QCD Anderson transition with overlap valence quarks on a twisted-mass sea†

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QCD Anderson transition

- potential relation between chiral restoration and deconfinement in hot QCD
- study localization properties of low-lying Dirac eigenmodes [1]:
  - localize above certain temperature
  - are separated from delocalized higher ones by *mobility edge*
  - delocalize when mobility edge vanishes as chiral transition is approached from above
  - produce chiral condensate via Banks-Casher relation in the chirally broken phase
- *Anderson transition* in condensed matter systems [2]:
  - describes metal-insulator transition in disordered solids
  - in metal phase delocalized low-lying eigenmodes of Hamiltonian provide conductivity
  - above critical disorder all eigenmodes localized, no conductivity

Lattice setup

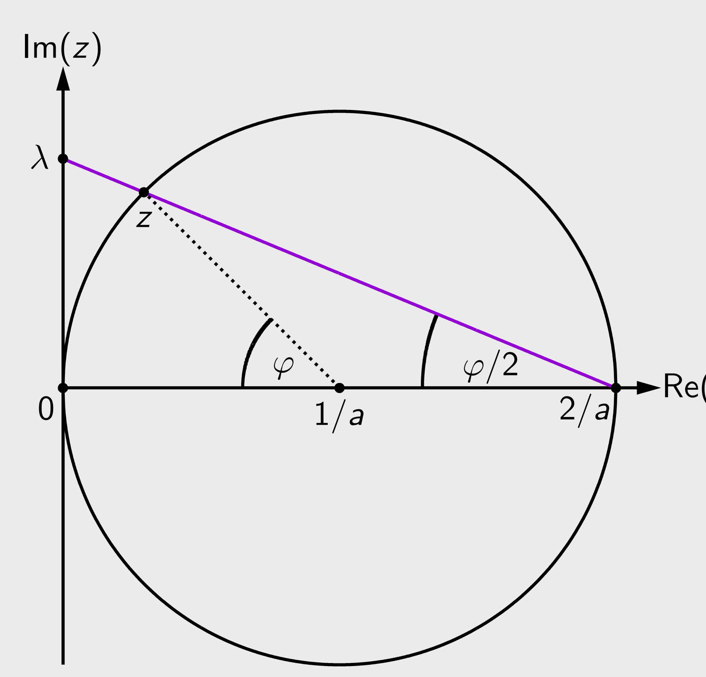
- chiral lattice fermions: massless overlap Dirac operator with Wilson kernel K

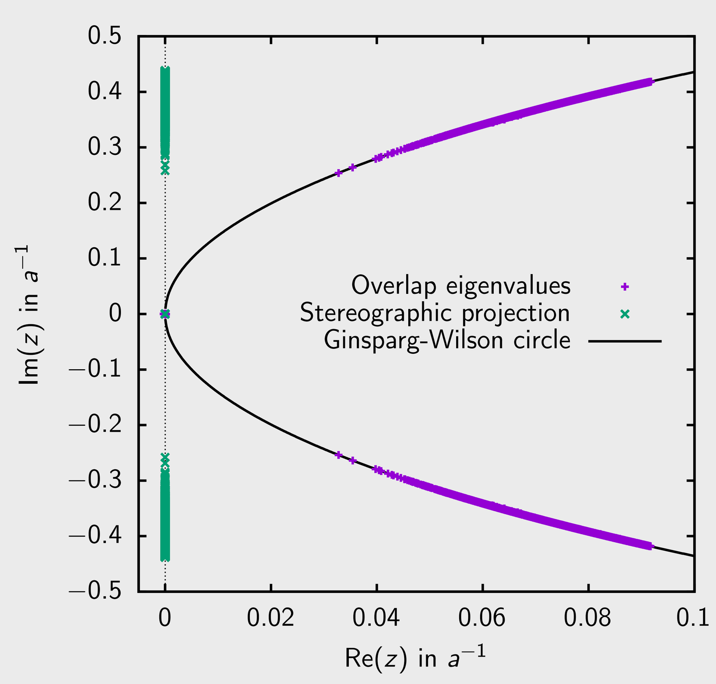
$$D = \frac{1}{a}(1 + \text{sgn}\, K)$$

- configurations from *twisted mass at finite temperature* collaboration [3]
  - twisted-mass Wilson fermions at maximal twist, Iwasaki gauge action
  - $N_f = 2 + 1 + 1$: two degenerate light plus physical strange and charm quarks

