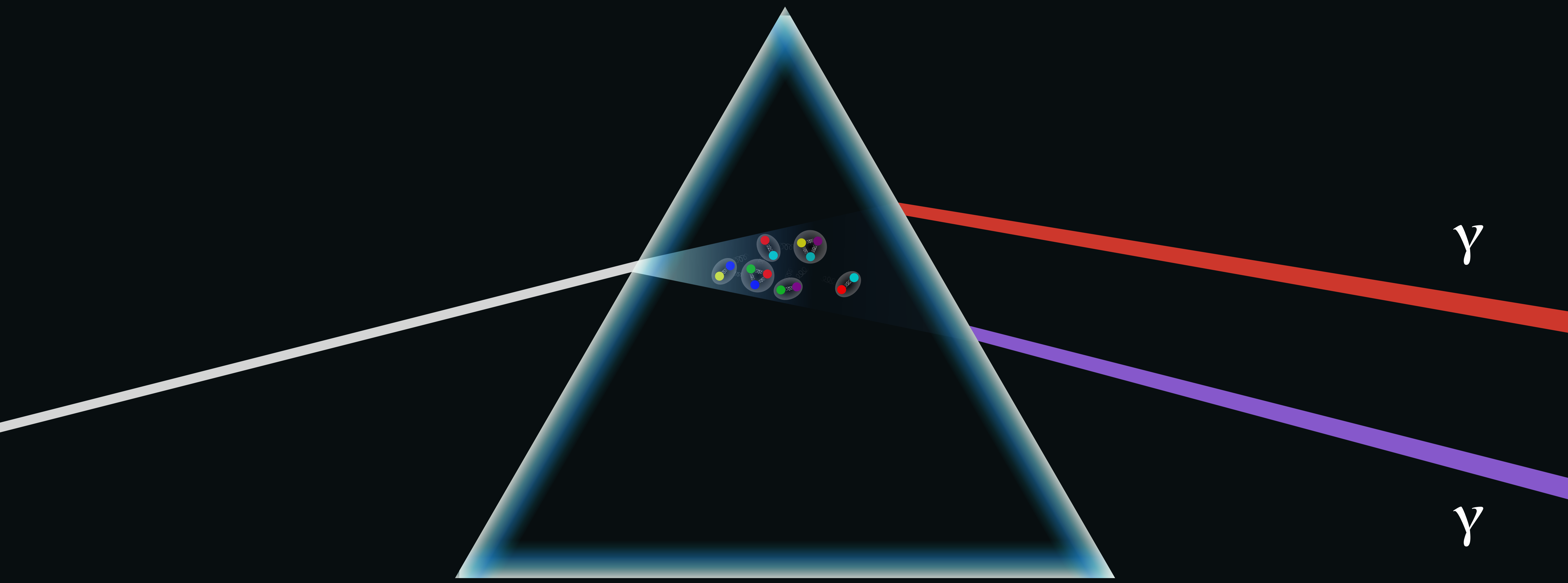
The Dark Side of the Proton



Franziska Hagelstein (JGU Mainz & PSI Villigen)

in collaboration with
Volodymyr Biloshytskyi, Vadim Lensky and Vladimir Pascalutsa

The Standard Model and Beyond

On the "bright" side

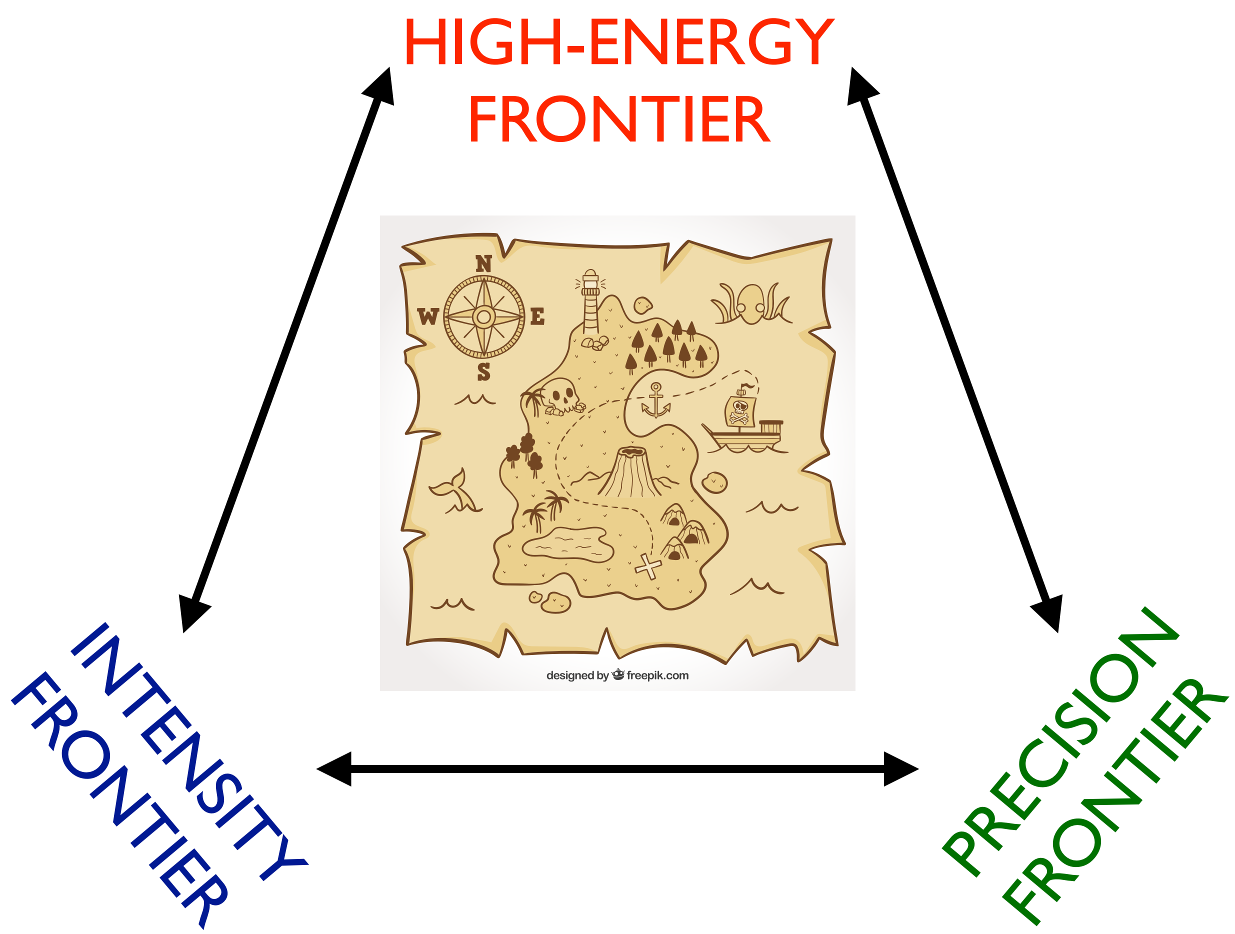
- Standard Model of Particle Physics (SM) — very successful
- Proton — difficult but calculable

On the "dark" side

- Evidence for New Physics — Physics beyond the SM (BSM)
- Hadrons (e.g., the proton) are often involved in "puzzles" — disagreement between SM predictions and experiments

NEW PHYSICS SEARCHES

- **Freeman Dyson:** *"If you look for nature's secrets in only one direction, you are likely to miss the most important secrets"*



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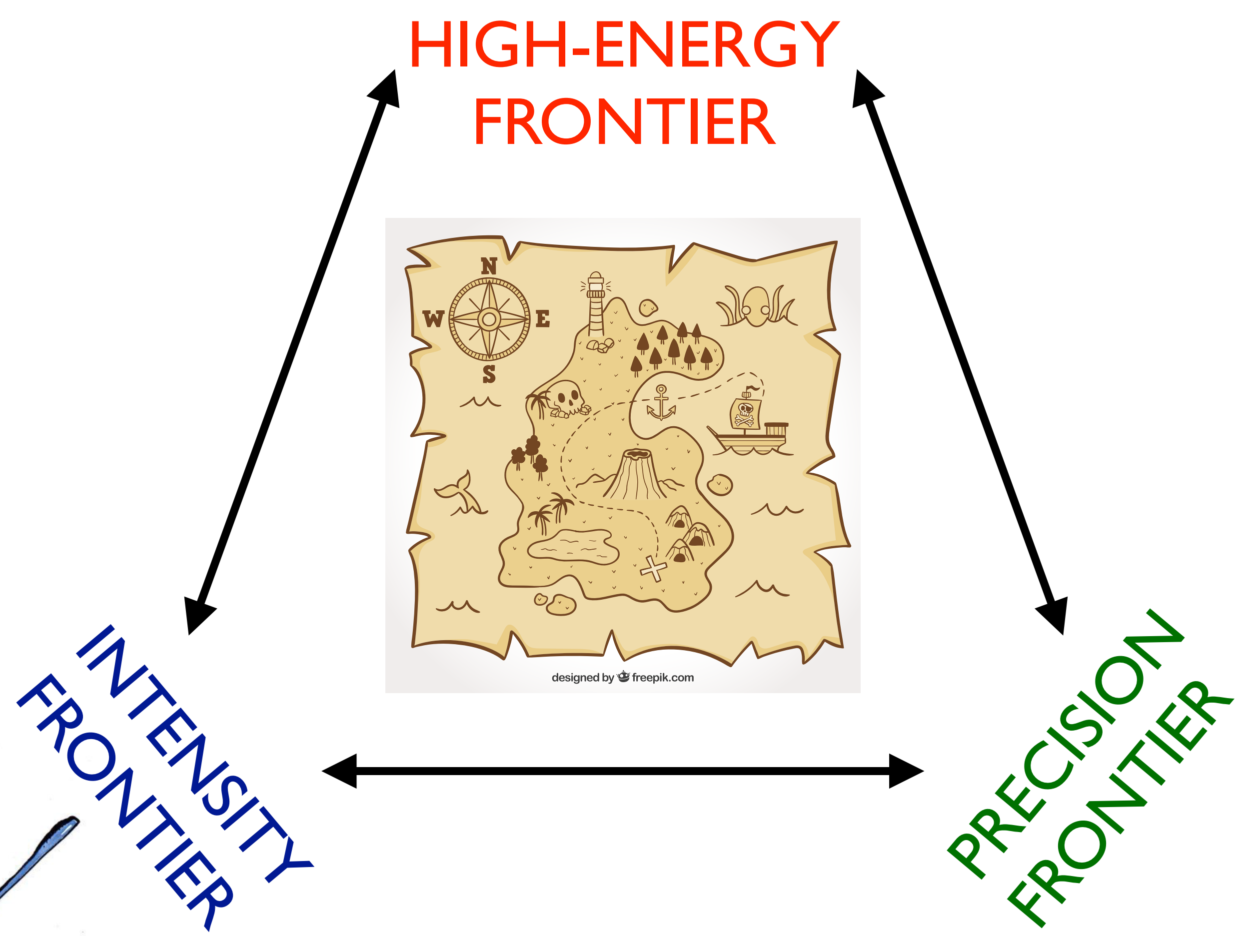

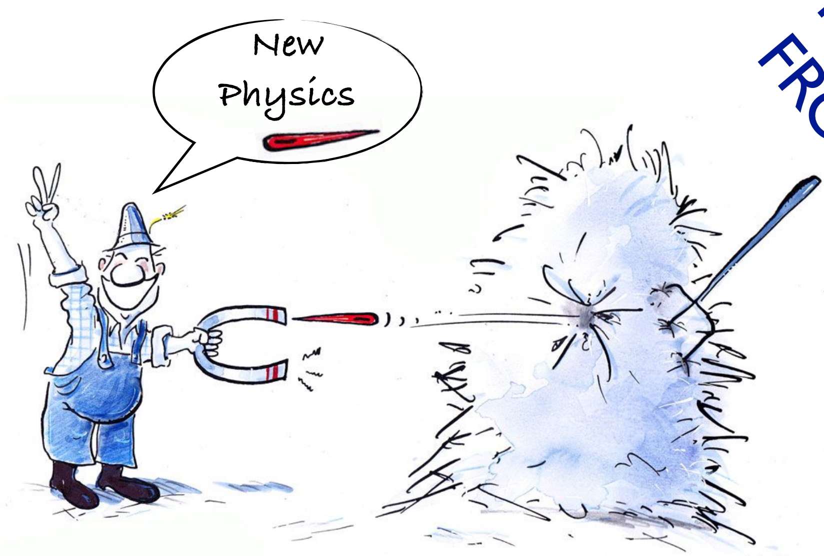

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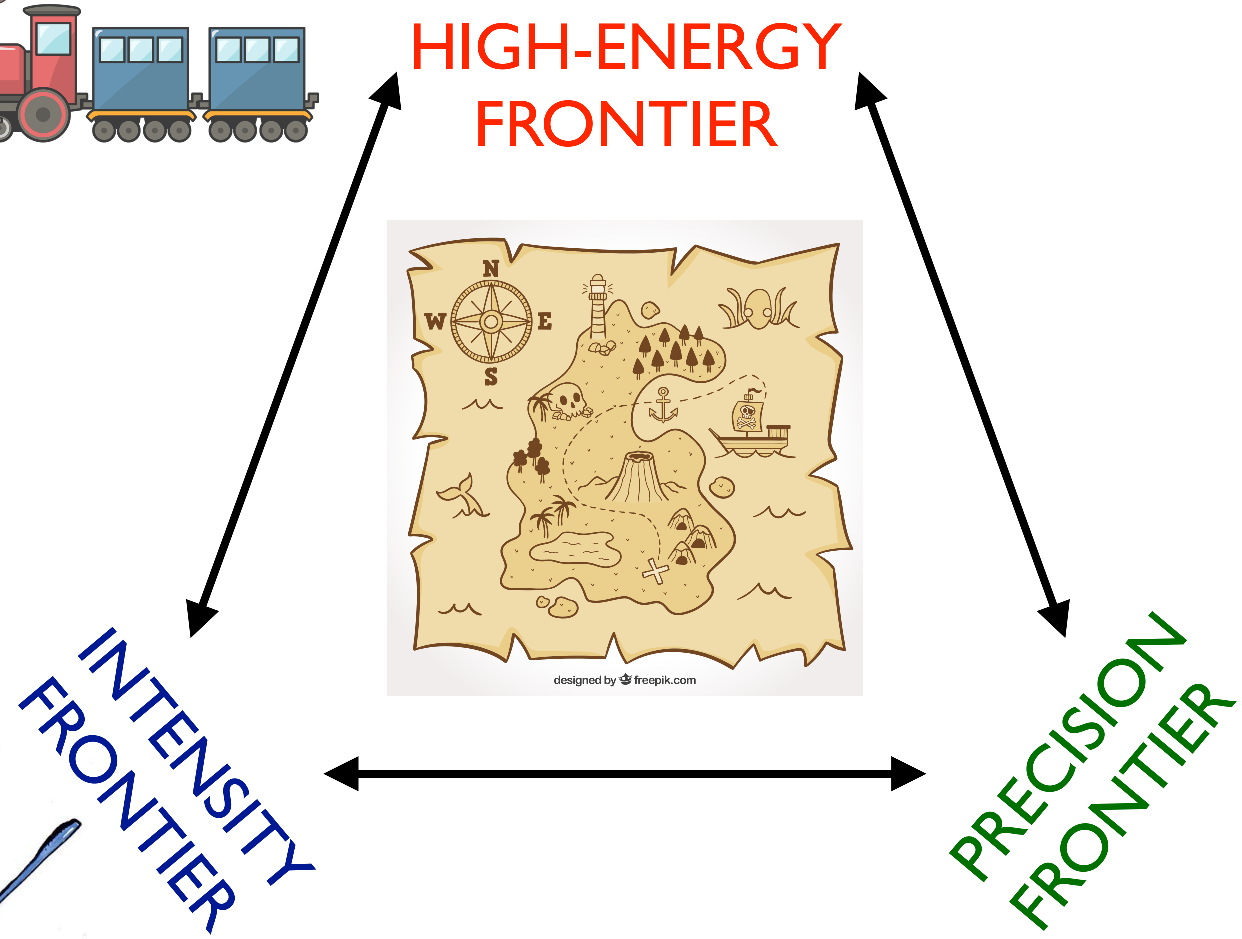
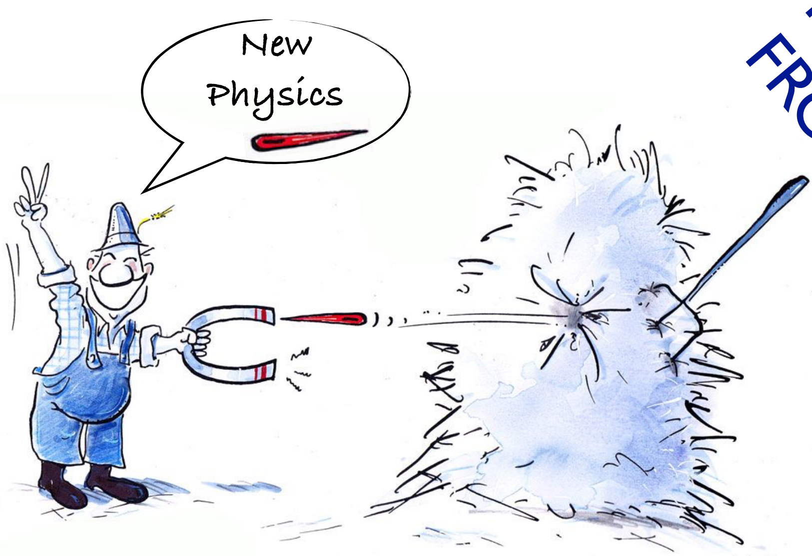
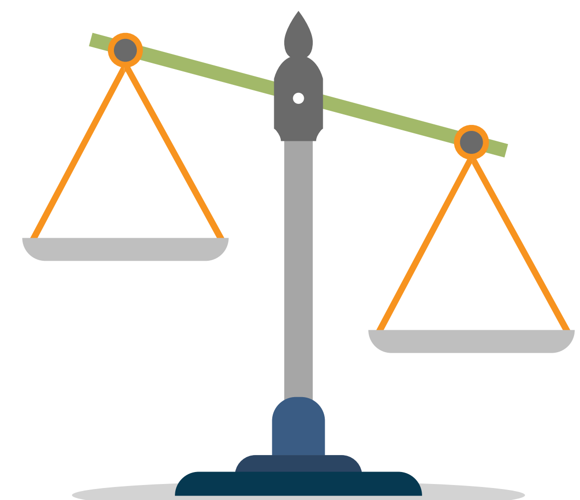




NEW PHYSICS SEARCHES

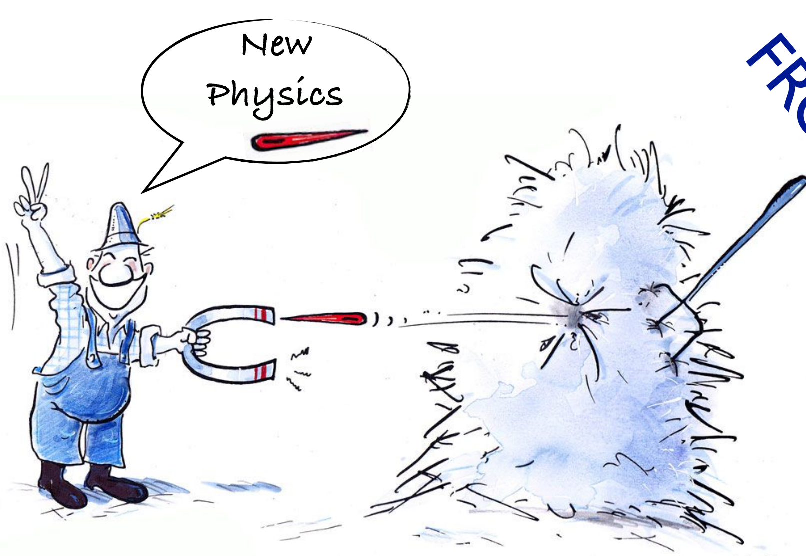
- **Freeman Dyson:** *"If you look for nature's secrets in only one direction, you are likely to miss the most important secrets"* … *"**For making important discoveries, high accuracy was more useful than high energy.**"*

