Exploring external field related phenomena in full lattice QCD

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CRC–TR 211
Strong-interaction matter under extreme conditions

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Strong magnetic fields in QCD governed systems

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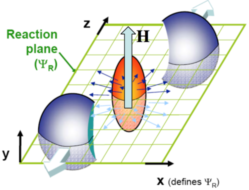
- off-central heavy-ion collisions

  [Kharzeev, McLerran, Warringa '07]

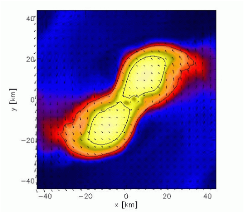
  impacts: EoS, anomalous transport, anisotropies, elliptic flow, ...



- neutron stars
  - magnetars [Duncan, Thompson '92]
  - neutron star mergers

    [Anderson *et al.* '08]

  impacts: EoS, mass-radius relation, cooling mechanism, ...

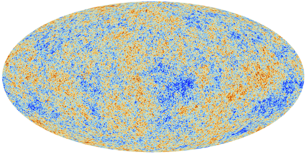


- early Universe

  [Vachaspati '91, Enqvist, Olesen '93]

  (generated through phase transitions in the electroweak epoch)

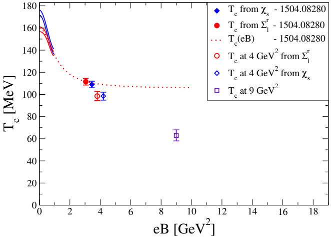
  impacts: EoS, ..., BSM properties?, ...



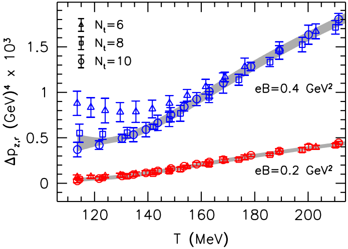
Impact on phase diagram and EoS

Investigated in detail: effects of uniform external magnetic fields

- Phase diagram:
  [Bali *et al* '11; D'Elia *et al* '10, '21; Endrődi '15; Bornyakov *et al* '13]
  - inverse magnetic catalysis
    [Bruckmann, Endrődi, Kovács '13]
  - effects on (de)confinement
    [Bonati *et al* '14, '16, '18; D'Elia *et al* '21]



- Equation of state: [Bali *et al* '14]



- Extensions to $\mu \neq 0$
  [Ding *et al* '21; Kolomoyets *et al* LAT21]

Other interesting (less explored) directions

- impact on hadron structure
  - spectrum [. . . ; Bali *et al* '17; Ding *et al* '20]
  - decay rates [Bali *et al* '18] (also: [Coppola *et al* '19, '20])

  (+ many studies on polarizabilities) (see also talk by Fukushima)

  relevance: neutron stars (stability analysis; cooling mechanisms; . . .)
- transport properties
  - standard coefficients σ_{em} [Astrakhantsev *et al* '20]
  - anomalous transport CSE [Puhr, Buividovich '17; Buividovich *et al* '21]

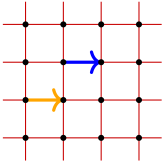
  relevance: Heavy-Ion collisions [Kharzeev *et al* '08; . . .]
- beyond the uniform field approximation?
  - temporal and spatially modulated fields

  relevance: Heavy-Ion collisions [Voronyuk *et al* '11; Deng, Huang '12; . . .]
- BSM physics?
  - axion physics (χ_{top})

  relevance: early Universe

Lattice QCD setup

[Wilson '74]



- lattice discretization: natural regulator for PI
- regularized path integral: (fermions integrated out)

$$\mathcal{Z} = \int [dU] \, \det\big(D[U]\big) \, e^{-S_{\mathrm{gluon}}[U]}$$

$D[U]$: discretized Dirac operator

$U \in \mathrm{SU}(3)$: link variables (parallel transporter on gauge manifold)

- finite volume: finite dimensional integral over compact group

  ⟶ can use Monte-Carlo methods for numerical integration

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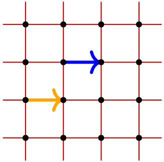

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- external magnetic fields:

  introduced via $\text{U}(1)$ link variables $u^q_\mu(x) = e^{ia\,Q_q\,A_\mu(x)}$

$$\mathbf{B} = \nabla \times \mathbf{A} = B_z \mathbf{e}_z \quad \text{with} \quad Q_d\,B_z = \frac{2\pi\,N_B}{L_x L_y}$$

