Whispering Gallery of a Neutral Particle with a Gravitational Shift

A few days ago, I received an email from one of the organizers of this conference discussing a **particular scientific question**. With this presentation, I am answering this letter.

To do this, I **slightly changed the initial title** of my talk to "Whispering Gallery of a Neutral Particle **with a Gravitational Shift**" and yesterday, at the last desk of the conference, I completely rewrote my talk.

NEUTRONS FOR SCIENCE

I am grateful to the organizers for the possibility to participate in the nice conference and always happy to come to this place (due to a different reason).

I would like to underline the excellent preparation of the meeting. Even a special wine with **a quantum interference pattern** (**the quantum technology frontier !**) has been specially produced for the participants of this meeting.

NEUTRONS FOR SCIENCE

I am grateful to the organizers for the possibility to participate in the nice conference and always happy to come to this place (due to a different reason).

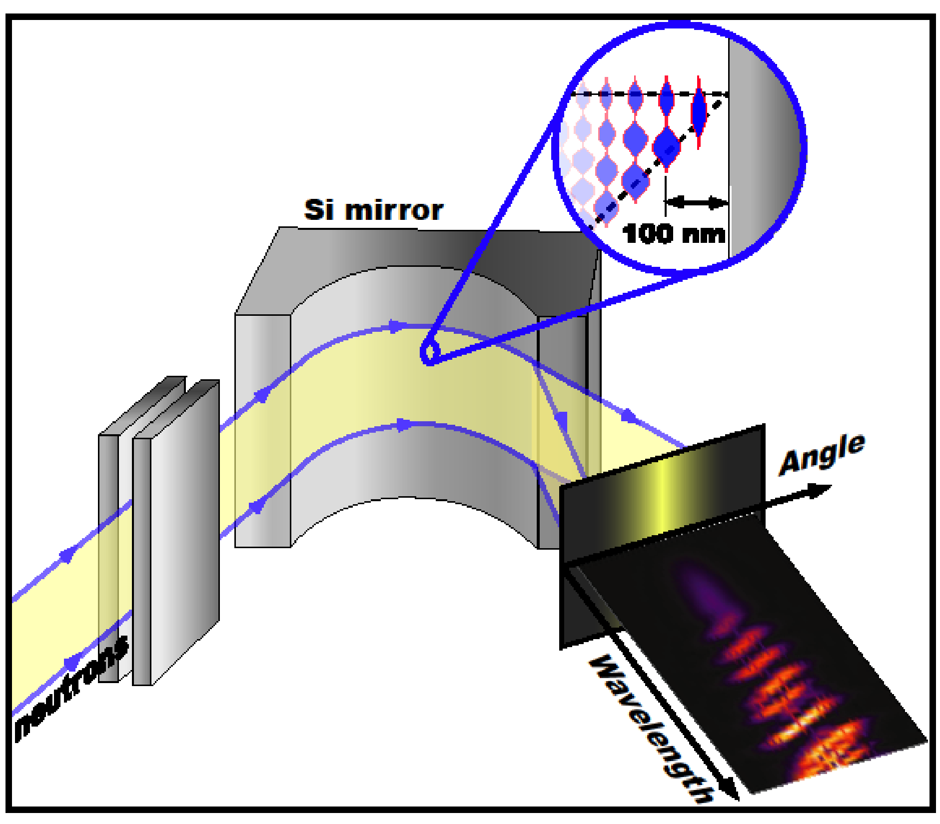
I would like to underline the excellent preparation of the meeting. Even a special wine with **a quantum interference pattern** (**the quantum technology frontier !**) has been specially produced for the participants of this meeting.

The whispering gallery states (**WGS**) of a neutral particle is a useful phenomenon for **searching for new physics** beyond the Standard Model using **precision quantum measurement technology** (you see, the topic of my talk is precisely in line with the title of the conference ...)

Anna Nesvijevskaia :

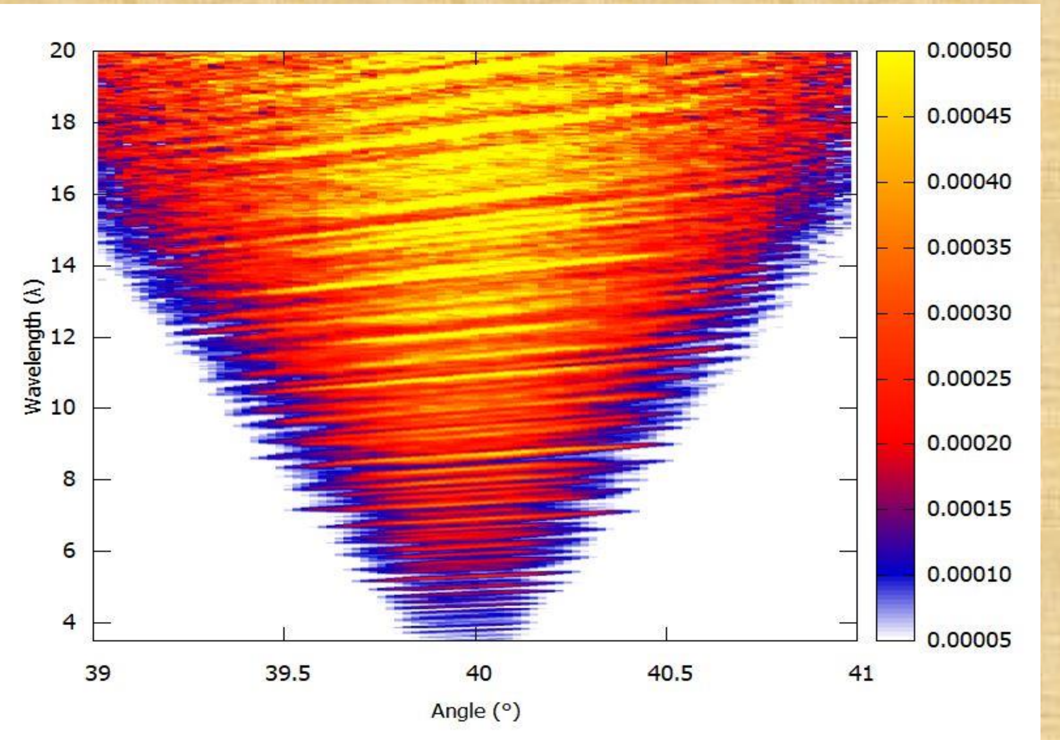
What is the **WGS** of a neutral particle?