SM prediction vs. experimental observation

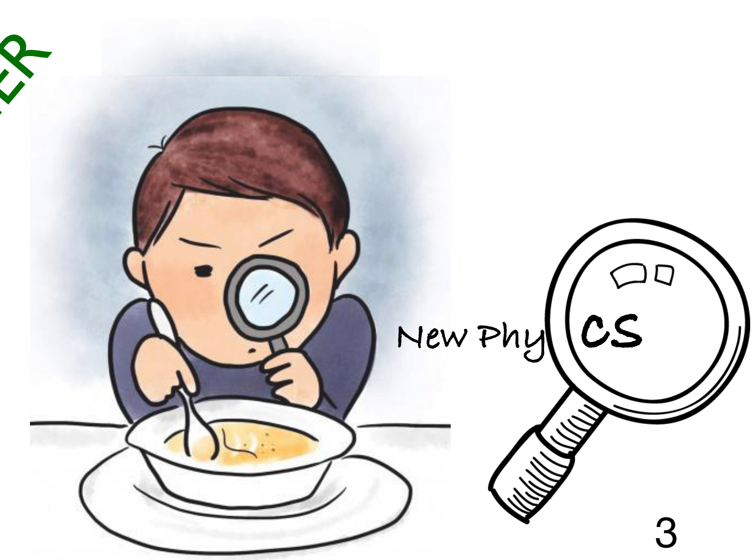


HIGH-ENERGY FRONTIER

INTENSITY FRONTIER

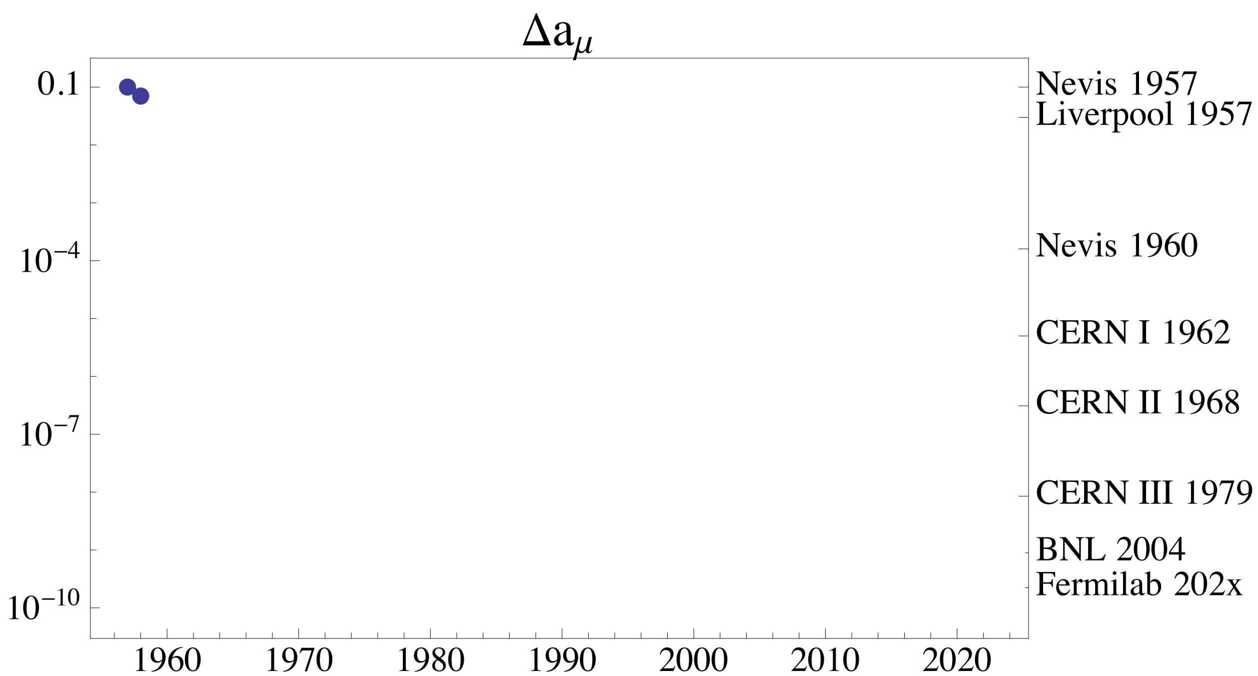


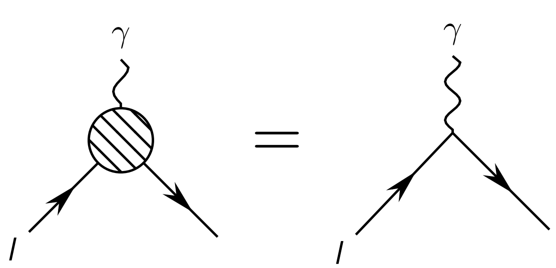
PRECISION FRONTIER



MUON g-2

see talk by
Peter Stoffer





- Anomalous magnetic moment of the muon:
  $a_\mu = (g-2)/2 = 0.0011659181$

MUON g-2

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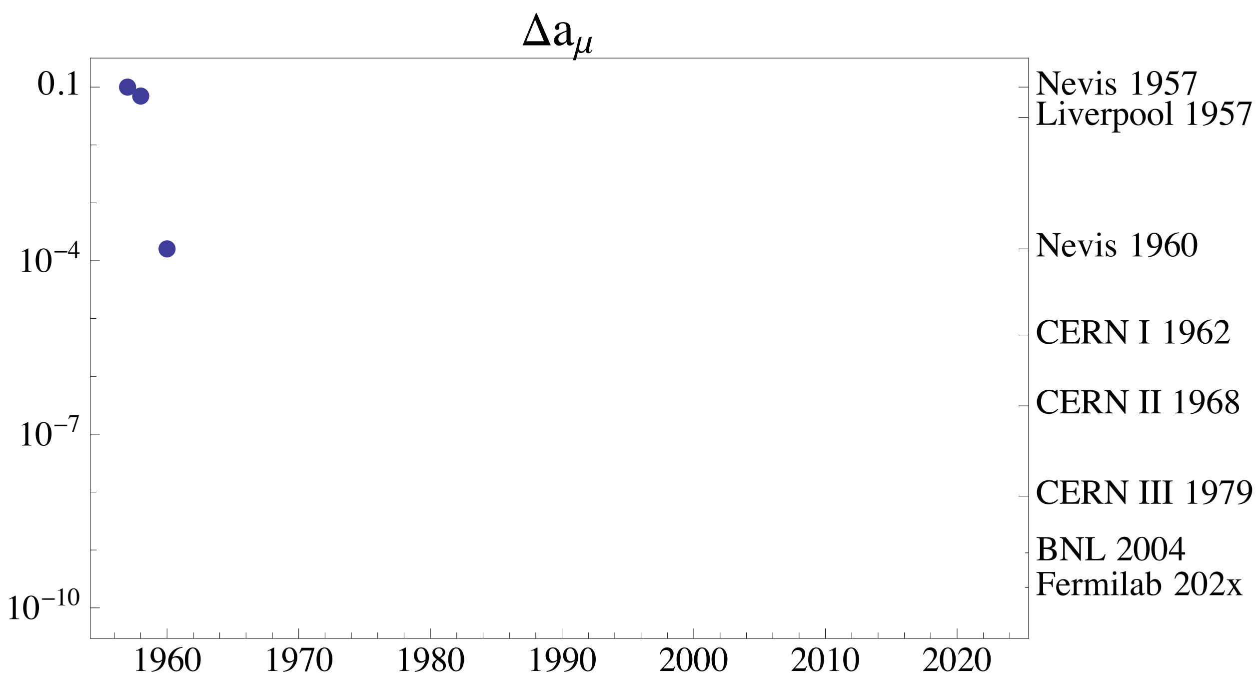
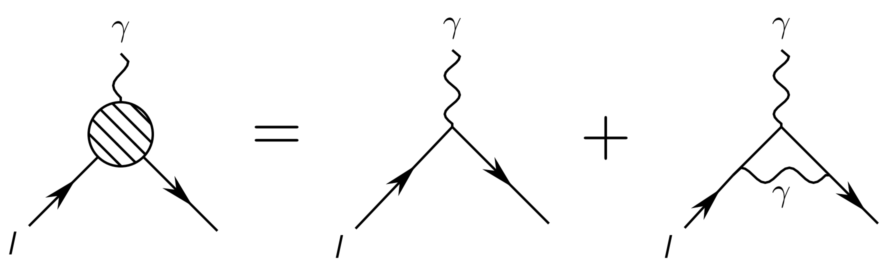


Schwinger term
$a_\mu = \alpha/2\pi$



1-loop QED [1 diagram]

MUON g-2

see talk by
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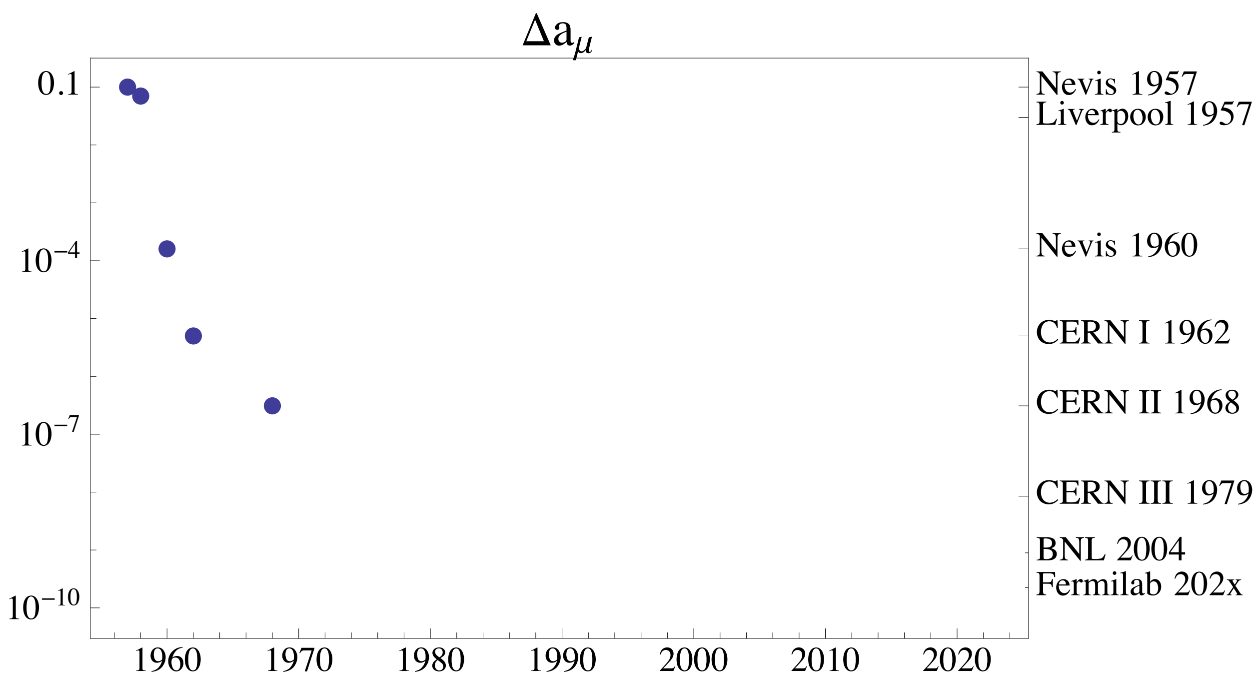

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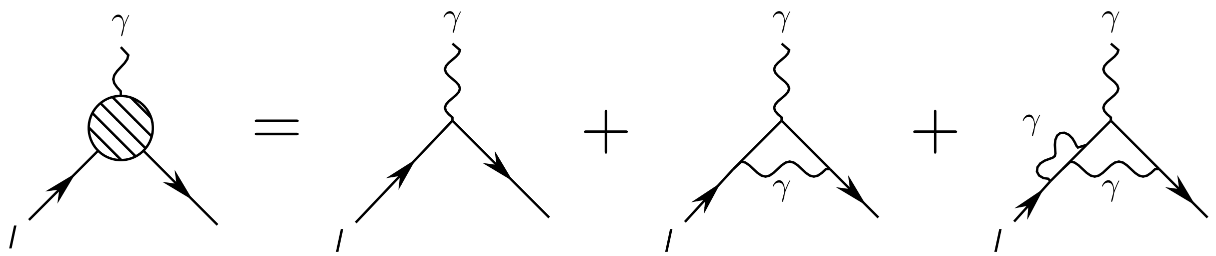

1-loop QED [1 diagram]
2-loop QED [7 diagrams]
3-loop QED [72 diagrams]

MUON g-2

see talk by
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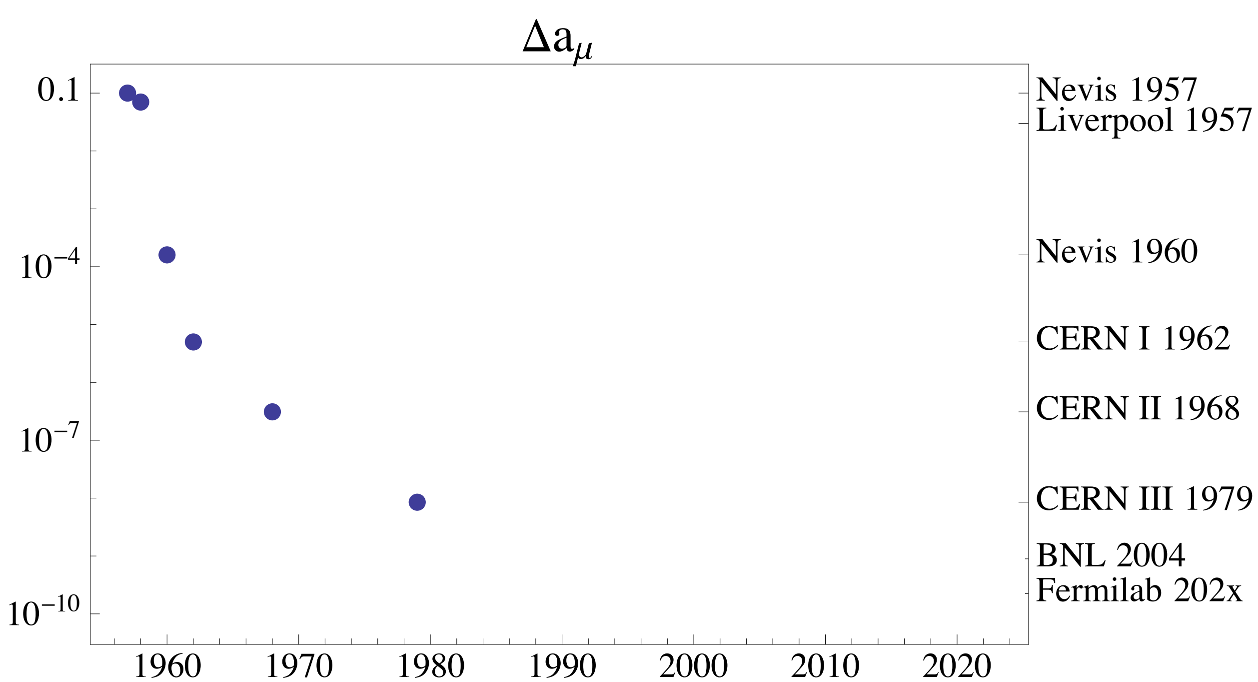

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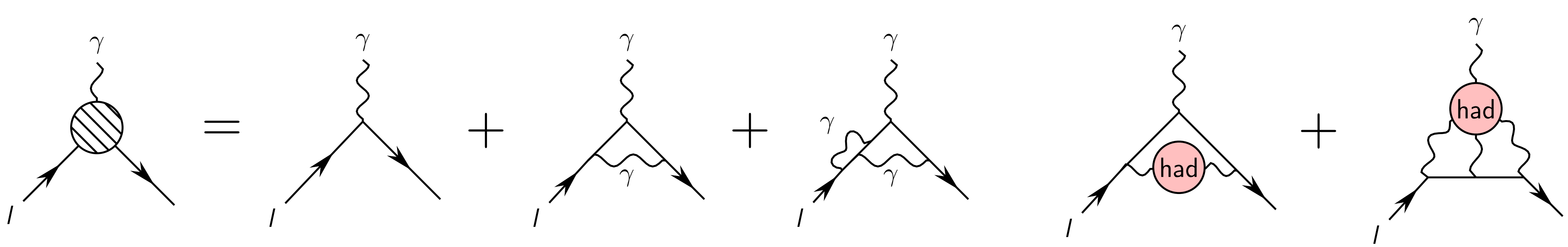

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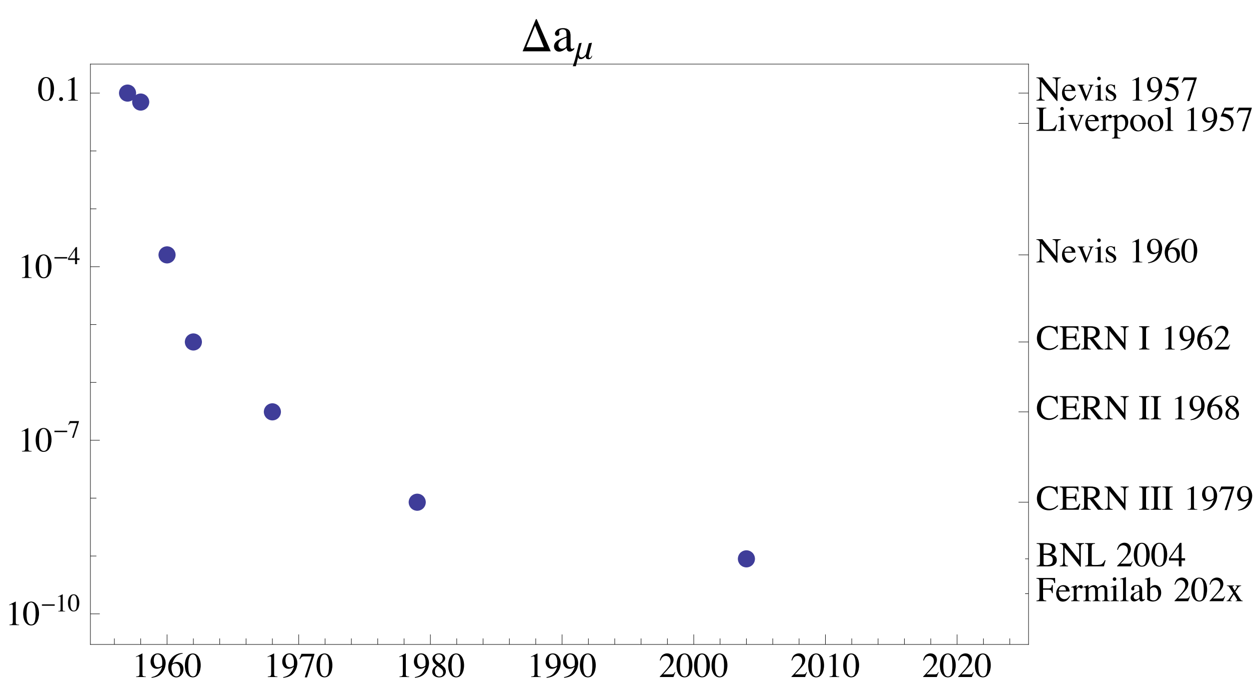

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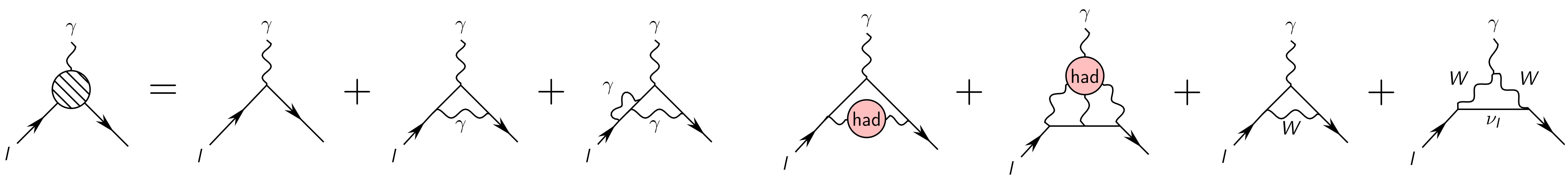

