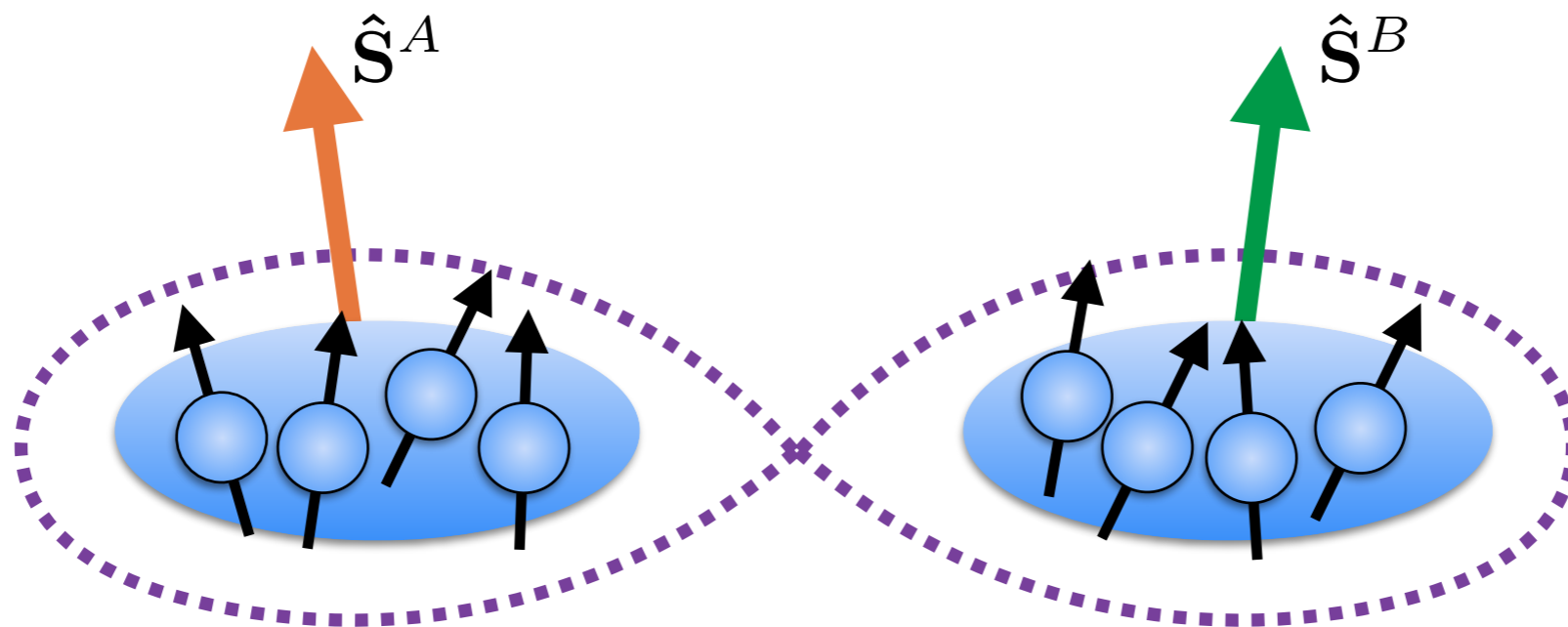


Einstein-Podolsky-Rosen experiment with two Bose-Einstein condensates



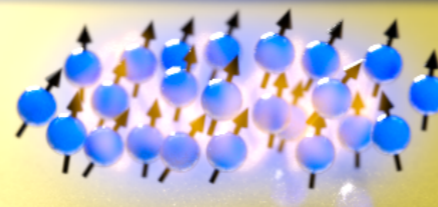
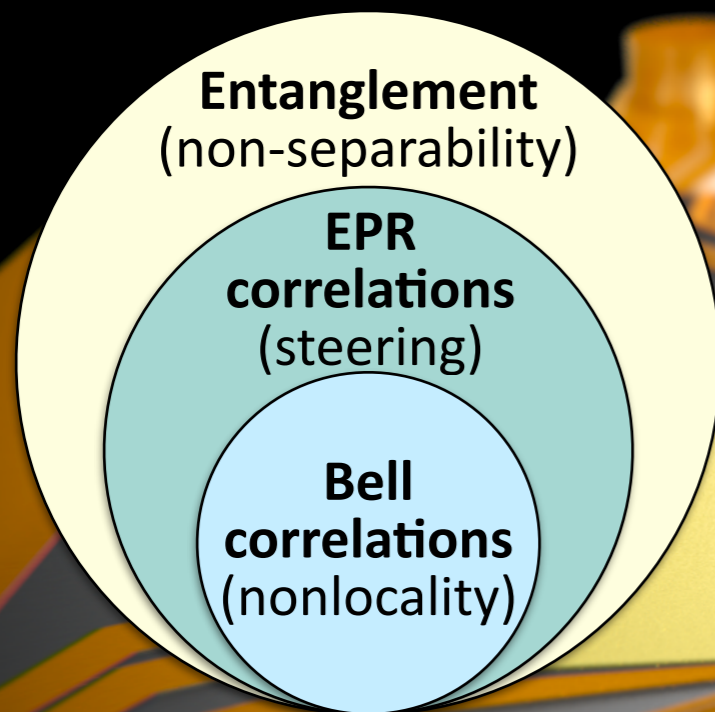
Many-particle entanglement and quantum metrology

Fundamental questions

- how to create and detect it
- how to quantify it
- classes of entangled states
- usefulness, robustness

Quantum metrology

- interferometry beyond standard quantum limit (SQL)
- field&force sensing on micrometer scale
- field imaging



Atomic spins in
Bose condensate

$$N = 10^2 - 10^3$$
$$\pm 5 \text{ atoms}$$


Einstein-Podolsky-Rosen experiment



Einstein-Podolsky-Rosen experiment




Einstein-Podolsky-Rosen experiment



A dark grey circle representing a particle, with a vertical green arrow pointing upwards on the left side and a vertical green arrow pointing downwards on the right side.

$$\frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

Einstein-Podolsky-Rosen experiment



A dark grey circle representing a particle, with a vertical green arrow pointing up on the left side and a vertical green arrow pointing down on the right side, indicating opposite spin components.

$$\frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

$$\text{var}(S_y)\text{var}(S_z) \geq \frac{1}{4} |\langle S_x \rangle|$$

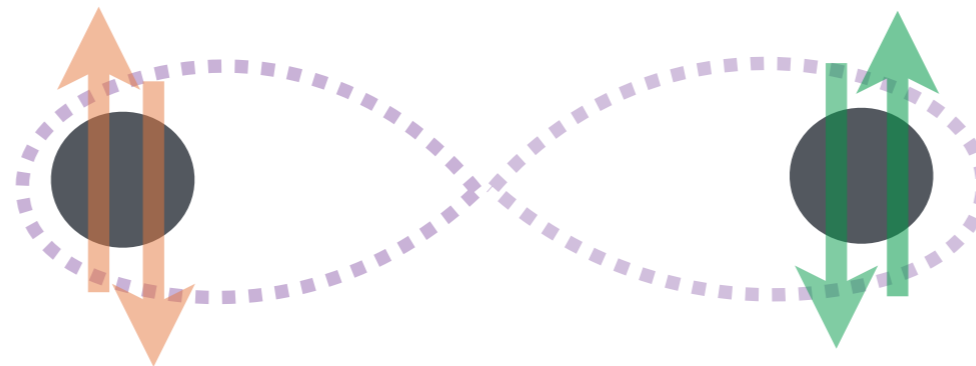
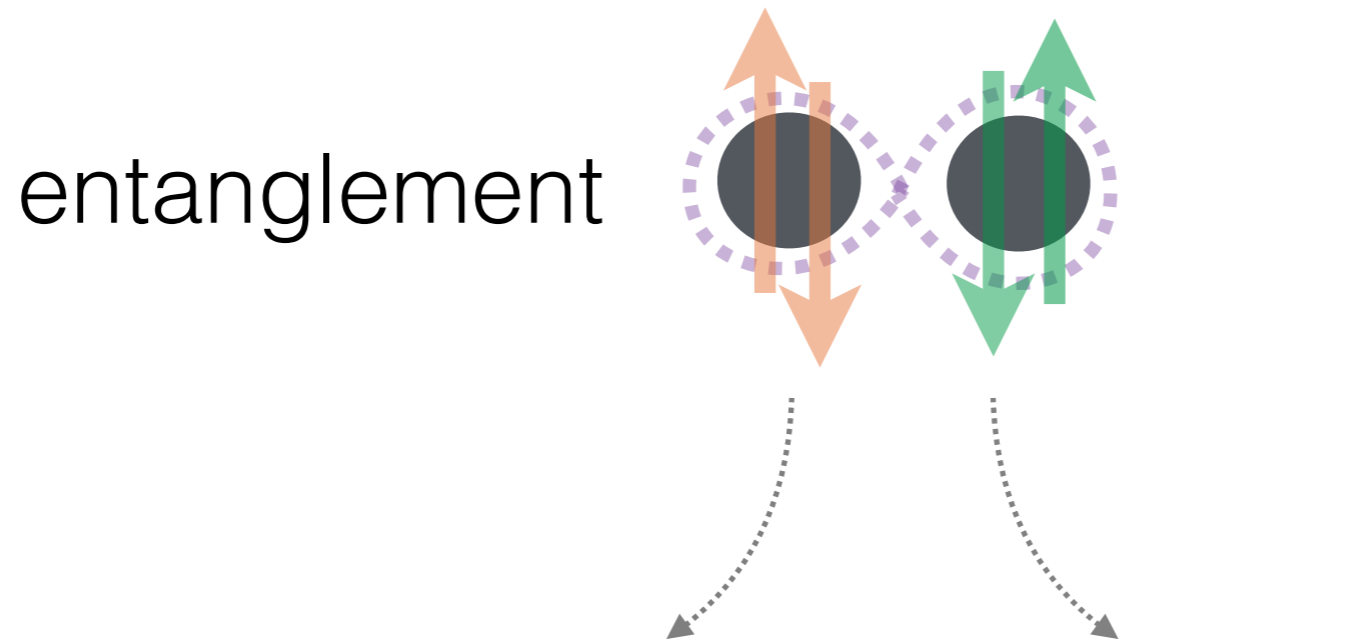
Einstein-Podolsky-Rosen experiment