- a **neutral particle** (otherwise electromagnetic interactions dominate, and the WGS phenomenon does not exist, or at least precision measurements are impossible),
- the **quantum reflection** of this neutral particle from the surface (it is not very important what the nature of this quantum reflection is (a wave may always get reflected from a surface…),
- a **curved mirror** (to provide an effective centrifugal force),
- certain **conditions on the parameters** of the problem (observation time, particle velocity and mass, mirror radius, etc) essentially determined by the uncertainty relations



[V.V. N., et al, *Neutron whispering gallery*, Nature Physics 6:114, 2010].

The scheme from [H. Rauch, *Neutrons in a whispering gallery*, Nature Physics 6:79, 2010].



a **typical interference pattern** (shown to everyone at various conferences, but not published yet…

New physics beyond the Standard Model, which can be studied using WGS, can be generalized as a search for **extra fundamental short-range interactions** (**depending** on the spin (axion-type) and **not depending** on the spin) between the **particle** and the **mirror**.

The WGS is a very competitive method over a certain range of characteristic distances of such interactions. Many modern physics problems could be probed:

- **dark matter**, **dark energy**, the origin of **gravity**, **matter-antimatter asymmetry**, theories with **extra light bosons**, extra **dimensions of space**, etc,

- also various applications in quantum mechanics studies, surface studies, precision spectroscopy, etc