Set of ensembles	N_s	N_t	T / MeV	T/T_{pc}	# conf.	modes/conf.
A370 $a = 0.0936(13)$ fm, $m_\pi = 364(15)$ MeV, $T_{pc} = 185(8)$ MeV	24	4	527(7)	2.85(13)	200	200
		5	422(6)	2.28(10)	200	160
		6	351(5)	1.90(9)	200	135
		7	301(4)	1.63(7)	150	115
		8	264(4)	1.42(6)	200	100
		9	234(3)	1.27(6)	200	90
		10	211(3)	1.14(5)	250	80
		11	192(3)	1.04(5)	200	75
		12	176(2)	0.95(4)	200	70
D370 $a = 0.0646(7)$ fm, $m_\pi = 369(15)$ MeV, $T_{pc} = 185(4)$ MeV	32	3	1018(11)	5.50(13)	120	400
		6	509(6)	2.75(7)	120	200
		14	218(2)	1.18(3)	160	85
		16	191(2)	1.03(2)	160	75
	40	18	170(2)	0.92(2)	20	150
	48	20	153(2)	0.83(2)	3	200
D210 $a = 0.0646(7)$ fm, $m_\pi = 213(9)$ MeV, $T_{pc} = 158(5)$ MeV	48	4	764(8)	4.83(16)	10	1000
		6	509(6)	3.22(11)	10	700
		8	382(4)	2.42(8)	10	500
		10	305(3)	1.93(6)	10	400
		12	255(3)	1.61(5)	10	350