Einstein-Podolsky-Rosen experiment

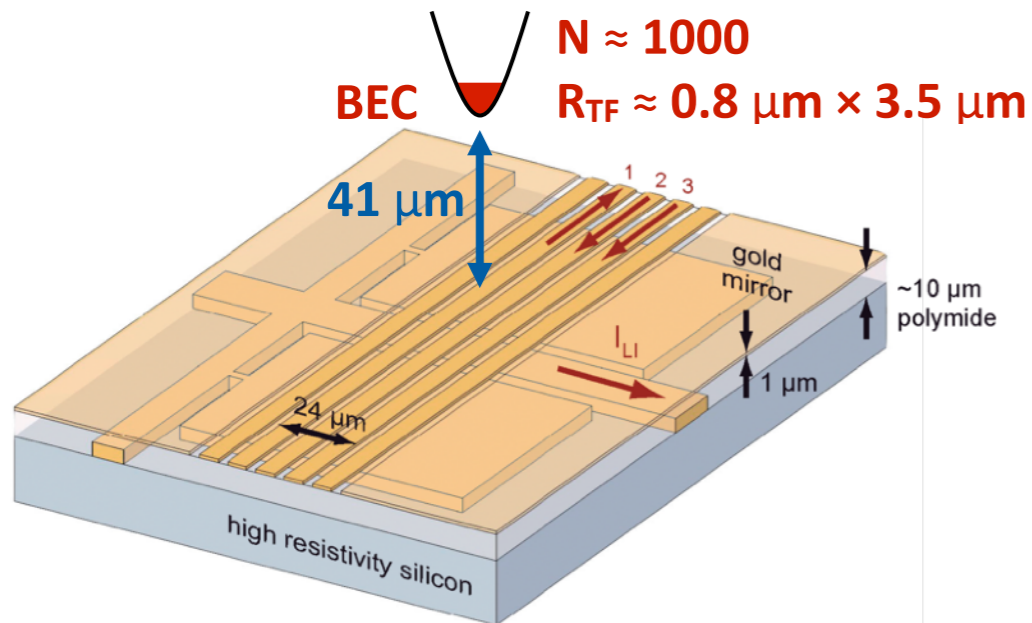
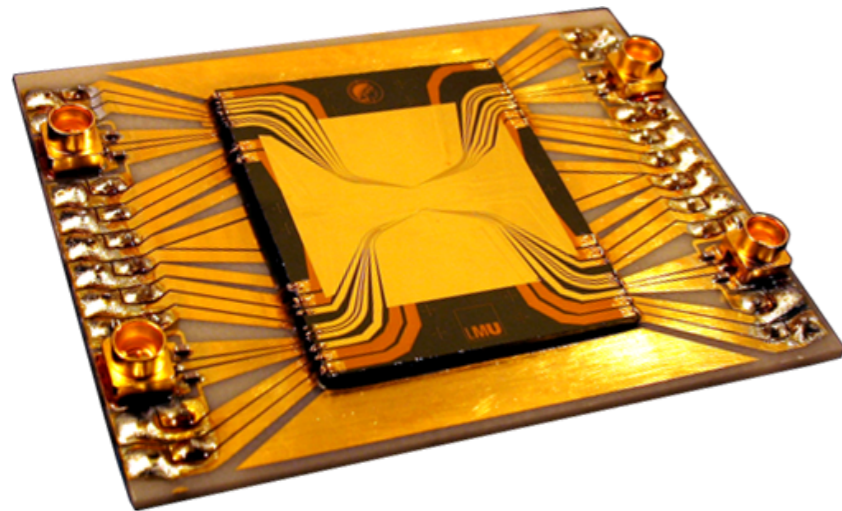


Einstein-Podolsky-Rosen experiment

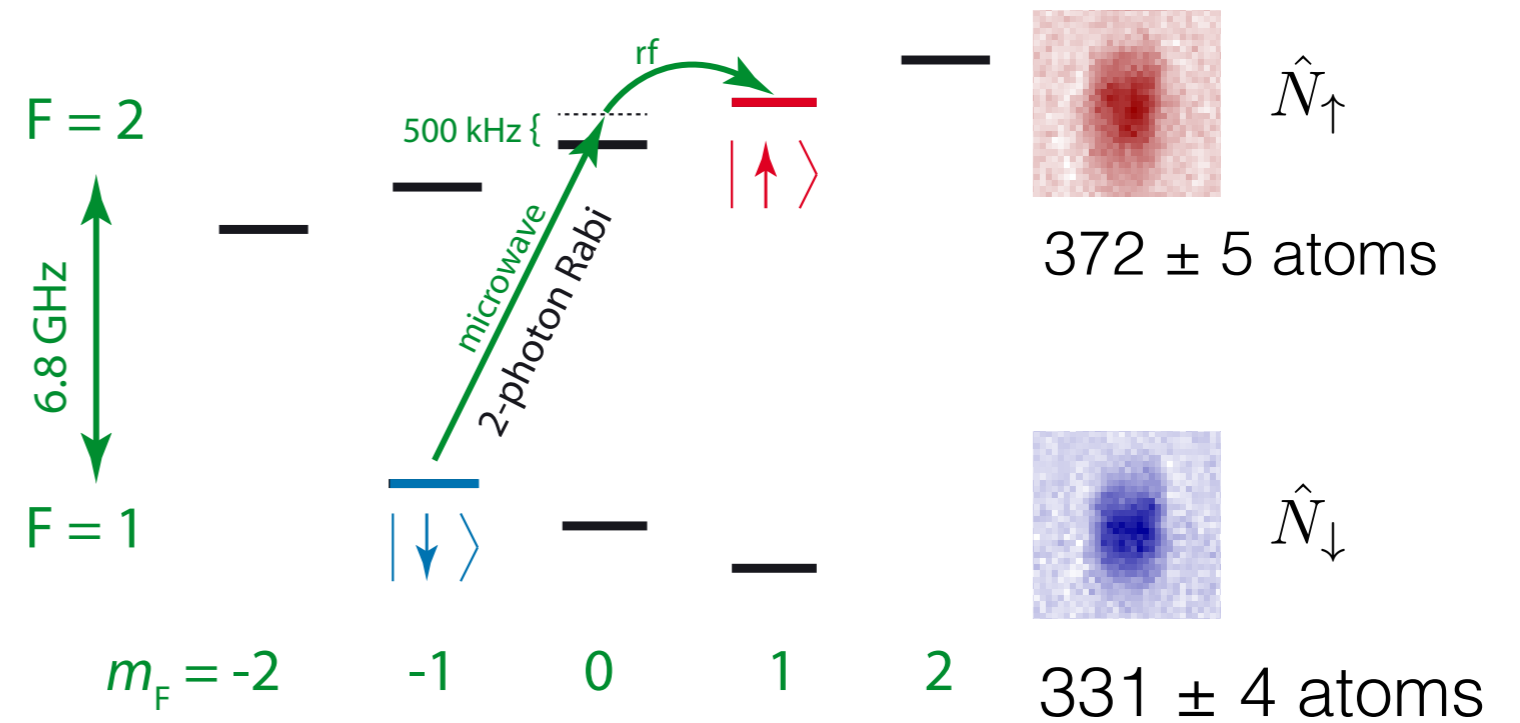


separate and measure

Two-component ^{87}Rb BEC on an atom chip

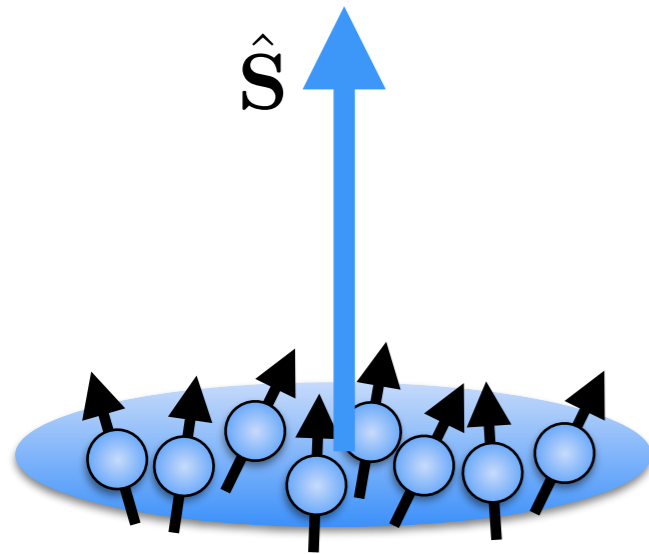


^{87}Rb ground state hyperfine structure:



- single spatial mode
- low susceptibility to magnetic field fluctuations
- spin rotations with very high fidelity
- very long coherence times (TACC, Syrte, Paris)
Li et al. PRL 125, 123402 (2020)