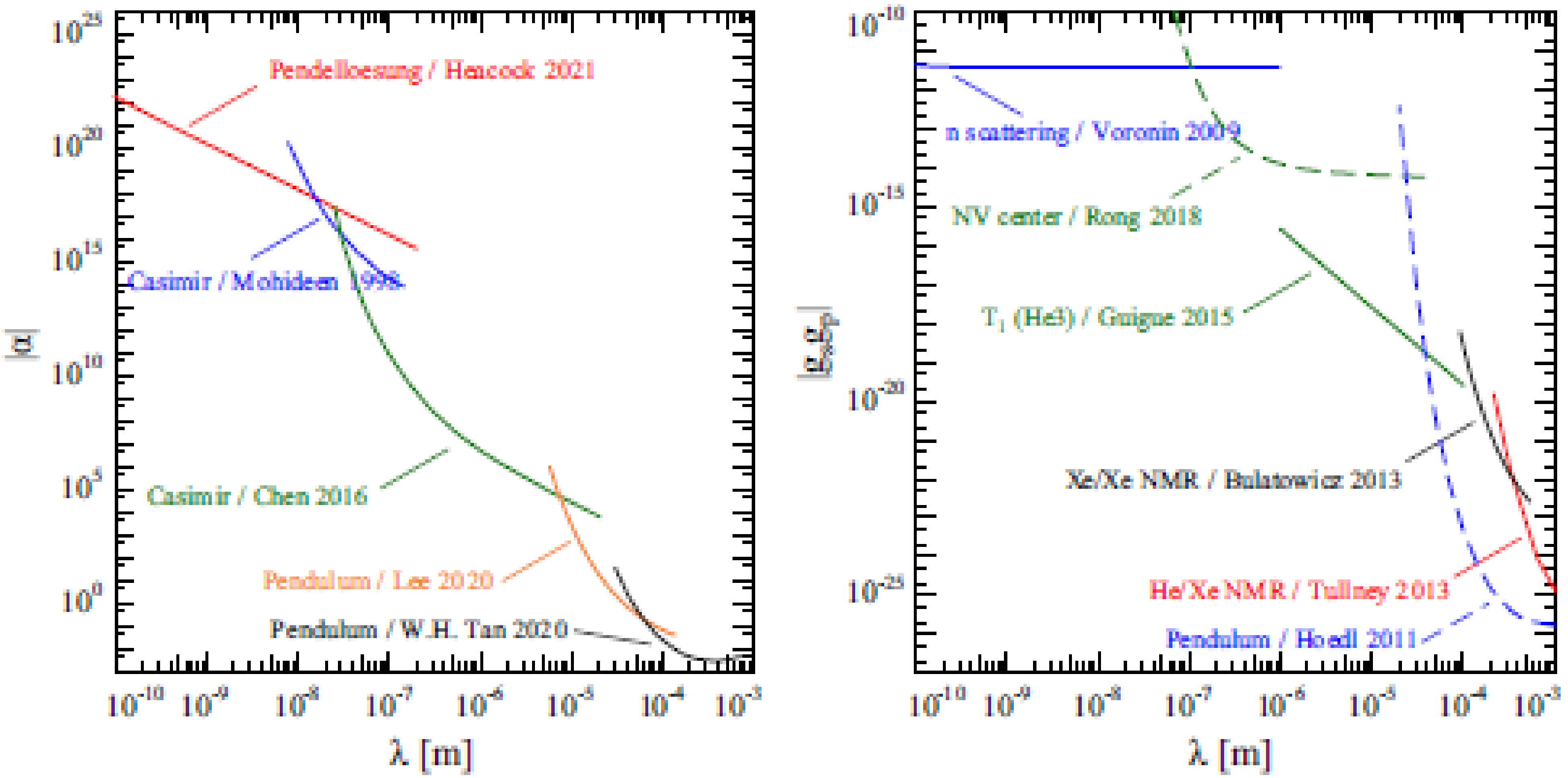


Figure 8: Left: Current most constraining limits for new spin-independent short-range forces. The first experiment (red line, [179]) uses free neutrons. At higher λ ranges, other techniques and probes are more sensitive [180, 181, 182, 183]. Right: Current most constraining limits for new spin-dependent interactions. Experiments [184, 185, 186, 187] use the neutron as the polarized particle, for the first of those it is a free neutron. In [188, 189], the polarized particle is an electron.

[H. Abele et al, *Particle physics at the European spallation source*, to be published in Physics Reports 2023] – the latest review.

Many of the features of **WGS** and gravitational quantum states (**GQS**) do not depend on which neutral particle is used: **neutron** (all first experiments), **hydrogen** atom (GRASIAN collaboration), **antihydrogen** atom (GBAR collaboration), **positronium**, **muonium** (gravity of antimatter?), **heavier neutral** atoms, molecules, nanoparticles, nanodroptets etc.

Measurements with each of these particles have their own motivation and optimal sensitivity areas.

Efficient quantum reflection results from almost **ideal sharp potential step** provided that the spatial size of the WGS is much larger than the spatial size of the diffuseness of the potential step, which is why WGS (and GQS) **depend little** on the properties of **the mirror**.

Moreover, the corresponding the small corrections (associated with the interaction of the particle with the mirror) can be **accurately calculated** (Serge Reynaud and collaborators) and **experimentally studied**.

If the particle **masses** are (nearly) **equal** (neutron, hydrogen, antihydrogen), the **WGS** are practically **identical**.

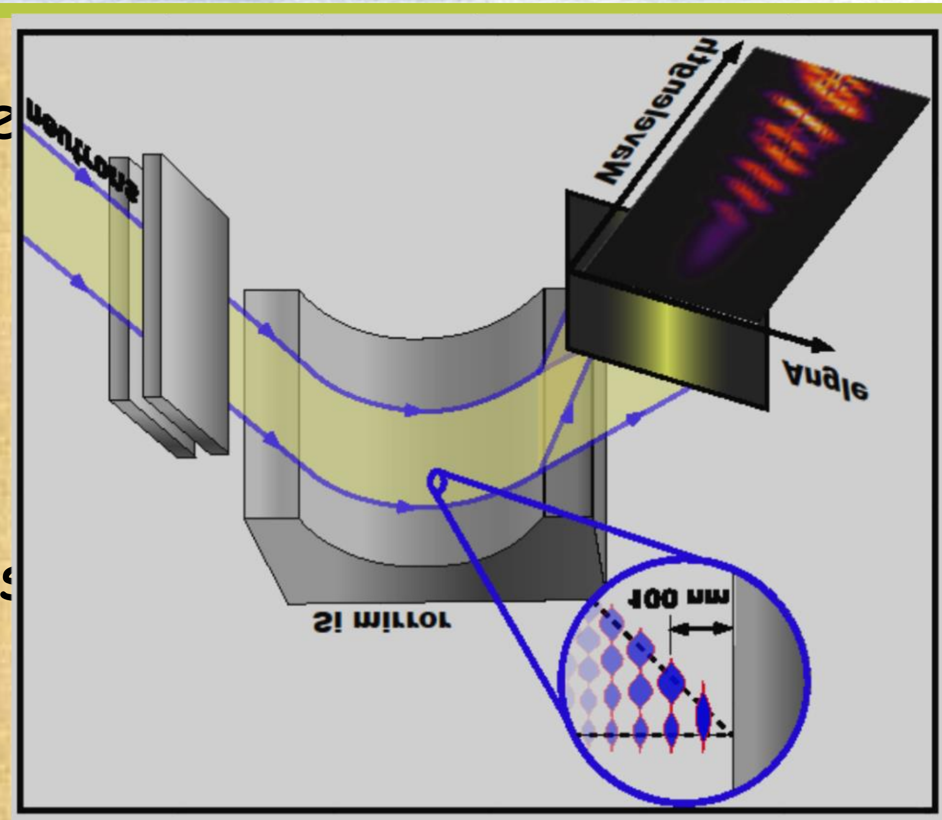
If the **mass is smaller** (muonium, positronium), the states have a larger spatial size and the **quantum reflection is more efficient**.

If the **mass is larger** (heavier atoms, molecules, nanoparticles), the quantum states spatial sizes are smaller, the reflection is less efficient, the systematic effects are larger, however, a better **sensitivity** to extra fundamental interactions **at smaller distances** may appear.

Of course, the optimal **methods of observation/detection** depend on which particle is used and must be considered on a case-by-case basis.

With the characteristic parameters of all the WGS experiments discusse or performed, **gravity is a small correction** (hence "**shift**").

Moreover, this correction is usually **suppressed** by choosing the mirror orientation: the gravity axis is **perpendicular** to the axis of the centrifugal force.



However, the gravitational effect in WGS may be **the goal** of the experiment (as in the case of antimatter particles).