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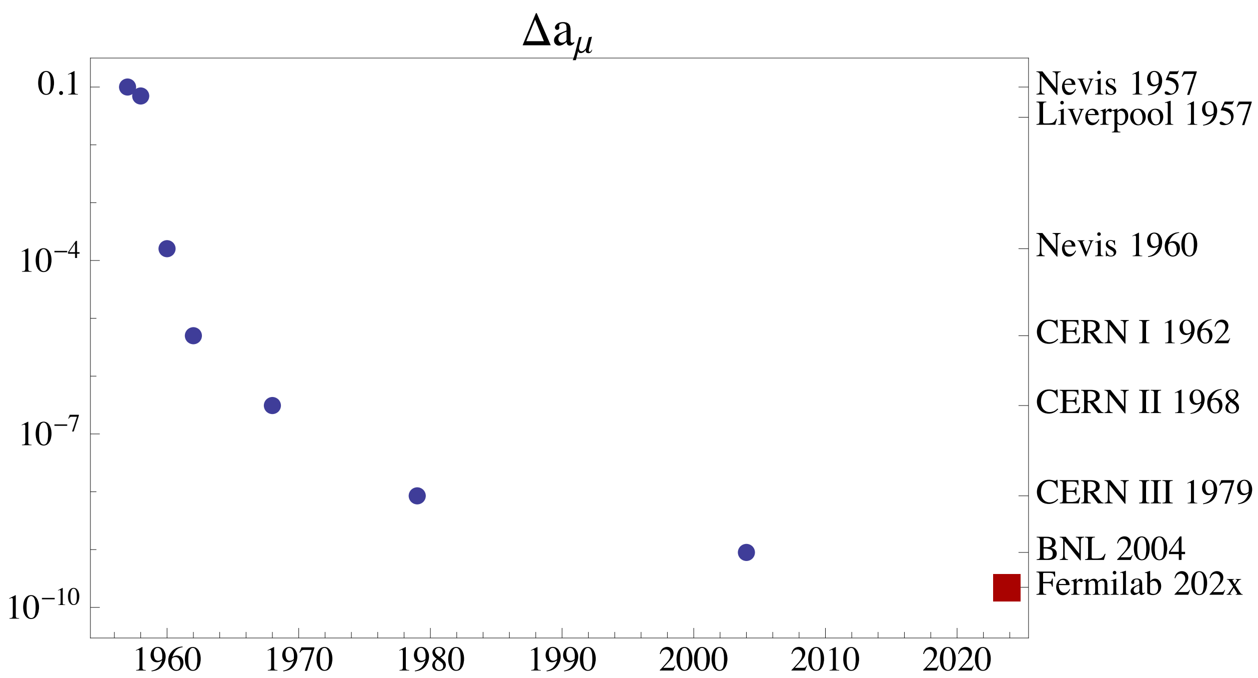
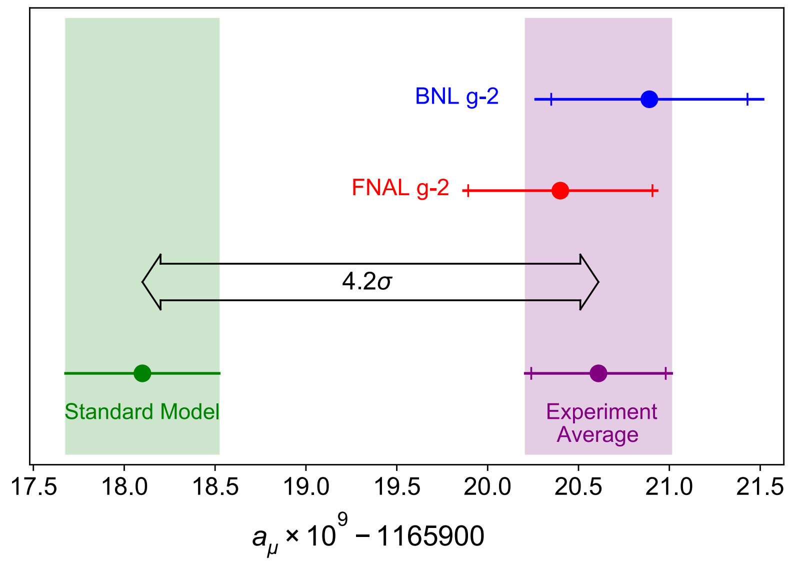
4-loop QED [891 diagrams]
5-loop QED [12 672 diagrams]

MUON g-2

see talk by Peter Stoffer



- Anomalous magnetic moment of the muon: $a_\mu = (g-2)/2 = 0.0011659181$
- Factor of 2 reduction of experimental uncertainty expected



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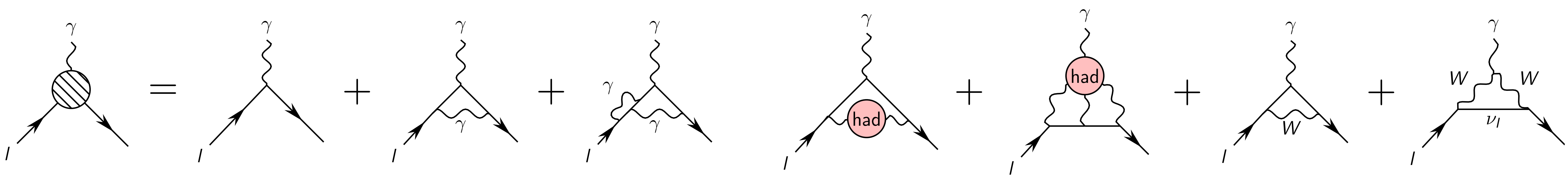

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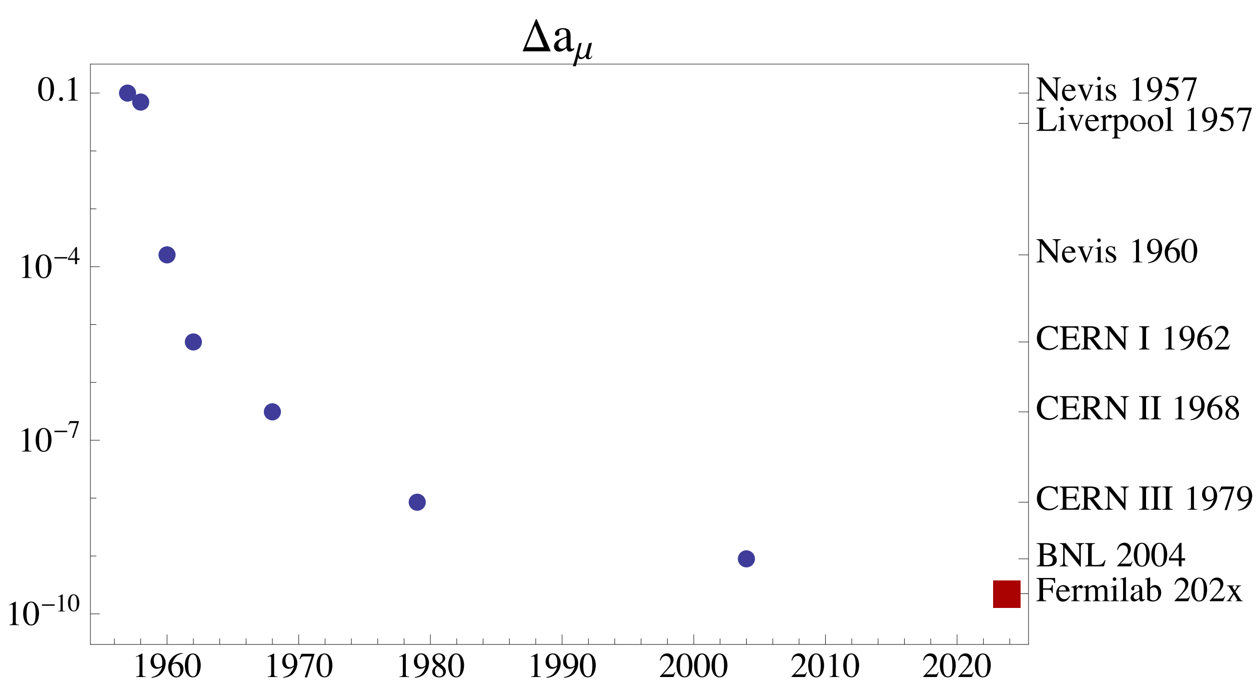

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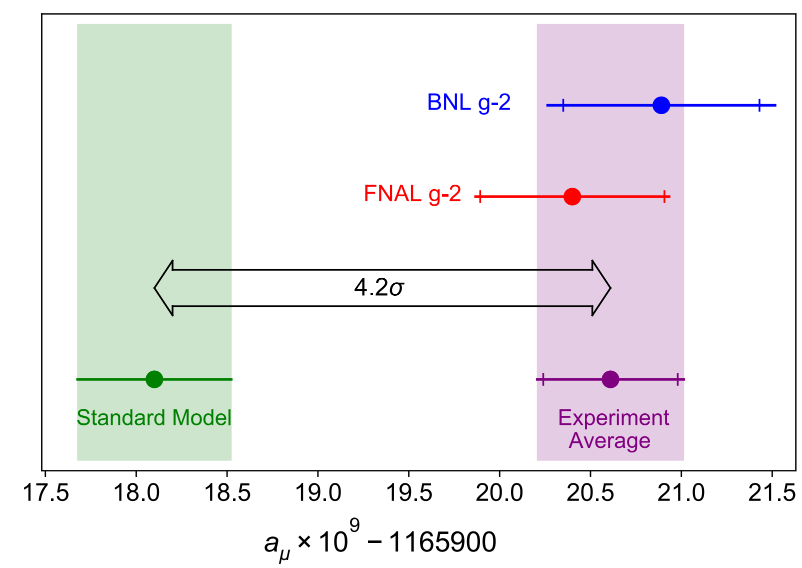

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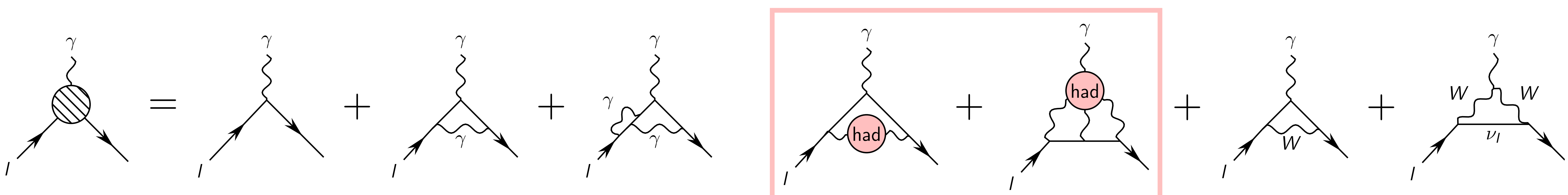

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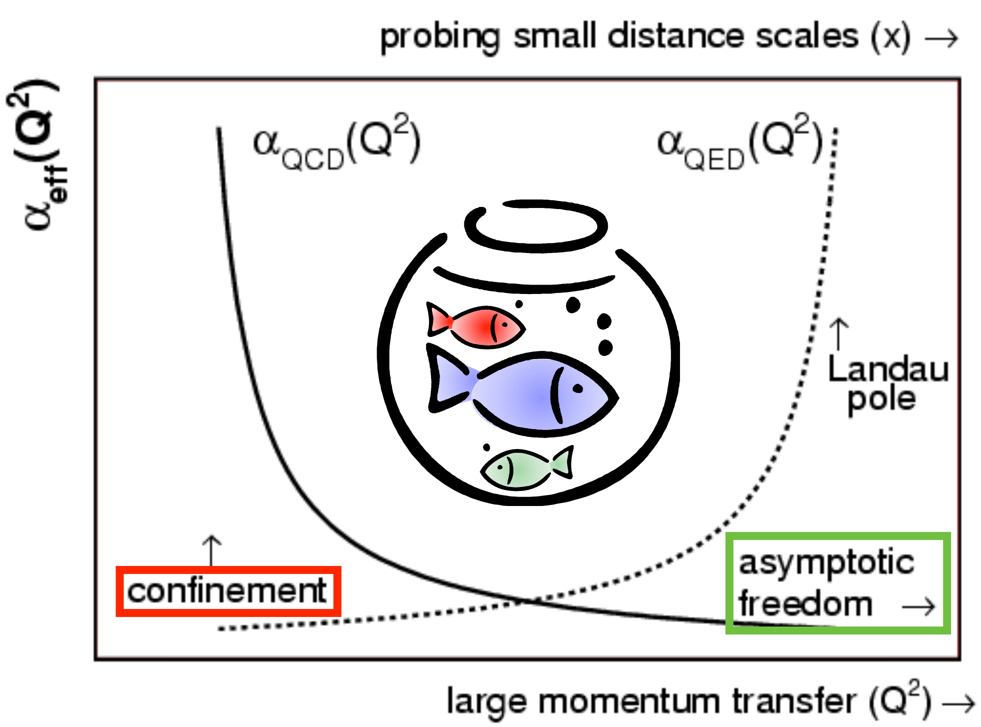
5-loop QED [12 672 diagrams]

- *Mismatch implies "New Physics" or insufficient understanding of the SM!*
- *SM prediction has to improve yet again!*

HADRONIC CORRECTIONS

- QCD is non-perturbative at low energies, due to strong coupling

running of the coupling constant

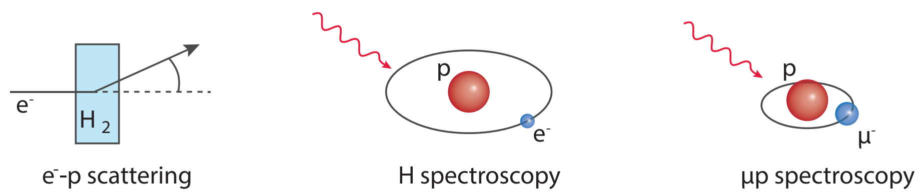


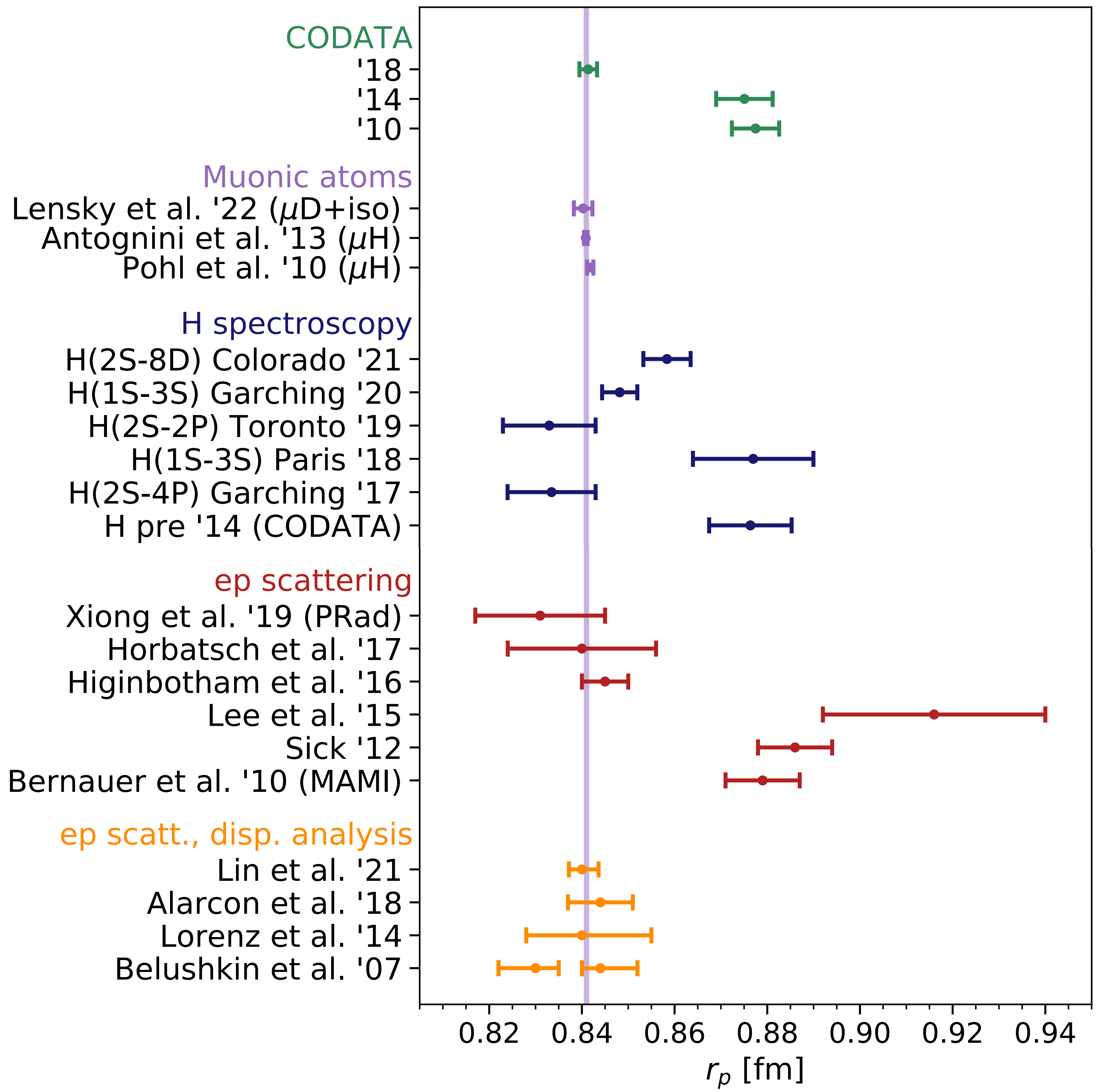
- Hadronic corrections are challenging to calculate:
  - Effective field theories, e.g., chiral perturbation theory (ChPT) or pionless EFT
  - Lattice QCD
  - Dispersion relations (unitarity, causality) — basis for data-driven evaluations

The Puzzling Proton

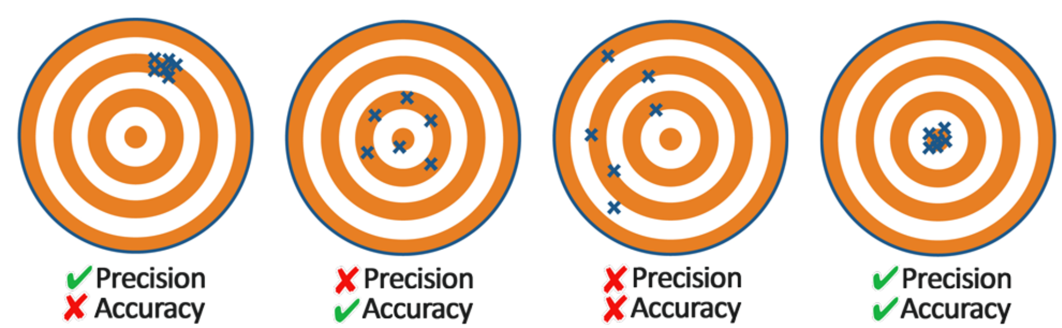
https://www.sciencenews.org

PROTON RADIUS PUZZLE





- Muonic atoms allow for PRECISE extractions of nuclear charge and Zemach radii
- CODATA since 2018 included the μH result for r_p
- Still open issues: H(2S-8D) and H(1S-3S)
- Precise and accurate!



LAMB SHIFT IN MUONIC ATOMS

	THEORY		EXPERIMENT	
	$\Delta E_{TPE} \pm \delta_{theo}\,(\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
μH	$33\ \mu$eV $\pm\ 2\ \mu$eV	Antognini et al. (2013)	$2.3\ \mu$eV	Antognini et al. (2013)
μD	$1710\ \mu$eV $\pm\ 15\ \mu$eV	Krauth et al. (2015)	$3.4\ \mu$eV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV $\pm$ 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV $\pm$ 0.25 meV -0.15 meV $\pm$ 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

μH: present accuracy comparable with experimental precision

μD, μ³He⁺, μ⁴He⁺: present accuracy factor 5-10 worse than experimental precision

$$r_p = 0.84087(12)_{\rm sys}(23)_{\rm stat}(29)_{\rm theory}\ \ {\rm fm}$$

(25) 2PE (mainly subtraction term)
(15) QED

$$r_d = 2.12562(5)_{\rm sys}(12)_{\rm stat}(77)_{\rm theory}\ \ {\rm fm}$$

basically only nuclear 2PE

$$r_\alpha = 1.67824(2)_{\rm sys}(13)_{\rm stat}(82)_{\rm theory}\ \ {\rm fm}$$

(70) 2PE (elastic 25, nuclear inelastic 36, nucleon inelastic 56)
(42) 3PE (inelastic contribution missing)
(4) QED

FROM PUZZLE TO PRECISION

- Several experimental activities ongoing and proposed:
  - 1S hyperfine splitting in μH (ppm accuracy) and μHe
  - Improved measurement of Lamb shift in μH, μD and μHe^+ possible ($\times 5$)
  - Medium- and high-Z muonic atoms
- **Theory Initiative** is needed!

Muonic Atom Spectroscopy Theory Initiative

- “Muonic atoms @ PSI2022” / PSI, October 2022
- “Nucleon Structure at Low-Q” / Crete, May 2023
- “PREN & μASTI” / JGU Mainz, June 2023
- Initials objectives:
  - Accurate theory predictions for light muonic atoms to test fundamental interactions by comparing to electronic atoms
  - Community consensus on SM predictions
  - Emphasis on the hyperfine splitting in μH
- Steering committee: Aldo Antognini, Carl Carlson, Franziska Hagelstein, Paul Indelicato, Krzysztof Pachucki, Vladimir Pascalutsa

Homepage and mailing list → https://asti.uni-mainz.de

JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Atomic Spectroscopy Theory Initiative

HOME SEARCH SITEMAP

HOME AIMS AND SCOPE MAILING LIST WORKING GROUPS PAST AND FUTURE WORKSHOPS PUBLICATIONS NEWS

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- Aims and Scope
- Mailing List
- Working Groups
- Past and Future Workshops
- Publications
- News

Muonic Atom Spectroscopy Theory Initiative

Inspired by the success of the Muon g-2 Theory Initiative we are launching the Muonic Atom Spectroscopy Theory Initiative (μASTI).

The initiative aims to support the experimental effort on the spectroscopy of light muonic atoms by improving the Standard Model theory predictions for the Lamb shift and hyperfine splitting in muonic hydrogen, deuterium, and helium, in order to match the anticipated accuracy of future measurements. An initial focus will be on the ground state hyperfine splitting in muonic hydrogen.