Spin-squeezing through atomic interactions

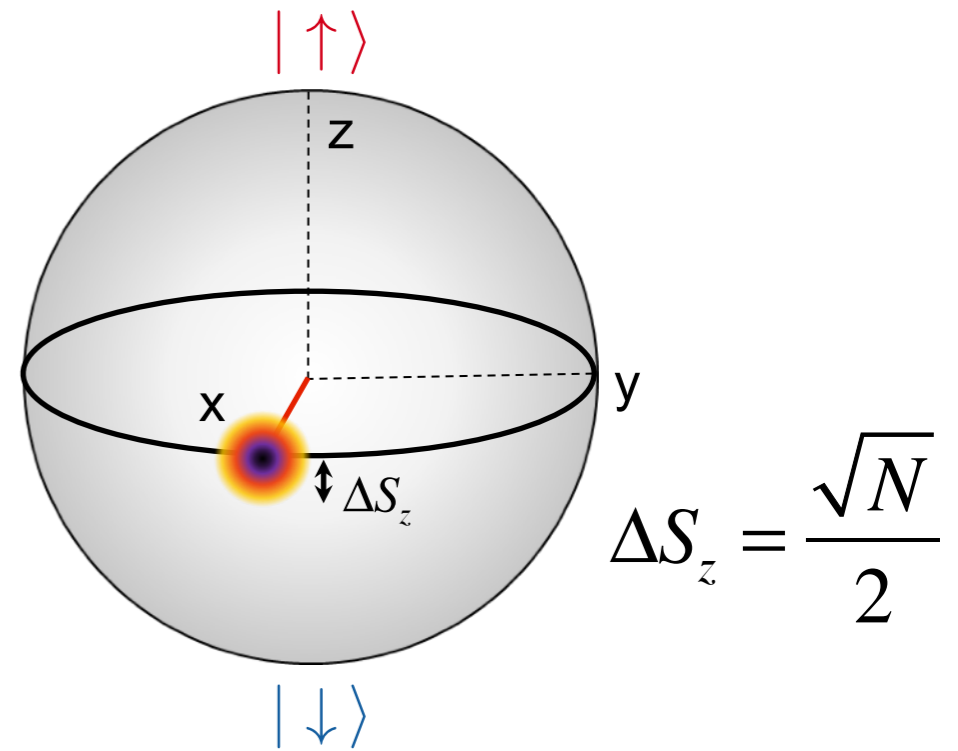


Ensemble of spins

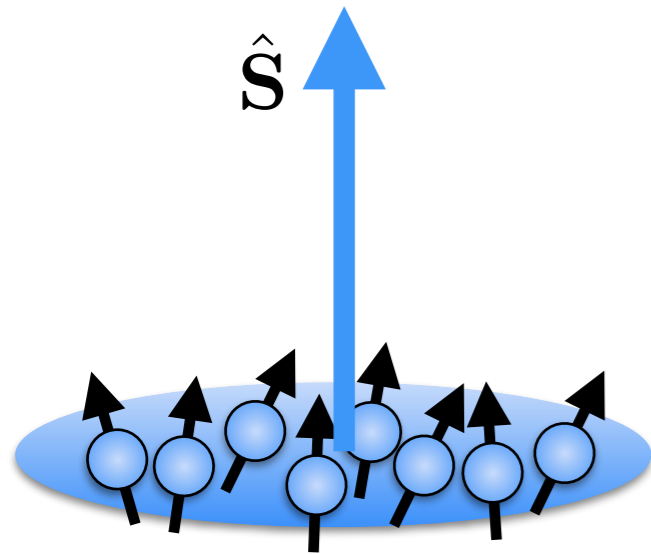
Collective spin:

$$\hat{S} = \sum_{i=1}^N \hat{s}_i, \quad S = \frac{N}{2}$$

$$\hat{S}_z = \frac{1}{2}(\hat{N}_\uparrow - \hat{N}_\downarrow)$$



Spin-squeezing through atomic interactions

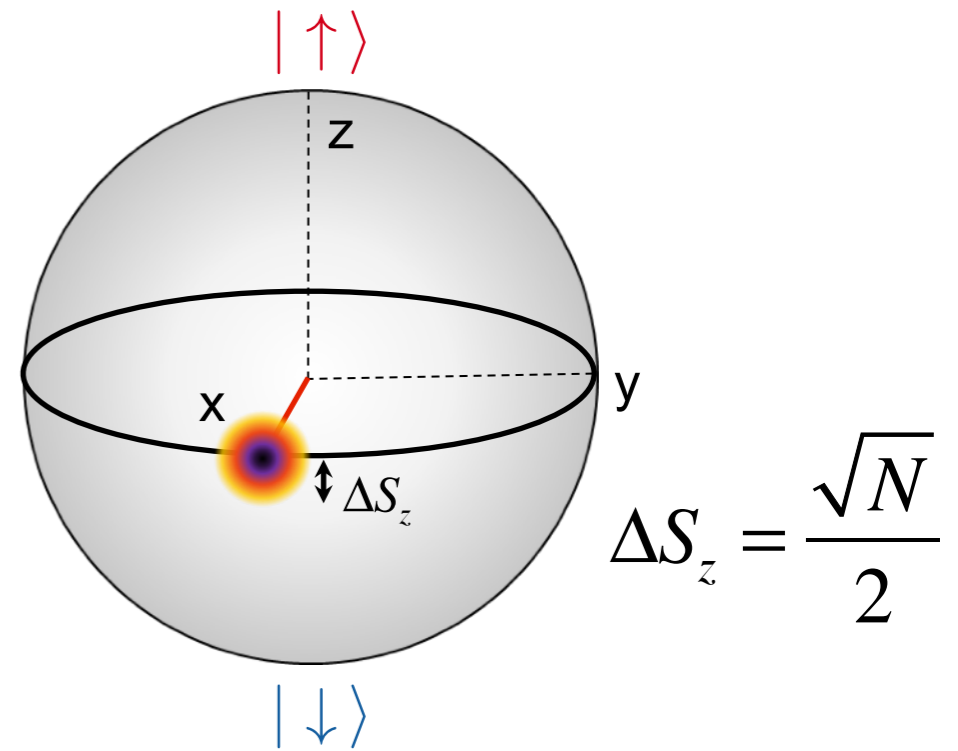


Ensemble of spins

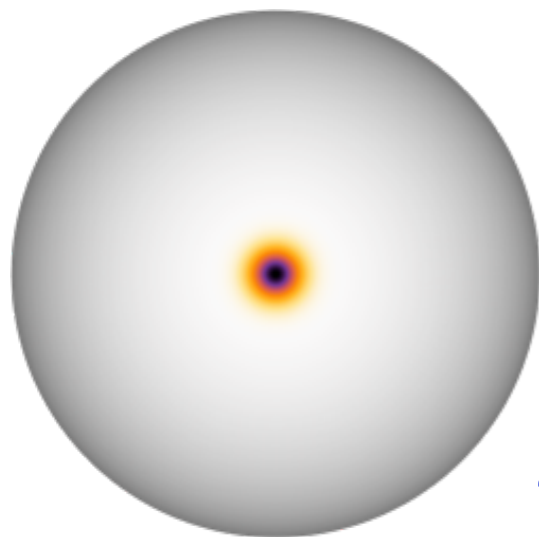
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$$\hat{S}_z = \frac{1}{2}(\hat{N}_\uparrow - \hat{N}_\downarrow)$$



Atomic collisions create spin squeezing

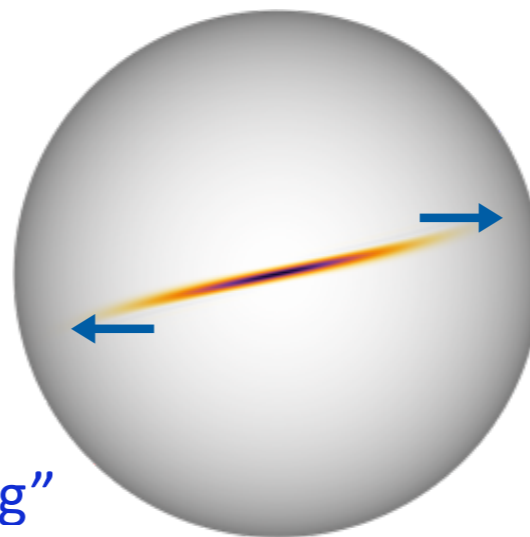


coherent spin state

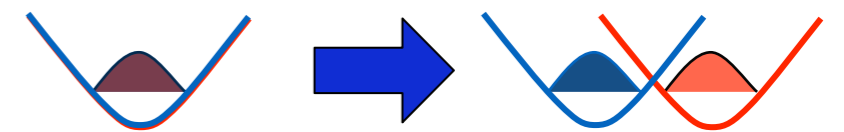
$$\hat{\mathcal{H}} = \chi \hat{S}_z^2$$



time evolution
"one-axis twisting"