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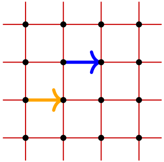

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Our lattice setup:

- $2+1$ flavours of 2-stout staggered fermions & improved gluon action
- physical quark masses

Contents

I Anomalous transport – CSE & CME

Eduardo Garnacho

Anomalous transport phenomena

Matter imbalances + (electro-)magnetic fields

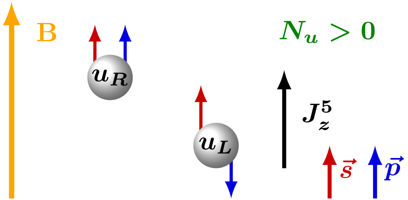
⟶ non-dissipative transport effects

existence of currents is related to the anomalous triangle diagram

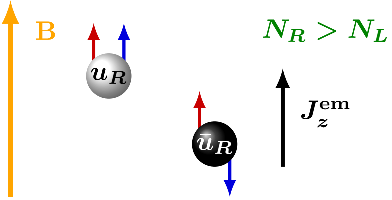
⟶ **anomalous transport phenomena** [review: Kharzeev *et al* '16]

Here: Chiral separation effect (CSE) & Chiral magnetic effect (CME)

CSE: $J_z^5 = C_{\mathrm{CSE}}\, QB_z\, \mu$



CME: $J_z^{\mathrm{em}} = C_{\mathrm{CME}}\, QB_z\, \mu_5$



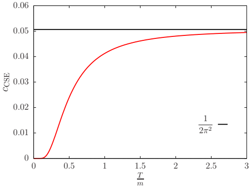
- Great experimental effort for detection of CME! (e.g. Star collaboration '21)
- What is the theoretical situation?

Expectations and previous results

- Analytic free results: (em perturbative; gluon interaction neglected)

- $C_{\rm CSE} \quad = 0 \xrightarrow{\frac{T}{m}\to\infty} \frac{1}{2\pi^2}$

  [Metlitski, Zhitnitsky '05]



- $C_{\rm CME} \quad = 0$ in equilibrium

  $= \frac{1}{2\pi^2}$ out-of equilibrium

  [Fukushima *et al* '08, Son, Surowka '09 Horvath *et al* '20]

  Controversial equilibrium results!

  → needs careful regularization

  [Buividovich '14, Landsteiner '16]

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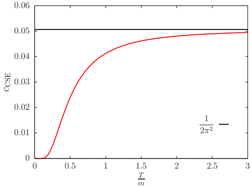

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- Lattice QCD results: (controversial, few, no physical point results)

- $\mathrm{QC_2D}$ with Wilson/domain wall: suppression of $C_{\rm CSE}$ at low T

  [Buividovich, Smith, von Smekal '21]

- Quenched with overlap: always large T free case result

  [Buividovich, Puhr '17]

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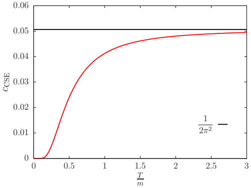

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  - full QCD with 2 identical flavours
    & quenched using Wilson

    $C_{\rm CME} \approx 0.01$ to 0.03

    [Yamamoto '11]

    (despite being in equilibrium?)

Lattice computation

We want to compute C_{CSE} & C_{CME} on the lattice:

- to directly compute currents: need simulations at $\mathbf{B} \neq 0$ and
  - C_{CSE} $\mu \neq 0$
    - ✗ sign problem
  - C_{CME} $\mu_5 \neq 0$
    - ✓ no sign problem generically
    - ✗ for staggered fermions
      (non-local representation of γ-matrices)

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- work around: compute coefficients via derivatives
  - $C_{\text{CSE}} = \left.\frac{d^2 \left\langle J_z^5 \right\rangle}{d\mu \, d(QB_z)}\right|_{\mu, B_z=0}$
  - $C_{\text{CME}} = \left.\frac{d^2 \left\langle J_z \right\rangle}{d\mu_5 \, d(QB_z)}\right|_{\mu_5, B_z=0}$

  but: computing $\mathbf{B}$ derivatives on the lattice is complicated
  (see [Bali, Endrődi, Piemonte '20])

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  but: computing $\mathbf{B}$ derivatives on the lattice is complicated (see [Bali, Endrődi, Piemonte '20])
- here: compute μ-der. at $\mathbf{B} \neq 0$ & extract $C_{\rm CSE/CME}$ from linear fit
  - $C_{\rm CSE}\, QB_z = \left.\frac{d \langle J_z^5 \rangle}{d\mu}\right|_{\mu=0}$
  - $C_{\rm CME}\, QB_z = \left.\frac{d \langle J_z \rangle}{d\mu_5}\right|_{\mu_5=0}$