The **upcoming kick-off event** for the Theory Initiative is organized as a joint meeting with the Proton Radius European Network (PREN) at the Johannes Gutenberg University Mainz (June 26-30, 2023).

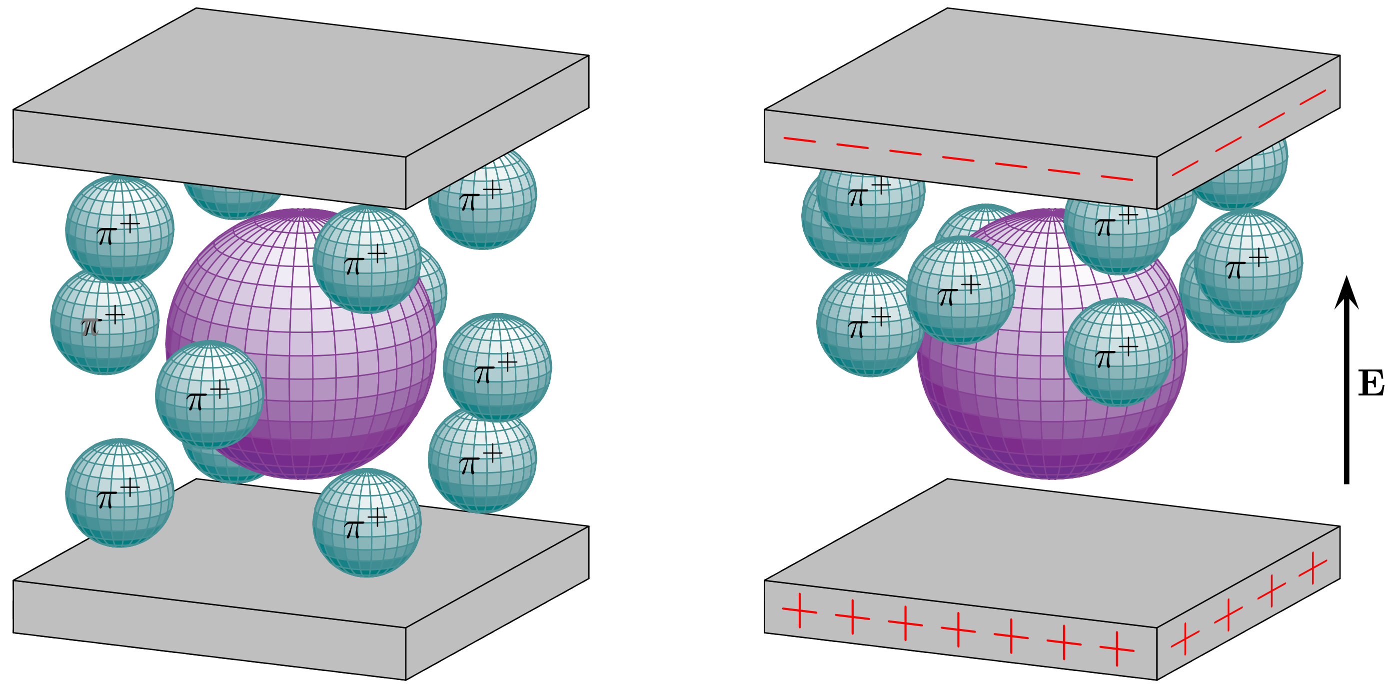
from PUZZLE
ASTI
to PRECISION

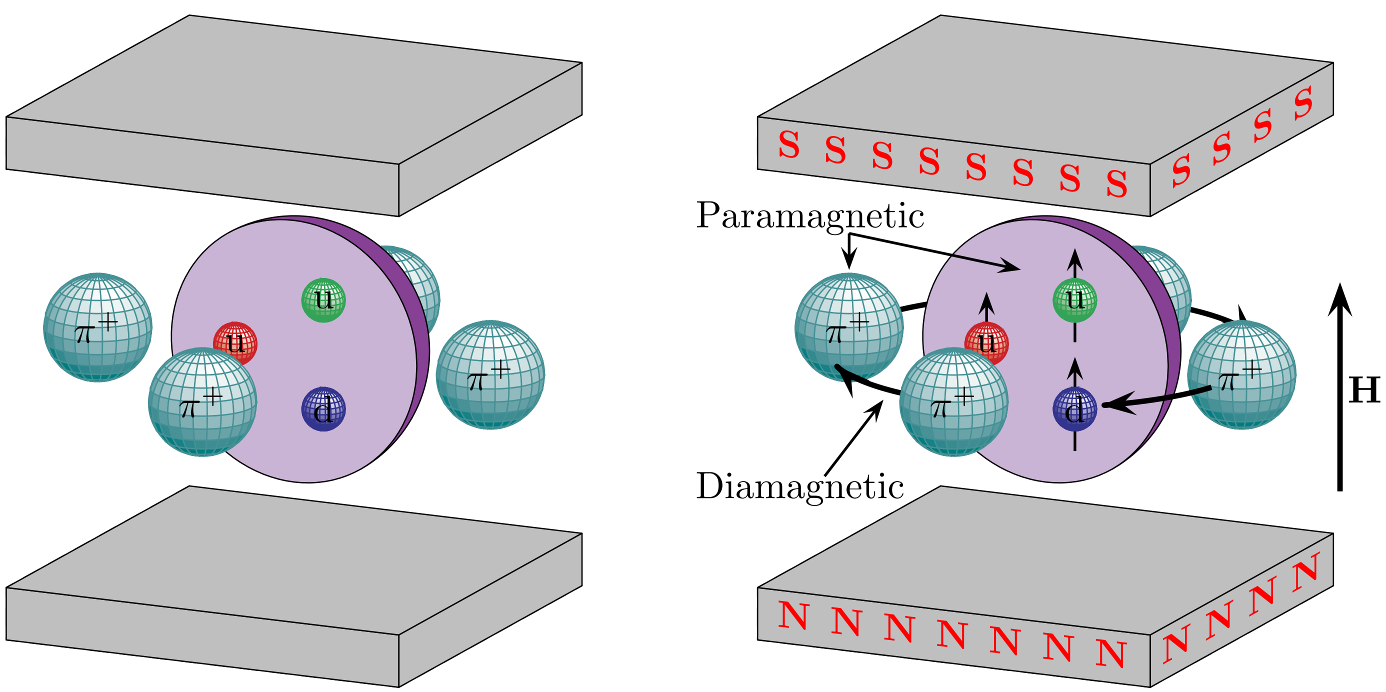
POLARIZABILITIES

Electric dipole polarizability:

$$\vec{P} = \alpha_{E1}\,\vec{E}$$

induced electric dipole polarization





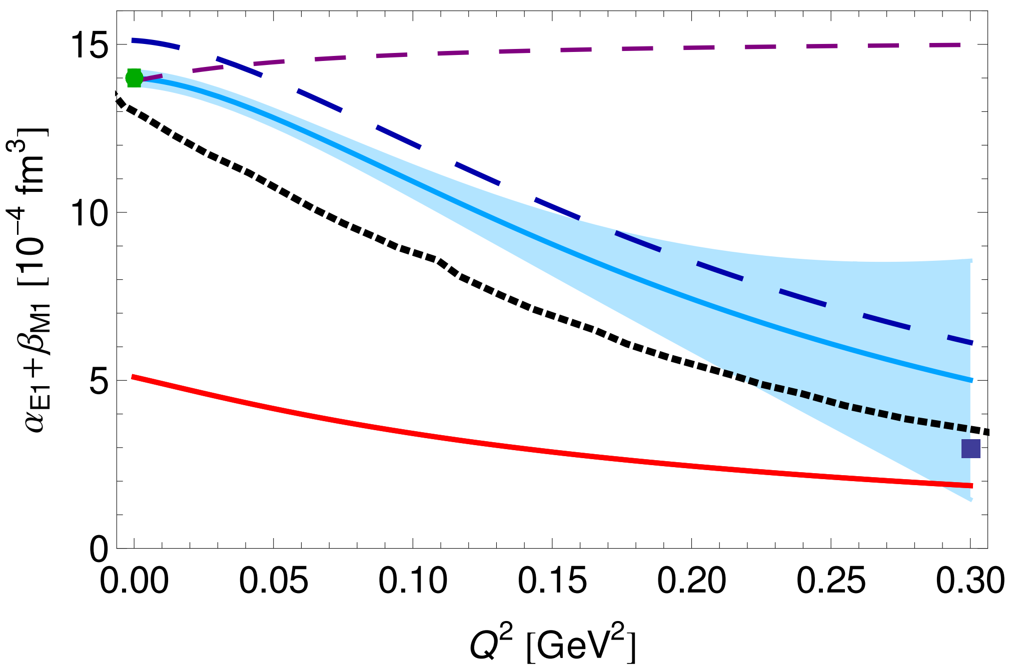
Magnetic dipole polarizability:

$$\vec{P} = \beta_{M1}\vec{H}$$

polarization induced by magnetic field

diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$

DIAMAGNETIC OR PARAMAGNETIC ?



NLO BChPT
LO BChPT
LO HBChPT
MAID

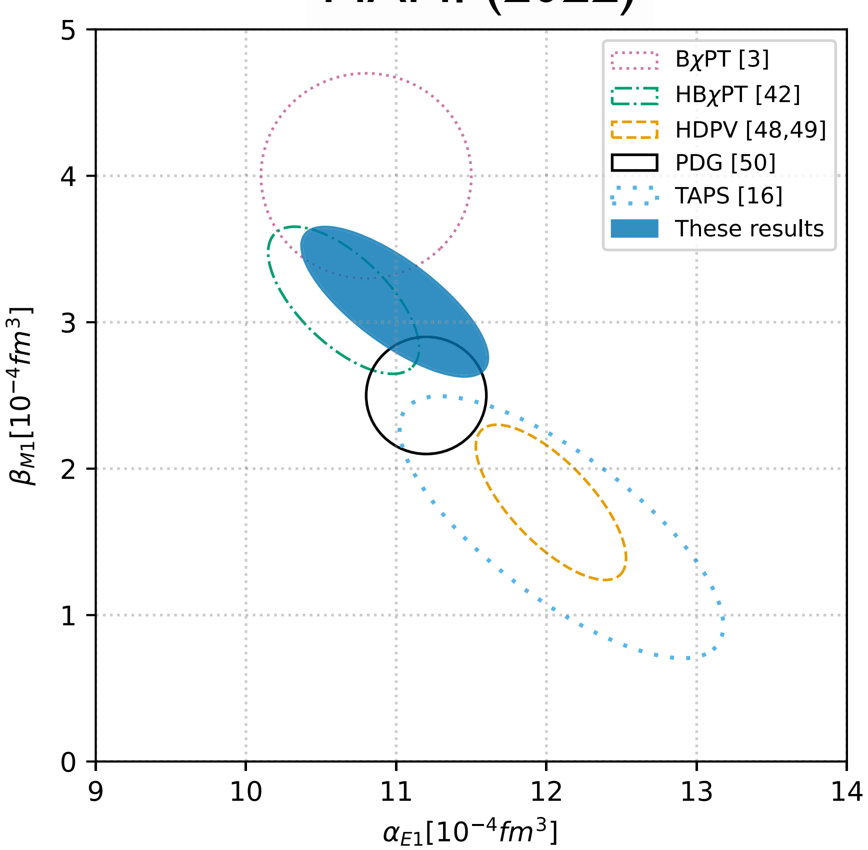
J. Alarcon, FH, V. Lensky and V. Pascalutsa,
Phys. Rev. D 102 (2020) 114026;
ibid. 102 (2020) 114006

- Baldin sum rule constrains the sum of electric and magnetic dipole polarizabilities through photoabsorption data

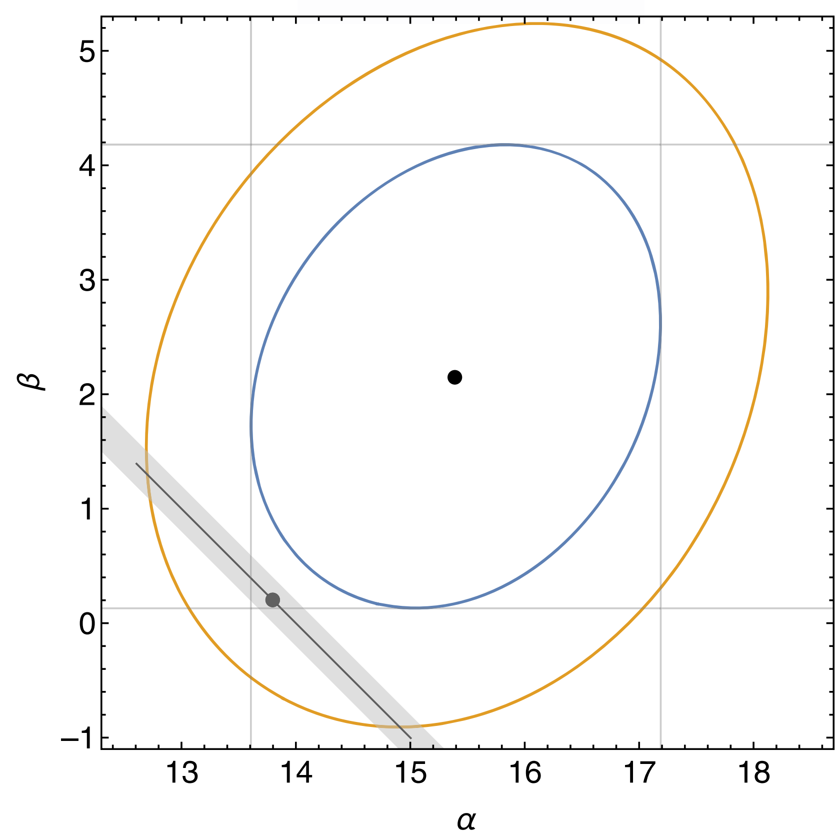
$$\alpha_{E1} + \beta_{M1}\,(\text{Baldin}) = 14.0(2) \times 10^{-4}\,\text{fm}^3$$

O. Gryniuk, FH, V. Pascalutsa,
Phys. Rev. D 92, 074031 (2015)

MAMI (2022)



HIGS (2022)



- Proton dia- or paramagnetic?

$$\beta_{M1}\,(\text{ChPT}) = 3.9(0.7) \times 10^{-4}\,\text{fm}^3$$

$$\beta_{M1}\,(\text{DR}) = 2.4(0.6) \times 10^{-4}\,\text{fm}^3$$

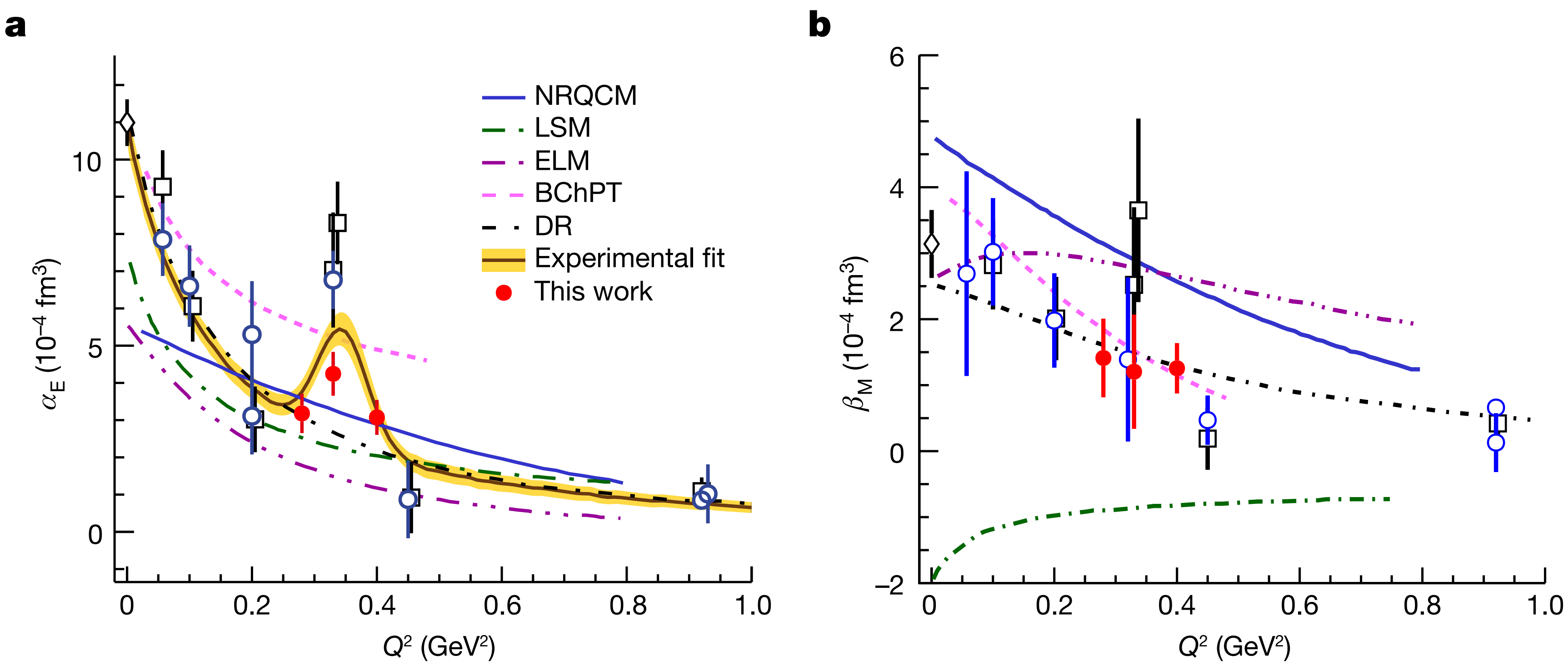
$$\beta_{M1}\,(\text{MAMI}) = 3.14(0.51) \times 10^{-4}\,\text{fm}^3$$

$$\beta_{M1}\,(\text{HIGS}) = 0.2(1.2) \times 10^{-4}\,\text{fm}^3$$

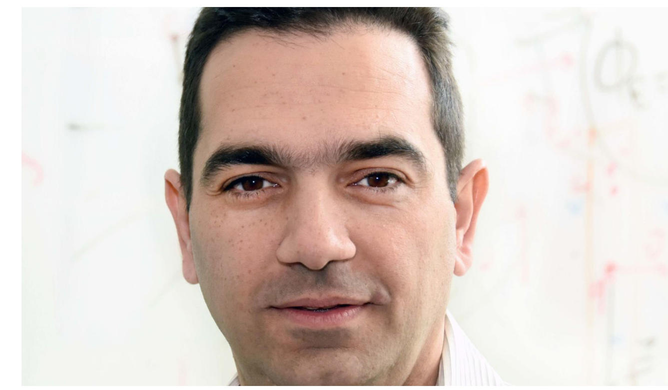
Eur. Phys. J. C75 (2015) 604
Phys. Rev. Lett. 129 (2022) 10, 102501
Phys. Rev. Lett. 128 (2022) 13, 132502
Phys. Rev. Lett. 128 (2022) 13, 132503

PROTON "STRECHINESS" ?

