Laboratoire Kastler Brossel
Physique quantique et applications

Precise determination of the fine structure constant and test of QED

Pierre Cladé

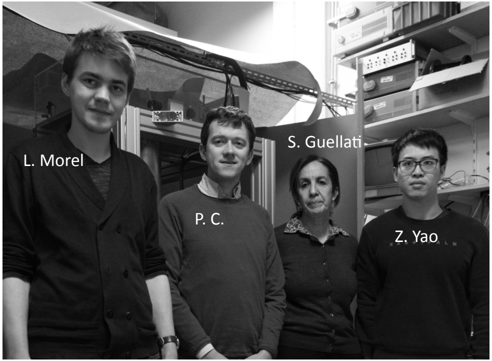
SORBONNE UNIVERSITÉ

CNRS

ENS ÉCOLE NORMALE SUPÉRIEURE

COLLÈGE DE FRANCE 1530

More than 25 years of work



PhD students (since 2000) and postdoc:

- C. Debavelaere
- C. Carrez
- L. Morel
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- P. Cladé
- R. Battesti

Permanent staff (2023):

- Pierre Cladé
- Saïda Guellati-Khelifa
- François Biraben (emeritus)

PhD and post-doc positions available

`pierre.clade@lkb.upmc.fr`

Anomalous magnetic moment of the electron

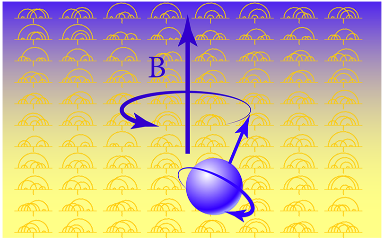
$a_e = \frac{g_e - 2}{2}$, with $g_e = \frac{\text{Larmor freq.}}{\text{cyclotron freq.}}$

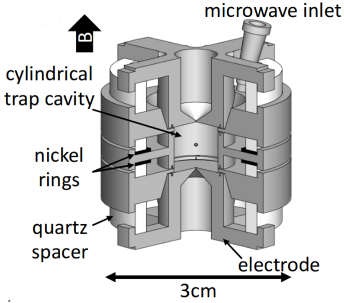
- Measurement: $\boxed{a_e(\text{exp}) = 0.00115965218059(13)}$ (group of G. Gabrielse, Phys. Rev. Lett. 130, 071801, 2023), improved by a factor of 2.
- QED calculation (Laporta PRB **772** 232 (2017); Aoyama *et. al.* Atoms 7 **28** (2019); ...)

$$a_e\,(\text{QED}) = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A^{(2n)}_{\mu,\tau} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

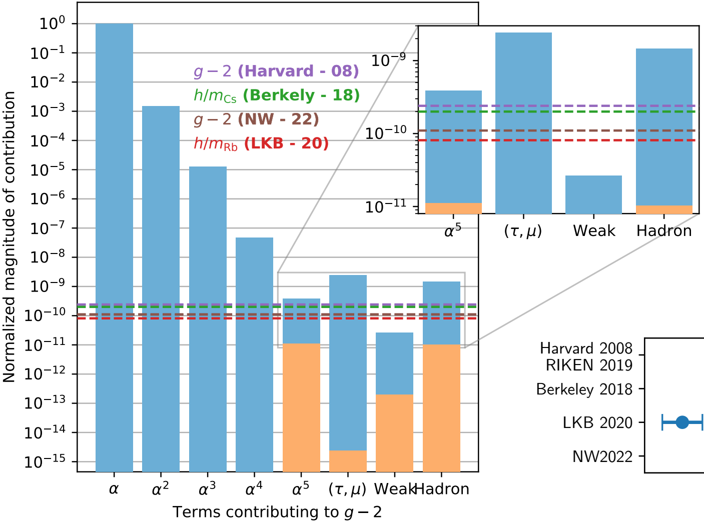
- Calculation of the A^{10} term : 12672 Feynman diagrams.
- Other contributions

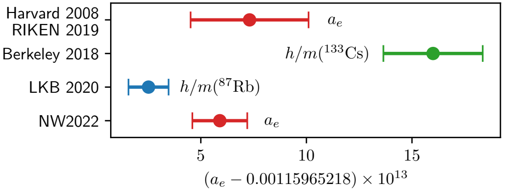
$$a_e\,(\text{theo}) = a_e\,(\text{QED}) + a_e\,(\text{Hadron}) + a_e\,(\text{Weak})$$



