Inhomogeneous meson condensation

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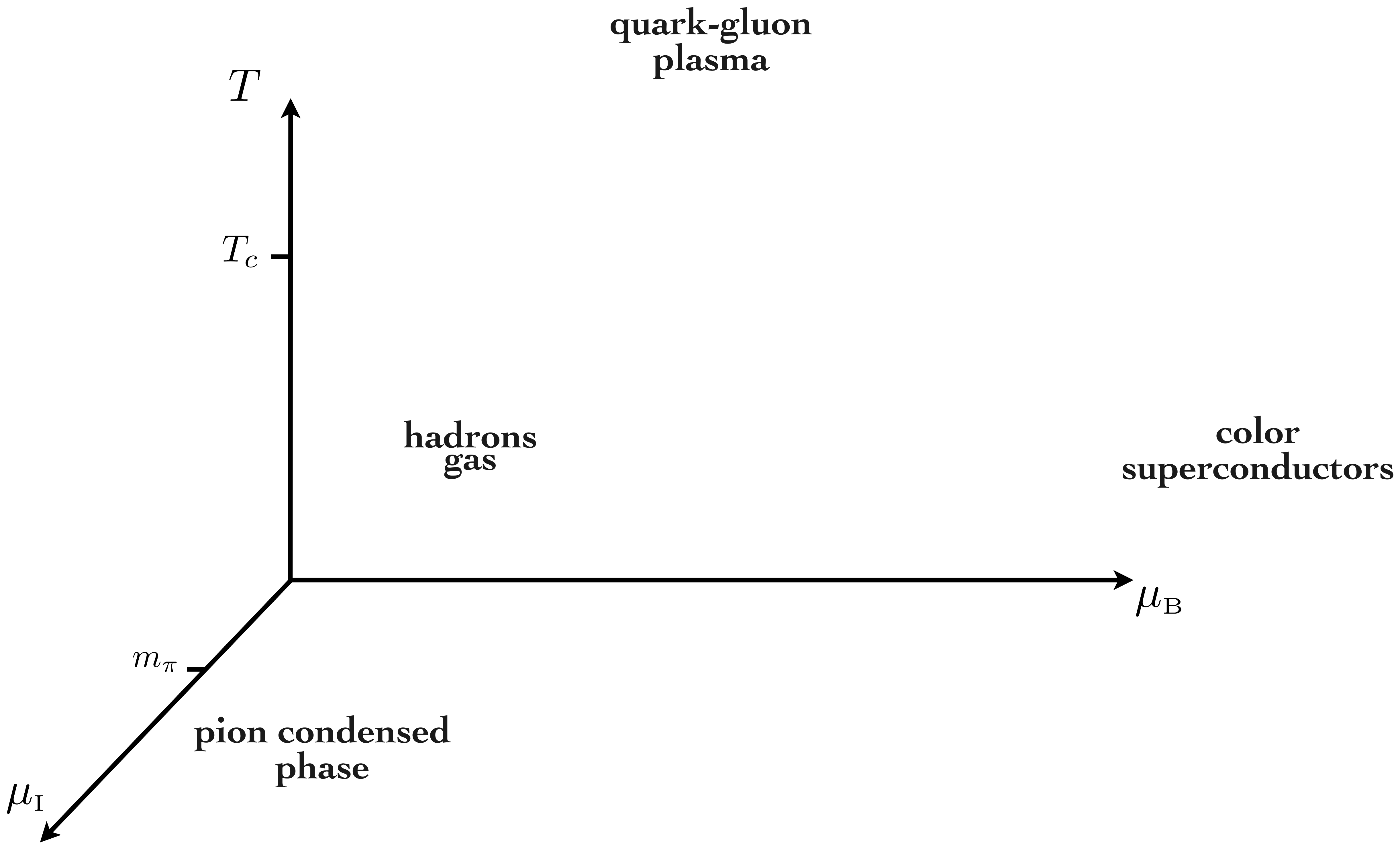
Outline

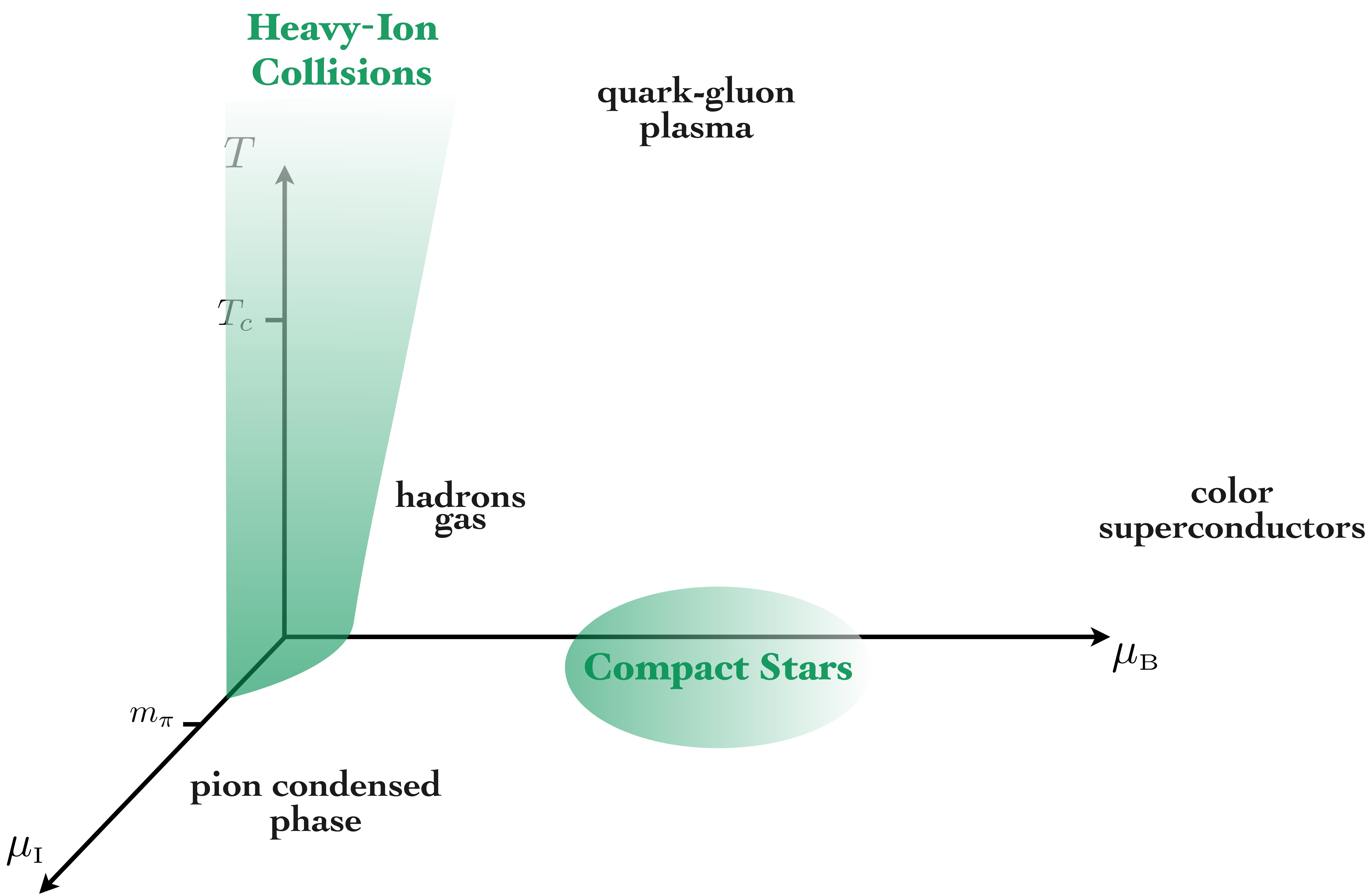
- **Phases of quark matter**
- **Meson condensation**
- **Chiral perturbation theory**
- **Supersolid of pions**
- **Outlook and conclusions**

Phases of quark matter

Phases of hadronic matter



Phases of hadronic matter



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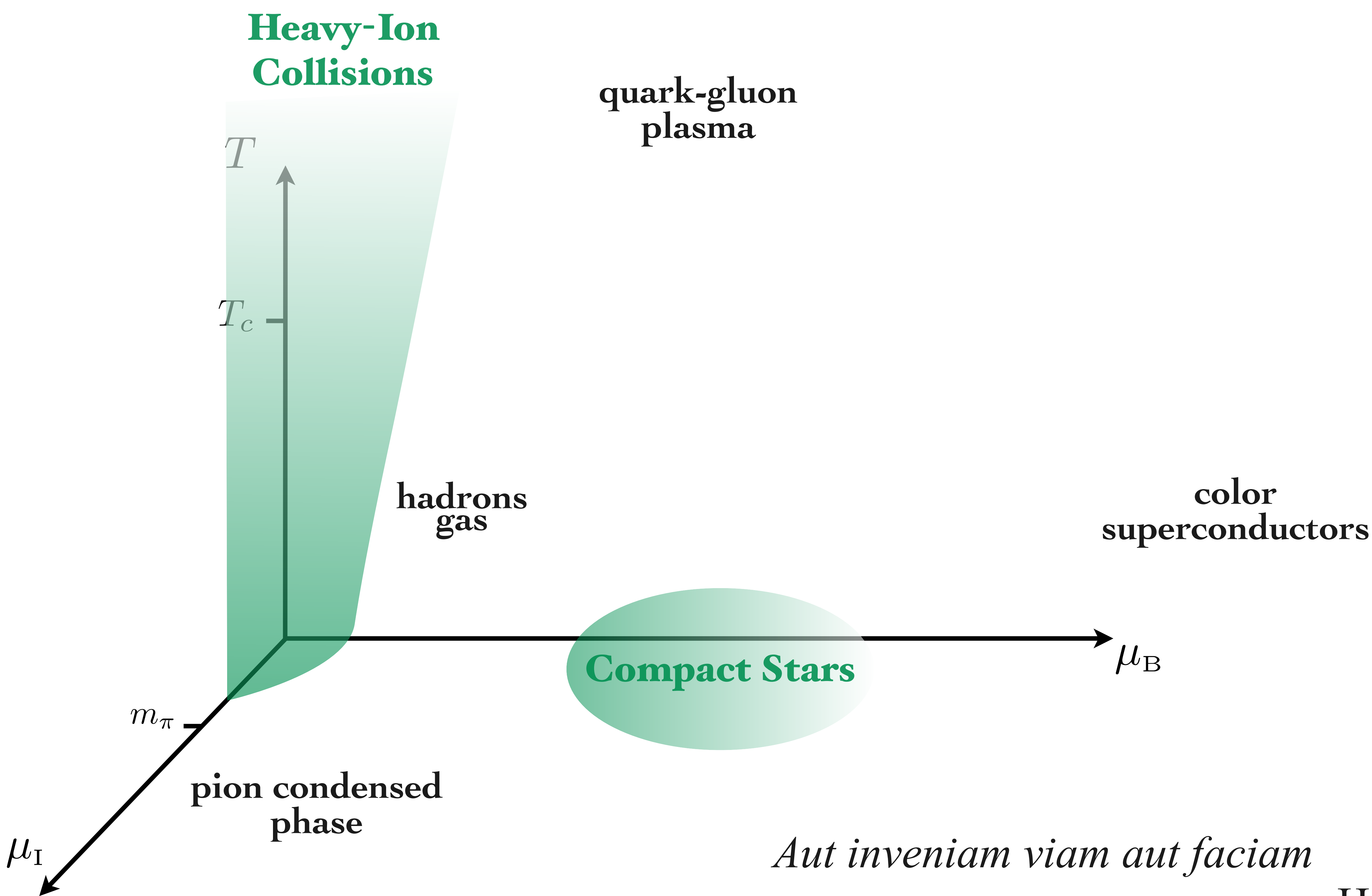