- Electric dipole polarizability extracted from virtual Compton scattering differs from theoretical expectation



nature



Nikolaos Sparveris: *"It is certainly puzzling for the physics of the strong interaction, if this thing persists … **So, the ball now is on the side of the [standard model] theory**."*

NewScientist

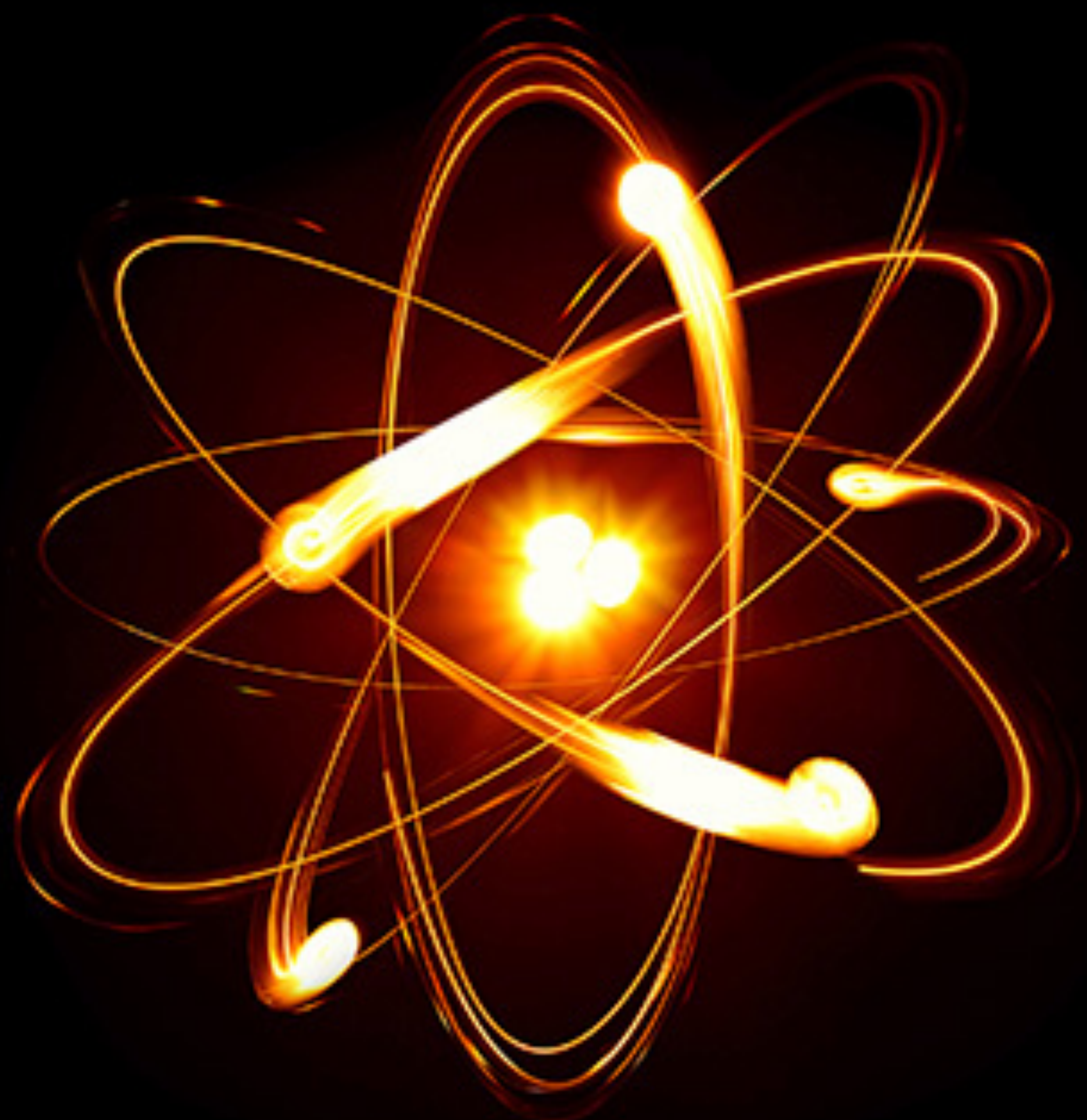
Judith McGovern: *"I don't think most people took [the 2000 result] really seriously, I think they assumed that it would go away, and, **if I'm quite honest, I think most people will still assume that it will go away**."*

ScienceNews

INDEPENDENT JOURNALISM SINCE 1921

Vladimir Pascalutsa: *"Usually, behaviors of these things are quite, let's say, smooth and there are no bumps … **don't want to kill the buzz, but yeah, I'm quite skeptical as a theorist that this thing is going to stay**."*

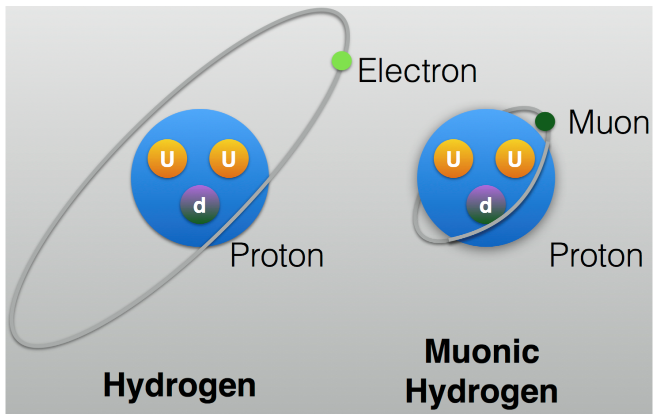
Nuclear Structure from Spectroscopy

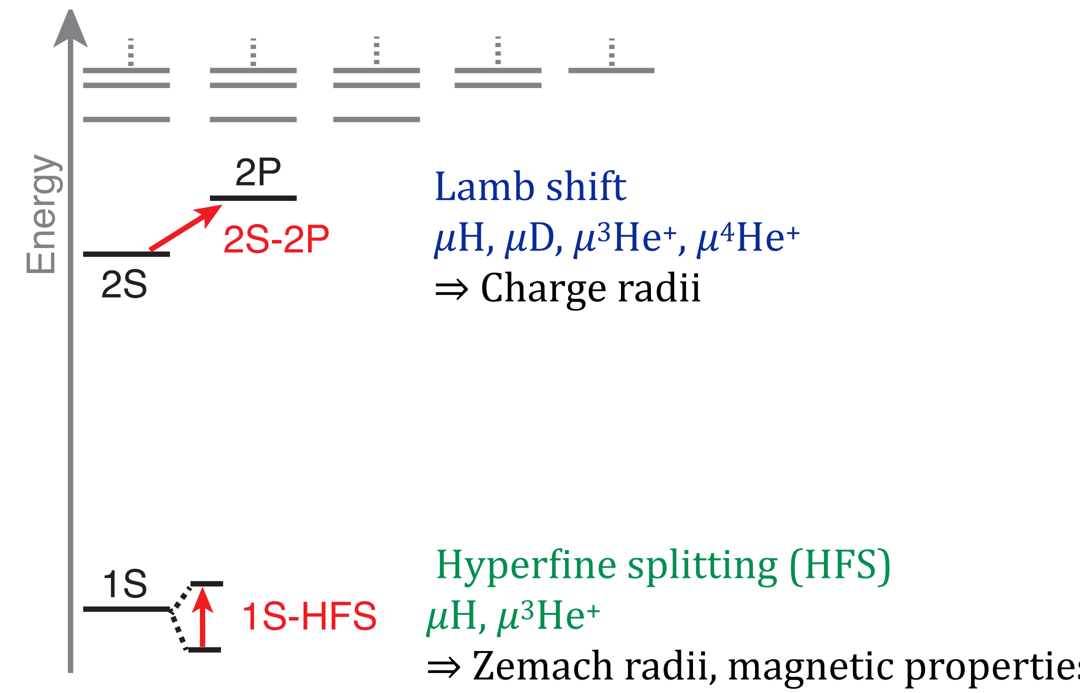


https://arstechnica.com

NUCLEAR STRUCTURE EFFECTS

Why muonic atoms ?





- **Lamb shift:**

wave function at the origin

$$\Delta E_{nl}(\mathrm{LO{+}NLO}) = \delta_{l0}\,\frac{2\pi Z\alpha}{3}\,\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R^3_{E(2)}\right]$$

NLO becomes appreciable in μH

- **HFS:**

$$\Delta E_{nS}(\mathrm{LO}+\mathrm{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

Fermi energy:

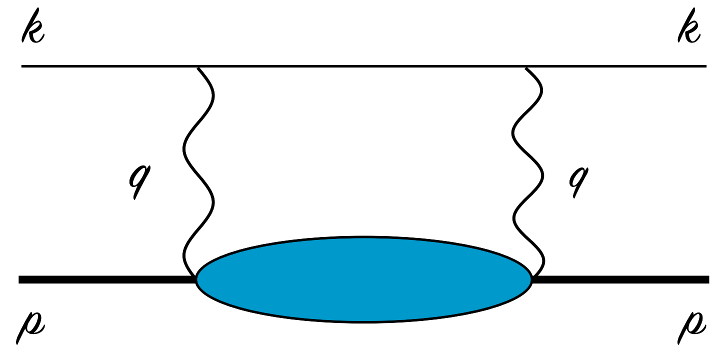
$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

with Bohr radius $a = 1/(Z\alpha m_r)$

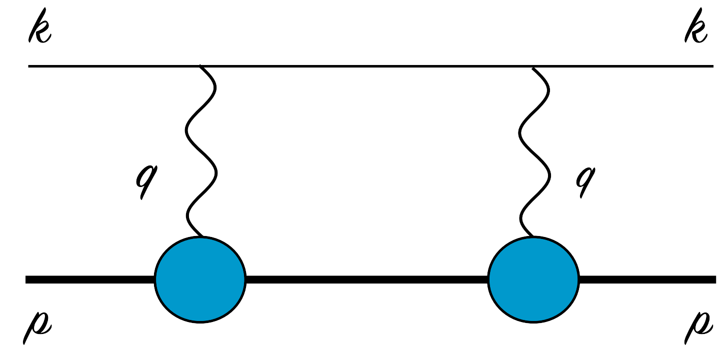
STRUCTURE EFFECTS THROUGH 2γ

- Proton-structure effects at subleading orders arise through multi-photon processes

forward two-photon exchange (2γ)



polarizability contribution
(non-Born VVCS)



elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

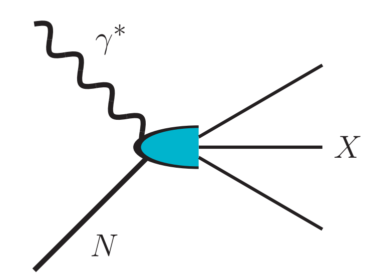
- "Blob" corresponds to doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

- Proton structure functions: $\boxed{f_1(x,Q^2),\ f_2(x,Q^2)}$, $\boxed{g_1(x,Q^2),\ g_2(x,Q^2)}$

Lamb shift

Hyperfine splitting (HFS)



2γ EFFECT IN THE LAMB SHIFT

wave function at the origin

$$\boxed{\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}}$$

dispersion relation & optical theorem:

$$T_1(\nu,Q^2) = \boxed{T_1(0,Q^2)} + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

- Caution: in the data-driven dispersive approach the $T_1(0,Q^2)$ subtraction function is modelled!

low-energy expansion:

$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior:

$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\,Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !

POLARIZABILITY EFFECT IN LAMB SHIFT

BChPT result is in good agreement with dispersive calculations !!!

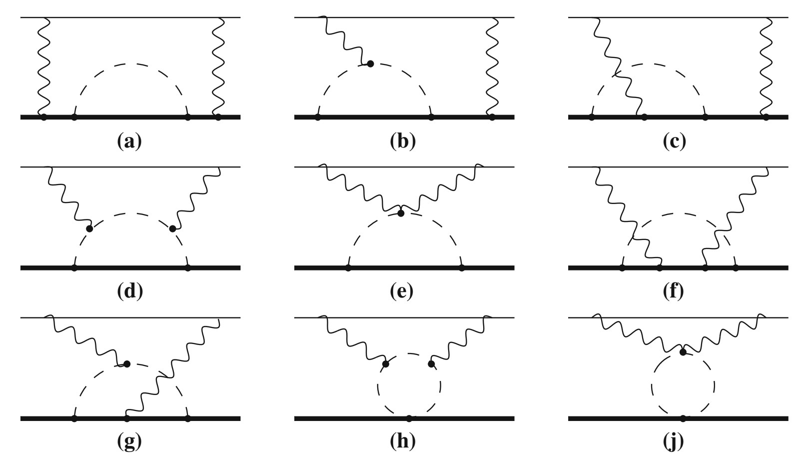
Agreement also for the contribution of the T_1 subtraction function !!!

Table 1 Forward 2γ-exchange contributions to the $2S$-shift in μH, in units of μeV.

Reference	$E_{2S}^{(\rm subt)}$	$E_{2S}^{(\rm inel)}$	$E_{2S}^{(\rm pol)}$	$E_{2S}^{(\rm el)}$	$E_{2S}^{\langle 2\gamma\rangle}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
(75) Carlson *et al.* '11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(76) Birse and McGovern '12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(77) Gorchtein et al.'13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(78) Hill and Paz '16					−30(13)
(79) Tomalak'18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(80) Alarcòn *et al.* '14			$-9.6^{+1.4}_{-2.9}$		
(81) Lensky *et al.* '17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the $\Delta(1232)$-isobar contribution.

LO BChPT prediction with pion-nucleon loop diagrams:



J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

POLARIZABILITY EFFECT IN LAMB SHIFT

BChPT result is in good agreement with dispersive calculations !!!

Agreement also for the contribution of the T_1 subtraction function !!!

Table 1 Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.

Reference	$E_{2S}^{(\text{subt})}$	$E_{2S}^{(\text{inel})}$	$E_{2S}^{(\text{pol})}$	$E_{2S}^{(\text{el})}$	$E_{2S}^{\langle 2\gamma\rangle}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
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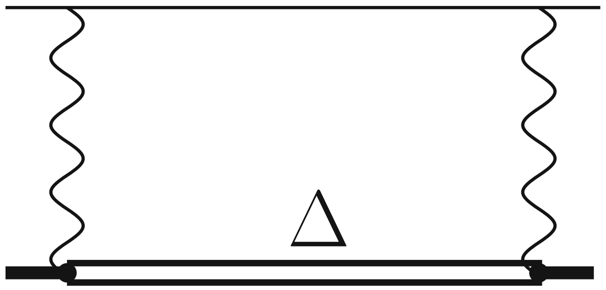
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the Δ(1232)-isobar contribution.

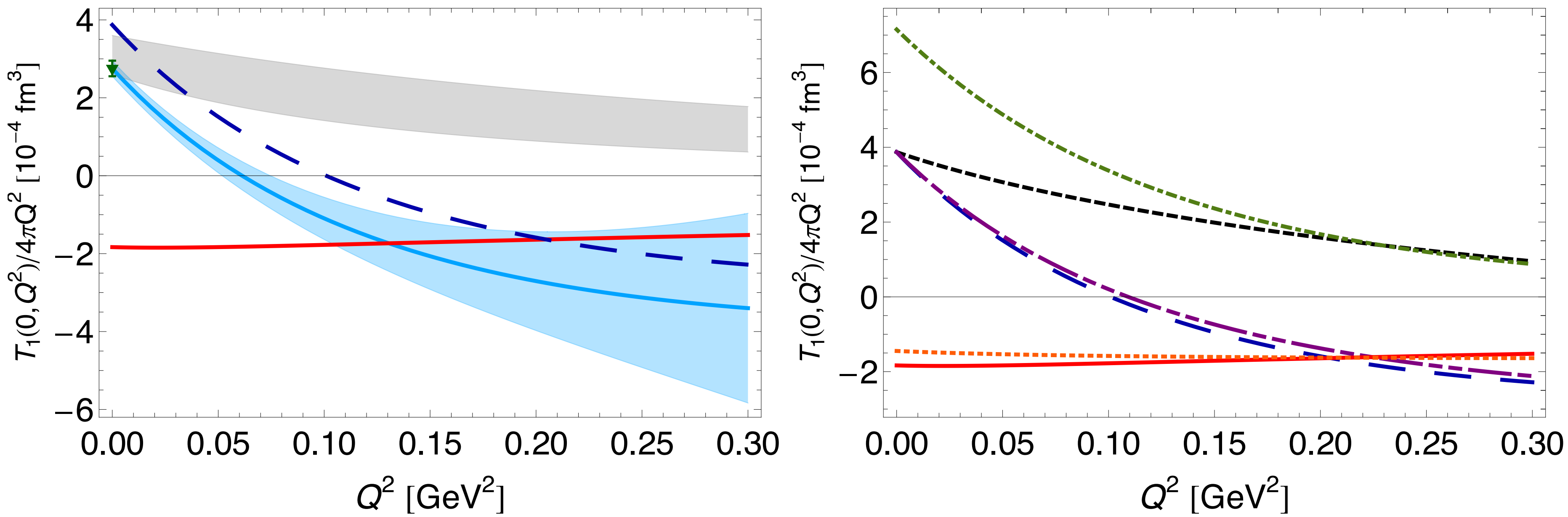
Δ prediction from Δ(1232) exchange:

- Uses large-N_c relations for the Jones-Scadron N-to-Δ transition form factors
- Small due to the suppression of β_{M1} in the Lamb shift but important for



V. Lensky, FH, V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **97** (2018) 074012

SUBTRACTION FUNCTION



NLO BChPT δ-exp.

total without g_M dipole

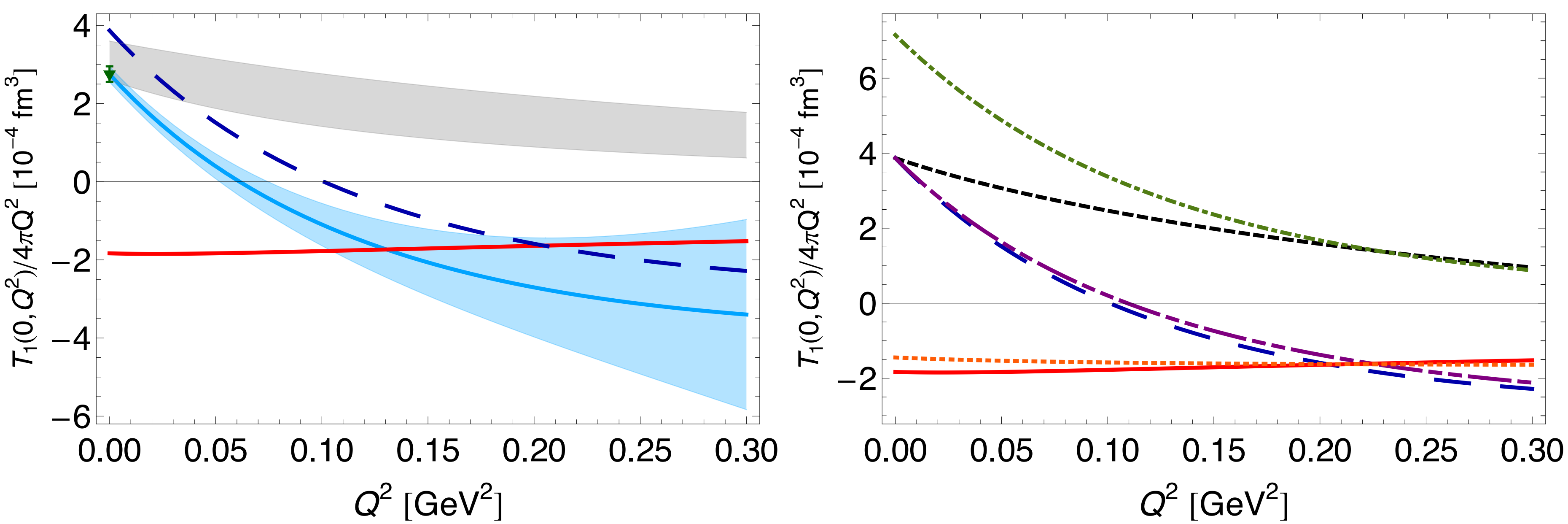
πN loops

πΔ loops

Δ-exchange

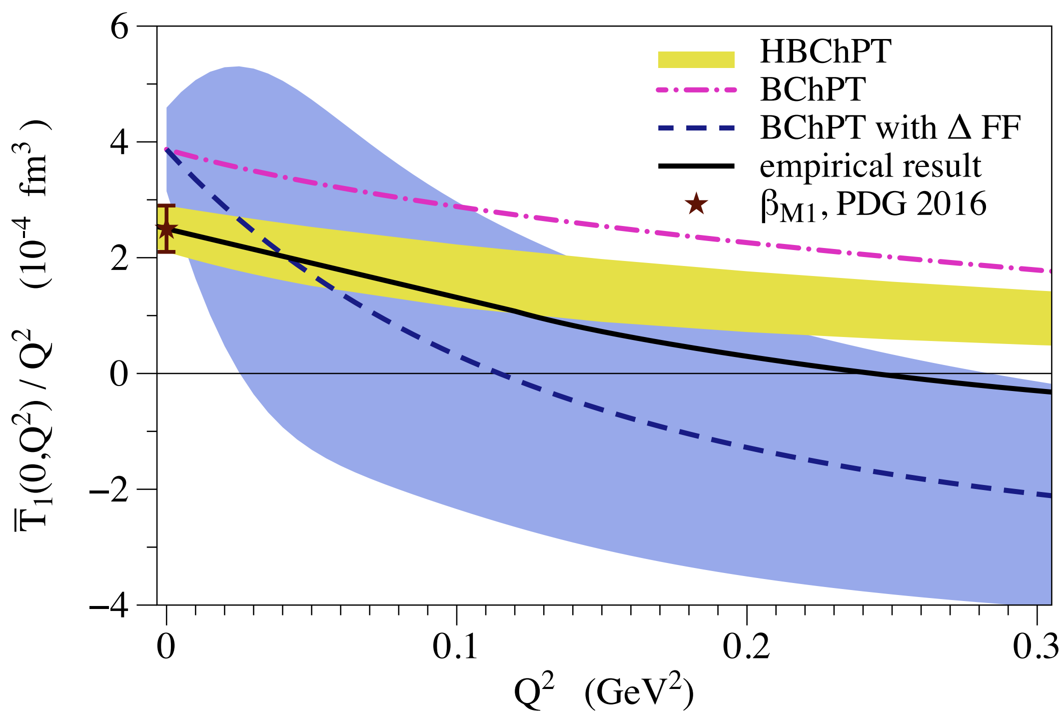
J. Alarcon, FH, V. Lensky and V. Pascalutsa, Phys. Rev. D **102** (2020) 114026; ibid. **102** (2020) 114006