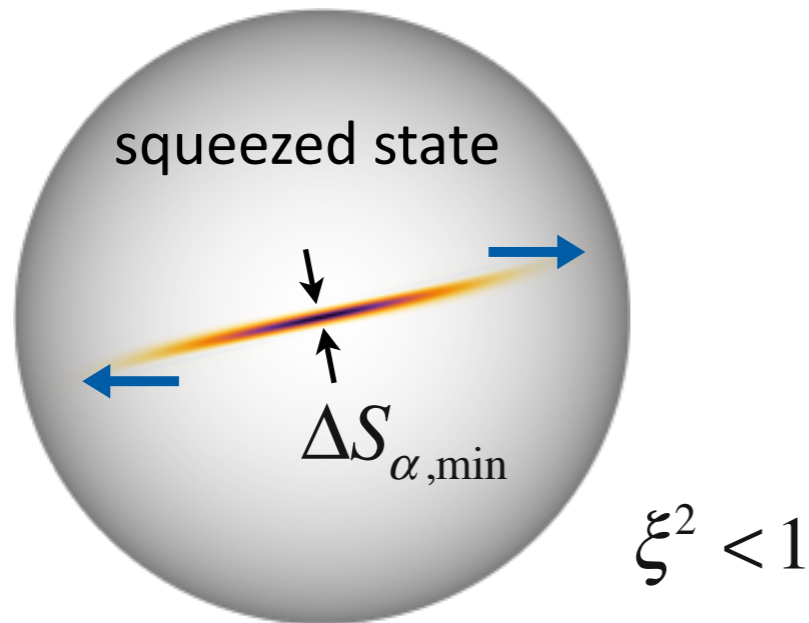
squeezed spin state



interaction tuning by
state dependent potentials

M. F. Riedel et al.,
Nature 464, 1170 (2010)

Entanglement in spin squeezed states



Spin-squeezing parameter

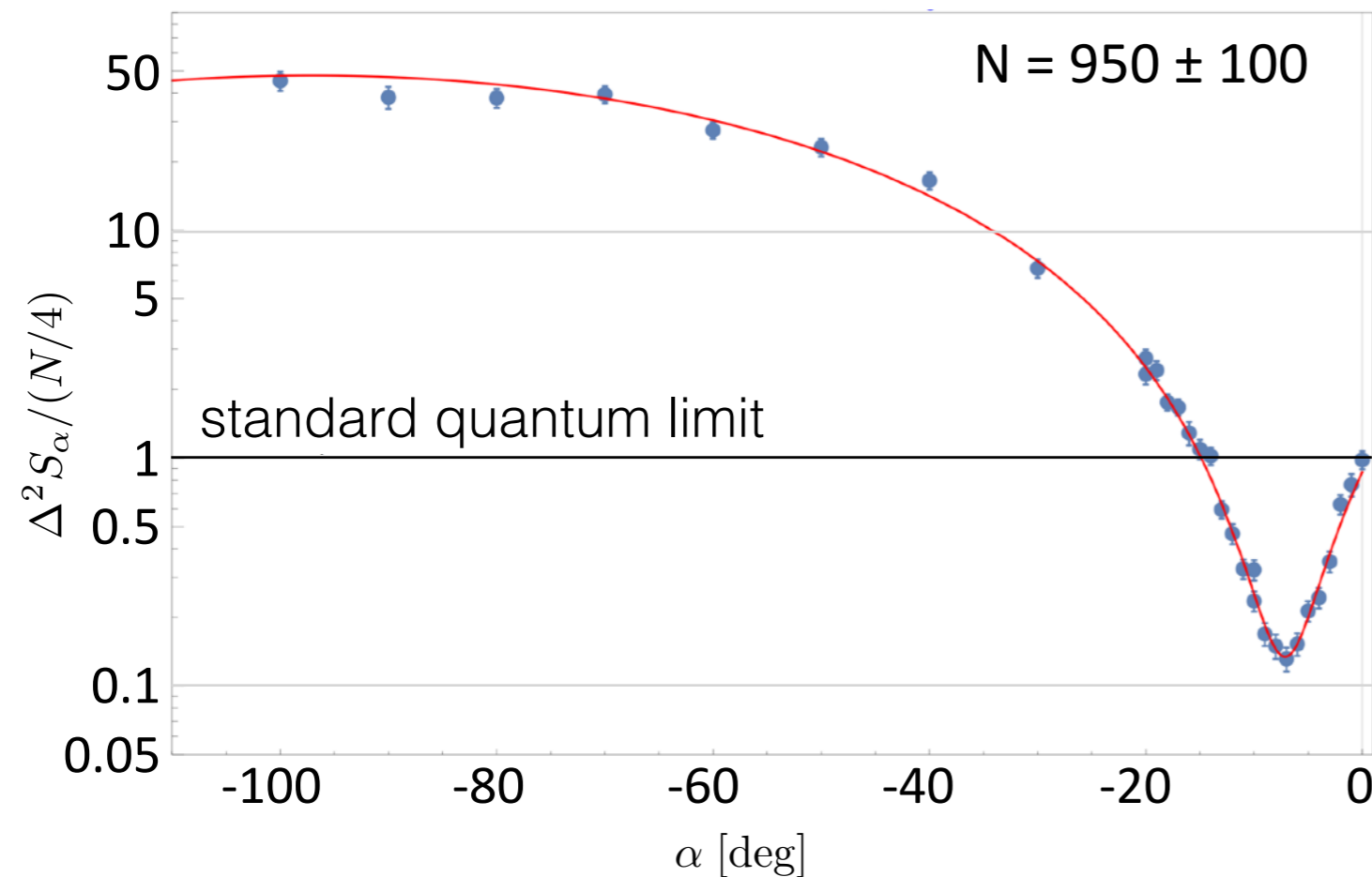
$$\xi^2 \equiv \frac{N (\Delta S_{\alpha, \min})^2}{\langle S_x \rangle^2}$$

entanglement witness

Kitagawa & Ueda PRA 47, 5138, 1993

Wineland et al. PRA 50, 67, 1994

Sørensen et al. Nature 409, 63–66, 2001



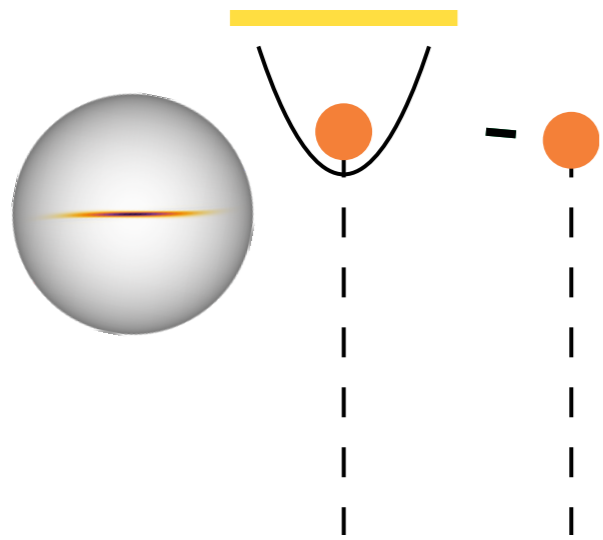
$$\xi^2 = -8.2 \pm 0.5 \text{ dB}$$




\Rightarrow entanglement

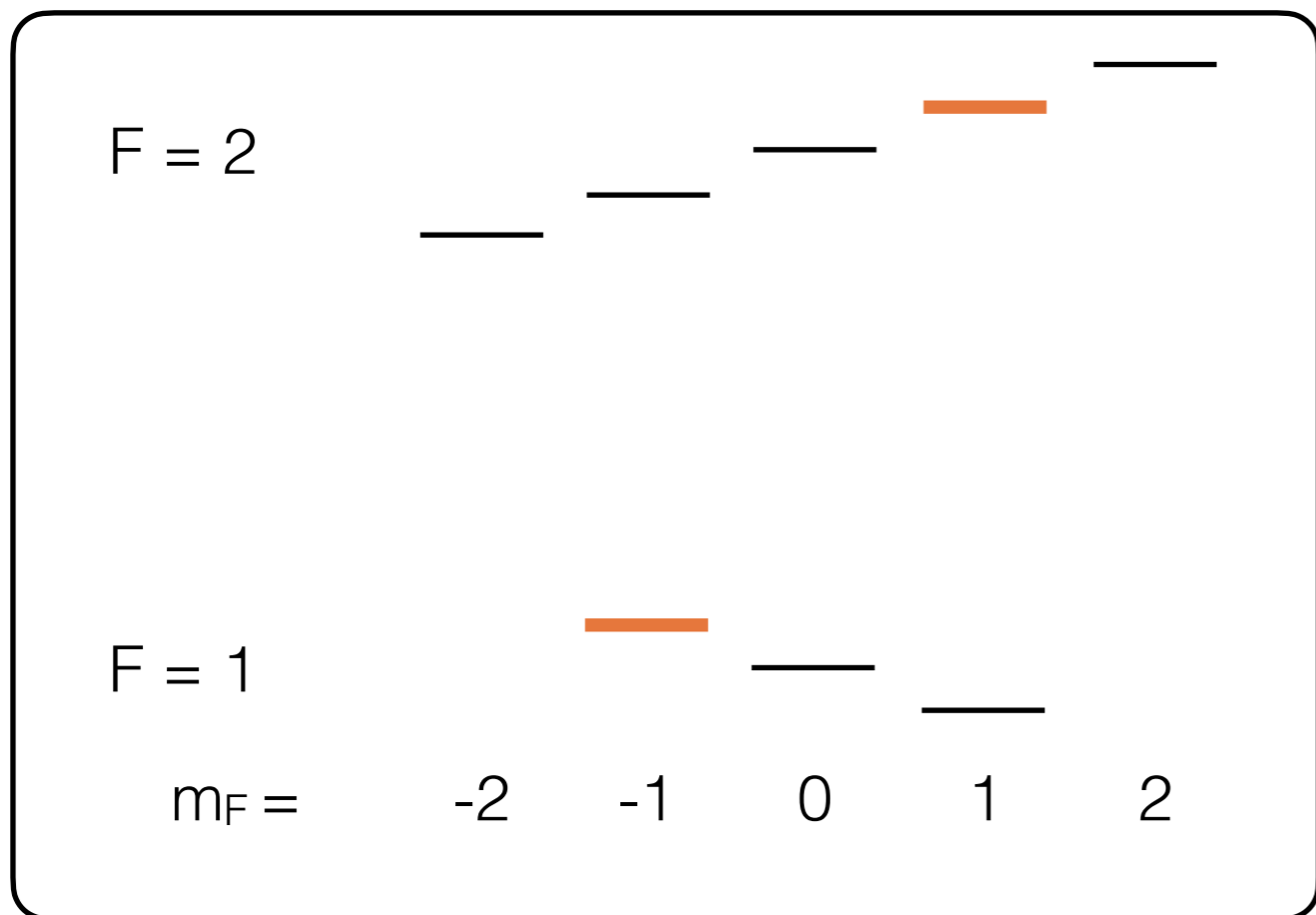
(Noise reduced by -8.7 ± 0.5 dB,
contrast $C = 94.9\%$)

Spin squeezed states possess
EPR-Entanglement and Bell correlations:
Fadel et al. Science 360, 409 (2018)
Kunkel et al. Science 360, 413 (2018)
Lange et al. Science 360, 416 (2018)
Schmied et al. Science 352, 441 (2016)