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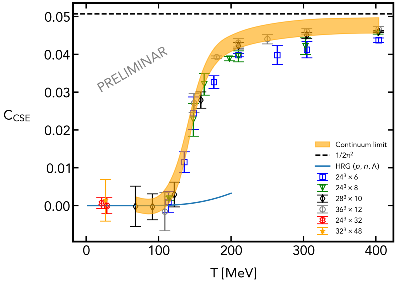
Full QCD lattice results



- C_{CSE} changes rapidly around T_c
- asymptotically for $T \to \infty$:
  approaches $m = 0$ analytic result
  [Metlitski, Zhitnitsky ’05]

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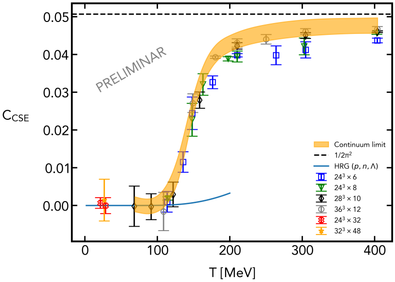

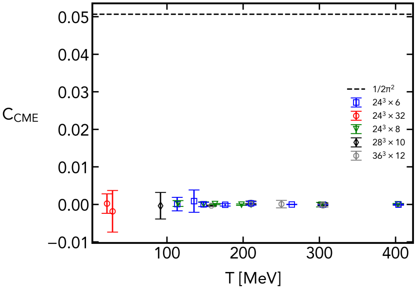

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  robust confirmation of the analytic equilibrium result

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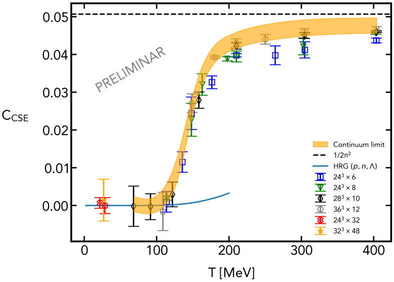

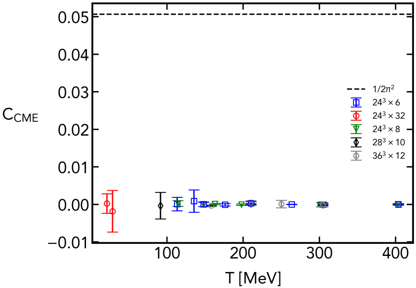

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▶ crucial to obtain correct results:
  - use conserved vector current
  - staggered: keep track of extra terms due to non-local γ-matrices

    (additional tadpole contributions to derivatives)

II QCD in non-uniform magnetic fields

Dean Valois [arXiv: 2305.19029]

Magnetic fields in Heavy-Ion collisions

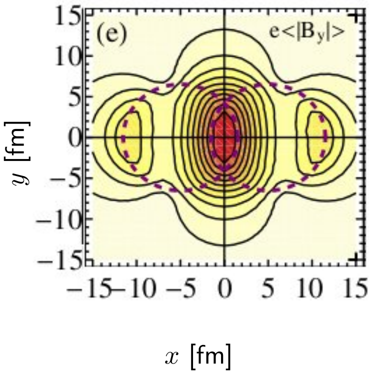
Previously: consider uniform $\mathbf{B}$-fields

→ good approximation when $\mathbf{B}$-fields are approximately constant on QCD scales

- Magnetic fields in Heavy-Ion collisions:
  - strongly space and time dependent
  - appear together with electric fields

  [Deng, Huang '12]

→ non-uniform on QCD scales



- What can we do on the lattice?
  - ✗ electric fields
  - ✗ time-dependent magnetic fields
  - ✓ spatially modulated magnetic fields

    but: without back reaction (background field)

→ consider as a first step towards more realistic field settings

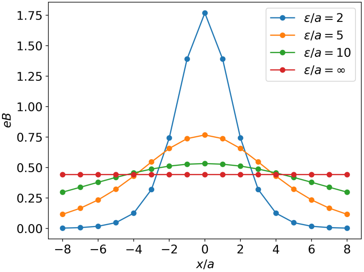
Non-uniform magnetic fields on the lattice