SUBTRACTION FUNCTION



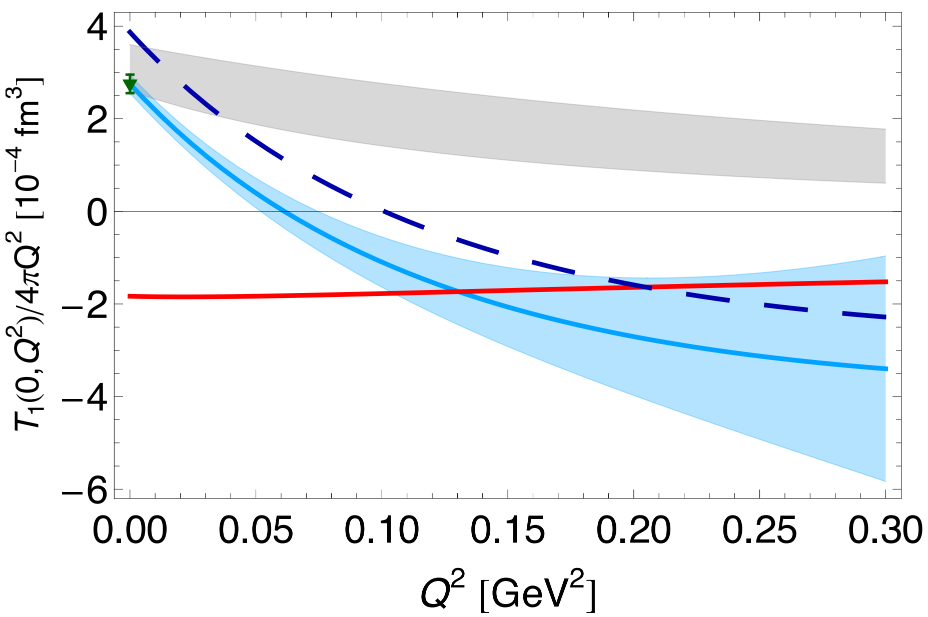
NLO BChPT δ-exp.
total without g_M dipole
πN loops
πΔ loops
Δ-exchange

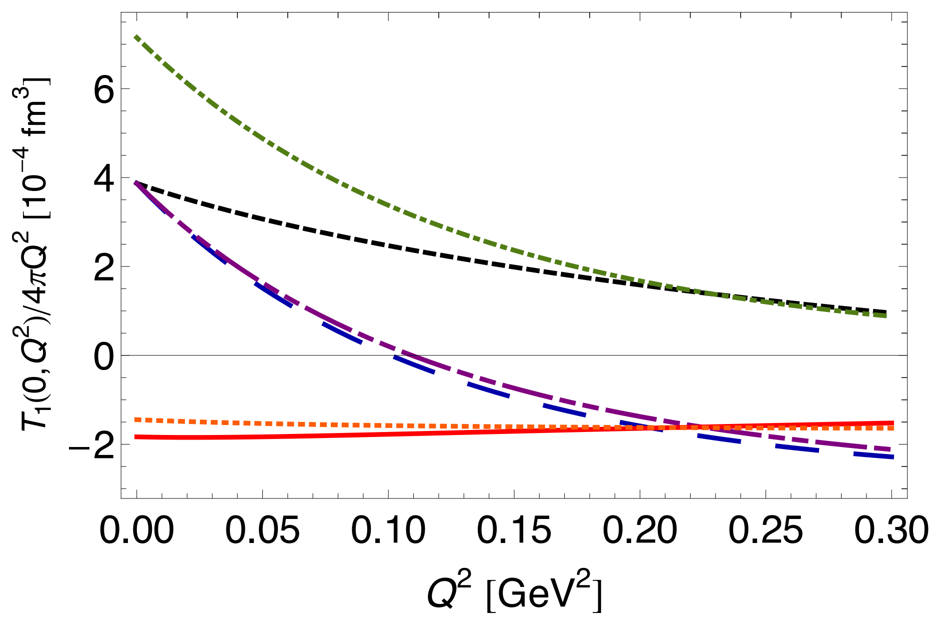
J. Alarcon, FH, V. Lensky and V. Pascalutsa,
Phys. Rev. D **102** (2020) 114026;
ibid. **102** (2020) 114006



V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen
Phys. Rev. D **97** (2018) 074012

SUBTRACTION FUNCTION





NLO BChPT δ-exp.
total without g_M dipole
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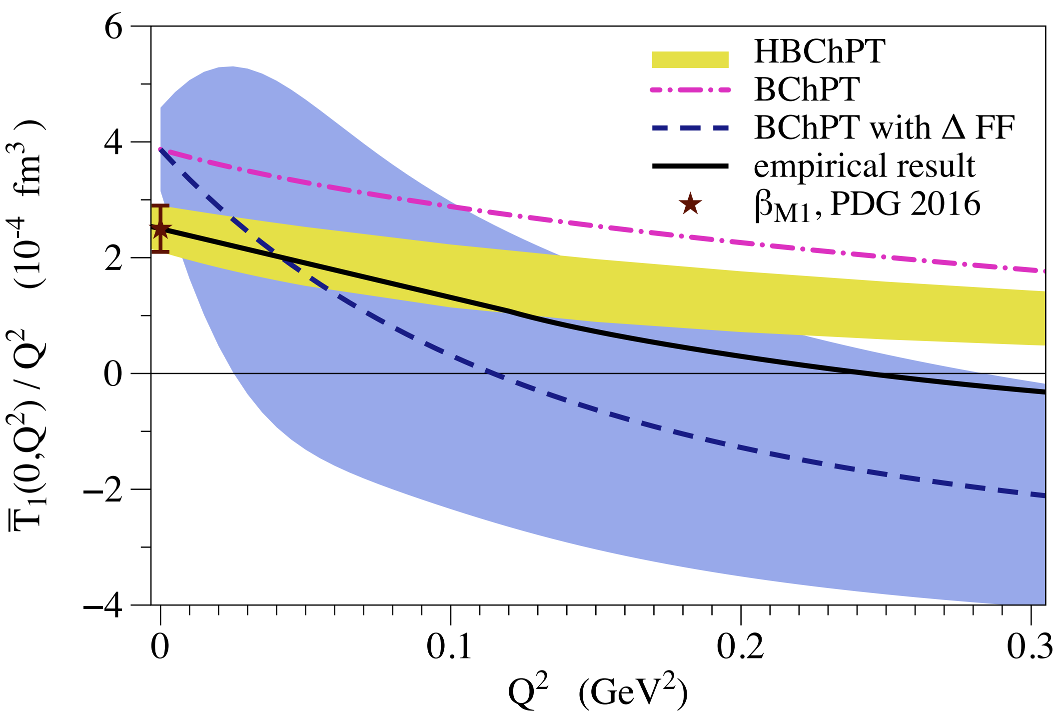
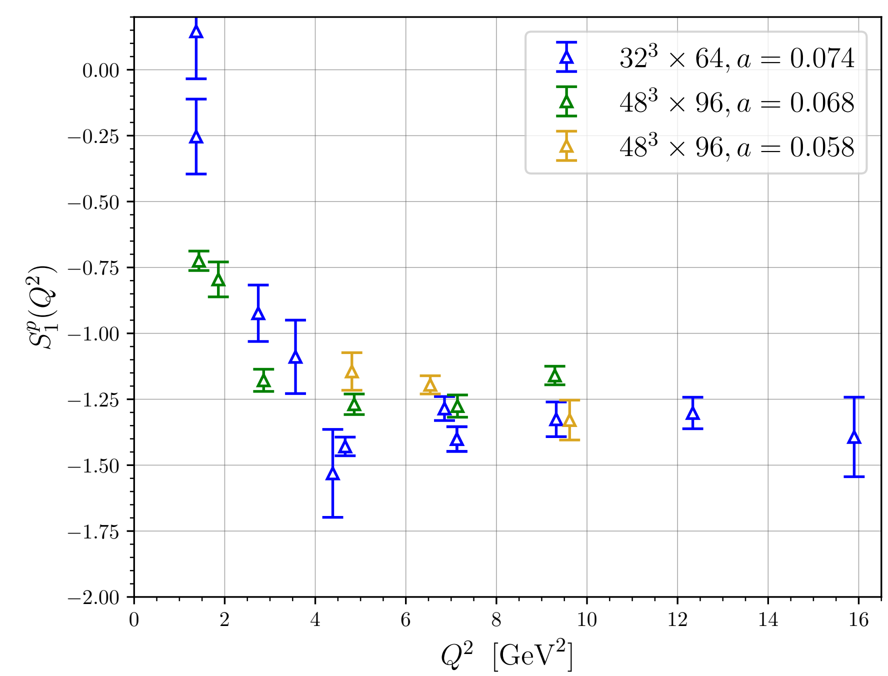
J. Alarcon, FH, V. Lensky and V. Pascalutsa, Phys. Rev. D **102** (2020) 114026; ibid. **102** (2020) 114006



V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen Phys. Rev. D **97** (2018) 074012

First lattice results by several groups!



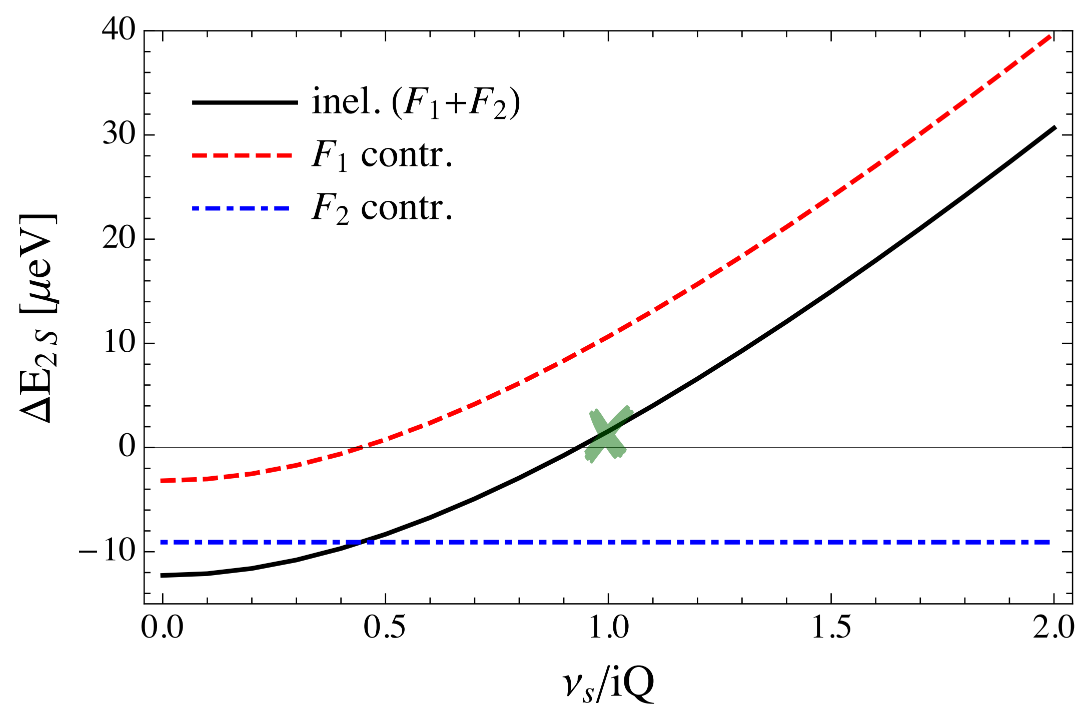
CSSM-QCDSF-UKQCD Collaboration, 2207.03040.

EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{\prime\,(\mathrm{subt})} = \frac{2\alpha m}{\pi}\phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3}\frac{2+v_l}{(1+v_l)^2}\,\overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1+4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



FH, V. Pascalutsa, Nucl. Phys. A **1016** (2021) 122323

based on Bosted-Christy parametrization:

$$\Delta E_{2S}^{(\mathrm{inel})}(\nu_s = 0) \simeq -12.3\,\mu\mathrm{eV}$$

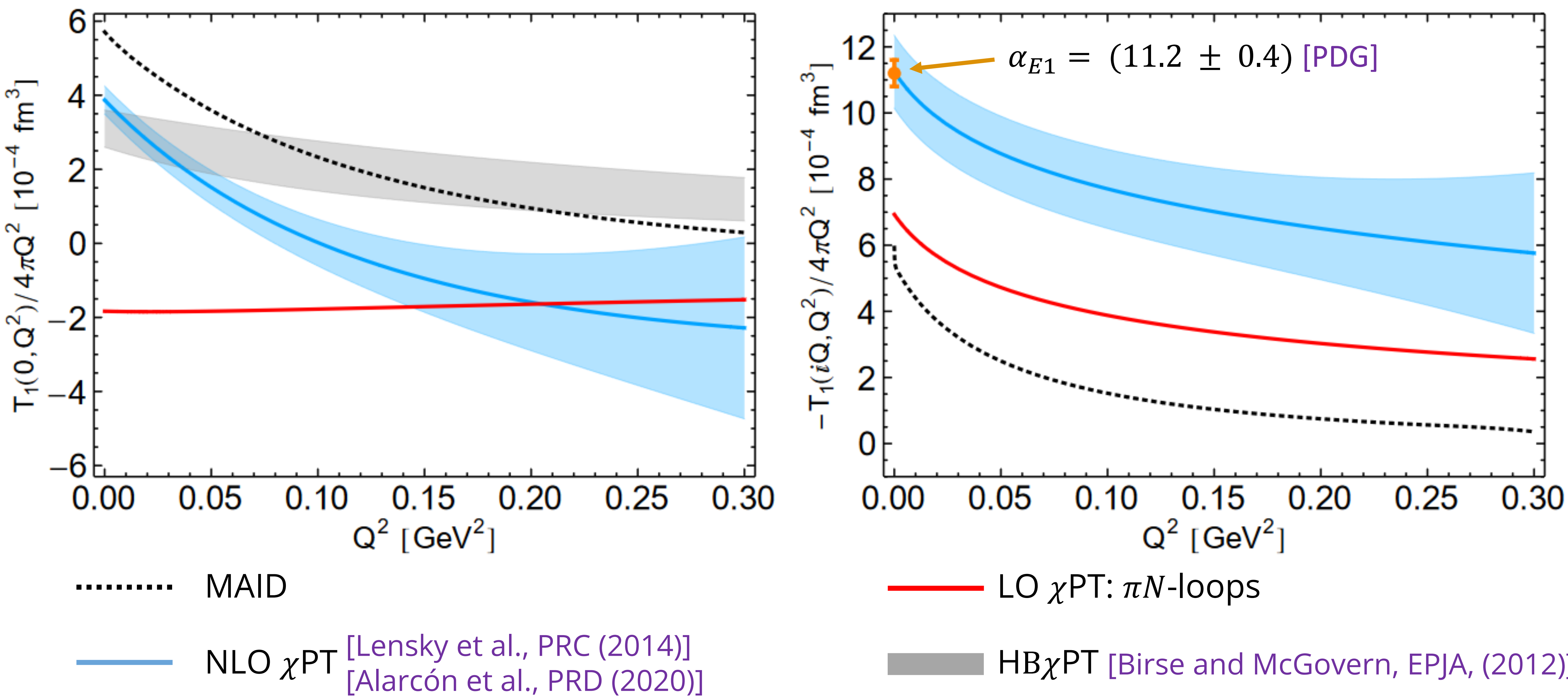
$$\Delta E_{2S}^{\prime\,(\mathrm{inel})}(\nu_s = iQ) \simeq 1.6\,\mu\mathrm{eV}$$

DATA-DRIVEN EVALUATION

- New integral equations for data-driven evaluation of subtraction functions
- High-quality parametrization of σ_L at $Q \to 0$ needed

$$T_1(0,Q^2) = \frac{2Q^2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{d}\nu}{\nu^2 + Q^2} \left[\sigma_T - \frac{\nu^2}{Q^2} \sigma_L \right] (\nu, Q^2)$$

$$T_L(iQ, Q^2) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \mathrm{d}\nu \, \nu^2 \, \frac{\sigma_L(\nu, Q^2)}{\nu^2 + Q^2}$$



HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_F(nS)$$

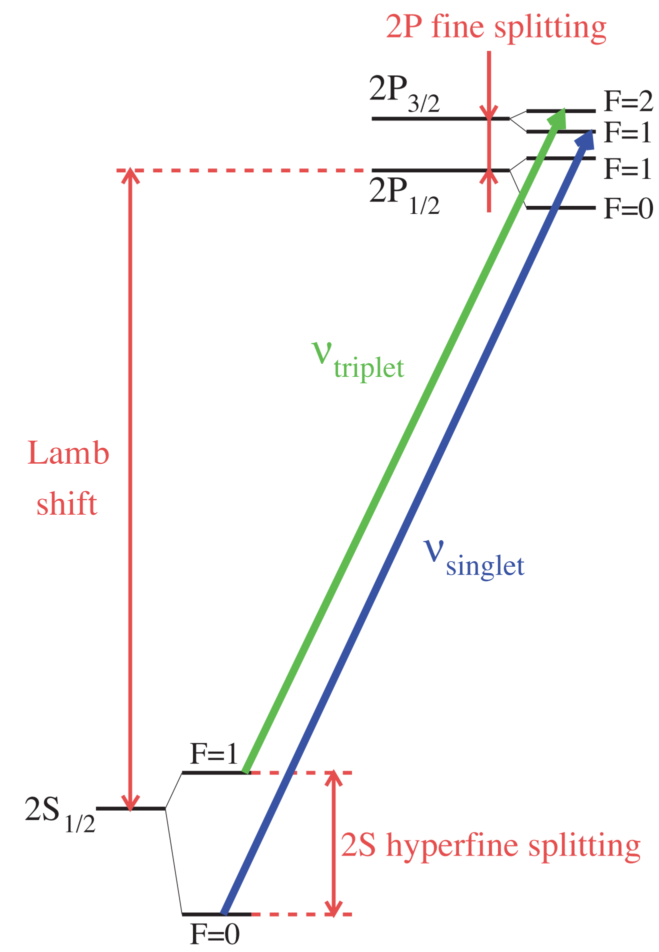
with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa} - 1 \right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\,\mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



⏰ Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ effect needed to narrow down frequency search range for experiment
- Zemach radius can help to pin down the magnetic properties of the proton

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

Experiment: HFS in μH, μHe^{+}, …

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\mathrm{pol.}}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Testing the theory

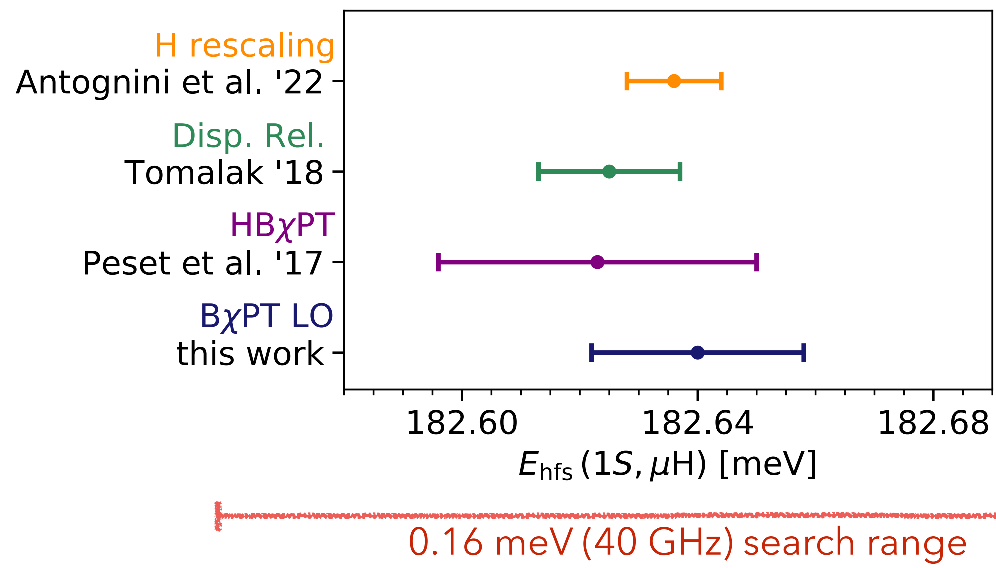
- discriminate between theory predictions for polarizability effect
  - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He^{+})

Electron and Compton Scattering



Predictions for the 1S HFS in μH are driven by the 1S HFS in H

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

see talk by Clara Peset

PLITTING

Experiment: HFS in μH, μHe$^+$, …

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extract E^{TPE}, $E^{\mathrm{pol.}}$ or R_Z

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Determine fundamental constants

Zemach radius R_Z

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering

HYPERFINE SPLITTING

The hyperfine splitting of μH (theory update):

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022)

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_{\mathrm{F}}} \underbrace{+1.350(7)}_{\text{QED+weak}} \underbrace{+0.004}_{\text{hVP}} \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_{\mathrm{F}}\left(1.01656(4)\,\Delta_{\text{recoil}} + 1.00402\,\Delta_{\text{pol}}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{meV}$$

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\text{pol.}}$ or R_Z

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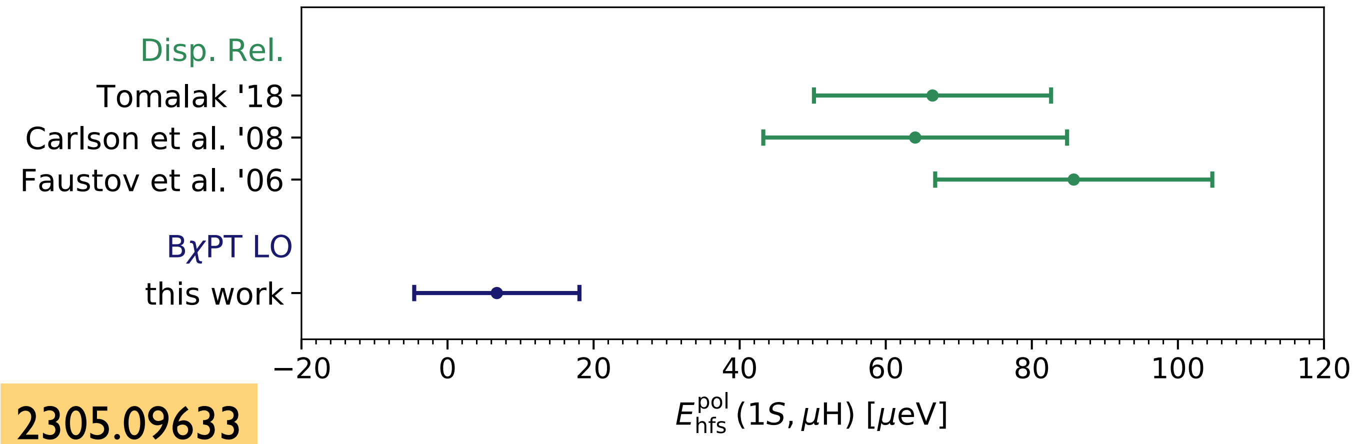
PLITTING



2305.09633

Experiment: HFS in μH, μHe$^+$, ...