Aut inveniam viam aut faciam

Hannĭbăl

Quark condensates

In each phase different quark condensates are realized

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Hadron gas

chiral condensate

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Quark-gluon plasma
no condensate

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Quark-gluon plasma
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Each condensate **breaks** or **locks** some symmetries of QCD

Meson condensation

A.B. Migdal, Zh.Eksp.Teor.Fiz. 61 (1971) 2209-2224

R.F. Sawyer, Phys.Rev.Lett. 29 (1972) 382-385

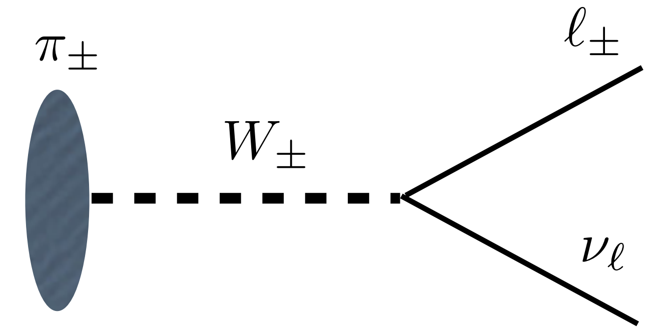
D.J. Scalapino, Phys.Rev.Lett. 29 (1972) 386-388

Review: MM, Particles 2 (2019) no.3, 411

In medium pions

Making the pion stable

pion decay in vacuum



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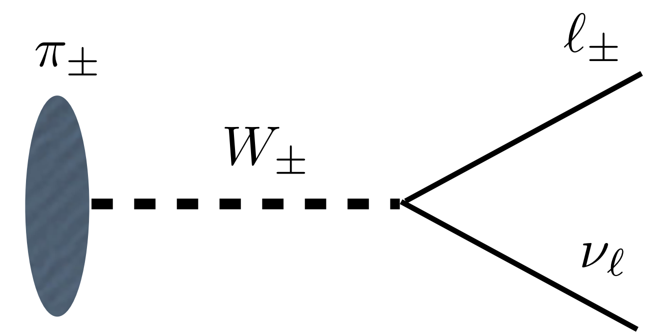

The pion decay can be Pauli blocked for a large lepton chemical potential

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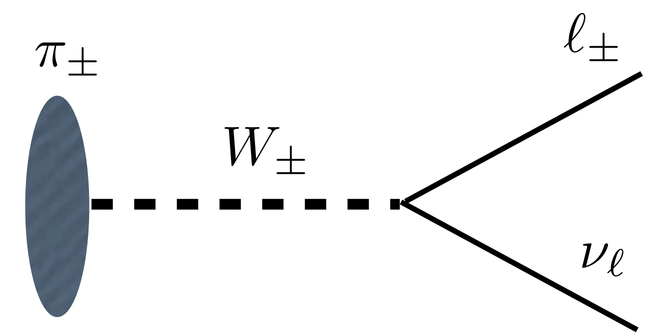

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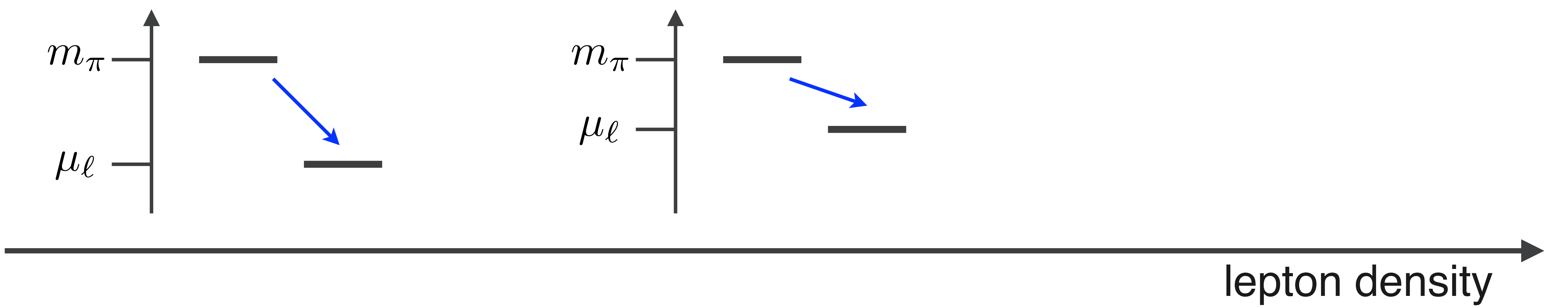

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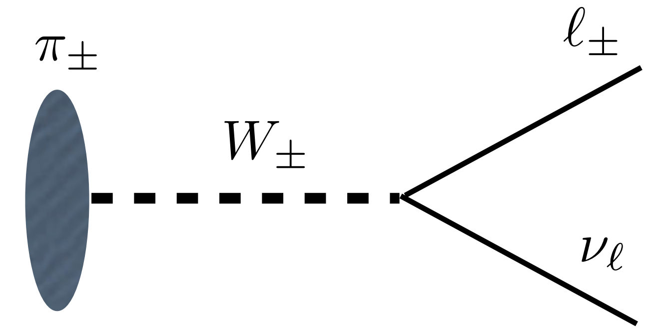

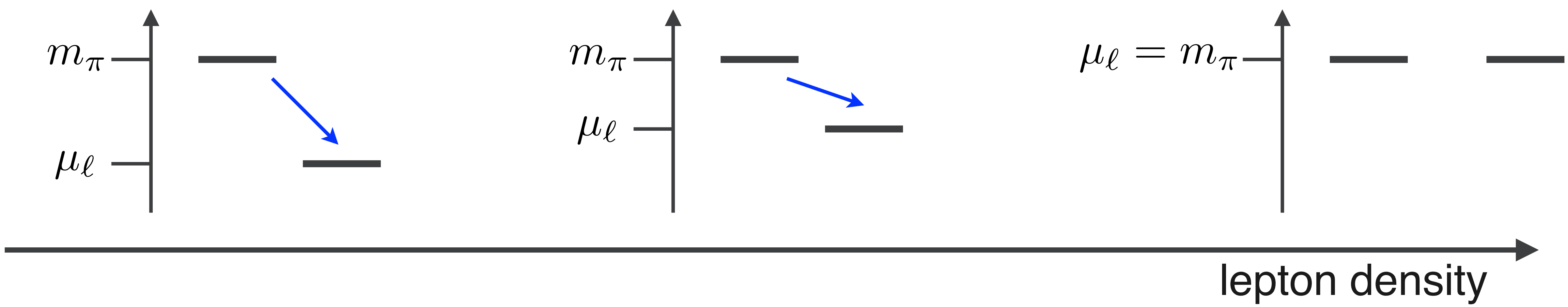

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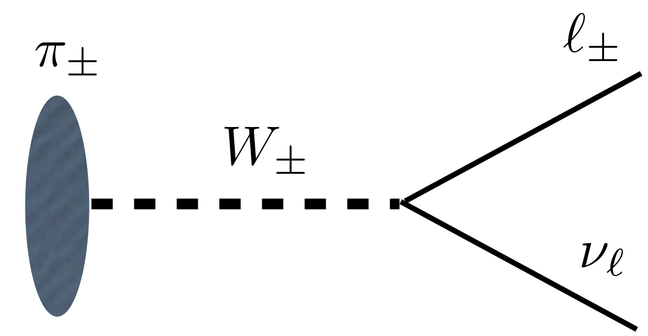
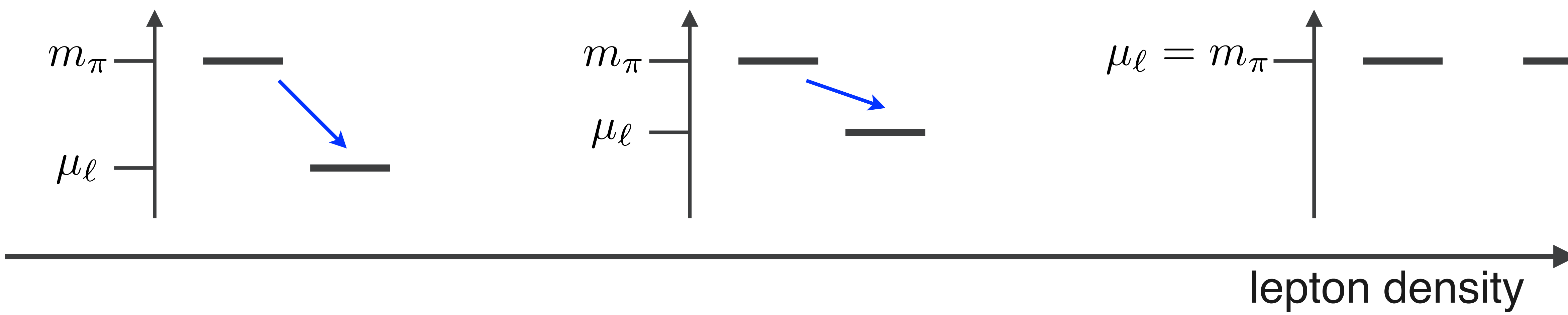




Energy spectrum splitting
Stark-like effect

$$E_{\pi^0} = \sqrt{m_\pi^2 + p^2}$$

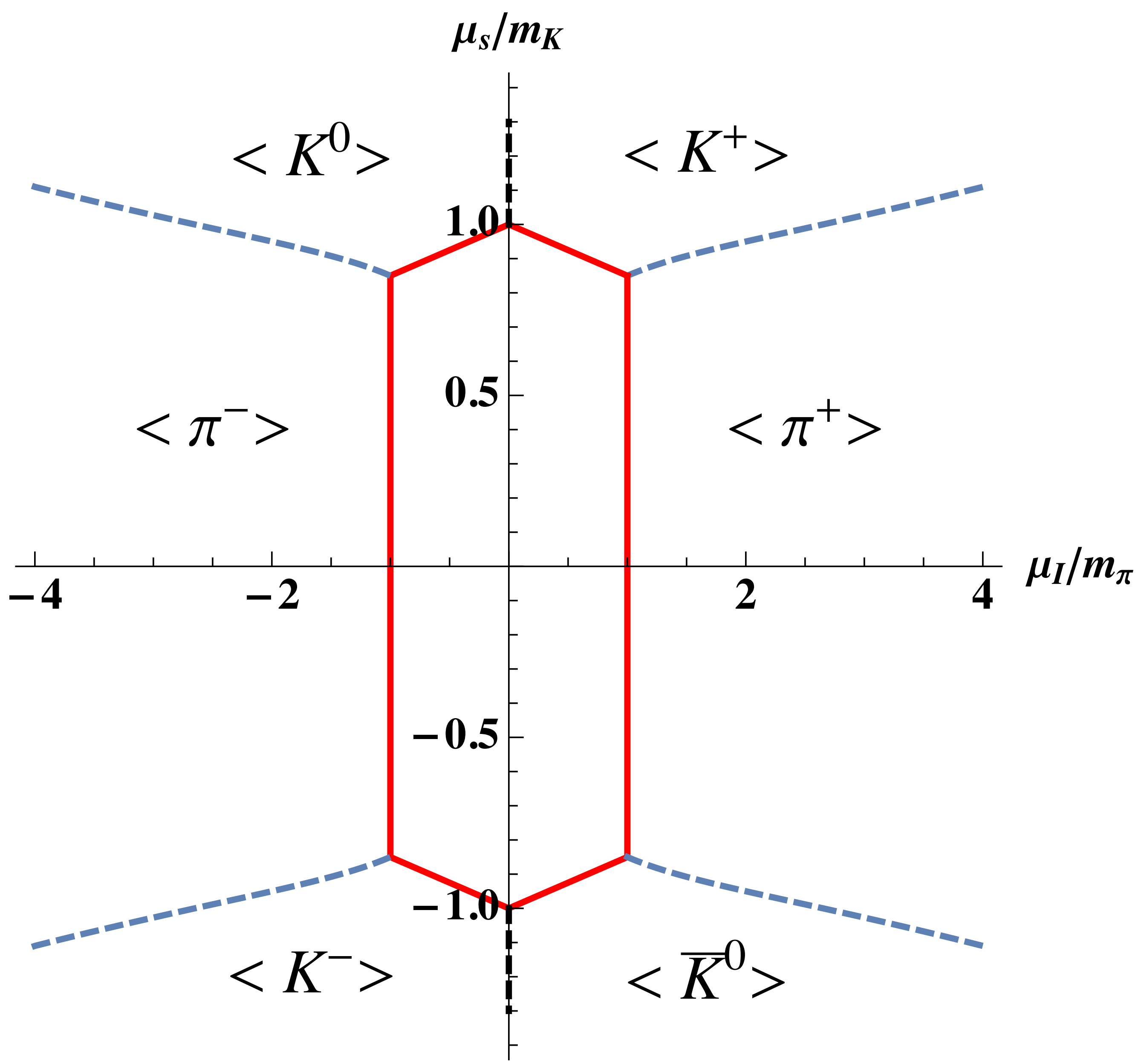
$$E_{\pi^-} = +\mu_I + \sqrt{m_\pi^2 + p^2}$$