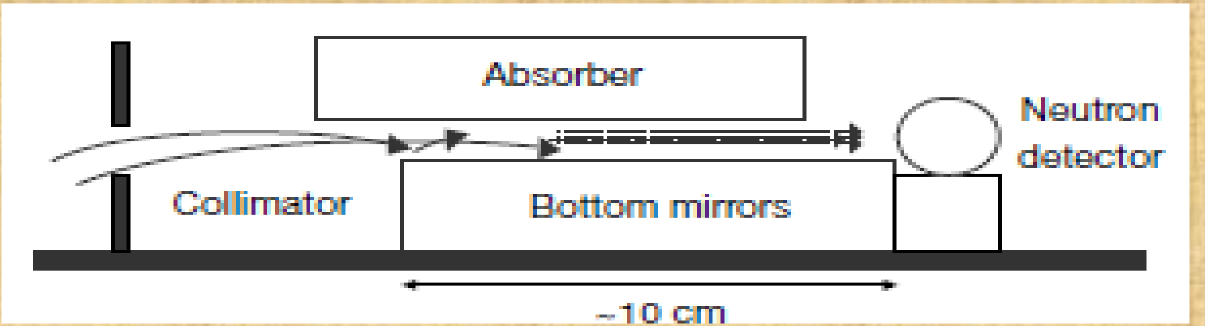
In this case, you need **just to rotate the mirror by 90 degrees** and examine this effect more closely.

[V.V. N. et al, *Quantum states of neutrons in the Earth's gravitational field*, Nature 415 (2002) 297]

First, let's analyze, for simplicity, **GQS** of ultracold neurons. This is an analogous phenomenon, differing only in the attraction of the particle particle to the surface:

gravity instead of centrifugal force

Similar triangular potential and theoretical formalism



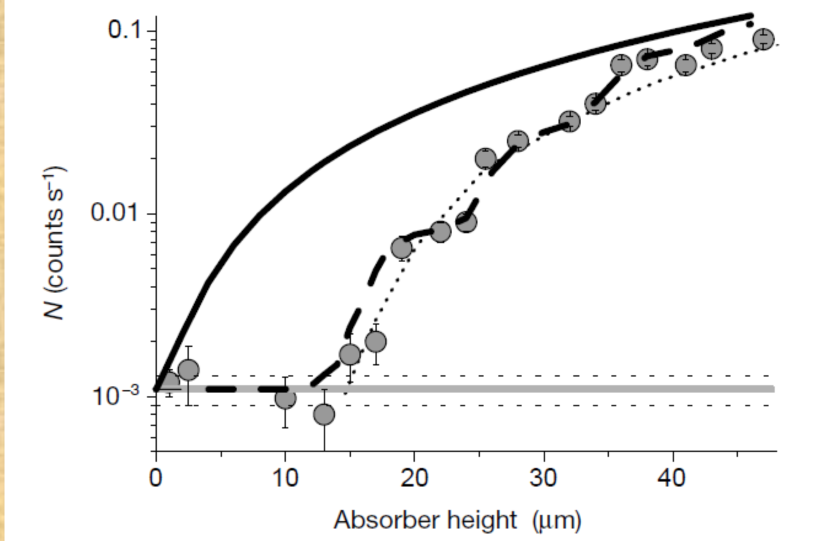


Figure 4 The neutron throughput versus the absorber height at low height values. The data points are summed up in intervals of 2 μm. The dashed curve corresponds to a fit using the quantum-mechanical calculation, in which all level populations and the height resolution are fitted from the experimental data. The solid curve is again the full classical treatment. The dotted line is a truncated fit in which it is assumed that only the lowest quantum state—which leads to the first step—exists.

- mass **1 at. un.**,
- typical energies, ε_0**~0.6 peV**,
- typical vertical sizes of GQS, l_0**~5.9 μm**,
- typical effective temperatures, **~10 nK**,
- typical times of formation of GQS is τ_0**~1.1 ms**

- The surface reflects neutrons **elastically** due to the condition $l_0 >> 1$ Å (a typical interatomic distance). It means that the thermal motions of individual nuclei/atoms are effectively averaged out, and

- **specularly** provided the surface is well-polished and homogeneous (on the spatial scale of the order of l_0),

- A **characteristic range** of raising the potential should be $<< l_0$ (the condition of sharpness of the reflecting potential). Otherwise, the raising part of potential modifies GQS, and this modification has to be known precisely.

Two uncertainty ratios:

- Vertically, for the uncertainty of **vertical position** and **vertical velocity** (energy) **l_0~5.9 μm** – **v_0~2.4 cm/s** (**ε_0~0.6 peV**).

- All these parameters of the problem are determined only by the value of gravitational acceleration at the local point of the measurement **g~9.81 m/c^2**. Parameters of **the mirror are not involved**, to a good accuracy.

- It is **difficult to change g** by a noticeable amount...

You can **tilt the mirror**.

In this case, the component of motion along the mirror ("horizontal") is still classical and is **separated** from the vertical component ("quantum"), in a properly designed experiment.

The **projection** of the gravitational field will **decrease**. However, this is **not very interesting**, because the most interesting area in terms of sensitivity to short-range interactions is "stronger" rather than "weaker" gravity.