Stereographic projection





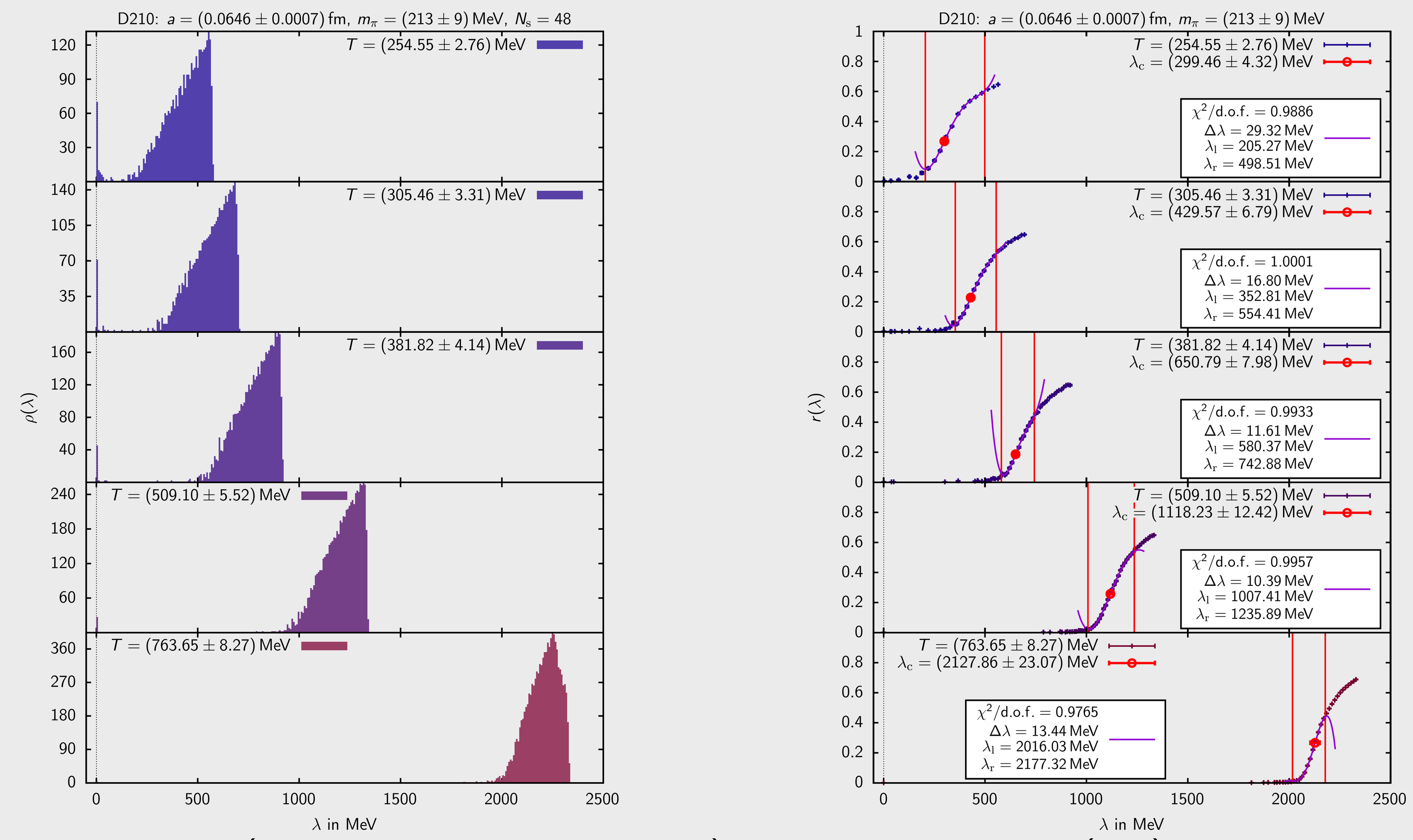
Localization measure

- relative eigenmode volume

$$r(\lambda) = \frac{P_2^{-1}(\lambda)}{|\Lambda|} \in [1/|\Lambda|, 1]$$

with *inverse participation ratio* $P_2(\lambda) = \sum_{i\in\Lambda}(v_\lambda(i)^\dagger v_\lambda(i))^2$ of eigenmode v_λ for eigenvalue λ, and $|\Lambda|$ number of lattice sites

Eigenvalue distributions and localization



Distributions of (stereographically projected) overlap eigenvalues (left) and bin-averaged relative eigenmode volumes as measure of localization (right) for D210 configurations.