$$E_{\pi^+} = -\mu_I + \sqrt{m_\pi^2 + p^2}$$

$$m_{\pi^+}^{\text{eff}} = m_\pi - \mu_I$$

→ At $\mu_I = m_\pi$ a massless mode appears:
pion condensation

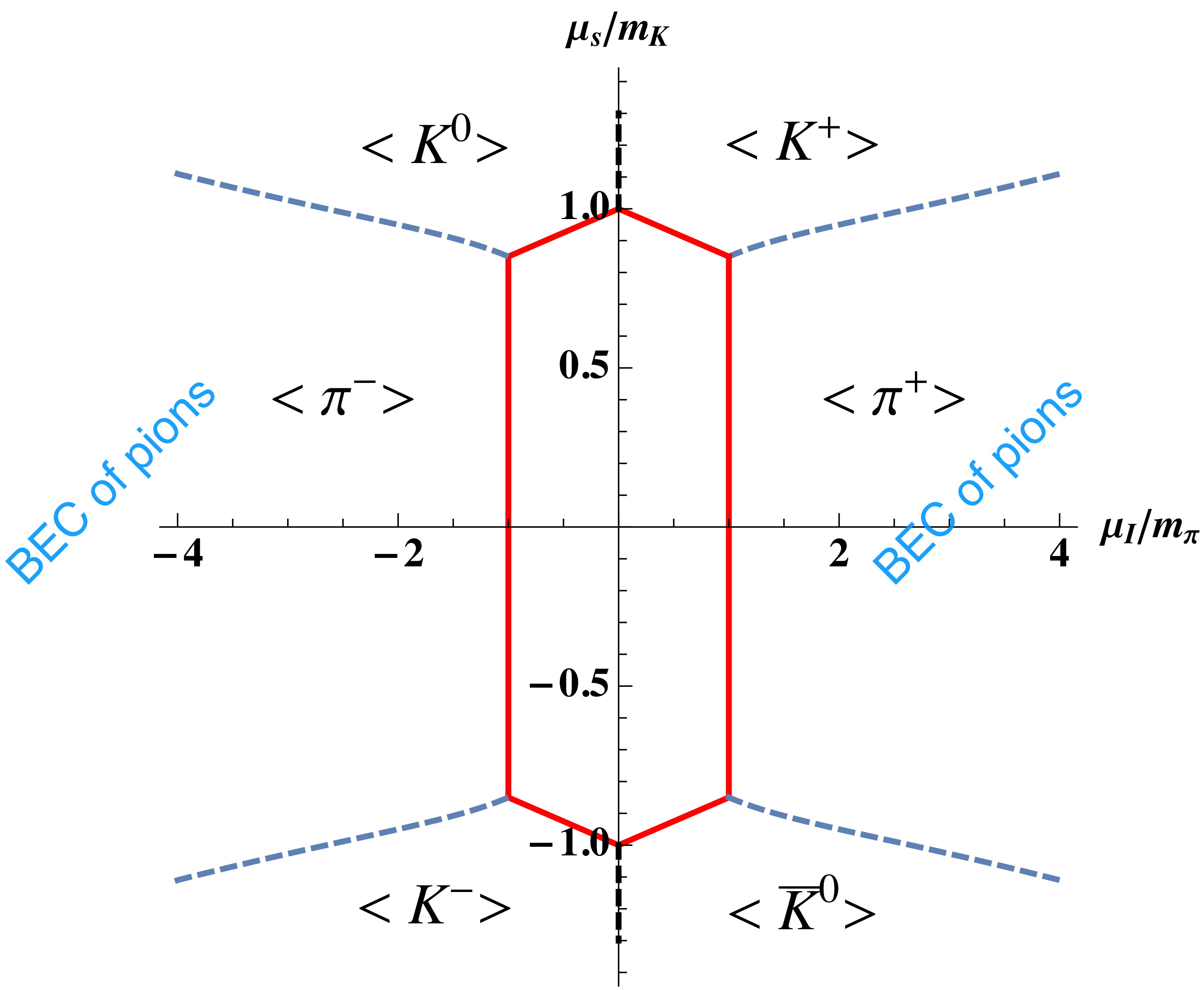
Phases driven by asymmetries



Dashed: first order
Solid: second order

Kogut and Toublan PhysRevD.64.034007

Phases driven by asymmetries



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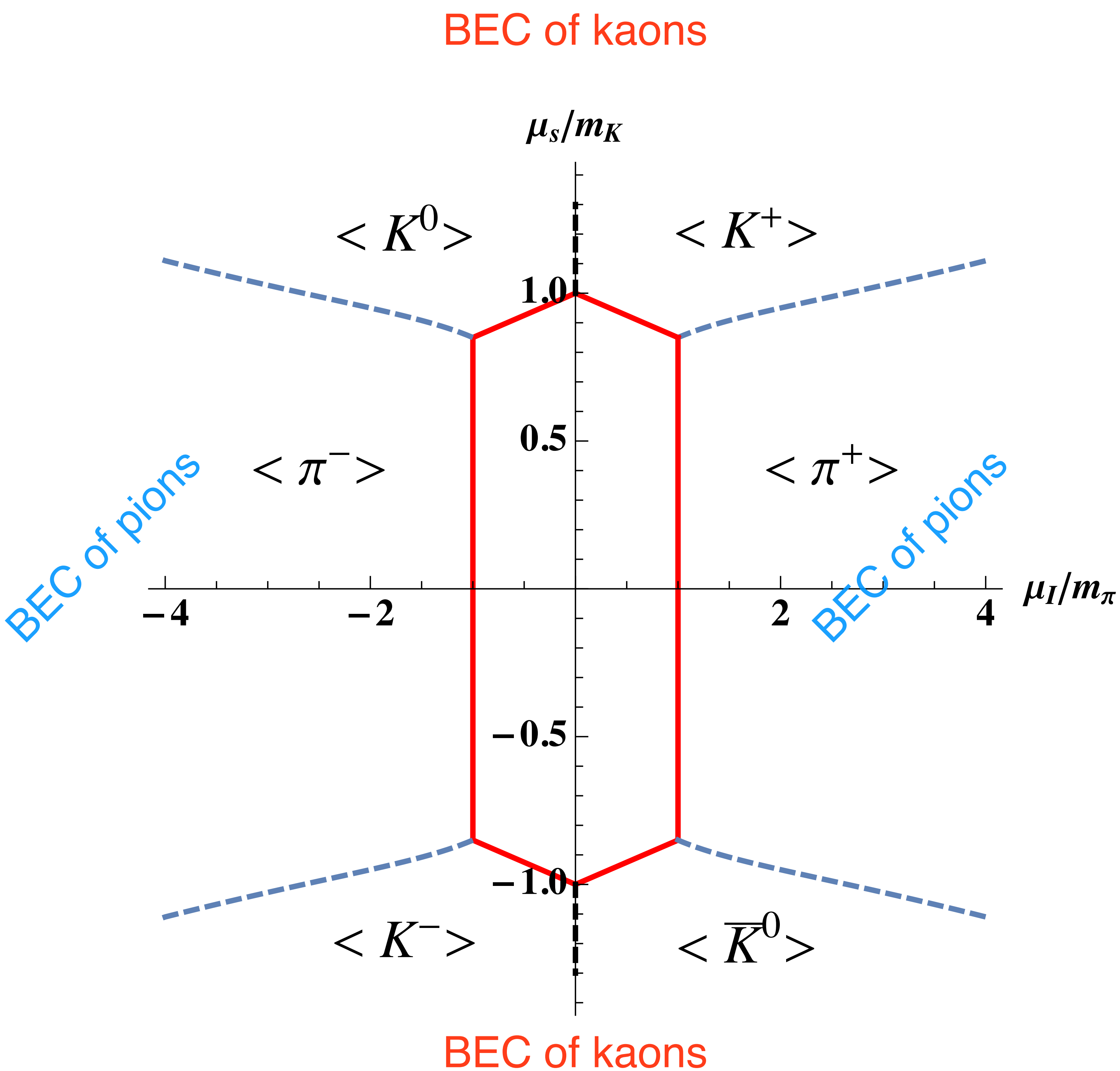

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Condensation in chiral perturbation theory (χPT)

χPT: realisation of hadronic matter preserving the global symmetries of QCD

Soft energy scales $\quad p \ll \Lambda_\chi \sim 1\,\mathrm{GeV}$

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Recipe

Variationally derive the **nonperturbative vacuum** and **expand** around that vacuum for small momenta.

Since we expand, we have a **control parameter**

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Since we expand, we have a **control parameter**

No baryons and vector mesons included

$$|\mu_B| \lesssim 940 \text{ MeV} \qquad |\mu_I| \lesssim 770 \text{ MeV}$$

Leading order pion Lagrangian

The $\mathcal{O}(p^2)$ Lagrangian density for pseudoscalar mesons

$$\mathscr{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) + \mathrm{Tr}(\Sigma^\dagger M + M^\dagger \Sigma)$$

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low energy constants
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For $SU(2)$ $\quad \Sigma = e^{i\alpha\cdot\sigma} = \mathbf{1}_2 \cos\alpha + i\sigma\cdot\mathbf{n}\sin\alpha$

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"Angular" fields: $n^1 = \sin\Theta\cos\Phi\ ,\quad n^2 = \sin\Theta\sin\Phi\ ,\quad n^3 = \cos\Theta$

"Radial" field: α

Homogeneous ground state

A general $\mathrm{SU}(2)$ static and homogeneous vev

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

fields become variational parameters

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Static Lagrangian

$$\mathscr{L}_0(\alpha, n_3) = f_\pi^2 m_\pi^2 \cos\alpha + \frac{f_\pi^2}{2}\mu_I^2 \sin^2\alpha(1 - n_3^2)$$

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Maximising the Lagrangian

for $\mu_I < m_\pi$	$\cos\alpha = 1$	$\mathcal{L}_0$ independent of $\boldsymbol{n}$
for $\mu_I > m_\pi$	$\cos\alpha_\pi = m_\pi^2/\mu_I^2$	$n_3 = 0$ residual $O(2)$ symmetry

BEC of pions

Rotated condensates

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \propto \cos\alpha$$
$$\langle \bar{d}\gamma_5 u + \text{h.c.} \rangle \propto \sin\alpha$$

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$$P = \frac{f_\pi^2 m_\pi^2}{2}\gamma^2 \left(1 - \frac{1}{\gamma^2}\right)^2$$