Testing the theory

- **discriminate between theory predictions for polarizability effect**
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- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
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Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

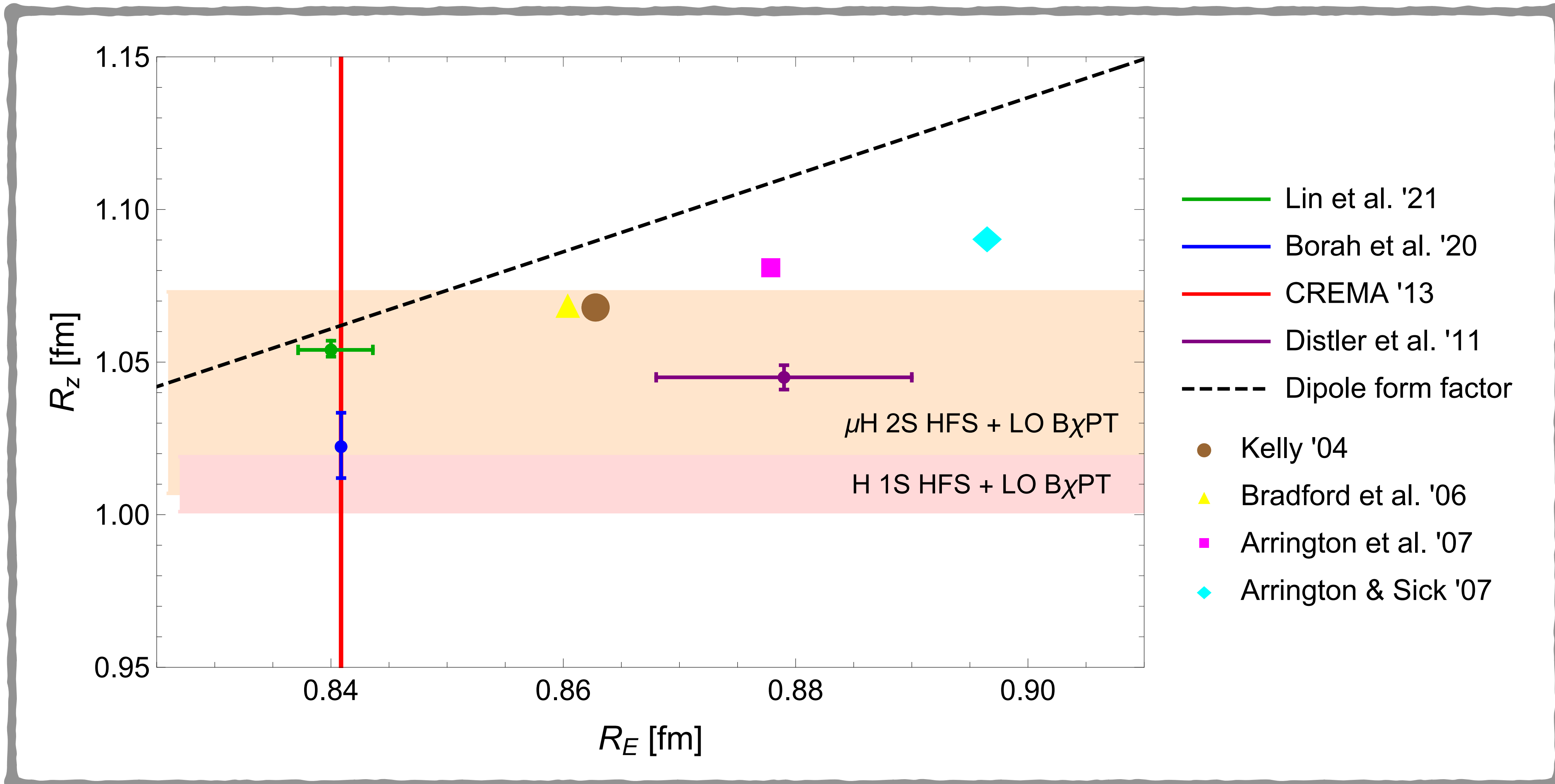
extract $E^{\rm TPE}$, $E^{\rm pol.}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Spectroscopy of ordinary atoms (H, He$^+$)

Electron and Compton Scattering



, μHe$^+$, …

Determine fundamental constants

Zemach radius R_Z

transitions with the help of full theory predictions: QED, weak, finite size, polarizability

combining HFS in H & μH

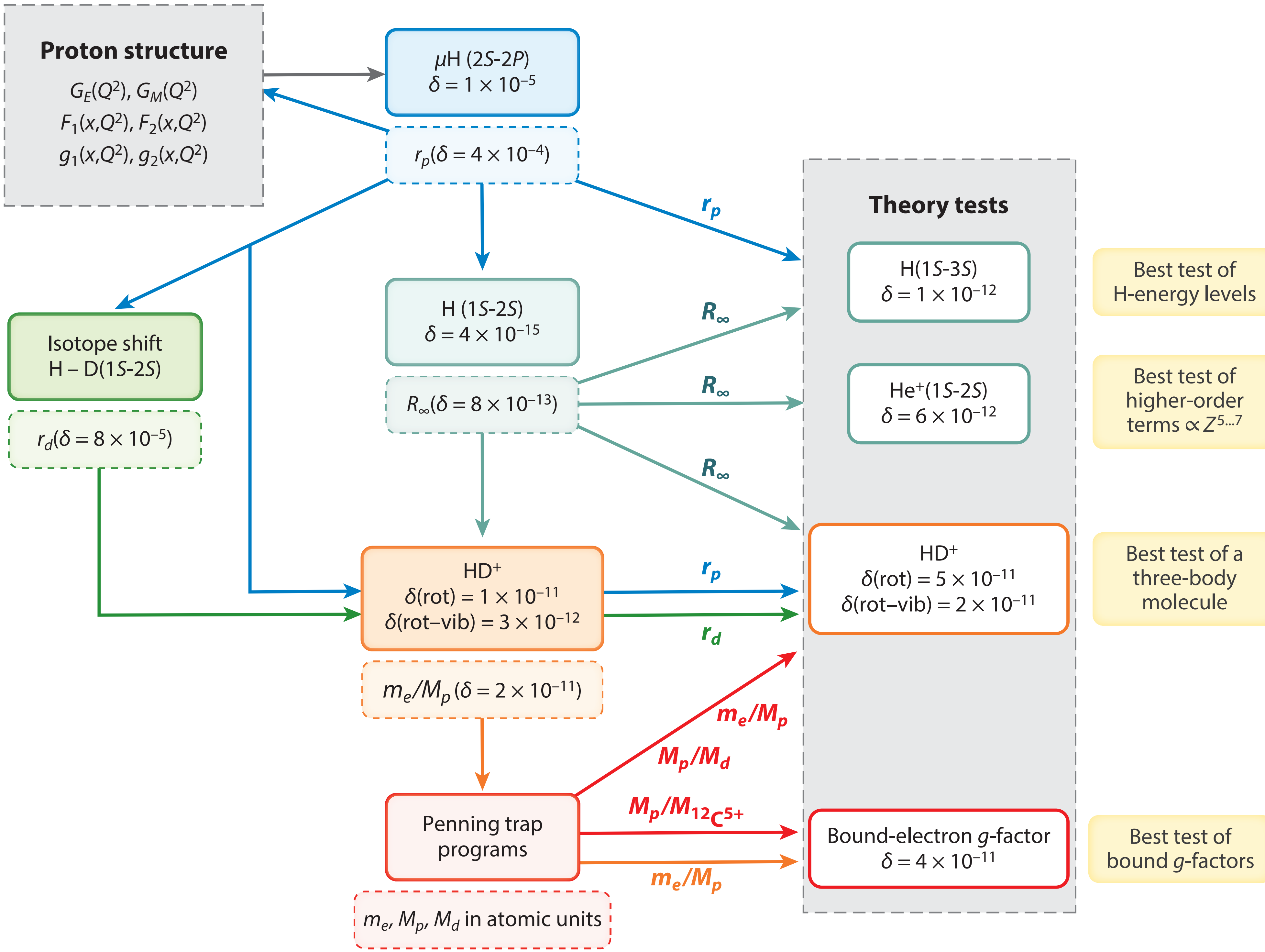
- test HFS theory
  - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Input for data-driven evaluations form factors, structure functions, polarizabilities

Spectroscopy of ordinary atoms (H, He$^+$)

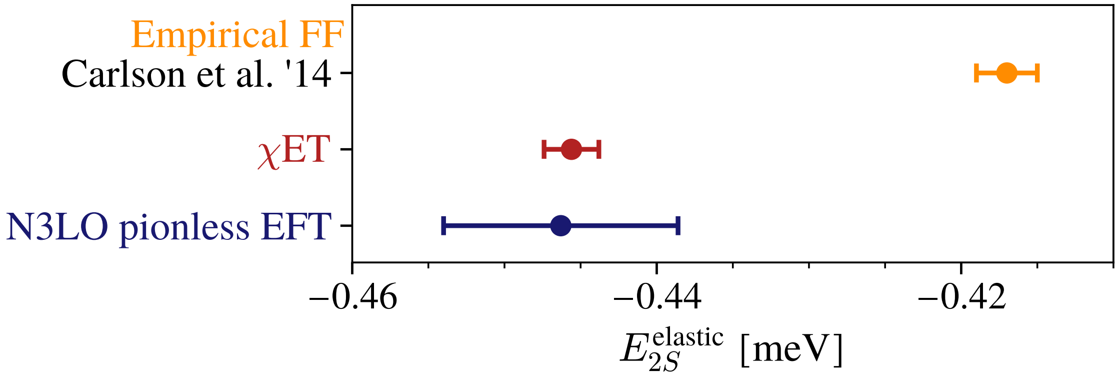
Electron and Compton Scattering

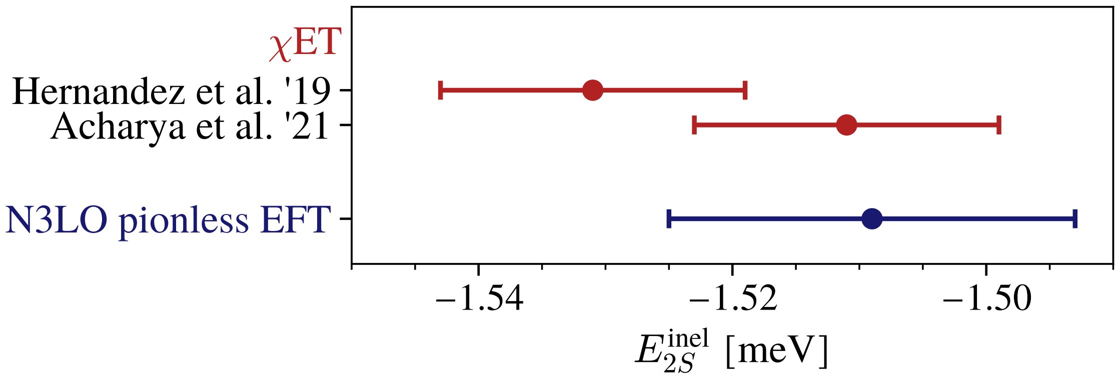
COMBINING μH, H, HE, HD+, …



A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418
A. Antognini, FH, et al., 2210.16929 (submitted as community input for the NuPECC Long Range Plan 2024)

2γ EFFECT IN μD LAMB SHIFT





	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not\pi$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)

V. Lensky, A. Hiller Blin, FH, V. Pascalutsa, 2203.13030
V. Lensky, FH, V. Pascalutsa, 2206.14756, 2206.14066

- N3LO pionless EFT + higher-order single-nucleon effects:

$$E_{2S}^{\text{elastic}} = -0.446(8)\,\text{meV}$$
$$E_{2S}^{\text{inel},L} = -1.509(16)\,\text{meV}$$
$$E_{2S}^{\text{inel},T} = -0.005\,\text{meV}$$
$$E_{2S}^{\text{hadr}} = -0.032(6)\,\text{meV}$$
$$E_{2S}^{\text{eVP}} = -0.027\,\text{meV}$$

- Elastic 2γ several standard deviations larger
- Inelastic 2γ consistent with other results
- Agreement with precise empirical value for the 2γ effect extracted with $r_d(\mu\text{H} + \text{iso})$

Thank you for your attention!

POLARIZABILITY EFFECT IN THE HFS

- Polarizability effect on the HFS is completely constrained by empirical information

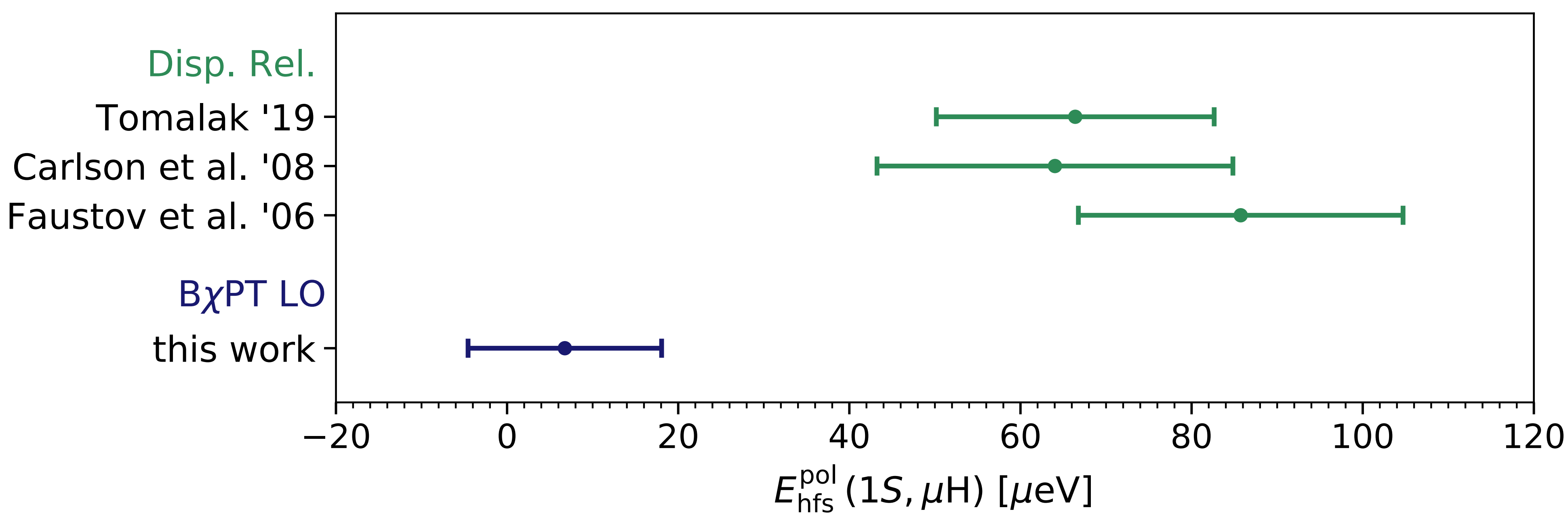
$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}\big(\delta_1 + \delta_2\big)$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\}$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

with $v_l = \sqrt{1+\frac{1}{\tau_l}}$, $v_x = \sqrt{1+x^2\tau^{-1}}$, $\tau_l = \frac{Q^2}{4m^2}$ and $\tau = \frac{Q^2}{4M^2}$

- BChPT calculation puts the reliability of dispersive calculations (and BChPT) to the test



2γ EFFECT IN THE μH HFS

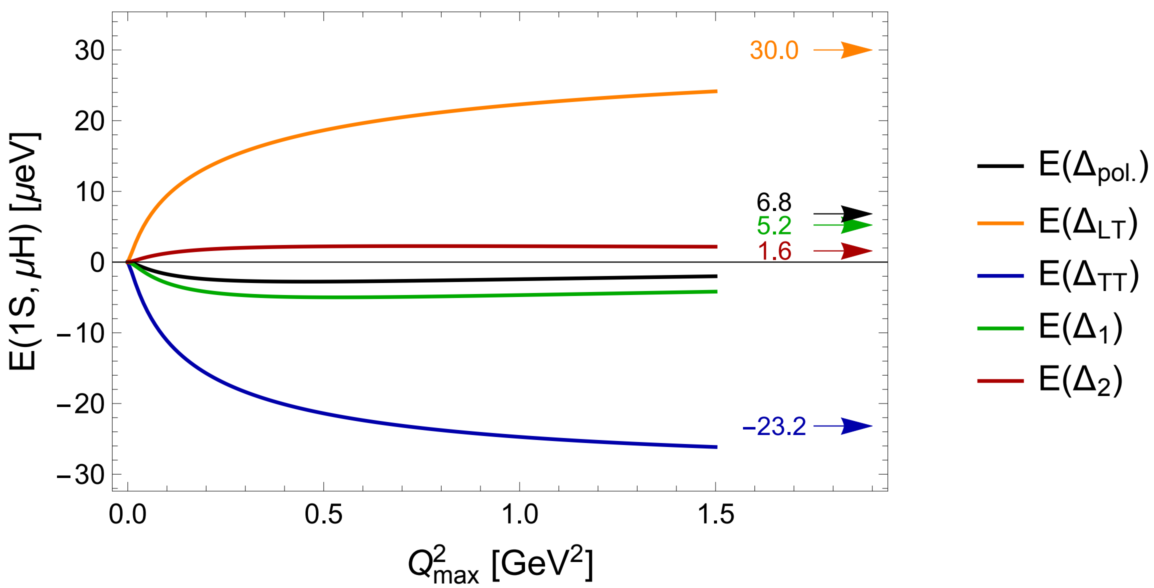
Table 1 Forward 2γ-exchange contribution to the HFS in μH.