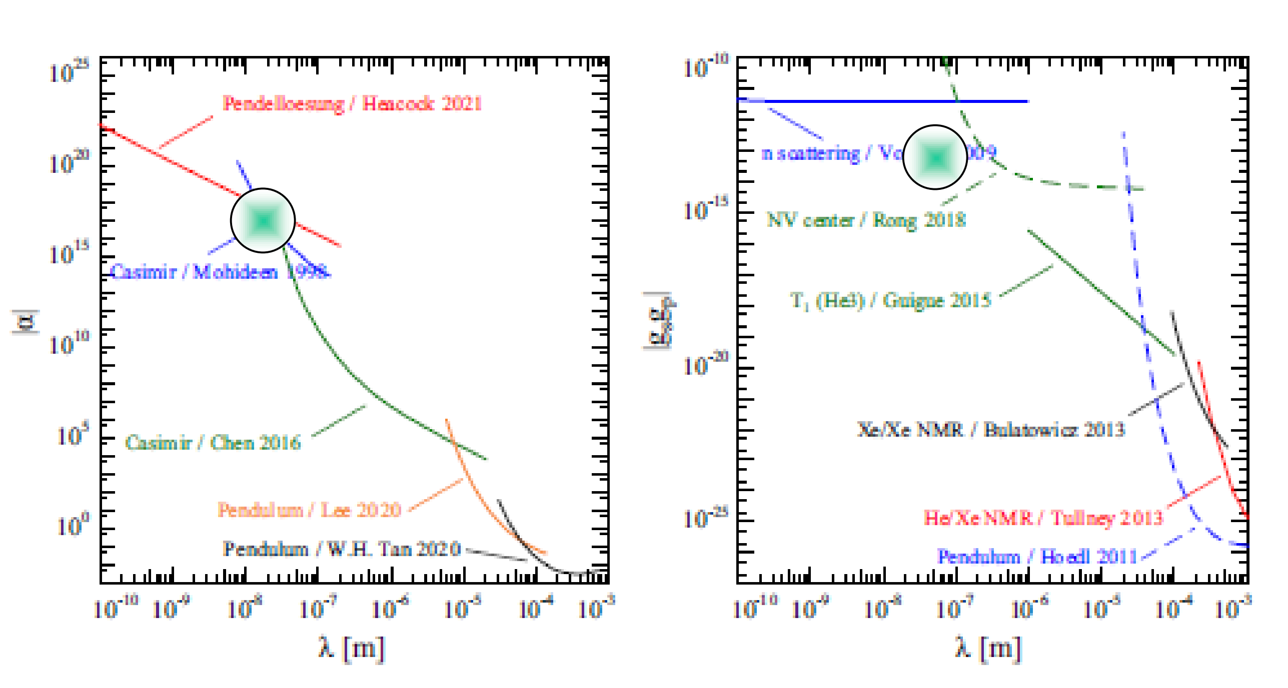


Figure 8: Left: Current most constraining limits for new spin-independent short-range forces. The first experiment (red line, [179]) uses free neutrons. At higher λ ranges, other techniques and probes are more sensitive [180, 181, 182, 183]. Right: Current most constraining limits for new spin-dependent interactions. Experiments [184, 185, 186, 187] use the neutron as the polarized particle, for the first of those it is a free neutron. In [188, 189], the polarized particle is an electron.

You can **move** the mirror vertically with a **constant acceleration**.

However,

- the **observation time** will be limited, and
- the systematics will increase dramatically, because instead of a "perfectly uniform and known" gravitational field, there will be a "**badly defined** 3-dimensional non-constant and poorly measured" acceleration.

You can **add a vertical magnetic field gradient**.

Indeed, the gradient will affect the magnetic moment of the neutral particle (opposite signs of the force for the two spin states of the particle).

It is effectively possible

- to **"increase"** the gravitational interaction,
- **"decrease"** the gravitational interaction,
- **compensate** for the gravitational interaction ("no gravity"),
- and even simulate **"anti-gravity"**.

The potential problem is the same as with the vertical acceleration of the mirror: it is **difficult to control** this effect with a reasonable accuracy.

- Horizontally: the uncertainty relation for the **energy** of the quantum state (**ε_0~0.6 peV**) and the **observation time** (**τ_0~1.1 ms**).

- An ultracold neutron with a typical velocity of **~5 m/s** travels **~0.55 cm** in **τ_0~1.1 ms**. To provide condition **$\tau \gg \tau_0$**, we need the mirror length of at least **5.5 cm**. A technically feasible length is, say, **55 cm**.

- The corresponding accuracy of experiments with neutrons, in the flow-through mode, is **~10^{-2}**. With certain efforts to take into account systematics, it may be improved to, say, **~10^{-3}**. Then there are insurmountable **problems**: the systematics following from the uncertainty relation, and too low densities of neutrons in the phase space. Since the required increase in sensitivity is **still 2-3 orders** of magnitude, the task is difficult.

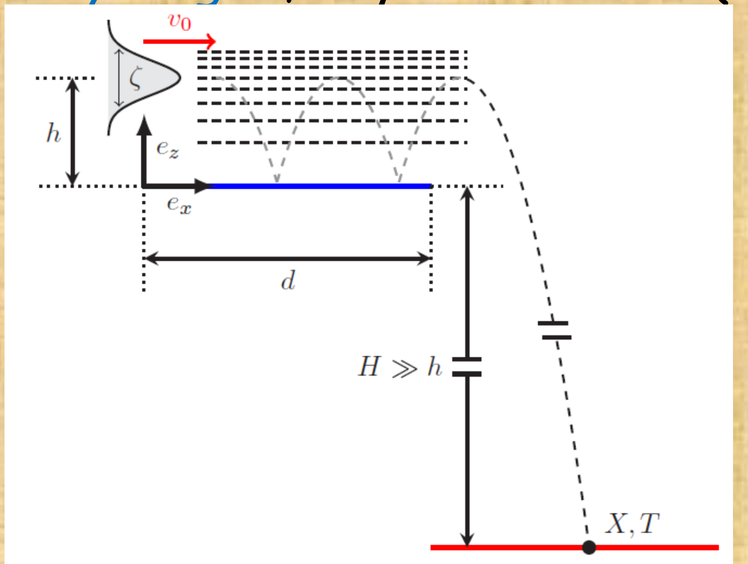
One found way out of all these problems:
instead of neutrons, use **hydrogen atoms** (GRASIAN).