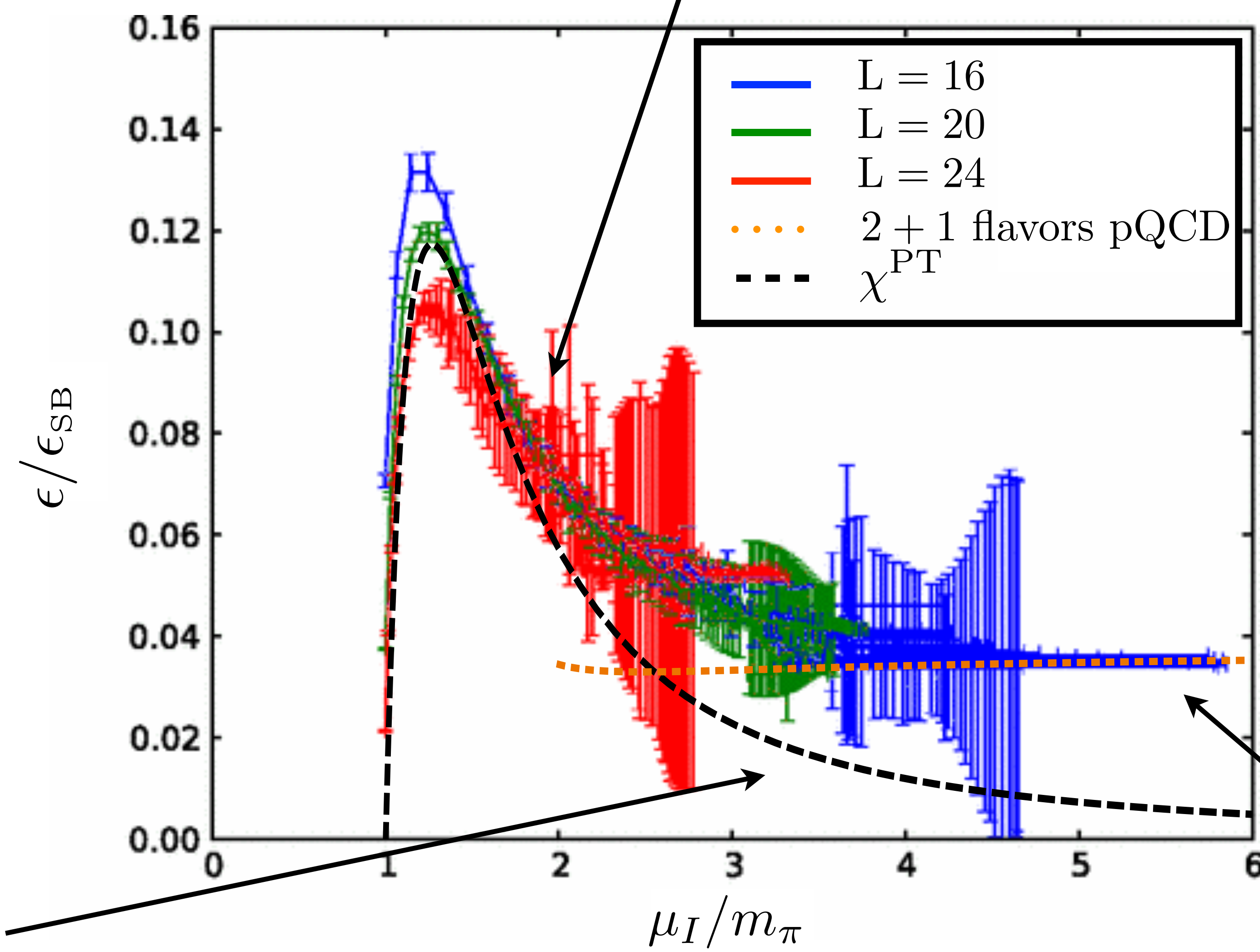
Ground state occupation number

$$n_I = f_\pi^2 m_\pi \gamma \left(1 - \frac{1}{\gamma^4}\right)$$

Energy density

Lattice QCD simulations

W. Detmold, K. Orginos, and Z. Shi,
Phys. Rev. D86, 054507 (2012)



$$\epsilon_{SB} = \frac{N_c N_f}{4\pi^2}\mu_I^4$$

factor $\sim \frac{1}{16}$ missing

χPT

S. Carignano, A. Mammarella, MM
Phys.Rev. D93 (2016) no.5, 051503

pQCD

T. Graf, et al.
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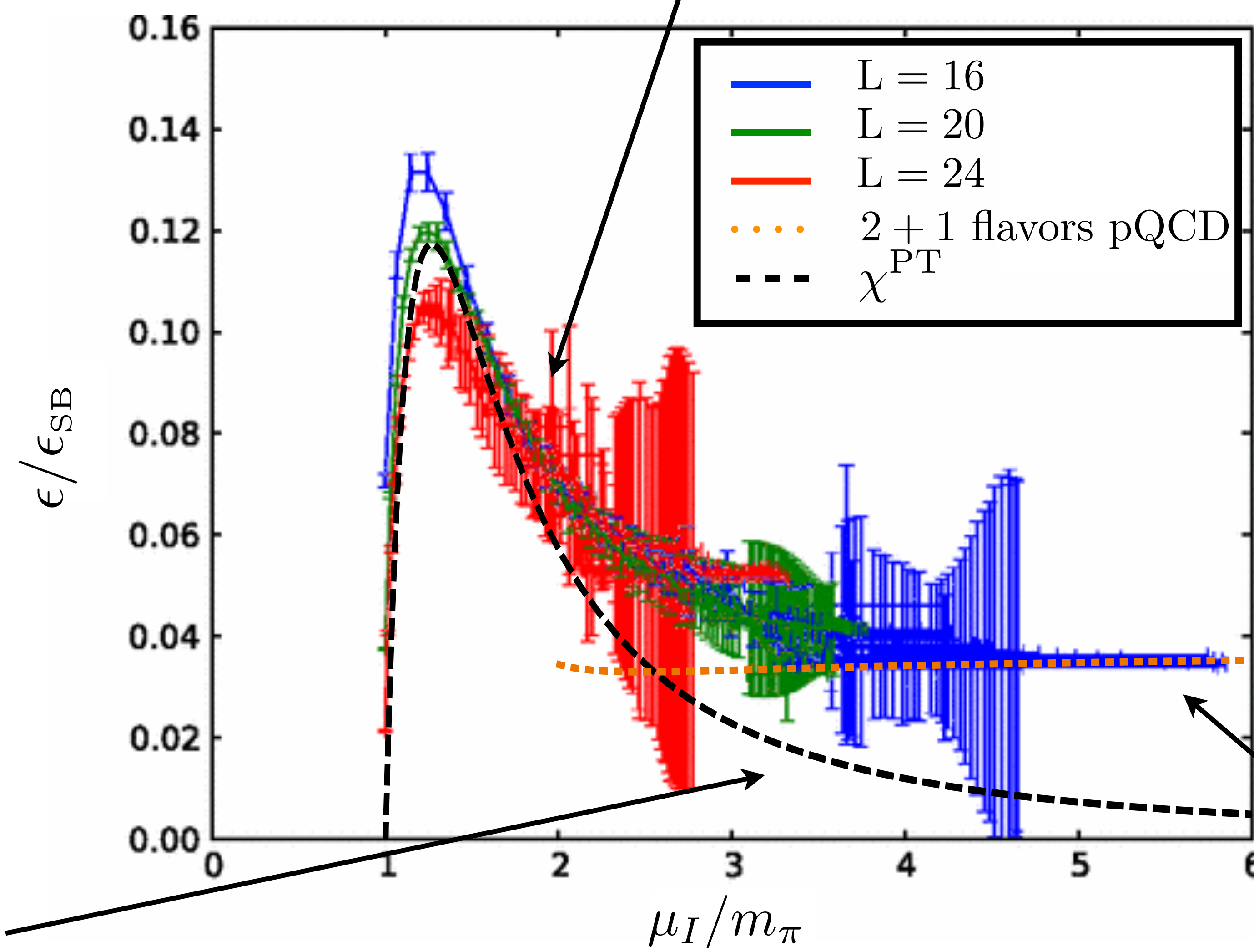

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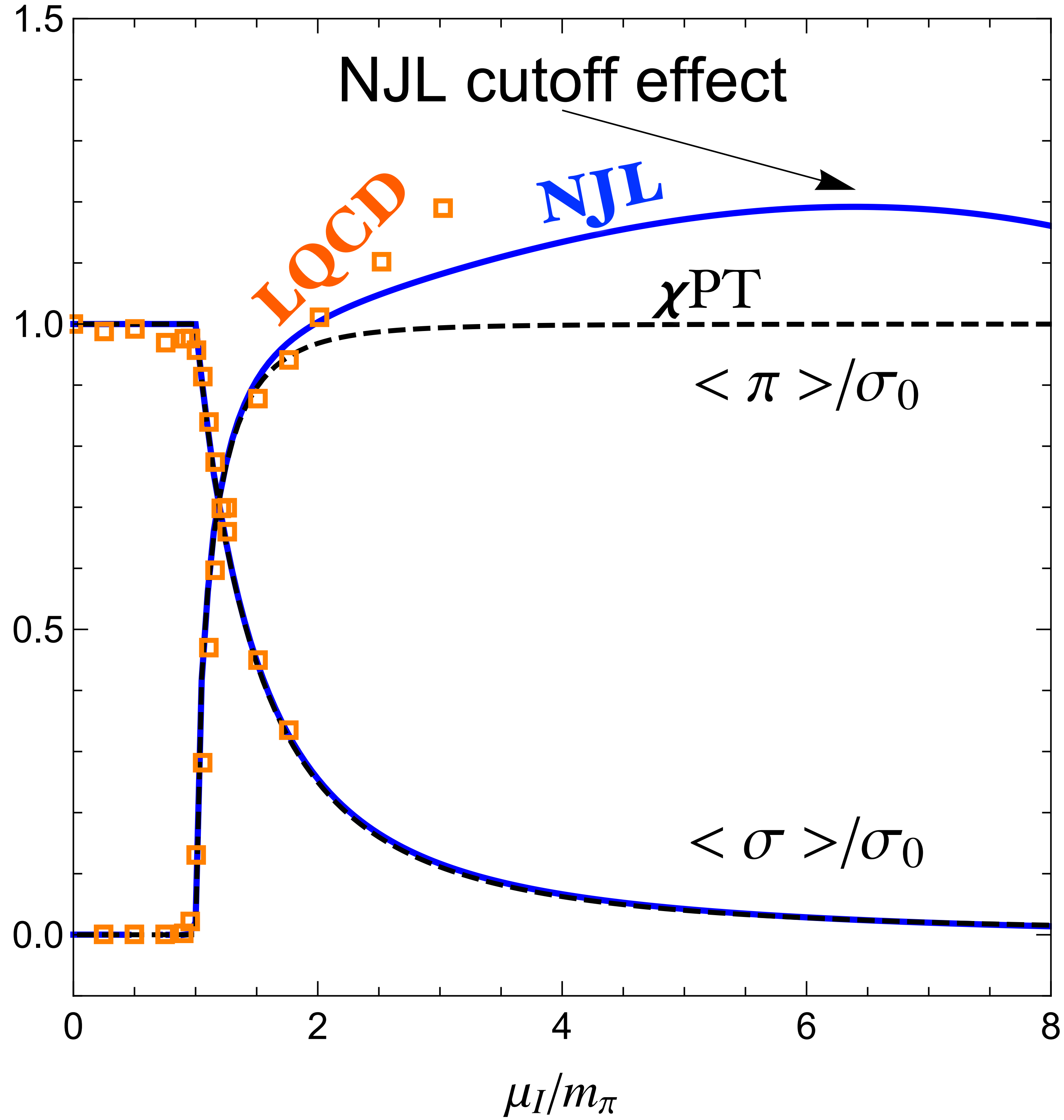
χPT gives an ANALYTIC expression for the peak

$$\mu_{I,\mathrm{LQCD}}^{\mathrm{peak}} = \{1.20, 1.25, 1.275\}m_\pi \qquad \mu_{I,\chi\mathrm{PT}}^{\mathrm{peak}} = (\sqrt{13}-2)^{1/2} m_\pi \simeq 1.276\, m_\pi$$