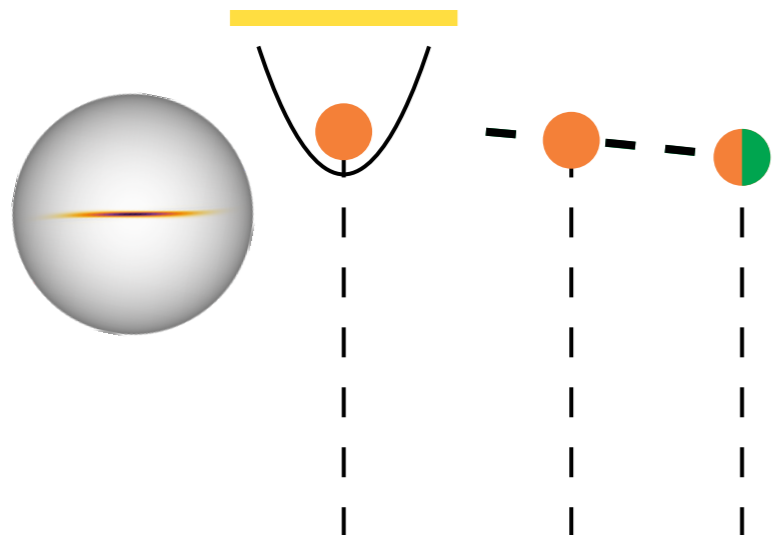
Splitting a BEC in two



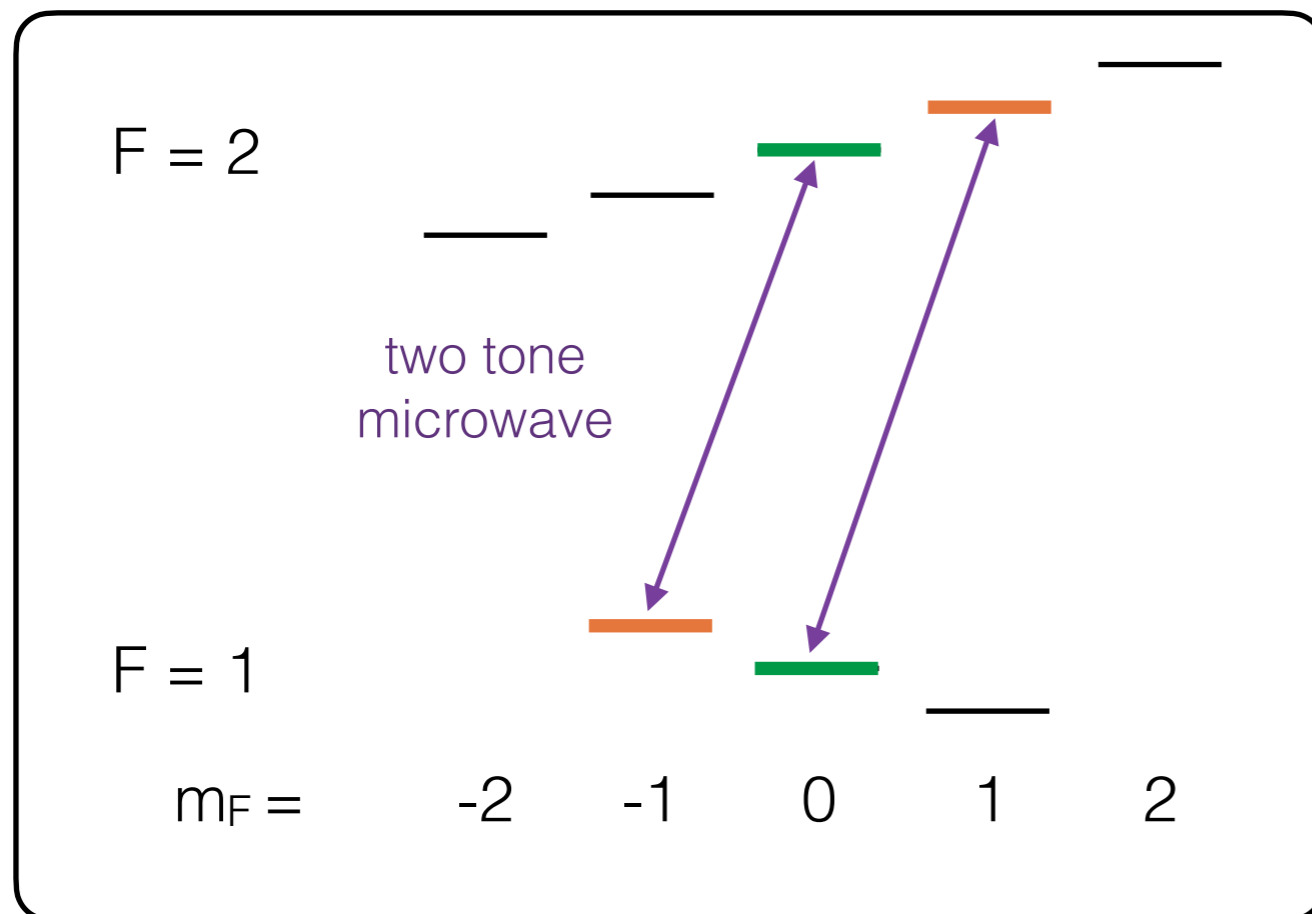
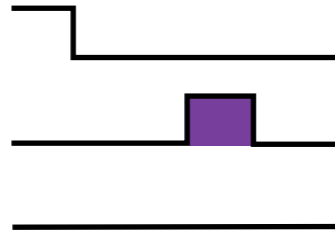
Trap 
Coupling 
Gradient 