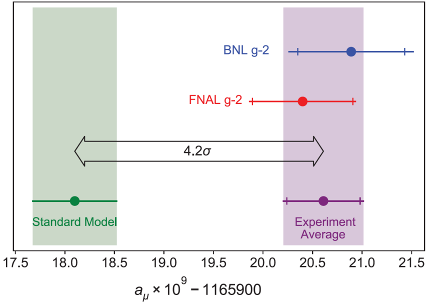


Test of a_e


Normalized magnitude of contribution
Terms contributing to g − 2
g − 2 (Harvard - 08)
h/m_Cs (Berkely - 18)
g − 2 (NW - 22)
h/m_Rb (LKB - 20)
α
α²
α³
α⁴
α⁵
(τ, μ)
Weak
Hadron


Harvard 2008
RIKEN 2019
Berkeley 2018
LKB 2020
NW2022
a_e
h/m(¹³³Cs)
h/m(⁸⁷Rb)
a_e
(a_e − 0.00115965218) × 10¹³

Muon a_μ discrepancy



$$\delta a_\mu = a_\mu(\text{exp}) - a_\mu(\text{theo})$$
$$= 2.51(0.59) \cdot 10^{-9}\ (4.2\sigma)$$

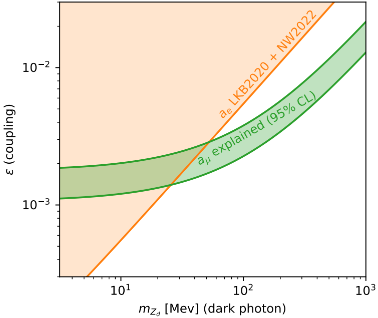
T. Aoyama *et al*, Physics Report **887**, p1-66, (2020)
B. Abi et al. (Muon g-2 Collaboration) Phys. Rev. Lett. 126, 141801 (2021)

One loop contribution of dark photon :

$$a_l = \frac{\alpha}{2\pi}\epsilon^2 F_V(m_{Z_d}/m_l)$$

$$F_V(x) = \int_0^1 dz \frac{2z(1-z)^2}{(1-z)^2 + x^2 z}$$

G. F. Giudice et al. JHEP 11, 113 (2012)



Fine structure constant and recoil velocity

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137.0}$$

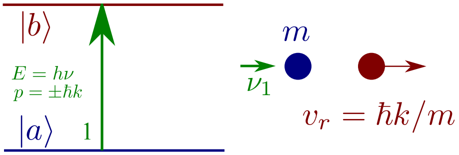
- Hydrogen spectroscopy: $hcR_\infty = \frac{1}{2}m_\mathrm{e}c^2\alpha^2$

$$\alpha^2 = \frac{2R_\infty}{c}\frac{h}{m_\mathrm{e}} = \frac{2R_\infty}{c}\frac{A_\mathrm{r}(\mathrm{X})}{A_\mathrm{r}(\mathrm{e})}\frac{h}{m_\mathrm{X}}$$

Rydberg constant	R_∞	2×10^{-12}
Relative mass of e	$A_\mathrm{r}(\mathrm{e})$	3.0×10^{-11}
Relative mass of atoms	$A_\mathrm{r}(\mathrm{X})$	$\simeq 6\times10^{-11}$

- Precision limited by $\frac{h}{m_\mathrm{X}}$ (or m_X in the new SI)

Recoil velocity



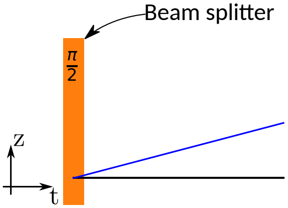
Rubidium atoms : $v_r = 6\,\mathrm{mm\,s^{-1}}$

Our goal is to measure v_r

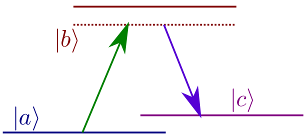
- Differential velocity sensor : atom interferometer
- Transfer a large number N of photon momenta : Bloch oscillation technique

Differential velocity sensor : atom interferometer

Interferometer based on counterpropagating Raman transitions



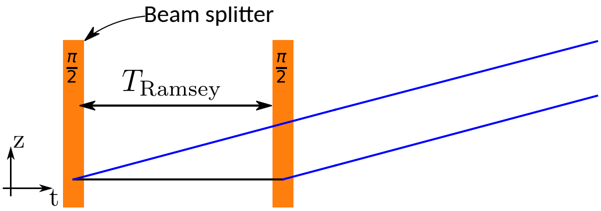
Raman transition



- No spontaneous emission
  $\rightarrow$ Coherent two level system
- Doppler sensitive ($\delta \sim k_R v$)
  with $k_R = k_1 + k_2$