Pion and chiral condensates



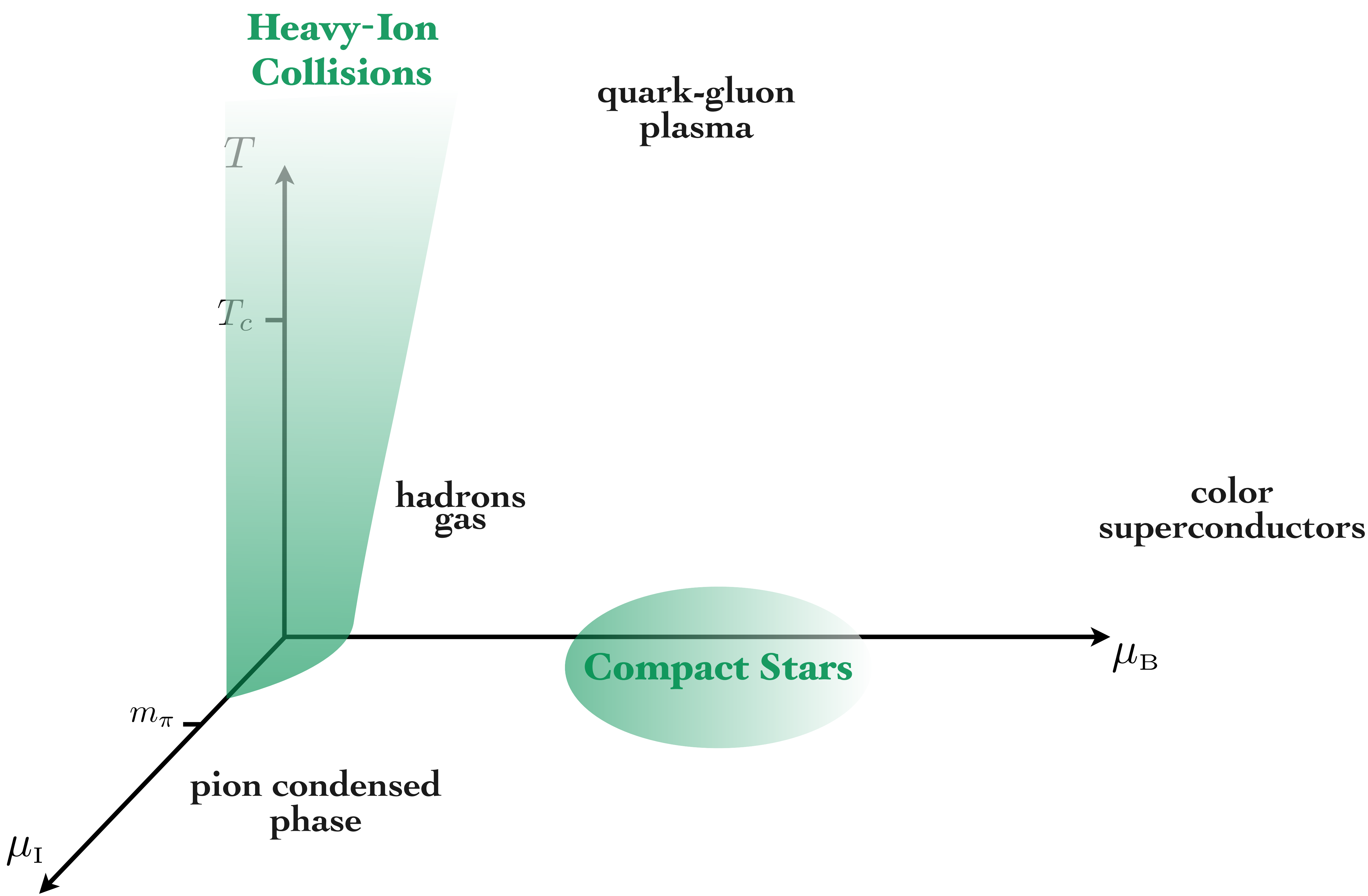
The three methods agree where they are supposed to work

More work to be done at large isospin

Brandt+ Phys. Rev. 2018, D 97, 054514

M.M. Particles 2 (2019) no.3, 411-443

Phases of hadronic matter



Supersolid of pions

F. Canfora, S. Carignano, M. Lagos, MM, A. Vera, Phys.Rev.D 103 (2021) 7, 076003

Variational parameters promoted to classical fields

Homogeneous phase $\bar{\boldsymbol{\Sigma}} = \mathbf{1_2} \cos\alpha + \boldsymbol{\sigma} \cdot \mathbf{n} \sin\alpha$

$$n^1 = \sin\Theta \cos\Phi \ , \quad n^2 = \sin\Theta \sin\Phi \ , \quad n^3 = \cos\Theta$$

Variational parameters α, Φ and Θ

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Classical fields α, Φ and Θ

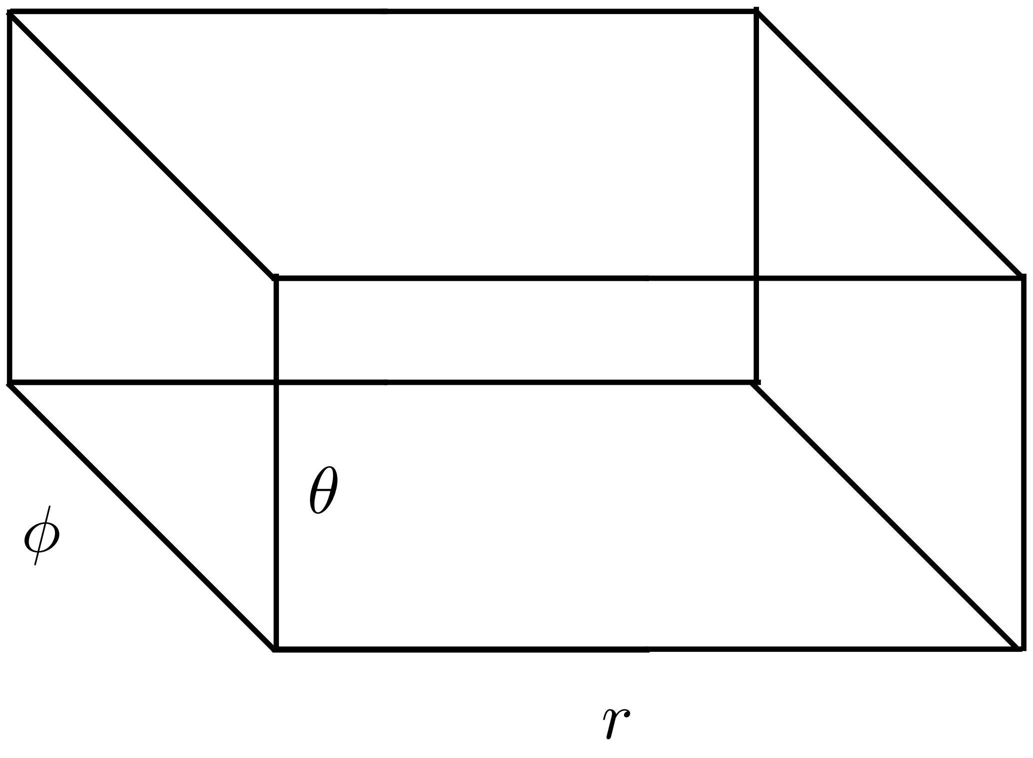
Pions in a box

Metric

$$ds^2 = dt^2 - \ell^2 \left(dr^2 + d\theta^2 + d\phi^2\right)$$

$$\ell = \frac{b}{m_\pi} \qquad 0 \le r \le 2\pi \ , \quad 0 \le \theta \le \pi \ , \quad 0 \le \phi \le 2\pi$$

$$V = 4\pi^3 \ell^3$$



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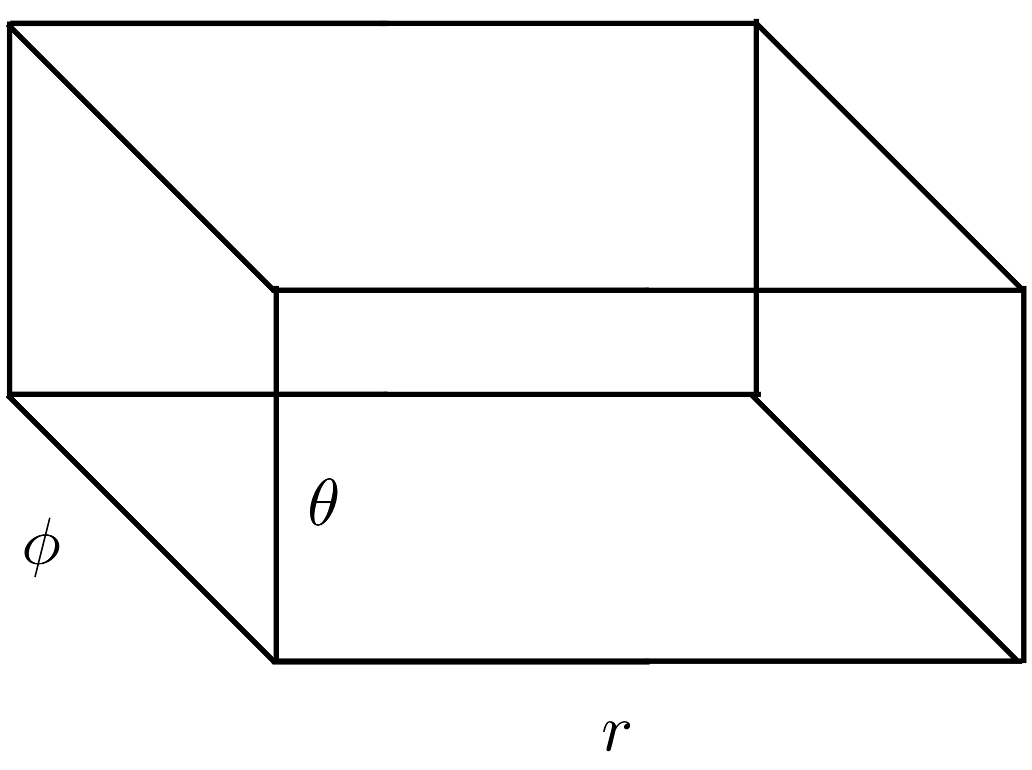

Boundary conditions

$$\Sigma(0,\theta,\phi) = \Sigma(2\pi,\theta,\phi)$$
$$n(r,0,\phi) = -n(r,\pi,\phi)$$
$$n(r,\theta,0) = n(r,\theta,2\pi)$$

different BCs can be easily implemented

Classical equations of motion

Classical fields α, Φ and Θ **obey Euler-Lagrange equations**

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$$\partial_\mu \partial^\mu \Phi = -(\partial_\mu \Phi - \mu_I \delta_{\mu_0}) \partial^\mu (\log(\sin^2 \alpha \sin^2 \Theta))$$

$$\partial_\mu \partial^\mu \Theta = -2 \cot \alpha \, \partial^\mu \Theta \partial_\mu \alpha + \frac{\sin 2\Theta}{2} K \,,$$

$$\partial_\mu \partial^\mu \alpha = -m_\pi^2 \sin \alpha + \frac{\sin(2\alpha)}{2} (\partial_\mu \Theta \partial^\mu \Theta + K \sin^2 \Theta)$$

where $K = (\partial_\mu \Phi - \mu_I \delta_{\mu_0})(\partial^\mu \Phi - \mu_I \delta^{\mu_0})$

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Hard to solve in general….