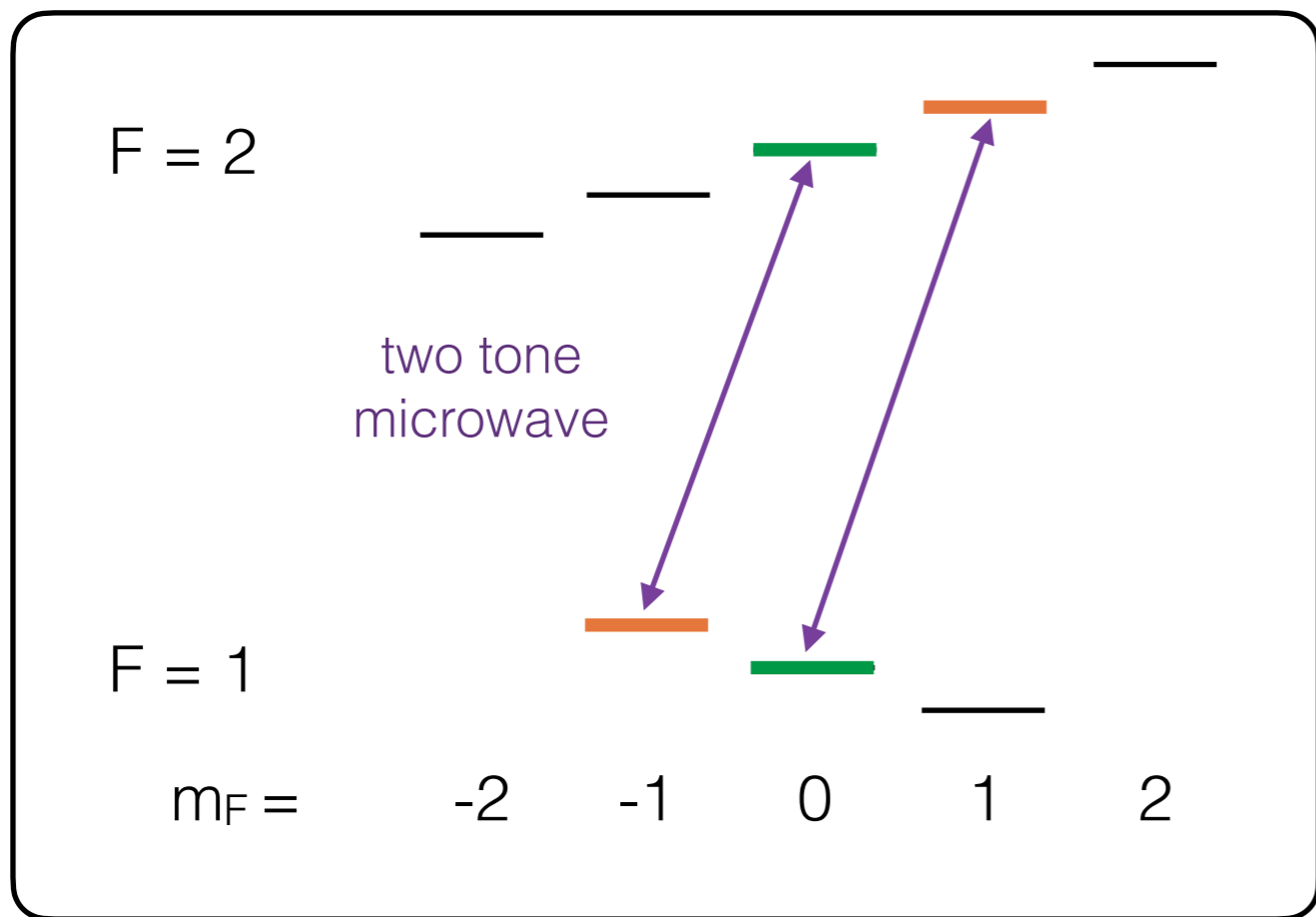
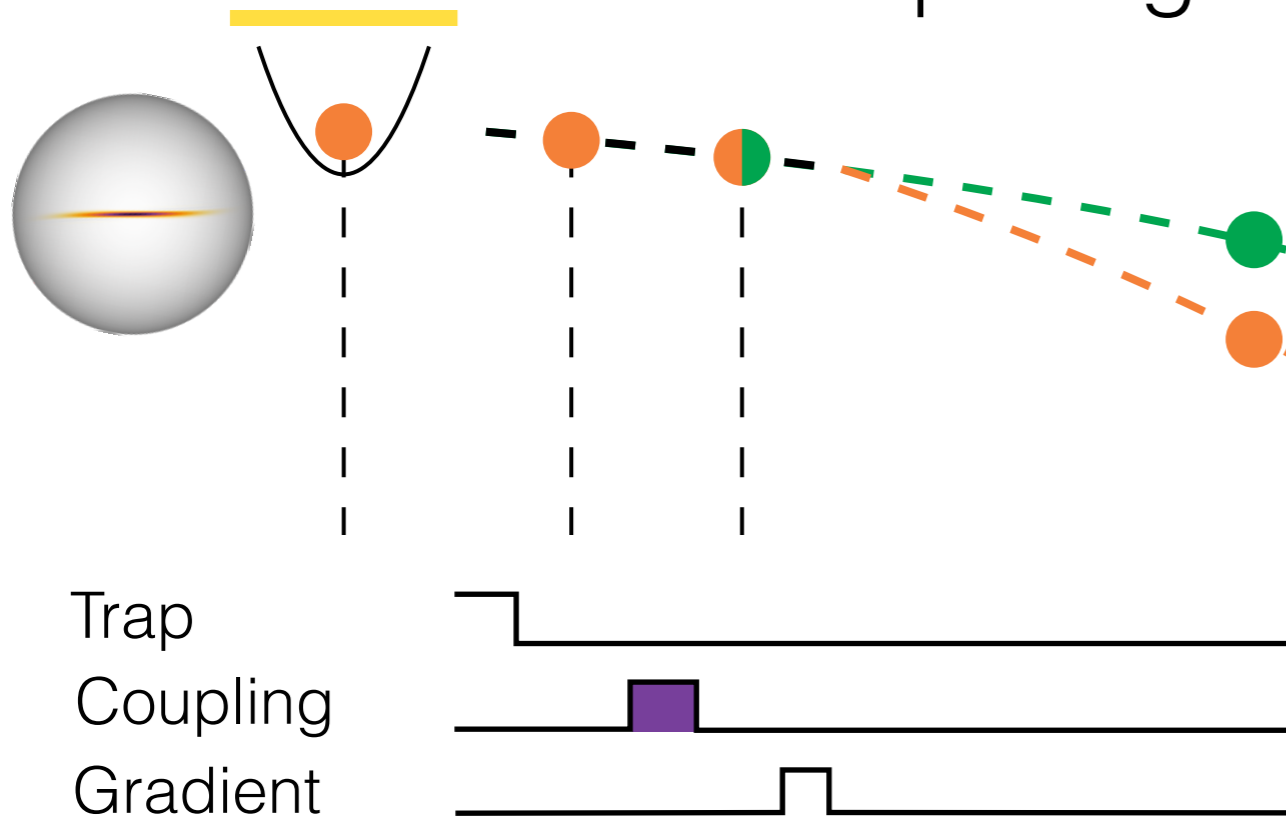
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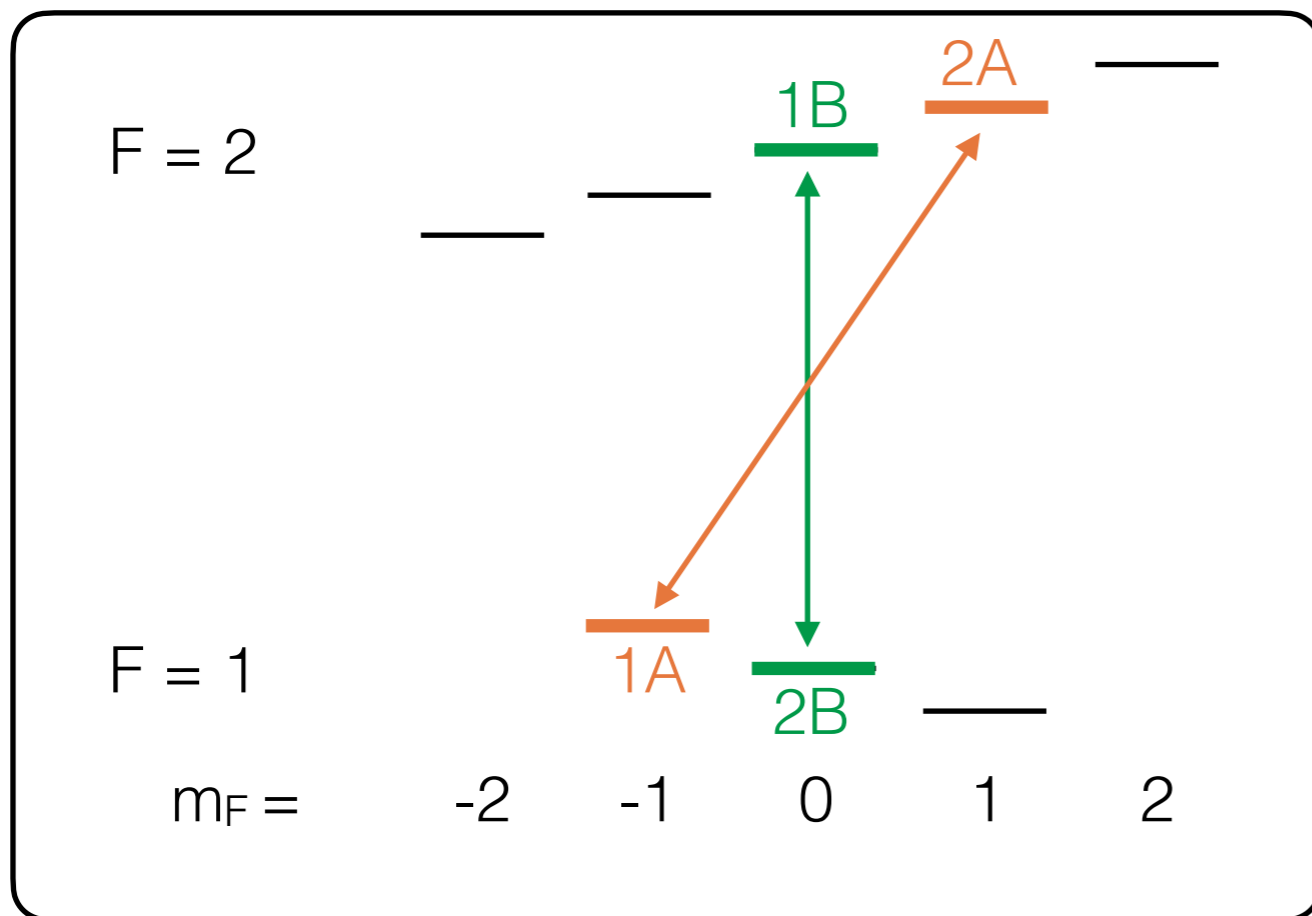
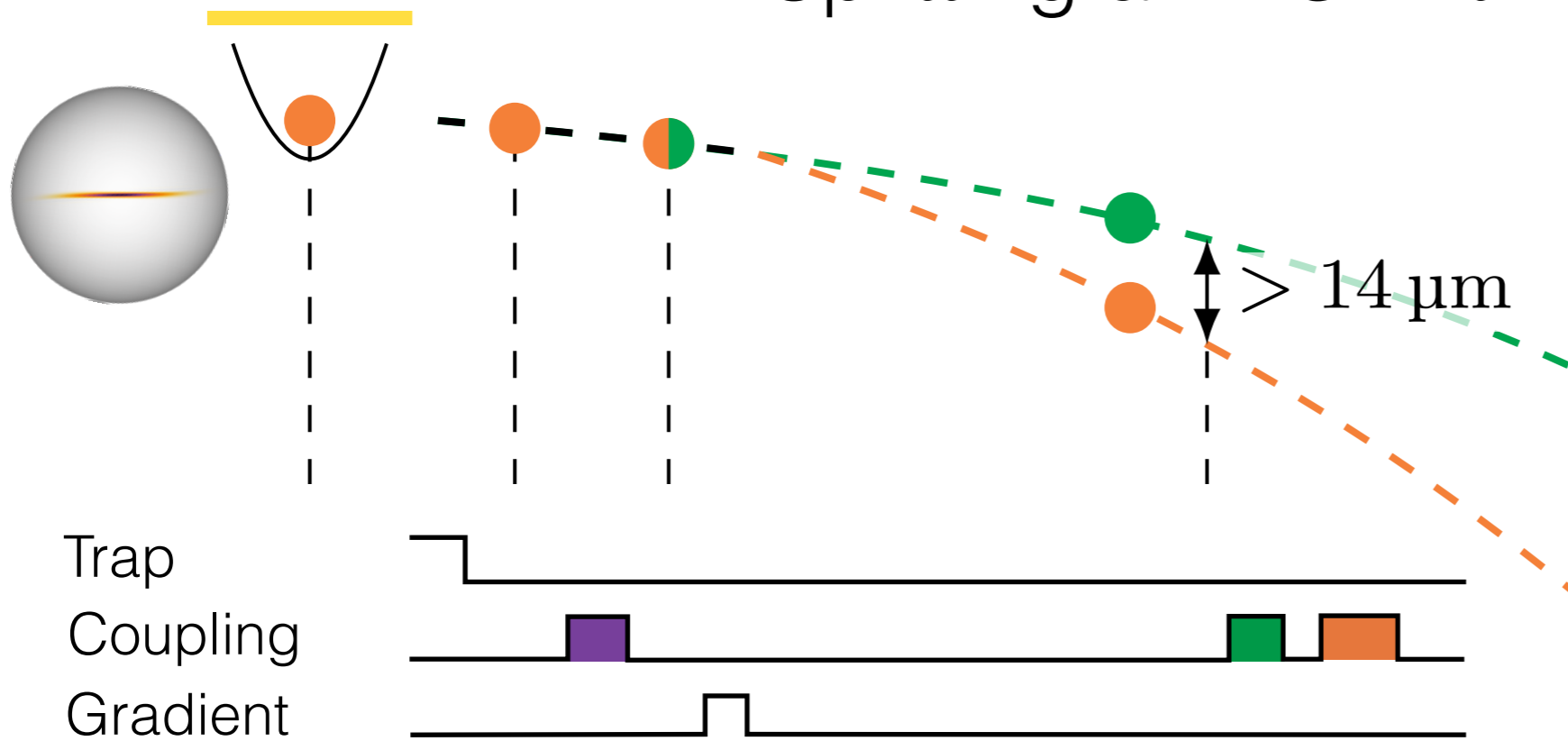