Differential velocity sensor : atom interferometer

Interferometer based on counterpropagating Raman transitions



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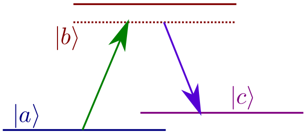

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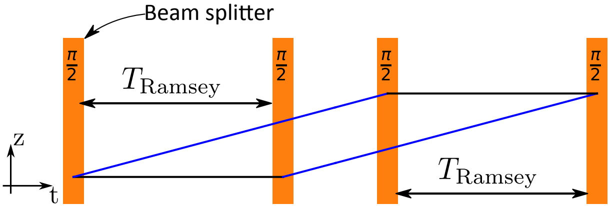

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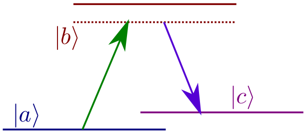

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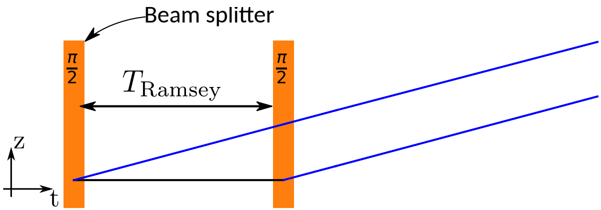
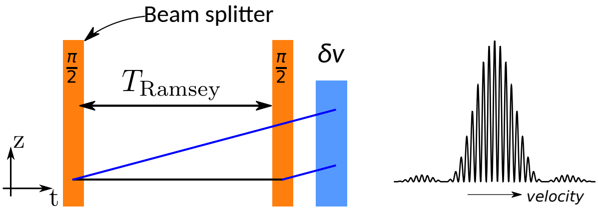
Differential velocity sensor



Phase difference between the two arms Φ

In the middle of the interferometer: $\Delta z = 2v_R T_{\mathrm{Ramsey}}$

Differential velocity sensor



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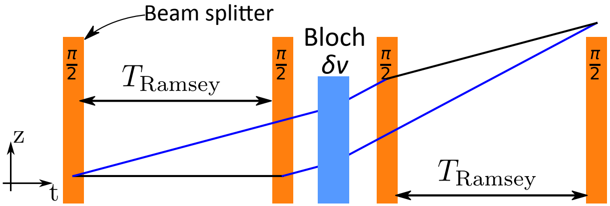
Atomic phase :

$$\Phi_{\text{at.}} = T_{\text{Ramsey}} k_R \delta v = \frac{\Delta z \delta v}{\hbar / m}$$

Laser phase = Doppler effect :

$$\phi_{\text{las.}} = 2\pi \delta f_R T_{\text{Ramsey}}$$

Differential velocity sensor



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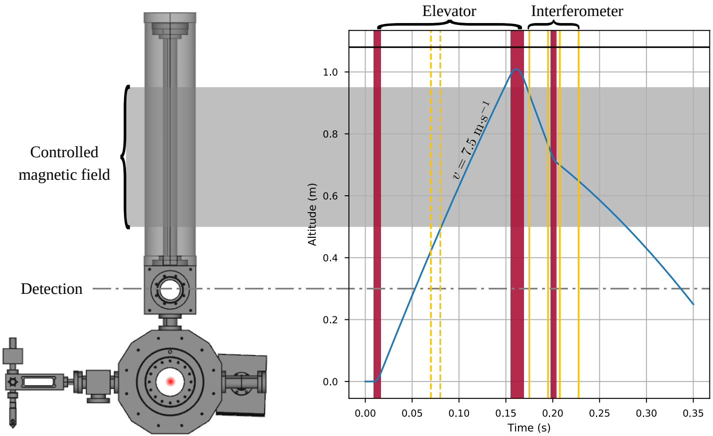
Laser phase = Doppler effect :

$$\phi_{\text{las.}} = 2\pi \delta f_R T_{\text{Ramsey}}$$

Probability to observe an atom in $|2\rangle$:

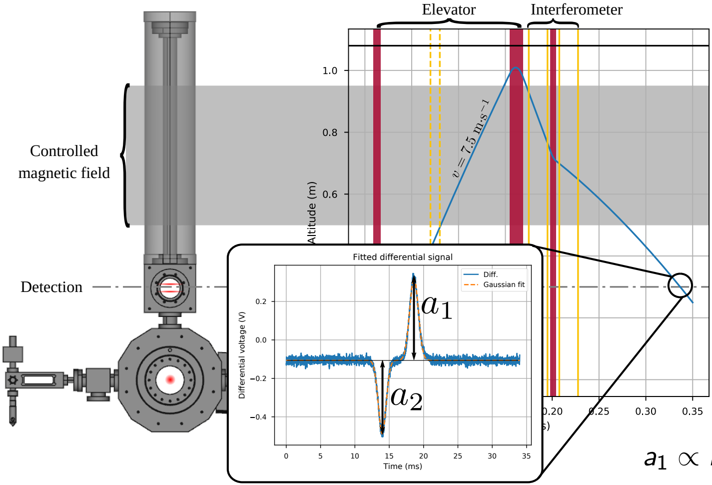
$$P_2 = \frac{1 + cos(\Phi_{\text{at.}} - \phi_{\text{las.}})}{2}$$

Experimental setup



Magneto-optical trap : 10^8 atoms at 2 μK

Experimental setup



$a_1 \propto N_1$

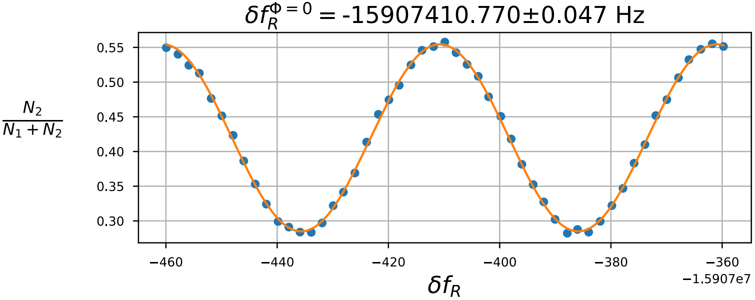
$a_2 \propto N_2$

$\Rightarrow P_2 = \frac{N_2}{N_1 + N_2}$

Magneto-optical trap : 10^8 atoms at $2\,\mu K$

Data acquisition

- ~ 1 point per second
- 50 points per spectra (~ 1 minute)
- $T_{\text{Ramsey}} = 20$ ms, Number of Bloch Oscillations $N_B = 500$

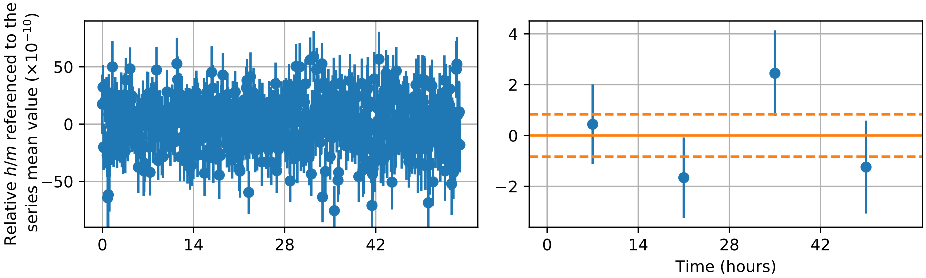


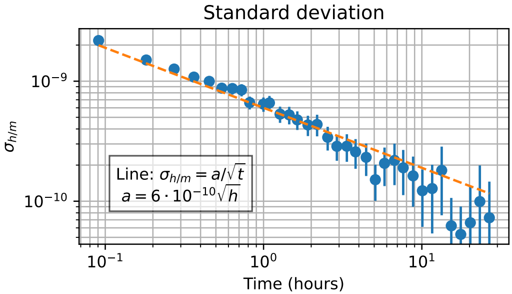
$$\Phi = T_{\text{Ramsey}} \left(k_R 2 N_B v_r - 2\pi \delta f_R \right)$$

- One photon momentum: ~ 15 kHz $\rightarrow$ 1000 photon momenta: ~ 15 MHz
- $\sigma_v = 0.047 Hz \sim 3 \cdot 10^{-6} v_r \sim 20$ nm·s^{-1} $\rightarrow 3 \cdot 10^{-9}$ on h/m

Statistical uncertainty (2020)

Stable and reliable device ⇒ Long measurement periods





Error budget

Source	Correction $[10^{-11}]$	Relative uncertainty $[10^{-11}]$
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of ^{87}Rb [16] : $86.909\,180\,531\,0(60)$		3.5
Relative mass of the electron [14] : $5.485\,799\,090\,65(16) \cdot 10^{-4}$		1.5
Rydberg constant [14] : $10\,973\,731.568\,160(21)\mathrm{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035\,999\,206(11)$		8.1

What is the momentum of a photon ?