Simplifying assumptions

$$\Phi \equiv \Phi(t, \phi) \qquad \Theta \equiv \Theta(\theta) \qquad \alpha \equiv \alpha(r)$$

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Solutions $$\Phi = \frac{a}{\ell}t - p\phi + \Phi_0 \ , \ \Theta = q\theta + \Theta_0 \qquad p, q \in \mathbb{Z} \text{ with } q \text{ odd}$$

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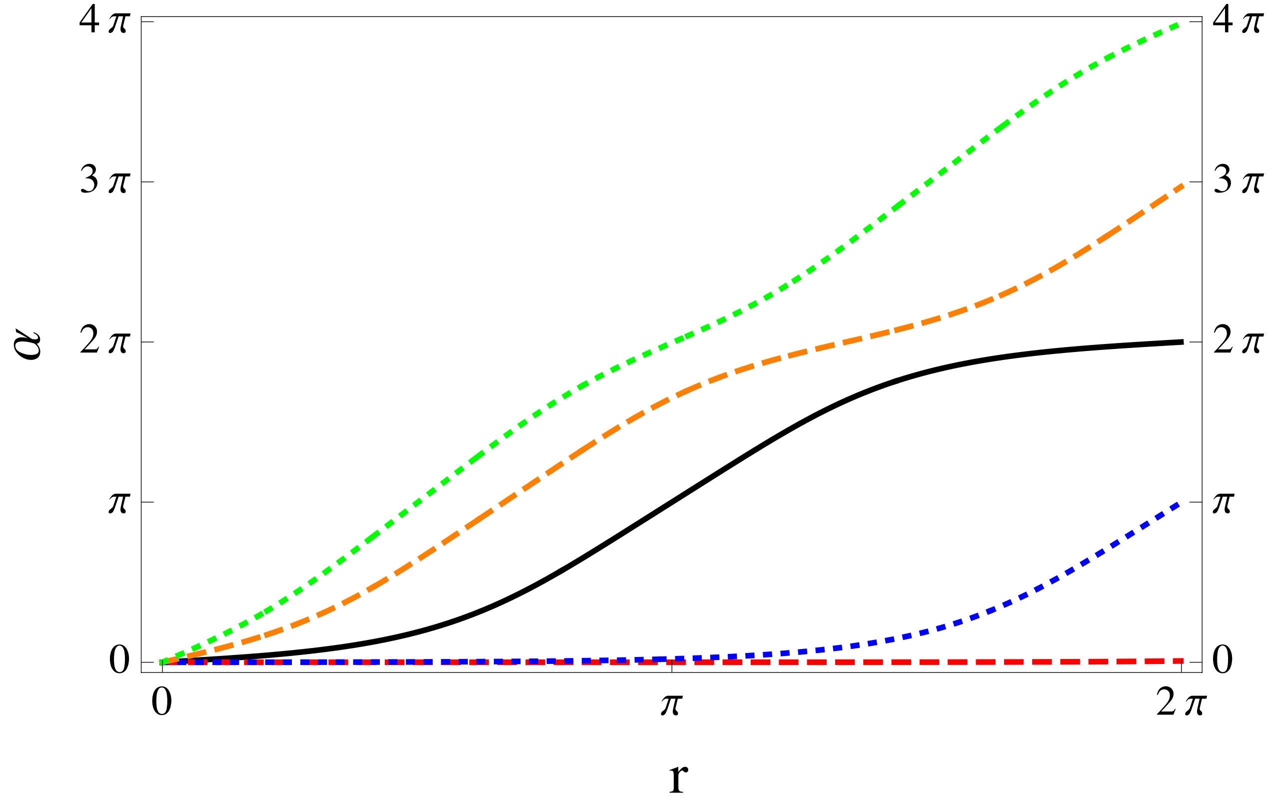
When $\mu_I = -p/\ell$ the Φ field becomes static and $K = 0$

Radial field behavior

$$\frac{\partial^2 \alpha}{\partial r^2} = m_\pi^2 \ell^2 \sin\alpha + \frac{q^2}{2}\sin(2\alpha) \qquad \alpha(0) = 0 \qquad \text{and} \qquad \alpha(2\pi) = n\pi$$



n is the winding number

Topological charge

$$B = \frac{\ell^3}{24\pi^2}\int_S dr d\theta d\phi \; \rho_{\rm m}$$

where $\rho_{\rm m} = \epsilon^{ijk}{\rm Tr}\left\{\left(\Sigma^{-1}\partial_i\Sigma\right)\left(\Sigma^{-1}\partial_j\Sigma\right)\left(\Sigma^{-1}\partial_k\Sigma\right)\right\}$

for the proposed solution $\rho_{\rm m} = \frac{3pq}{\ell^3}\sin(q\theta)\frac{\partial}{\partial r}\left(\sin(2\alpha) - 2\alpha\right),$

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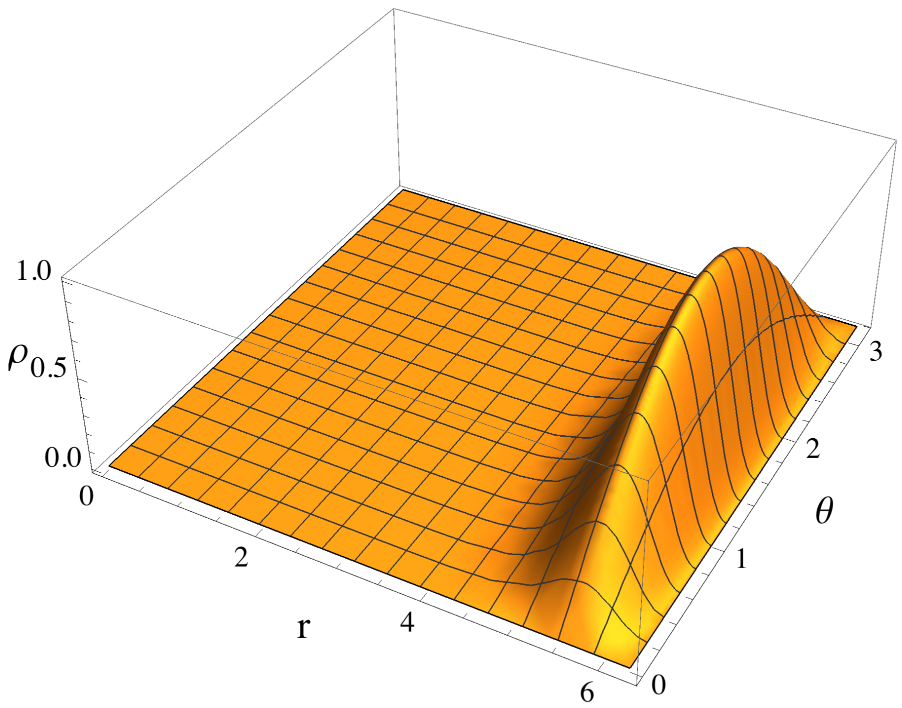
The topological charge protects the soliton from decay

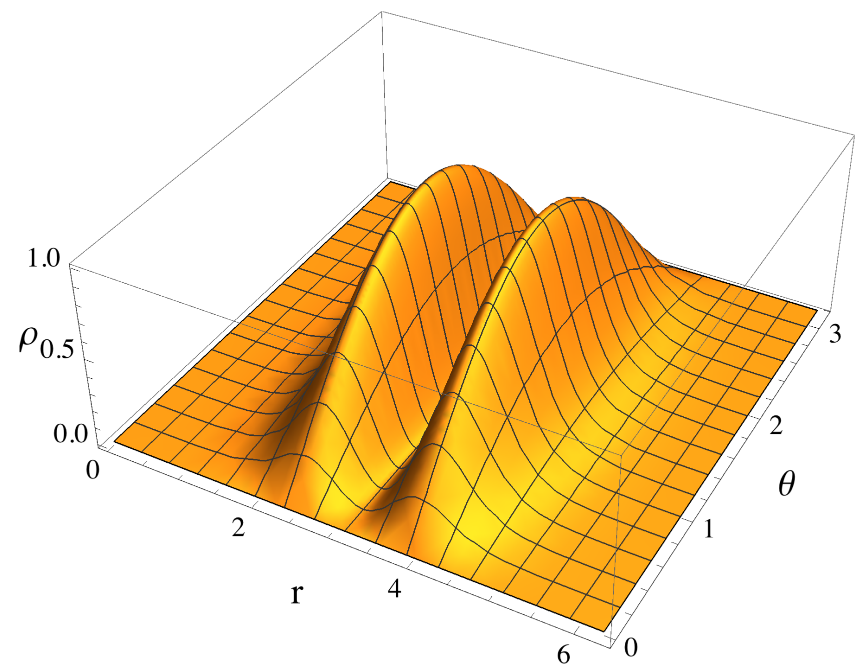
Topological charge

The topological charge depends on the boundary conditions.

$B = 1$



$B = 2$



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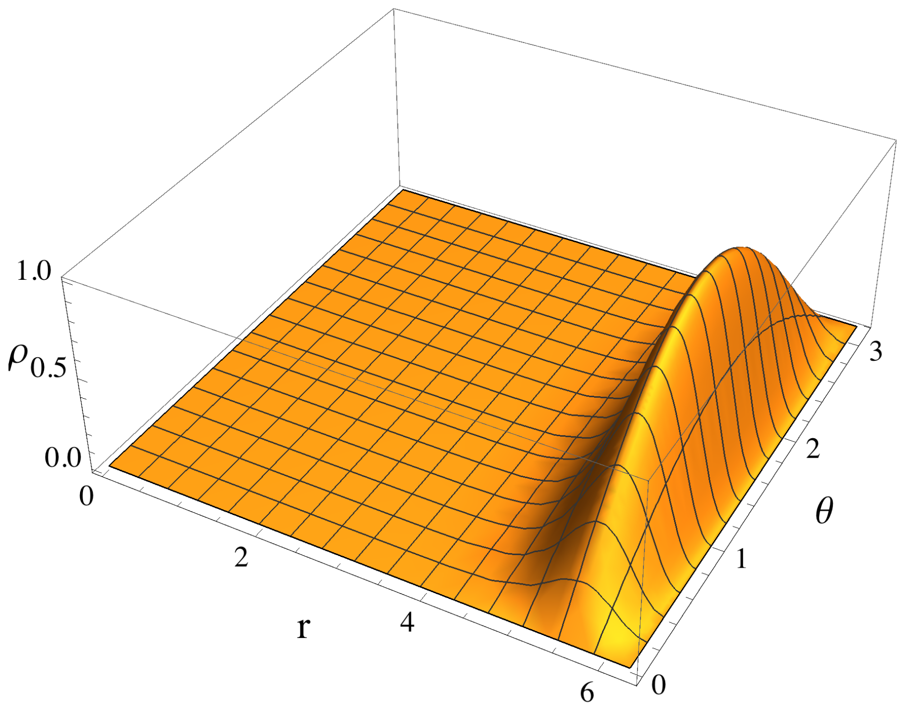
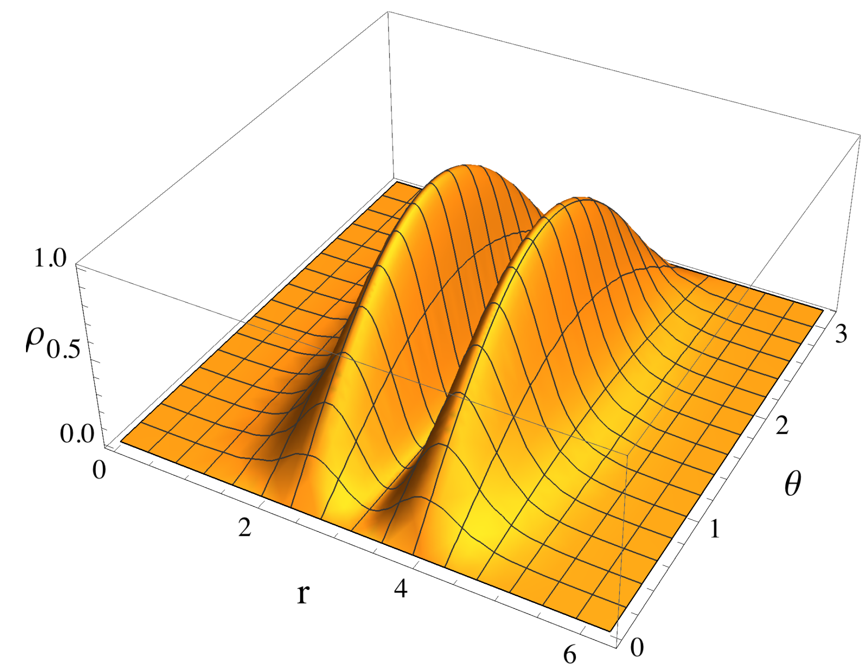


$B = 2$



Periodic structure of baryons in a superfluid of charged pions: a supersolid-like structure
We have to prove that it is rigid.