- choice of field profile:

$$B_z = B_0 \cosh^{-2}\left(\frac{x}{\epsilon}\right)$$

  - resembles expected HIC profiles [Voronyuk *et al* '11; Deng, Huang '12]
  - allows for analytical treatment in free case [Dunne '04; Cao '18]

  → can be used in model studies



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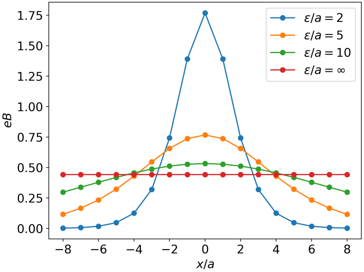

- representation in terms of link variables:

$$u_x(x,y) = \begin{cases} \exp\left(-i2\pi N_B\left(\frac{y}{L_y}+\frac{1}{2}\right)\right) & x = \frac{L_x}{2} - a \\ 1 & \text{otherwise} \end{cases} \qquad u_t(x,y) = 1$$

$$u_y(x,y) = \exp\left(iaQB_0\epsilon\left[\tanh\left(\frac{x}{\epsilon}\right) + \tanh\left(\frac{L_x}{2\epsilon}\right)\right]\right) \qquad u_z(x,y) = 1$$

quantization condition: $Q_d B_0 = \frac{\pi N_B}{L_y \epsilon \tanh\left(\frac{L_x}{2\epsilon}\right)}$ with $N_B \in \mathbb{Z}$

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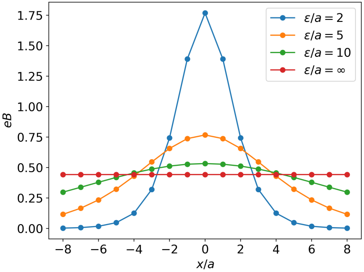

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- Setup in simulations: use $\epsilon = 0.6$ fm

Local observables

- renormalized local observables:

chiral condensate $\Sigma(x, B_0, T) = \frac{2m_{ud}}{m_\pi^2 f_\pi^2} \left[\left\langle \bar{\psi}\psi(x) \right\rangle_{B_0,T} - \left\langle \bar{\psi}\psi(x) \right\rangle_{0,0} \right]$

Polyakov loop $P_R(x, B_0, T) = W(a, T) \left\langle P(x) \right\rangle_{B_0,T}$

(with W from [Bruckmann *et al* '13])

- Do the observables simply follow the uniform field behaviour?

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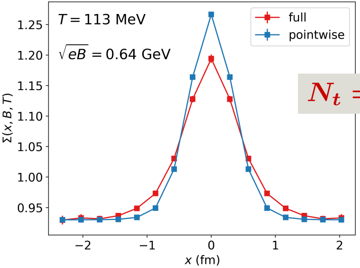

$N_t = 6$

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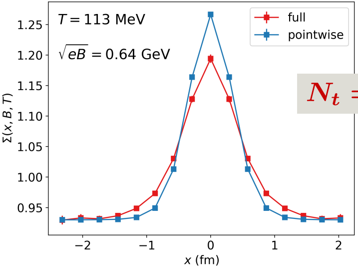

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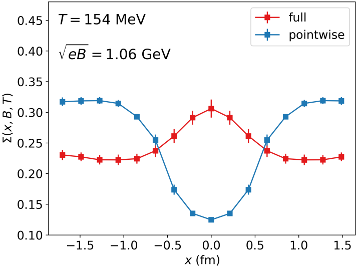

→ inverse magnetic catalysis lost?

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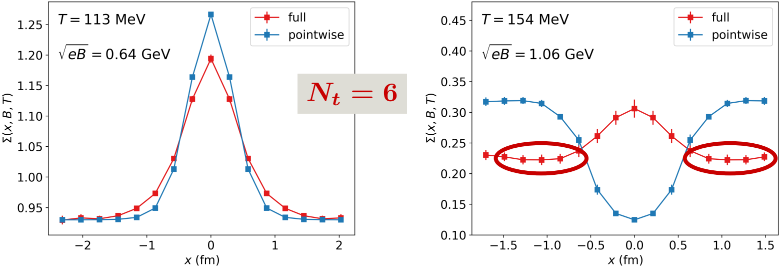
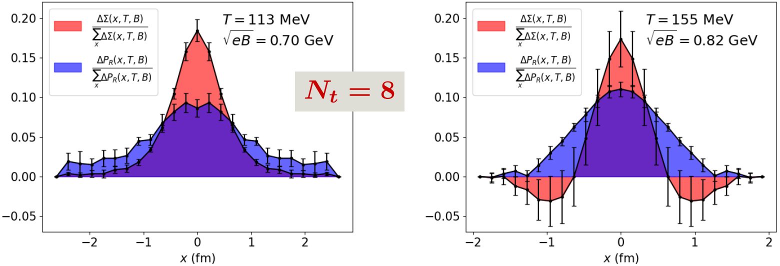