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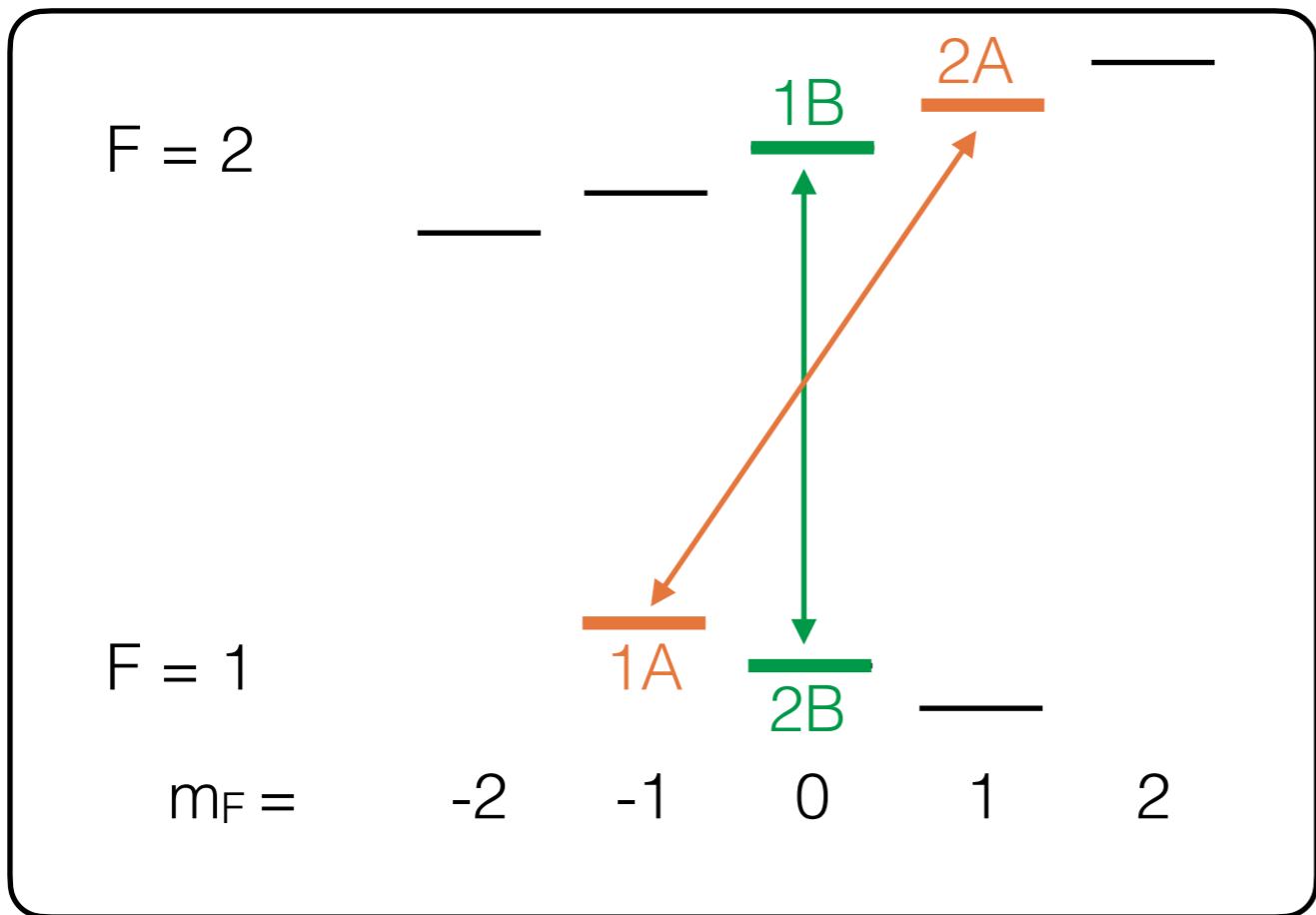
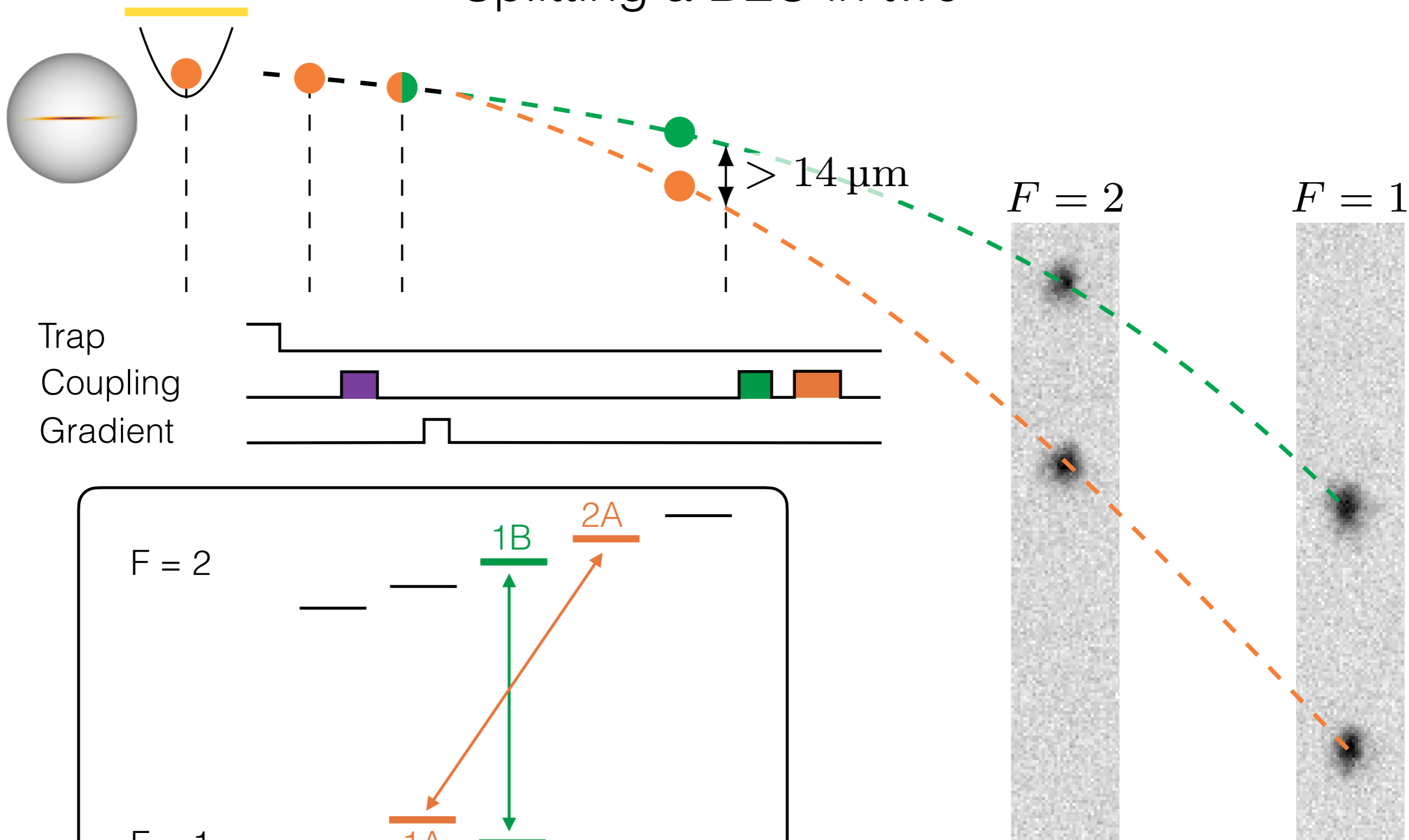
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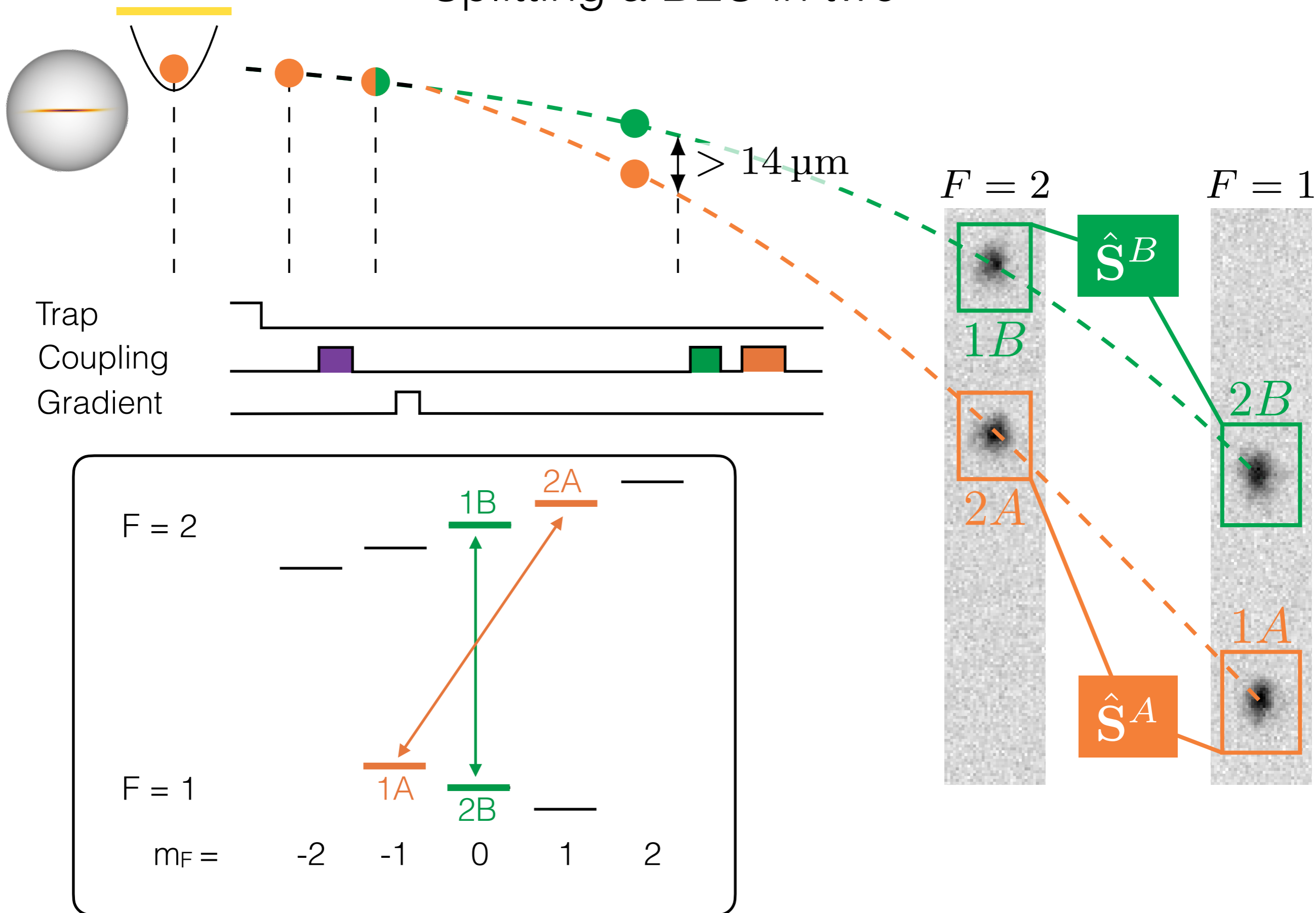
Splitting a BEC in two