Outlook

- Add fluctuations on the top of the background
- Do pions mediate the interaction between solitons?
- Study of shear waves
- Numerical solutions or more general modulations
- Add vector bosons

Conclusions

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- The meson condensed phase is a portal to QCD

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Conclusions

- The meson condensed phase is a portal to QCD

- It may be realized in compact stars or in exotic stars

- Inhomogeneous phases can be linked to baryons

Thanks for your attention!

massimo@lngs.infn.it

Backup slides

Alternative descriptions

Why is the theory so complicated?
Pions are no more charge conjugate fields, they mix etc..

At the lowest order in derivatives and close to the phase transition mapping to a Gross-Pitaevskii Lagrangian

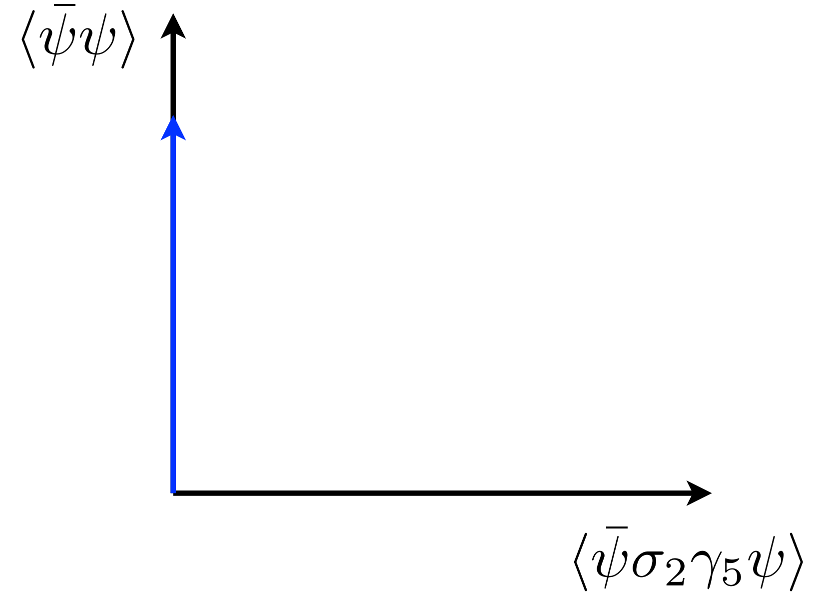
$$\mathcal{L}_{\rm GP} = f_\pi^2 m_\pi^2 + i\psi^* \partial_0 \psi + \mu_{\rm eff}\, \psi^* \psi - \frac{g}{2} |\psi^* \psi|^2 + \psi^* \frac{\nabla^2}{2M} \psi$$

$$\mu_{\rm eff} = \frac{\mu_I^2 - m_\pi^2}{2\mu_I}\,, \qquad g = \frac{4\mu_I^2 - m_\pi^2}{12 f_\pi^2 \mu_I^2}\,, \qquad M = \mu_I$$

S. Carignano, L. Lepori, G. Pagliaroli, A. Mammarella and M.M Eur.Phys.J. A53 (2017) no.2, 35

Depicting the pion condensation

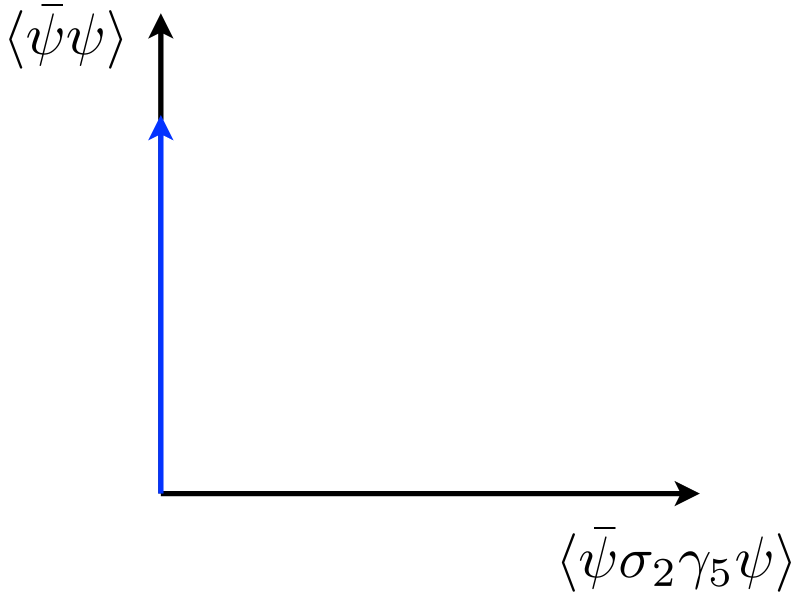
$\mu_I = 0$



Depicting the pion condensation

$$\mu_I = 0$$

$$\mu_I < m_\pi$$



Depicting the pion condensation

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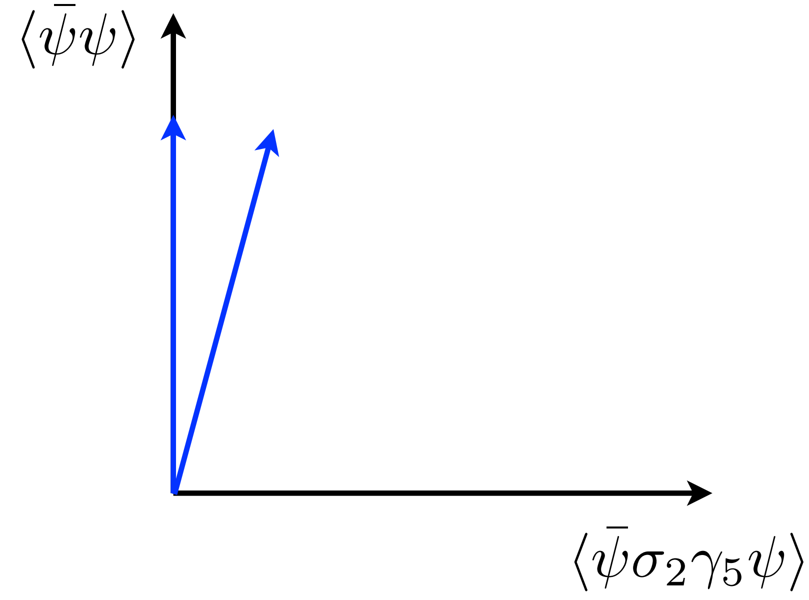

Depicting the pion condensation

$$\mu_I = 0$$

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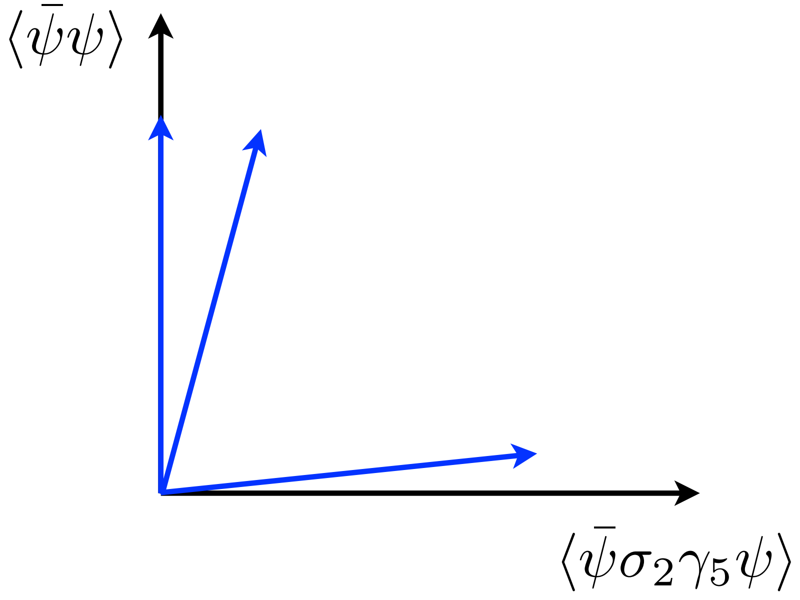
$$\mu_I > m_\pi$$

$$\mu_I \gg m_\pi$$



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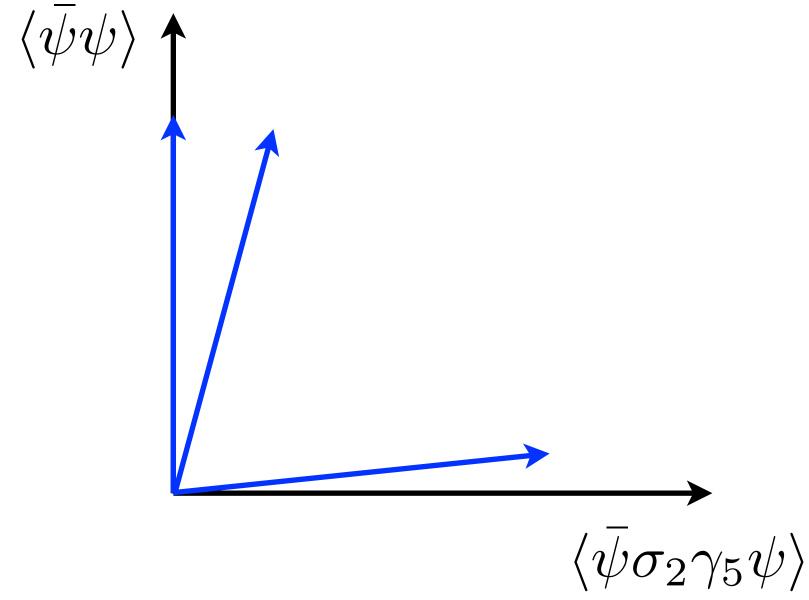

The condensate is "rotated"

Scalar condensate $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle \propto \cos\alpha$

Pseudo scalar condensate $\langle\bar{d}\gamma_5 u + \text{h.c.}\rangle \propto \sin\alpha$

Symmetry breaking path

massless quarks

$$\psi_L \to U_L \psi_L$$
$$\psi_R \to U_R \psi_R$$

$$\underbrace{SU(3)_L \times SU(3)_R}_{\supset\, U(1)_Q} \times U(1)_B$$

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$\langle \bar{\psi}\psi \rangle \neq 0$

Spontaneous chiral symmetry breaking

invariant under locked chiral rotations $U_L = U_R$

$$\underbrace{SU(2)_I \times U(1)_Y}_{\supset U(1)_Q} \times U(1)_B$$

Meson octet (Pseudo) Nambu-Goldstone bosons (massive quarks)

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(Pseudo) Nambu-Goldstone bosons
(massive quarks)

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$\mu_s \neq 0$

Explicit symmetry breaking

“normal phase” symmetry

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(no mass degeneracy)

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“normal phase” symmetry

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Meson octet
(no mass degeneracy)