- Photon (plane wave) : $p = \hbar k = h\nu/c$
- Poynting vector (density of momentum) / density of photons
- $\vec{k}_{\text{eff}} = \vec{\nabla}\phi$ (phase of the electric field $E(\vec{r}, t) = A(\vec{r}, t)e^{i\phi(\vec{r})}$).
- Correction to the plane wave model: $k_{\text{eff,z}} = k + \delta k$

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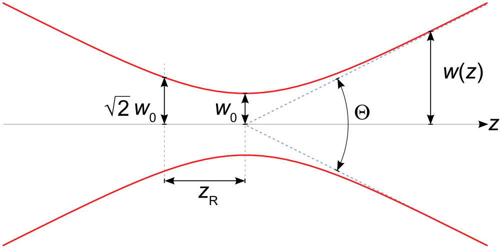
Gaussian beam (Gouy phase):



$$\frac{\delta k}{k} = -\frac{2}{k^2 w^2(z)}\left(1 - \frac{\langle r^2\rangle}{w^2(z)}\right) - \frac{\langle r^2\rangle}{2R^2(z)}$$

Arbitrary beam:

$$\delta k_{\text{rel}} = \frac{\delta k}{k} = -\frac{1}{2}\left|\left|\frac{\vec{\nabla}_\perp \phi}{k}\right|\right|^2 + \frac{1}{2k^2}\frac{\Delta_\perp A}{A}$$

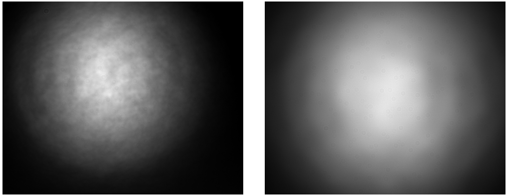
(Paraxial approximation)

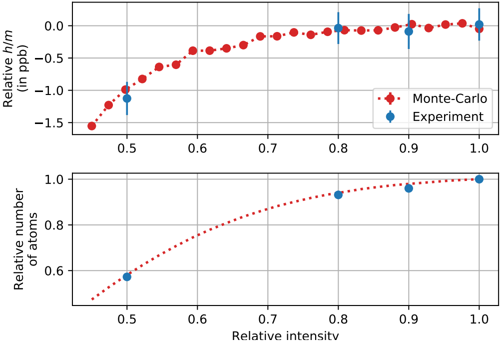
Systematic effect on the current experiment

Effect $\propto \mathrm{Cor}(I, P(I)) \times \mathrm{Cor}(I, k_{\mathrm{eff}}(I))$

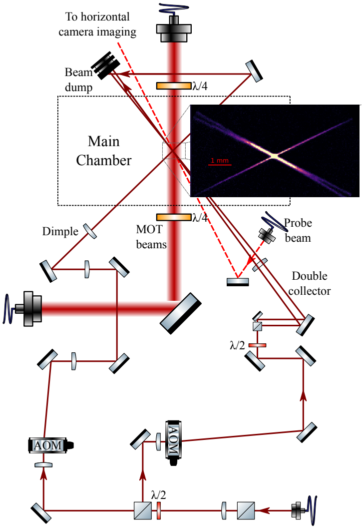
- Fluctuations are reduced by letting the beam propagate.
- Monte Carlo simulation.
- Comparison with experiment as a function of the number of atoms