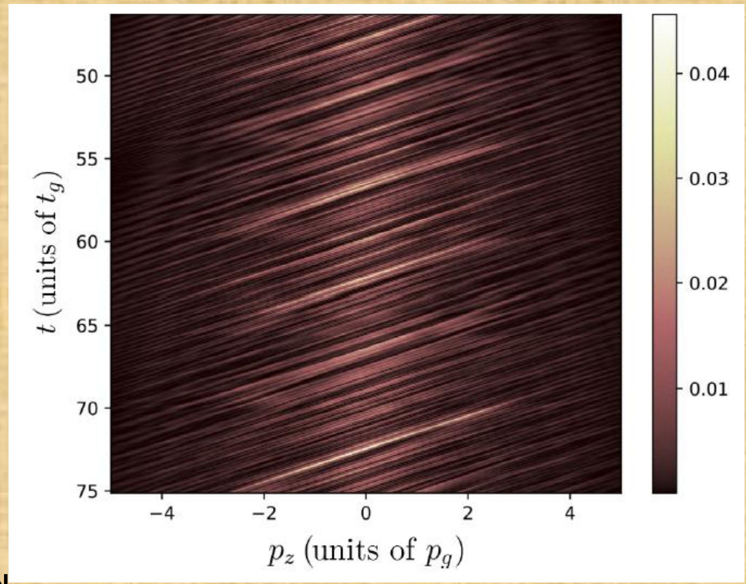
- the **density** of hydrogen atoms in the phase space can be many orders of magnitude higher, which solves the problem of statistical sensitivity,
- it is easier to implement the trapping of atoms in the horizontal direction ([V.V. N. et al, *A magneto-gravitational trap for studies of gravitational quantum states*, Europ. Phys. J. C 80 (2020) 520], [J. Ahokas, et al, *A large octupole magnetic trap for research with atomic hydrogen*, Rev. Sci. Instr. 93 (2022) 023201]), thus increasing the **observation time** by orders of magnitude and solving the problem of systematic effects.

An option for a very **competitive** search for fundamental interactions at distances of **~1-10 μm**.

Even the **density of antihydrogen** atoms in phase space can be much larger than typical UCN densities.

Provided that **100-1000 antihydrogen atoms** were produced in the GBAR experiment ([P. Perez et al, *The GBAR antimatter gravity experiment*, Hyperf. Inter. 233 (2015) 21]), a precise and systematics-free measurement of **g** for them seems to be straightforward … [P.P. Crepin et al, *Quantum interference test of the equivalence principle on antihydrogen*, Phys. Rev. A 99 (2019) 042119]





NEUTRONS FOR SCIENCE

What about extra fundamental interactions at **shorter distances**?

Whispering Gallery!

"Any" acceleration magnitude and better (than with mirror acceleration or magnetic field gradient) control over it.

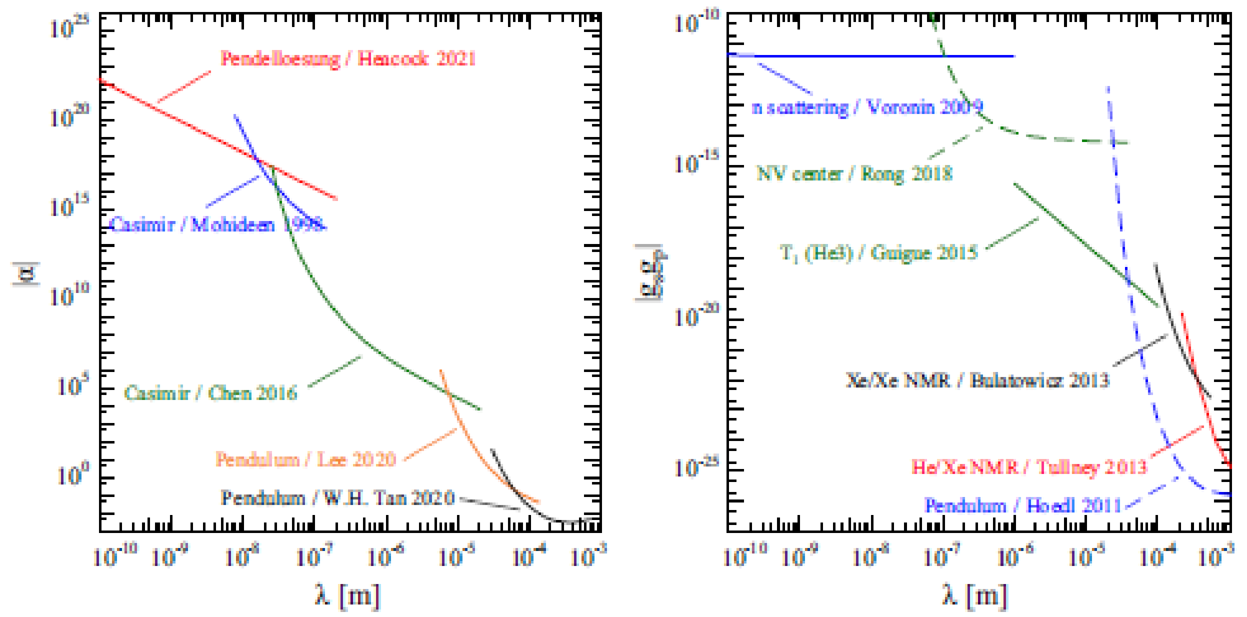


Figure 8: Left: Current most constraining limits for new spin-independent short-range forces. The first experiment (red line, [179]) uses free neutrons. At higher λ ranges, other techniques and probes are more sensitive [180, 181, 182, 183]. Right: Current most constraining limits for new spin-dependent interactions. Experiments [184, 185, 186, 187] use the neutron as the polarized particle, for the first of those it is a free neutron. In [188, 189], the polarized particle is an electron.

