal–μ_q–directions (right)

Schematic phase structure for large N_c (conclusion)



see: Phys. Rev. D **106**, 116016

Thank you for your attention!