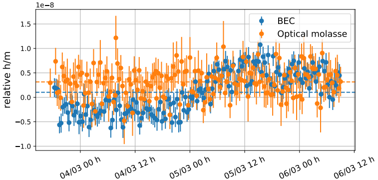
30 cm   3 m

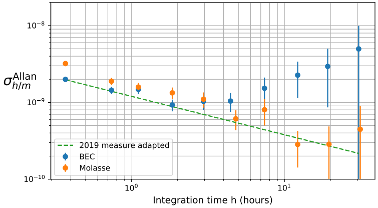


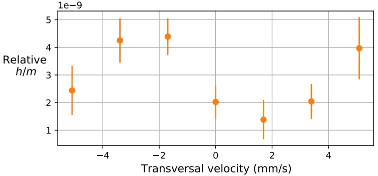


Work in progress : measurement with a BEC

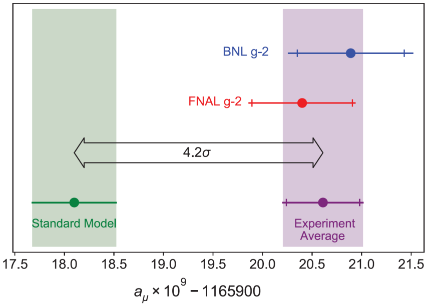








Muon a_μ discrepancy



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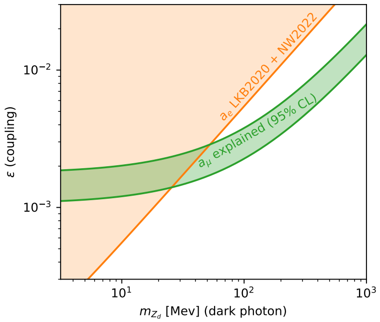

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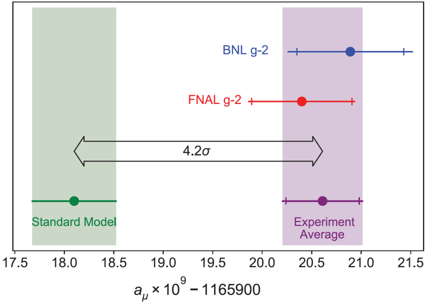

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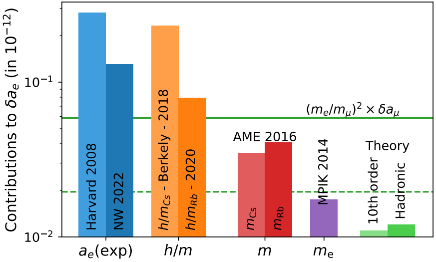
Naive scaling $\left|\frac{\delta a_e}{\delta a_\mu}\right| = \left(\frac{m_e}{m_\mu}\right)^2 \sim 2.3 \cdot 10^{-5}$

$\delta_{a_e} \sim 5.8 \cdot 10^{-14}$ (0.05ppb)

G. F. Giudice et al. JHEP 11, 113 (2012)
F. Terranova et al., PRA 89, 052118 (2014)



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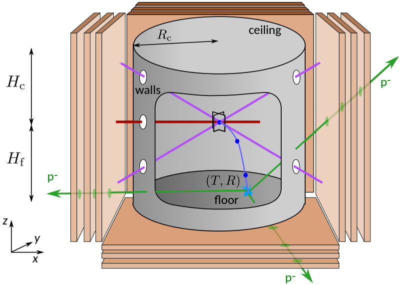
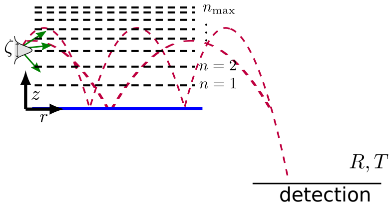
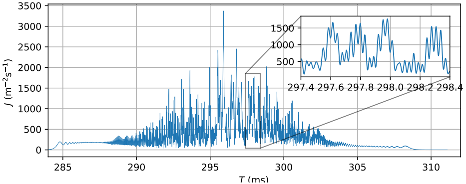
Quantum interference with antihydrogen

- Principle of the GBAR project : $\bar{H}^+$ trapped in a ion trap. $\bar{H}$ produced by photodetachment (Walz and Hänsch, *Gen. Relat. and Grav.*, **36**, 561 (2004))
- Measurement base on time of flight : $\simeq 1 \times 10^{-2}$ for 1000 events.
- Improvement based on quantum interferences antihydrogen (Crépin *et. al.*, Phys. Rev. A, 042119, 2019: Rousselle *et. al.*, Phys. Rev. D. **76** 209 2022)
- Quantum reflection of atoms (Casimir-Polder potential)



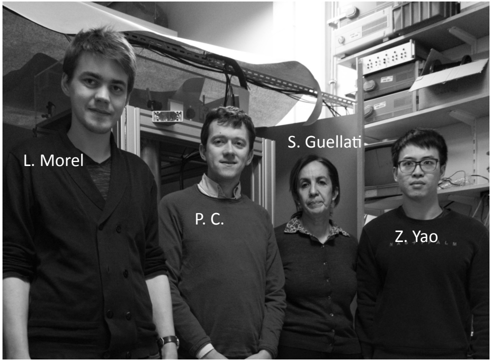


Relative accuracy : 1×10^{-5} for 1000 events.



LKB

More than 25 years of work



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- François Biraben (emeritus)

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PhD and post-doc positions available

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