The **shape of a curved mirror** is not known "exactly". But there is a good test experiment to control it: **X-ray whispering gallery** (no short-range forces and much higher statistical sensitivity). Very **good experimental data** are being analyzed now.

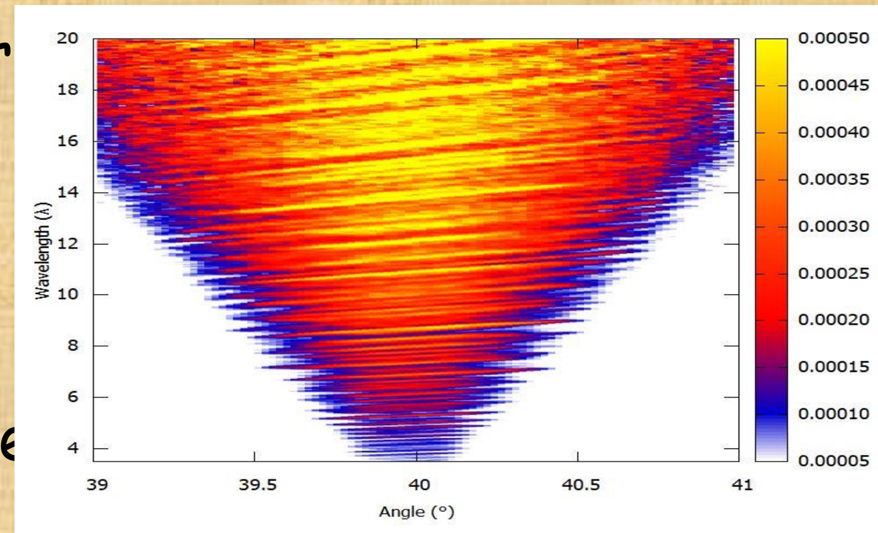
Excellent result from WGS experiments with both **polarized and unpolarized neutrons**.

Competitive (best) constrains for characteristic distances of **~10 nm**.

Significant potential for a **further increase in sensitivity** (no fundamental limitation of sensitivity improvement is found yet).

On a longer time-scale, it is possible to replace neutrons with **hydrogen atoms or heavier atoms** (GRASIAN) to further increase the sensitivity.

Very well defined interference patter



What will happen to them if we add
a gradient of a magnetic field or
a **gravitational field** in a direction
(close to) perpendicular to the surfac

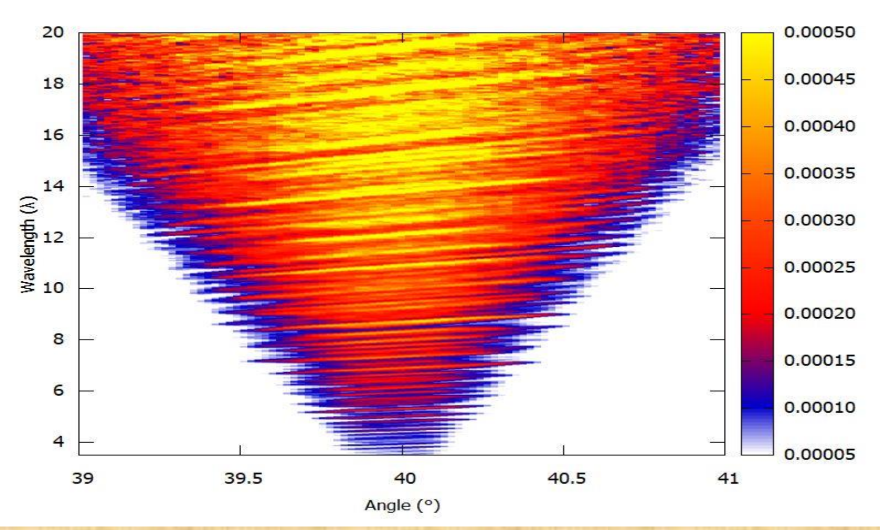
The interference pattern is deformed (shifted).

As always in interferometric experiments, a **high sensitivity to additional interactions** is expected.

Is it possible to measure this effect, for example, with **neutrons**?

In order of magnitude, the relative shift of the interference pattern lines is equal to the ratio of **g** to the centrifugal acceleration, or **~10^{-6}** (I don't want to bore you with coefficients of the order of one).

A typical statistical accuracy for neutrons with the velocity of **~10^3 m/s** is 1/N x ΔS/S **~10^{-5}**, where N~10^2 (the number of interference lines) and ΔS/S is a typical statistical sensitivity for a line position.