→ inverse magnetic catalysis lost?

but: $\Sigma(x, B_0, T)$ develops 'dips' around T_c

Local inverse magnetic catalysis

- interplay between sea and valence effects



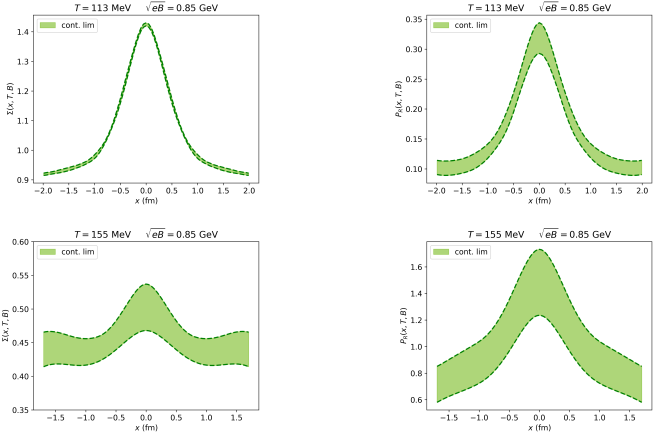
- valence effects – follow the magnetic field profile
- sea effects – are smeared out

→ leads to dips and loss of inverse magnetic catalysis in middle

(can also be seen in an explicit analysis of the individual effects
– see [arXiv: 2305.19029])

Continuum results

- Continuum limit from $N_t = 6, 8, 10$ and 12



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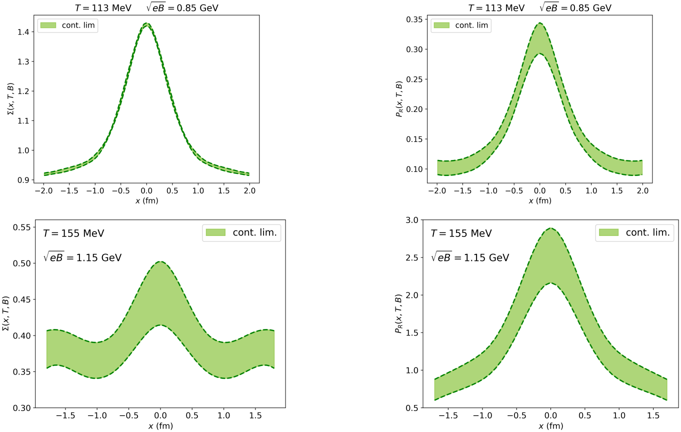

- new phenomena in non-uniform magnetic fields:

Polyakov loop:	center points appears deconfined
chiral condensate:	chiral symmetry breaking appears stronger in center

III Impact on topology

Javier Hernández

The axion mass and the topological susceptibility

- Axions: solution to strong CP problem [Peccei, Quinn '77]

  QCD with θ-term: $\mathcal{L} = \mathcal{L}_{\rm QCD} + \theta\, Q_{\rm top}$

  add pseudoscalar axion: $\mathcal{L} = \mathcal{L}_a + \frac{a}{f_a} Q_{\rm top}$

→ neutron electric dipole moment – but $|\theta| < 10^{-10}$ [Abel *et al* '20]

- a: massless pseudoscalar
- $\mathcal{L}_a$ so that $\langle a \rangle = -\theta f_a$

→ CP-odd effects vanish in expectation values

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- Dynamically generated axion mass:

$$m_a^2 = \frac{\delta^2}{(\delta a)^2} \log Z = \frac{\langle Q_{\rm top} Q_{\rm top} \rangle}{f_a^2} = \frac{\chi_{\rm top}}{f_a^2}$$

  → depends on topological susceptibility $\chi_{\rm top}$

The axion mass and the topological susceptibility

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  → depends on topological susceptibility χ_{top}

- Is χ_{top} affected by primordial $\mathbf{B}$-fields?