Mobility edge estimates and extrapolation

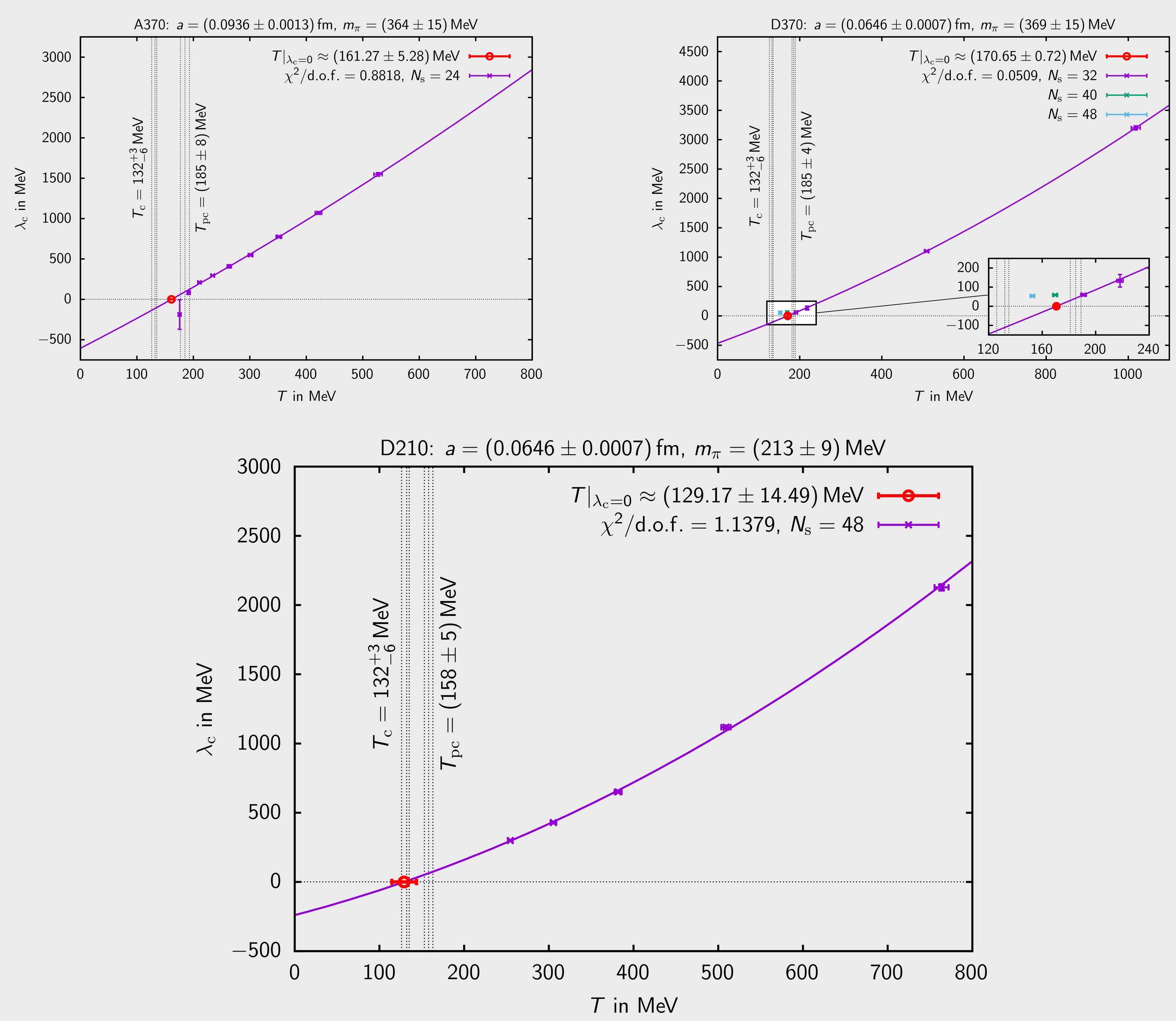
- inflection point λ_c from fit with Taylor polynomial

$$r(\lambda) = r_c + b(\lambda - \lambda_c) + c(\lambda - \lambda_c)^3 + d(\lambda - \lambda_c)^4$$

- Anderson transition temperature T_0 from second fit

$$\lambda_c(T) = b(T - T_0) + c(T - T_0)^2$$

Anderson transition estimate



Extrapolated mobility edges: towards physical point (bottom) Anderson transition temperature T_0 coincides with chiral phase transition temperature T_c from [4].

References

[1] M. Giordano and T. G. Kovacs, *Localization of Dirac Fermions in Finite-Temperature Gauge Theory*, Universe **7**, 194 (2021).

[2] F. Evers and A. D. Mirlin, *Anderson transitions*, Rev. Mod. Phys. **80**, 1355–1417 (2008).

[3] F. Burger, E.-M. Ilgenfritz, M. P. Lombardo, and A. Trunin, *Chiral observables and topology in hot QCD with two families of quarks*, Phys. Rev. D **98**, 094501 (2018).

[4] H. T. Ding *et al.*, *Chiral Phase Transition Temperature in (2+1)-Flavor QCD*, Phys. Rev. Lett. **123**, 062002 (2019); A. Y. Kotov, M. P. Lombardo, and A. Trunin, *QCD transition at the physical point, and its scaling window from twisted mass Wilson fermions*, Physics Letters B **823**, 136749 (2021).

† R. Kehr, D. Smith, and L. von Smekal, arXiv:2304.13617 [hep-lat], submitted to Phys. Rev. D.