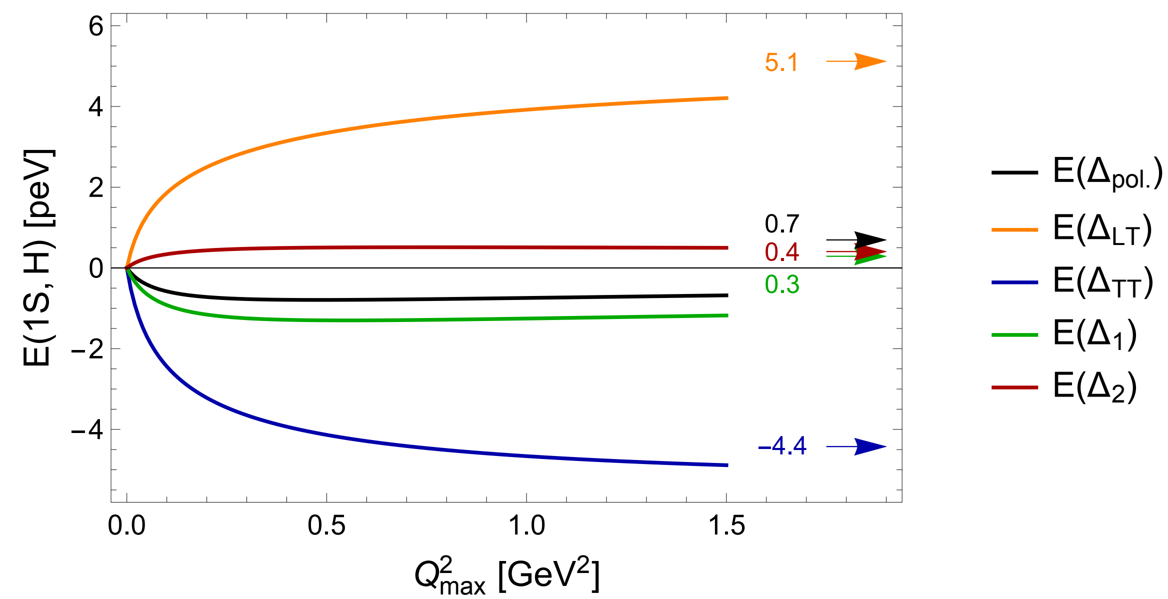
Reference	Δ_Z [ppm]	Δ_{recoil} [ppm]	Δ_{pol} [ppm]	Δ_1 [ppm]	Δ_2 [ppm]	$E^{\langle 2\gamma \rangle}_{1S\text{-hfs}}$ [meV]
DATA-DRIVEN						
Pachucki '96 (1)	−8025	1666	0(658)			−1.160
Faustov et al. '01 (9)[a]	−7180		410(80)	468	−58	
Faustov et al. '06 (10)[b]			470(104)	518	−48	
Carlson et al. '11 (11)[c]	−7703	931	351(114)	370(112)	−19(19)	−1.171(39)
Tomalak '18 (12)[d]	−7333(48)	846(6)	364(89)	429(84)	−65(20)	−1.117(19)
HEAVY-BARYON χPT						
Peset et al. '17 (13)						−1.161(20)
LEADING-ORDER χPT						
Hagelstein et al. '16 (14)			37(95)	29(90)	9(29)	
+ $\Delta(1232)$ EXCIT.						
Hagelstein et al. '18 (15)			−13	84	−97	

[a] Adjusted values: Δ_{pol} and Δ_1 corrected by −46 ppm as described in Ref. 16.
[b] Different convention was used to calculate the Pauli form factor contribution to Δ_1, which is equivalent to the approximate formula in the limit of $m = 0$ used for H in Ref. 11.
[c] Elastic form factors from Ref. 17 and updated error analysis from Ref. 16. Note that this result already includes radiative corrections for the Zemach-radius contribution, $(1+\delta_Z^{rad})\Delta_Z$ with $\delta_Z^{rad} \sim 0.0153$ (18, 19), as well as higher-order recoil corrections with the proton anomalous magnetic moment, cf. (11, Eq. 22) and (18).
[d] Uses r_p from μH (20) as input.

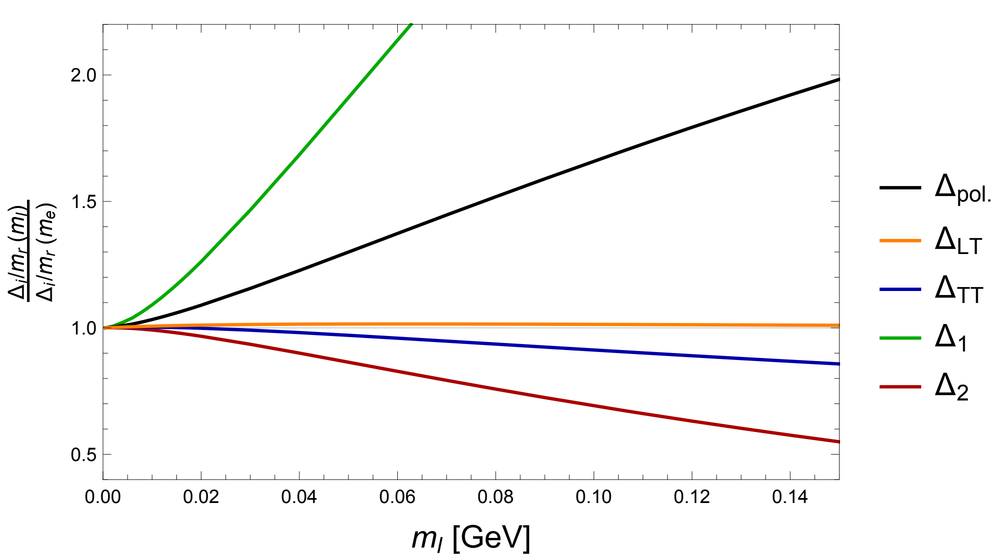
POLARIZABILITY EFFECT FROM BCHPT

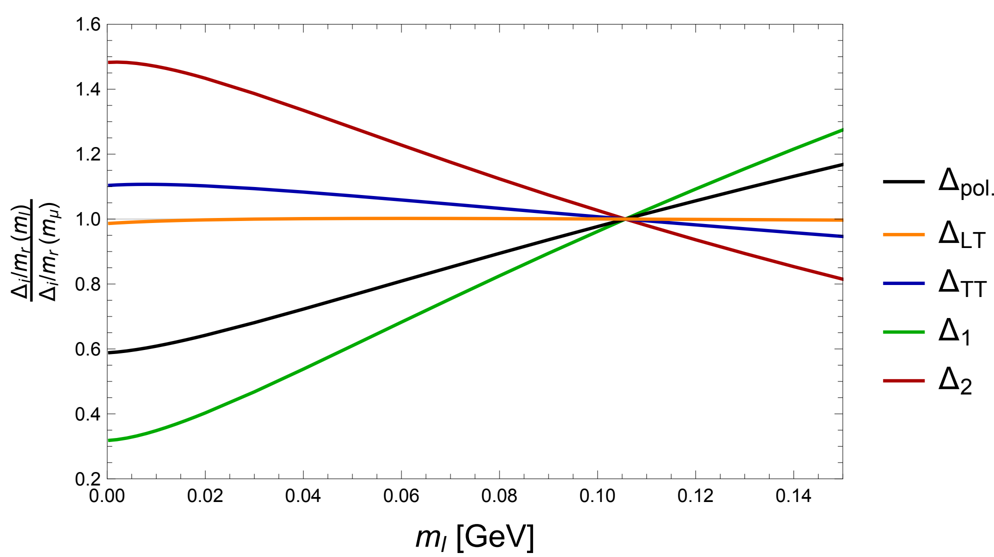
- LO BChPT result is compatible with zero
  - Contributions from σ_{LT} and σ_{TT} are sizeable and largely cancel each other





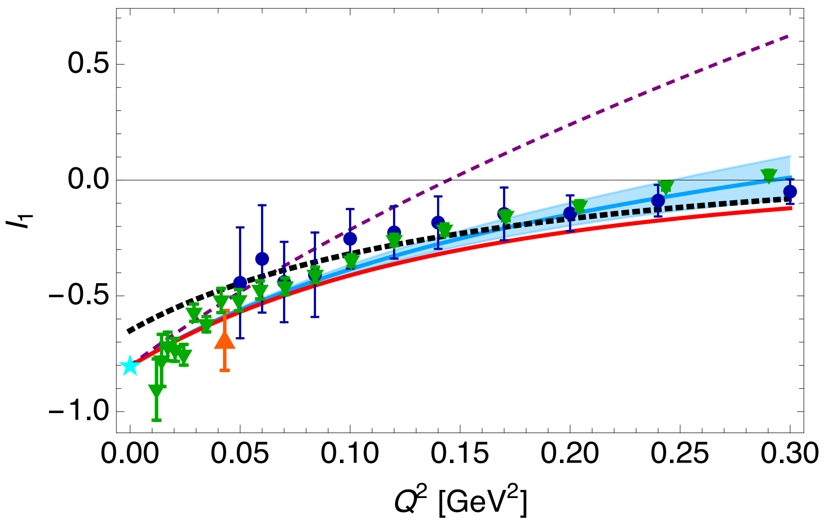
- Are the data-driven evaluations/uncertainties affected by cancelations?
- Scaling with lepton mass of the lepton-proton bound state

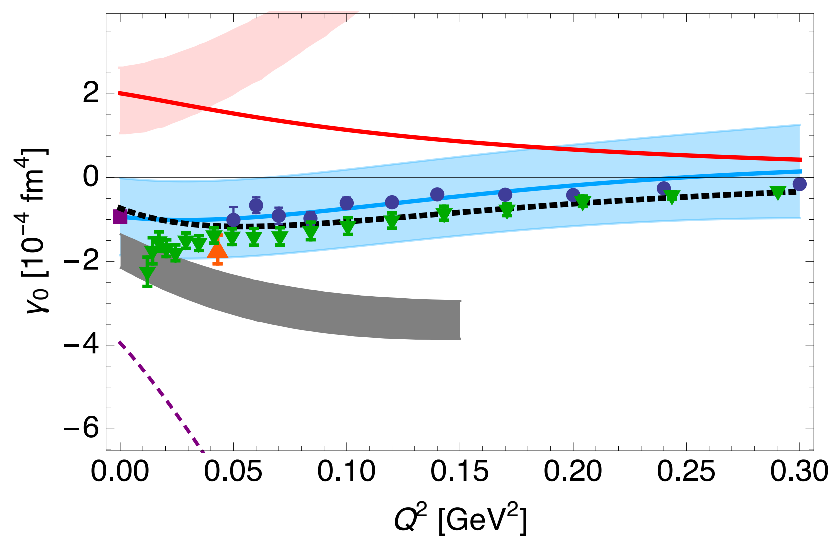


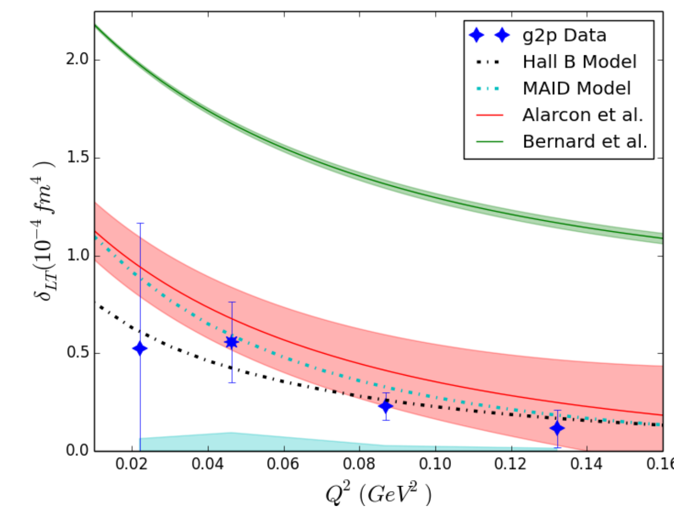


DATA-DRIVEN EVALUATION

- Empirical information on spin structure functions from JLab Spin Physics Programme



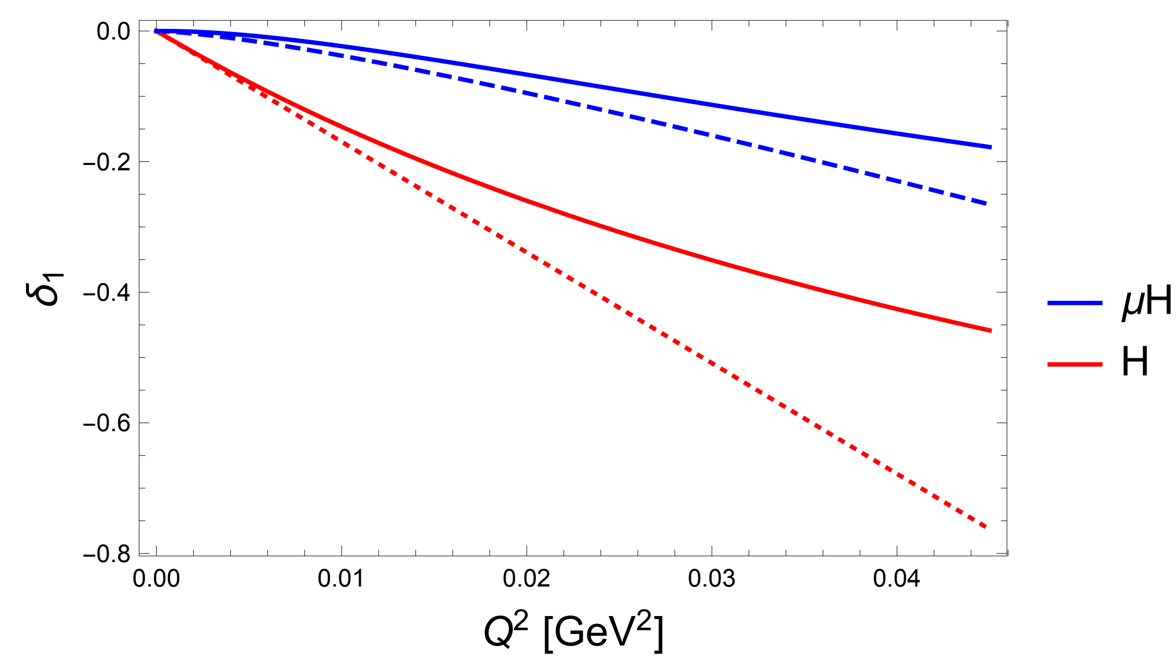




- Low-Q region is very important → cancelation between $I_1(Q^2)$ and $F_2(Q^2)$

$$\delta_1(\mathrm{H}) \sim \left(\underbrace{-\frac{3}{4}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -2.19} + \underbrace{18M^2 c_{1B}}_{\to 3.54} \right) Q^2_{\mathrm{max}} = 1.35(90),$$

$$\delta_1(\mu\mathrm{H}) \sim \left[\underbrace{-\frac{1}{3}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -1.45} + \underbrace{8M^2 c_1}_{\to 2.13} \underbrace{-\frac{M^2}{3\alpha}\gamma_0}_{\to 0.18} \right] \int_0^{Q^2_{\mathrm{max}}} \mathrm{d}Q^2 \beta_1(\tau_\mu) = 0.86(69)$$



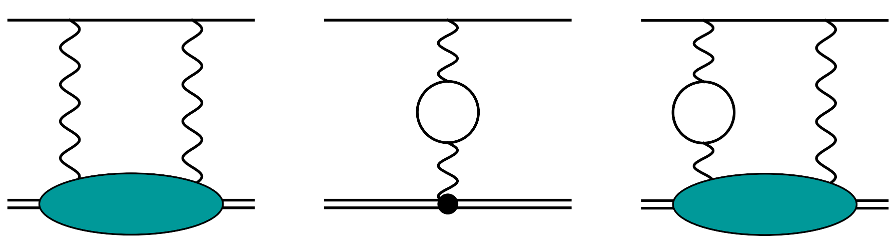
THEORY OF HYPERFINE SPLITTING

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418

The hyperfine splitting of μH (theory update):

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}} \underbrace{+1.350(7)}_{\text{QED+weak}} \underbrace{+0.004}_{\text{hVP}} \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\left(1.01656(4)\,\Delta_\text{recoil} + 1.00402\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{meV}$$

- 2γ + radiative corrections $\Longrightarrow$ differ for H vs. μH and 1S vs. 2S



$+ \ldots$

The hyperfine splitting of H (theory update):

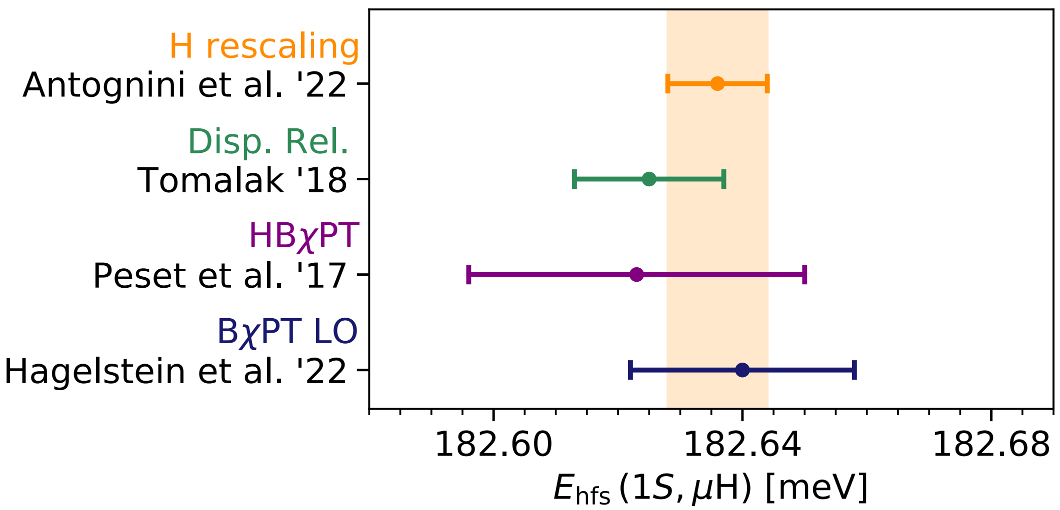
$$E_{1S\text{-hfs}}(\mathrm{H}) = \Big[\underbrace{1\,418\,840.082(9)}_{E_\mathrm{F}} \underbrace{+1\,612.673(3)}_{\text{QED+weak}} \underbrace{+0.274}_{\mu\text{VP}} \underbrace{+0.077}_{\text{hVP}}$$
$$\underbrace{-54.430(7)\left(\frac{r_{Zp}}{\text{fm}}\right) + E_\mathrm{F}\left(0.99807(13)\,\Delta_\text{recoil} + 1.00002\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{kHz}$$

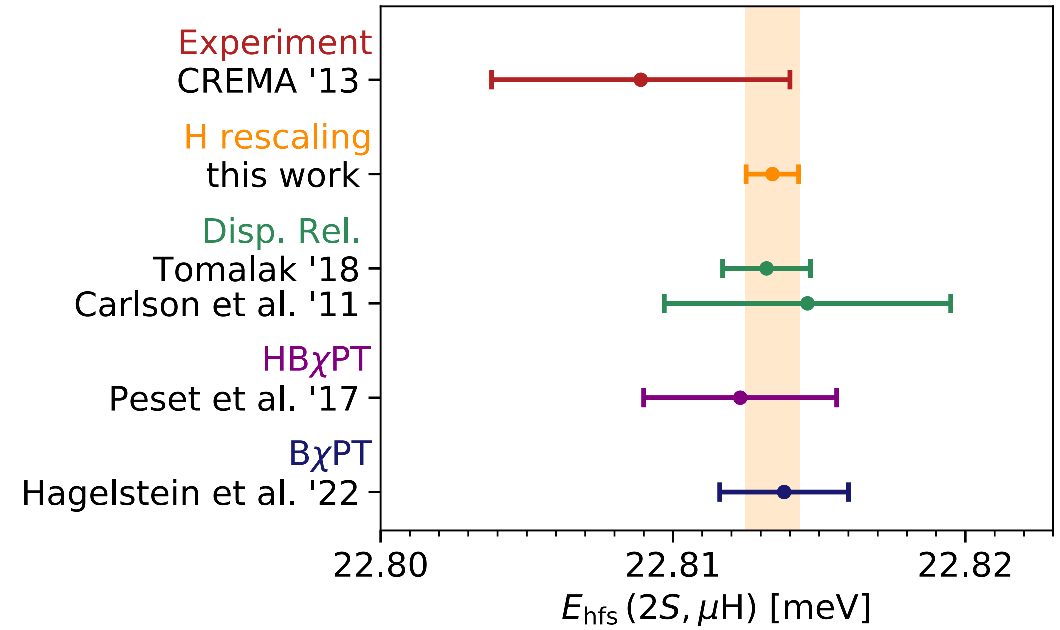
High-precision measurement of the "21cm line" in H:

$$\delta\left(E^\text{exp.}_{1S-\text{hfs}}(\mathrm{H})\right) = 10\times 10^{-13}$$

Hellwig et al., 1970

IMPACT OF H 1S HFS





- Leverage radiative corrections $E^{\rm Z+pol}_{1S-\rm hfs}(\mathrm{H}) = E_{\rm F}(\mathrm{H})\left[b_{1S}(\mathrm{H})\,\Delta_{\rm Z}(\mathrm{H}) + c_{1S}(\mathrm{H})\,\Delta_{\rm pol}(\mathrm{H})\right] = -\,54.900(71)\,\mathrm{kHz}$ and assume the non-recoil $\mathcal{O}(\alpha^5)$ effects have simple scaling $\dfrac{\Delta_i(\mathrm{H})}{m_r(\mathrm{H})} = \dfrac{\Delta_i(\mu\mathrm{H})}{m_r(\mu\mathrm{H})}, \quad i = \mathrm{Z, pol}$

1. Prediction for μH HFS from empirical 1S HFS in H

$$E^{\rm Z+pol}_{nS-\rm hfs}(\mu\mathrm{H}) = \frac{E_{\rm F}(\mu\mathrm{H})\,m_r(\mu\mathrm{H})\,b_{nS}(\mu\mathrm{H})}{n^3 E_{\rm F}(\mathrm{H})\,m_r(\mathrm{H})\,b_{1S}(\mathrm{H})}\,E^{\rm Z+pol}_{1S-\rm hfs}(\mathrm{H}) - \frac{E_{\rm F}(\mu\mathrm{H})}{n^3}\,\Delta_{\rm pol}(\mu\mathrm{H}) \quad \underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{=\,-\,6\times10^{-5}\ \text{for n = 1} \quad =\,-\,5\times10^{-5}\ \text{for n = 2}}$$

2. Disentangle Zemach radius and polarizability contribution
3. Testing the theory