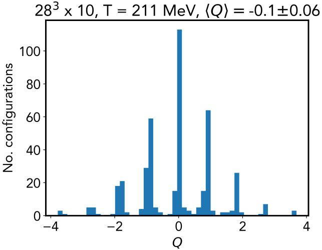
  chiral perturbation theory: χ_{top} mildly enhanced with $\mathbf{B}$ [Adhikari '22]

Lattice setup

- discretization of $Q_{\rm top}$:

  we use the clover discretization of $F_{\mu\nu}$

  but: we also test improved definitions



- Wilson flow: to suppress UV fluctuations
  - flow until $Q_{\rm top}$ distribution clusters around integer values

- zero mode reweighting: to remove main lattice artifact

  Atyiah-Singer index theorem: $\#$ zero modes $= Q_{\rm top}$ $(= N_R - N_L)$

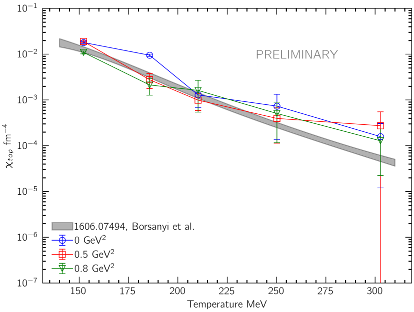
  massless staggered operator: no zero modes

  → cure by reweighting [Borsanyi *et al* '16]

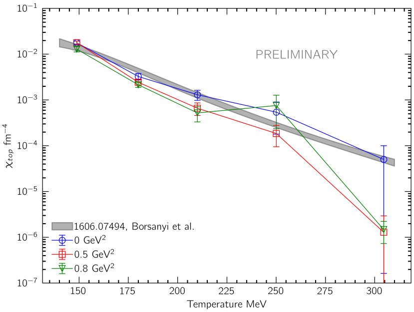
reweighting factor:
$$W = \prod_f \prod_{i=1}^{2|Q_{\rm top}|} \prod_{\sigma=\pm} \left(\frac{m_f}{i\sigma\lambda_i + m_f} \right)^{\frac{1}{4}}$$

Topological susceptibility at $\mathrm{B} \neq 0$

- Comparison between χ_{top} at $B_z \neq 0$ and $B_z = 0$:



$N_t = 8$



$N_t = 12$

- only mild B_z dependence – difficult to resolve

work in progress . . .

IV The axion-photon coupling

Javier Hernández

The axion photon coupling from the lattice

- another important parameter for axion physics in the early Universe:

axion-photon coupling: $g_{a\gamma\gamma} = g^{0}_{a\gamma\gamma} + g^{\mathrm{QCD}}_{a\gamma\gamma}$

direct coupling to photons
model dependent

non-perturbative QCD part
model independent

Chiral perturbation theory: $g^{\mathrm{QCD}}_{a\gamma\gamma} = -1.92(4)\,\frac{\alpha_{\mathrm{em}}}{2\pi f_a}$ [Cortona *et al* '16]

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- Lattice setup:

introduce electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields so that $\mathbf{E}\cdot\mathbf{B} \neq 0$

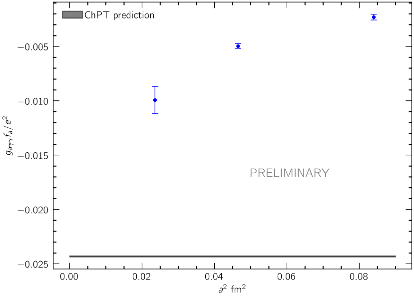
$$\rightarrow \quad \frac{T}{V}\left\langle Q_{\text{top}}\right\rangle = g^{\text{QCD}}_{a\gamma\gamma} f_a \mathbf{E}\cdot\mathbf{B}$$

(for homogeneous, static and sufficiently weak fields)

- Problem: $\mathbf{E}$-fields induce sign problem
  cure via analytic continuation to imaginary $\mathbf{E}$

First results in full lattice QCD

- first results for $g_{a\gamma\gamma}^{\mathrm{QCD}}$ at $T \approx 0$:



- result is negative – as expected from chiral perturbation theory
- still large lattice artifacts visible

→ need to implement improvement (zero mode reweighting) & include a finer lattice

work in progress ...

Conclusions

- computed C_{CSE} and C_{CME}
  - C_{CSE} changes rapidly around T_c
  - C_{CME} vanishes in equilibrium

  exact form of C_{CSE} might be important for chiral magnetic wave

- first study of QCD in non-uniform B-fields
  - new interplay of local deconfinement and chiral symmetry breaking

- $\mathbf{B}$-fields and axion physics
  - topological susceptibility: only mildly affected by $\mathbf{B}$-fields
  - axion-photon coupling: compute by including imaginary $\mathbf{E}$-fields still need to work on continuum limit