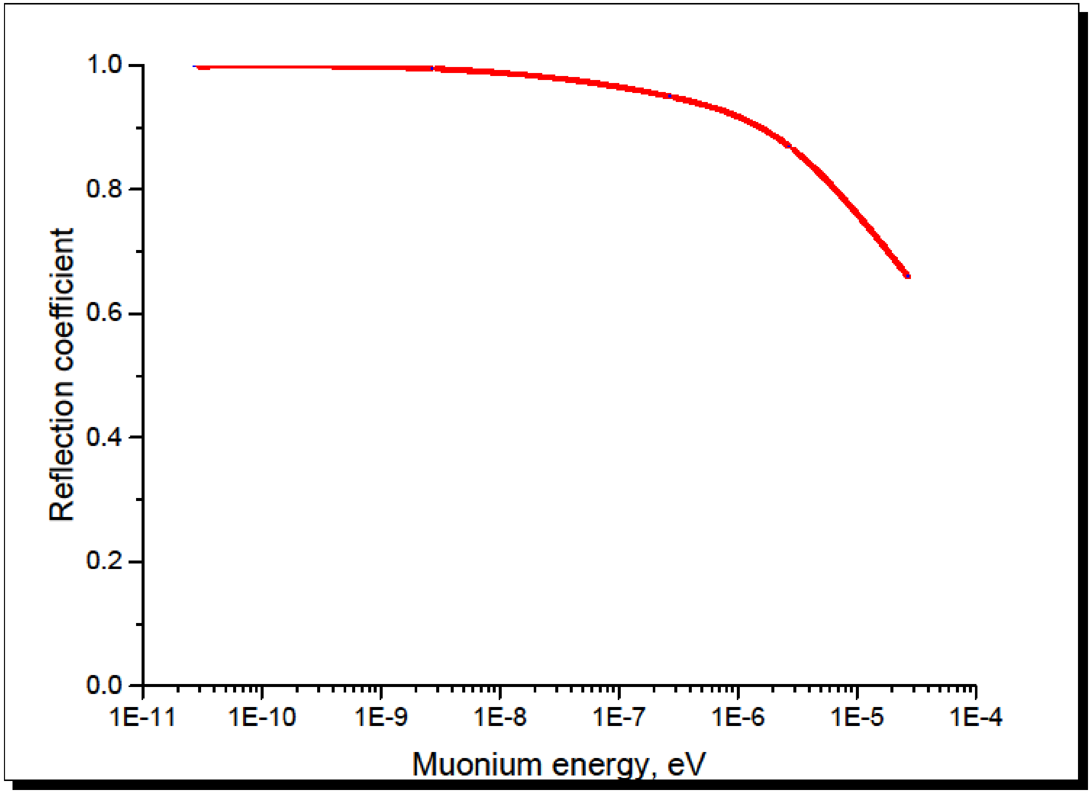
The **sensitivity** on top of the interference pattern (the slowest neutrons in the standard spectrum of cold neutrons) is already **sufficient**!

In a typical neutron experiment with a silicon mirror, the **radius** of the mirror was equal $R = 2.5\ cm$. The radium is chosen so to provide **a few sub-barrier WGS**. The **energy of WGS** is $E_{WGS} = \frac{\hbar}{\tau_{WGS}}$, where $\hbar$ is the reduced Planck constant, and τ_{WGS} the characteristic **time of formation of WGS** equal $\tau_{WGS} = \sqrt[3]{\frac{2\hbar}{ma^2}}$, where m is the neutron mass and a is the centrifugal acceleration equal $a = \frac{V^2}{R}$, where V is the neutron longitudinal **velocity**. The characteristic size of WGS is $l_{WGS} = \sqrt[3]{\frac{\hbar^2}{2am^2}}$.

Can a similar measurement be performed with **muonium**?

In order to evaluate such an experiment, we will use the same formulas but apply the following **scaling factors**: the ratio of **particle masses** is $\frac{m_{neutron}}{m_{muonium}} \sim 8.9$. First, we estimate **the length** of the cylindrical mirror surface L, which corresponds to the condition that most muonium atoms disappear $L = 3 \cdot V_{muonium} \tau_{muonium} \sim 4\ cm$, where $V_{muonium} = 6300 \pm 200\ m/s$ is the **muonium velocity** and $\tau_{muonium} \sim 2.2\ \mu s$ the **muon's lifetime**. Note about the same length as it is for neutrons, however, the need to use another detection scheme (annihilation on the mirror surface). A **spatial resolution** needed should be significantly better than $3 \cdot V_{muonium} \tau_{muonium}$, i.e. a fraction of millimeter.

The reflectivity of muonium from a helium surface is shown here.



The effective **potential height** is $E_{QR,muonium} \sim 1\ \mu eV$.

In order to have a few WGS, the centrifugal acceleration has to be $a_{muonium} \sim 1.5 \cdot 10^9 g$ and the mirror radius $R_{muonium} \sim 2.7\ cm$ (close to neutron experiments). Statistics is also comparable to that in neutron experiments.

Thus, WGS of muonium can be observed. A gravitational shift probably not, due to the **too high velocity** $V_{muonium} = 6300 \pm 200\ m/s$.

Conclusion

The methods of WGS and GQS have passed a long way: from an issue of **textbooks** to the **observation** with UCNs and possible **extension** to **other particles** (**atoms**, **antiatoms**, etc), also to the **applications** (**studies of gravitational interaction**, **short-range interactions**, physics **beyond Standard Model**, **extensions of quantum mechanics**, **surface studies**, **spectrometry**, etc).

Interesting results have been obtained using these methods; Perspectives for further **progress are clear**.

A gravitational shift of WGS of neutrons **can and should** be measured; WGS of muonium **can** be measured; to measure a gravitational shift of WGS of muonium, the muonium **velocity** has to be **further reduced** by an order of magnitude.

Simulation

I simulated (approximately) an interference pattern that can be measured using the current parameters of the muonium beam. The result is about the following.