Spontaneous phase locking

$\mu_I > m_\pi$

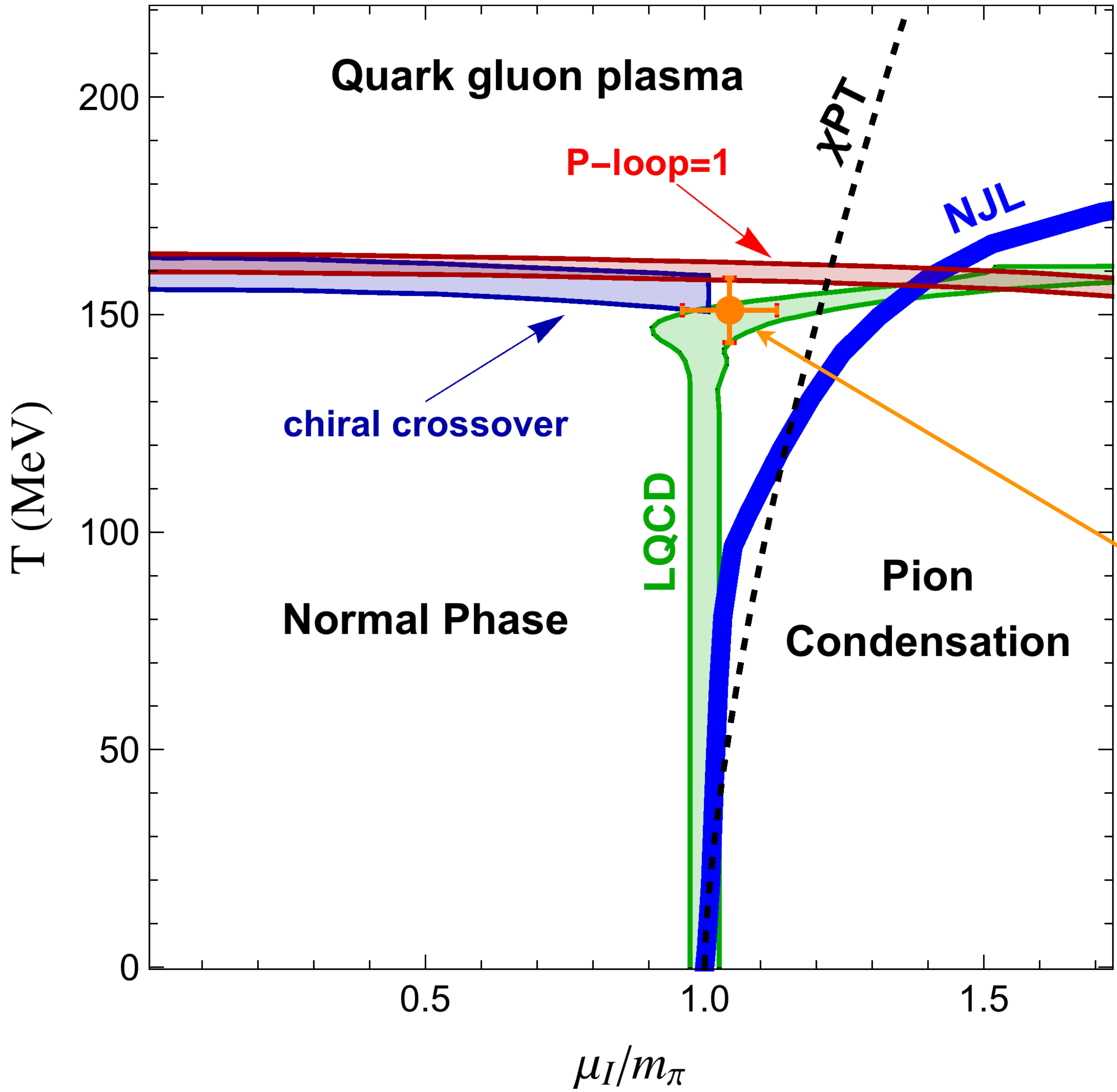
or

$\mu_s > m_k - \frac{m_\pi}{2}$

$$\underbrace{U(1) \times U(1)_B}_{\not\supset U(1)_Q}$$

One NGB

Phase diagram



Qualitative similar behavior at low T

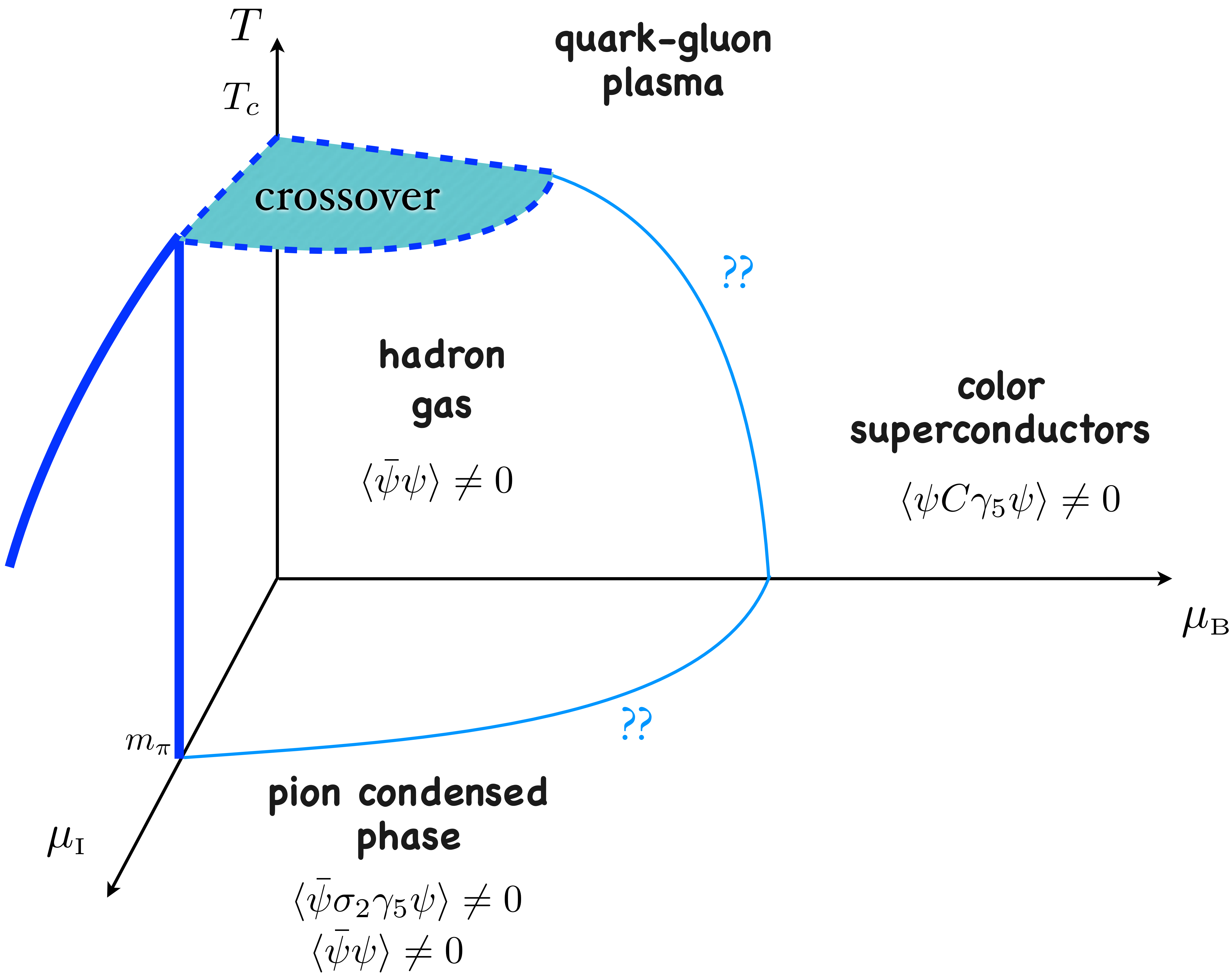
Not clear the origin of the discrepancies

(pseudo) tricritical point

Brandt+ Phys. Rev. 2018, D 97, 054514

M.M. Particles 2 (2019) no.3, 411-443

Revisiting the QCD phase diagram



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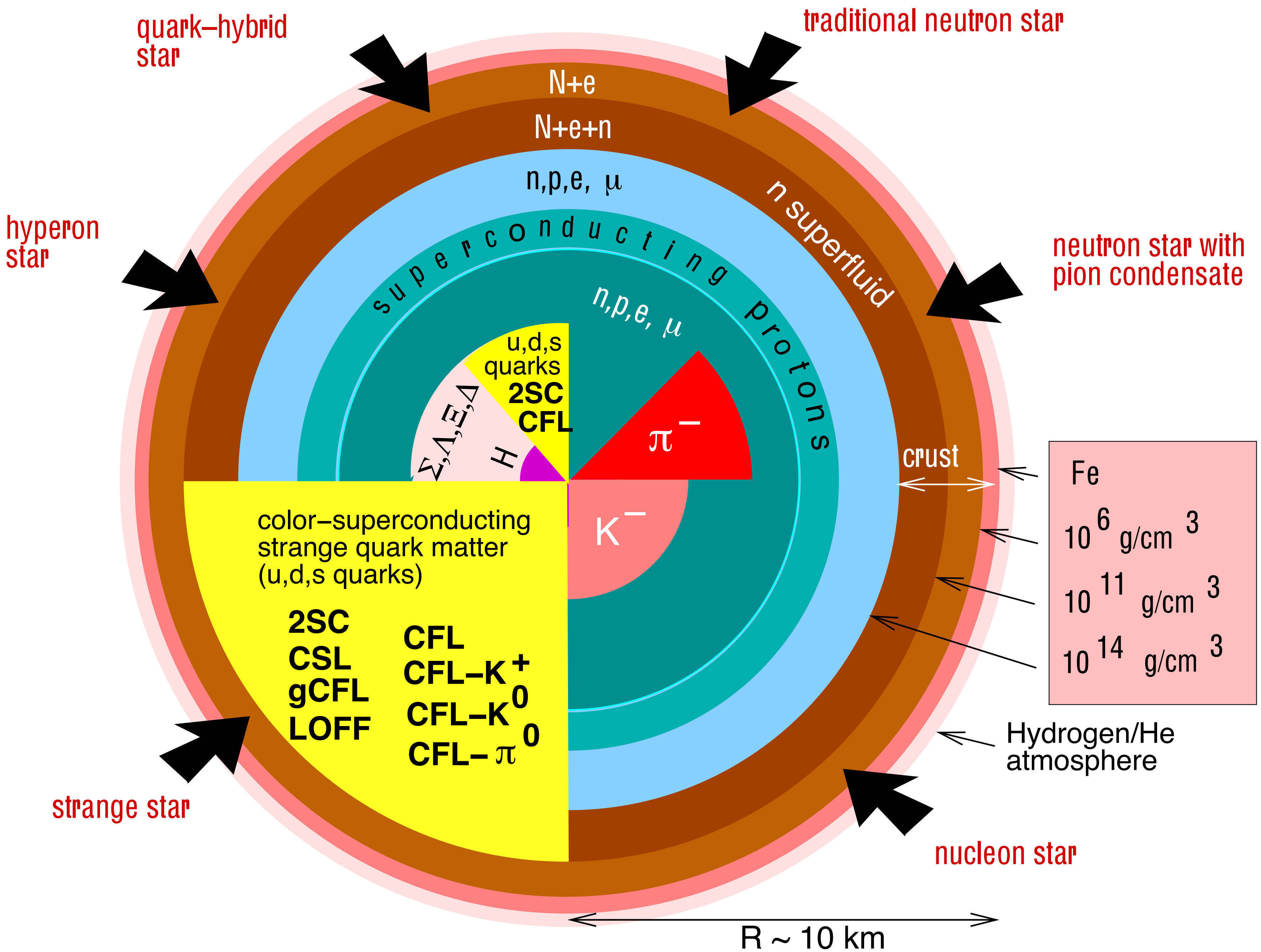
High baryonic density: Compact stars



$$M_\odot \lesssim M \lesssim 2M_\odot$$

$$R \sim 10 \text{ km}$$

$$T \lesssim 10^6 \text{ K}$$

F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193