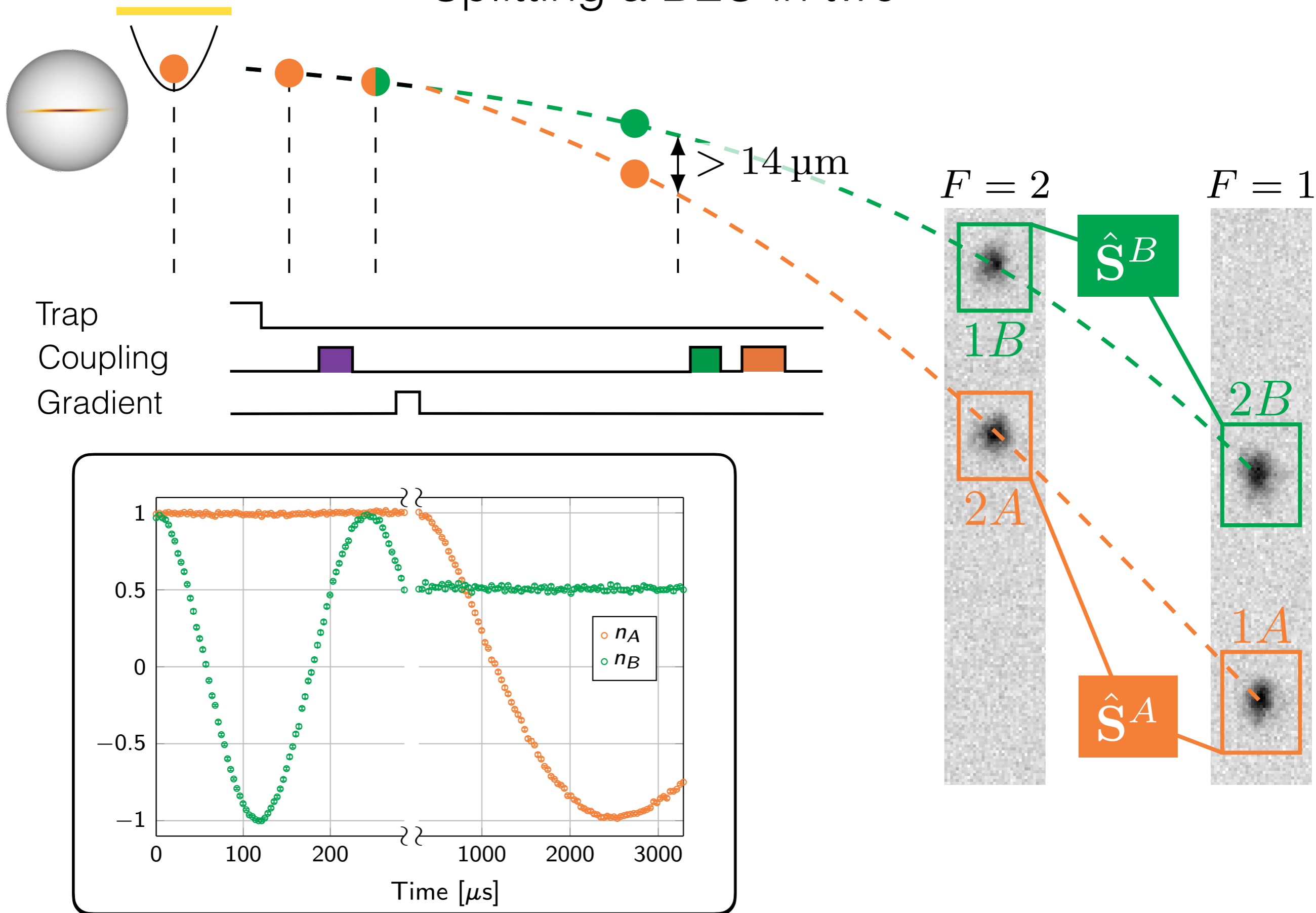
Splitting a BEC in two



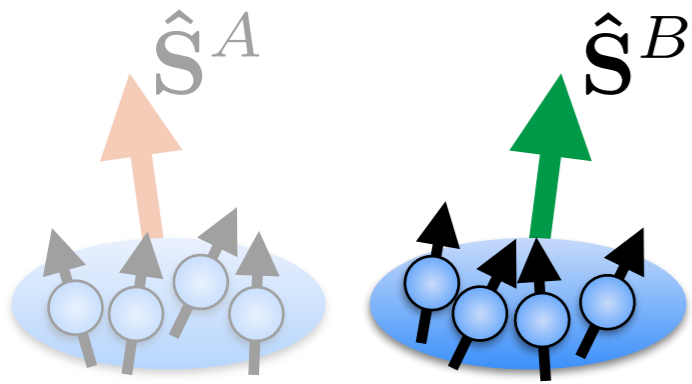
Splitting a BEC in two



Splitting a BEC in two



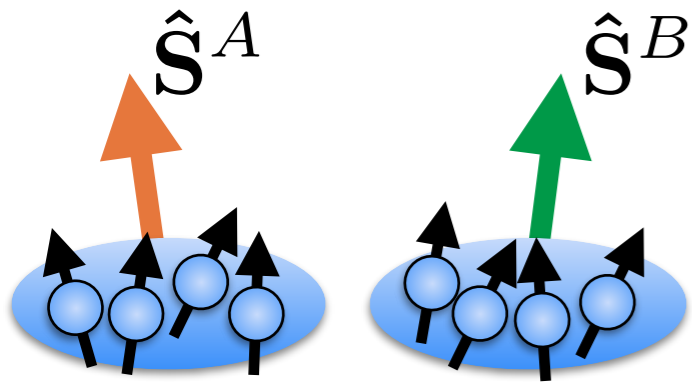
Einstein-Podolsky-Rosen entanglement



Heisenberg uncertainty relation for System **B**:

$$\text{Var}(\hat{S}_z^B) \text{Var}(\hat{S}_y^B) \geq \frac{1}{4} |\langle \hat{S}_x^B \rangle|^2$$

Einstein-Podolsky-Rosen entanglement

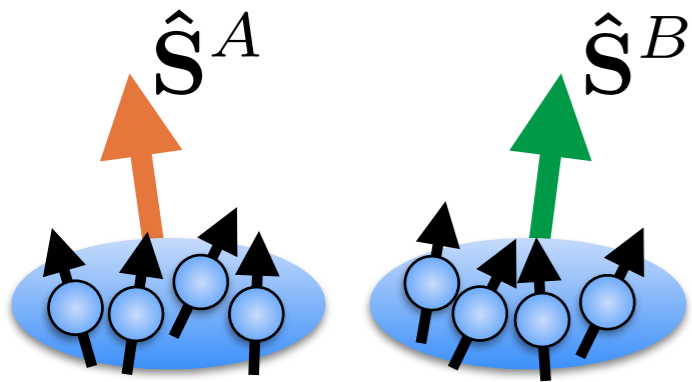


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Entanglement of \hat{S}^A and \hat{S}^B allows: $\text{Var}(g_z \hat{S}_z^A + \hat{S}_z^B) < \text{Var}(\hat{S}_z^B)$
inferred variance

Einstein-Podolsky-Rosen entanglement



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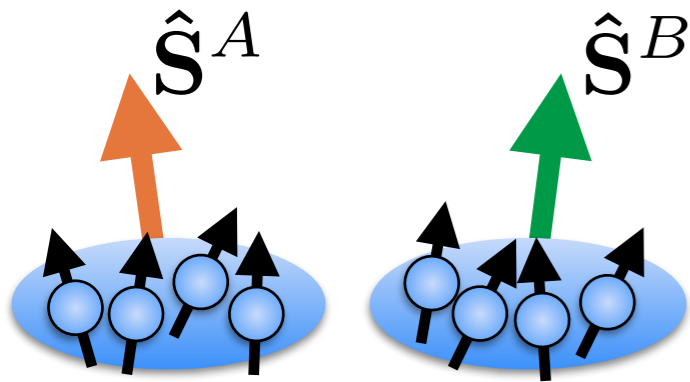
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inferred variance

EPR paradox:

$$\text{Var}_{\text{inf}}(\hat{S}_z^B) \text{Var}_{\text{inf}}(\hat{S}_y^B) < \frac{1}{4} |\langle \hat{S}_x^B \rangle|^2$$

inferred uncertainty product can beat the Heisenberg limit

Einstein-Podolsky-Rosen entanglement



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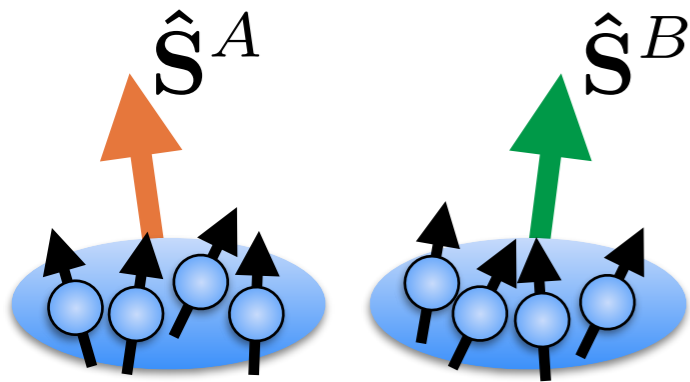
$$\text{Var}_{\text{inf}}(\hat{S}_z^B) \text{Var}_{\text{inf}}(\hat{S}_y^B) < \frac{1}{4} |\langle \hat{S}_x^B \rangle|^2$$

inferred uncertainty product can beat the Heisenberg limit

EPR criterion:

$$\mathcal{E}_{\text{EPR}} = \frac{4 \text{Var}(g_z \hat{S}_z^A + \hat{S}_z^B) \text{Var}(g_y \hat{S}_y^A + \hat{S}_y^B)}{|\langle \hat{S}_x^B \rangle|^2} \geq 1$$

Einstein-Podolsky-Rosen entanglement



Heisenberg uncertainty relation for System **B**:

$$\text{Var}(\hat{S}_z^B) \text{Var}(\hat{S}_y^B) \geq \frac{1}{4} |\langle \hat{S}_x^B \rangle|^2$$

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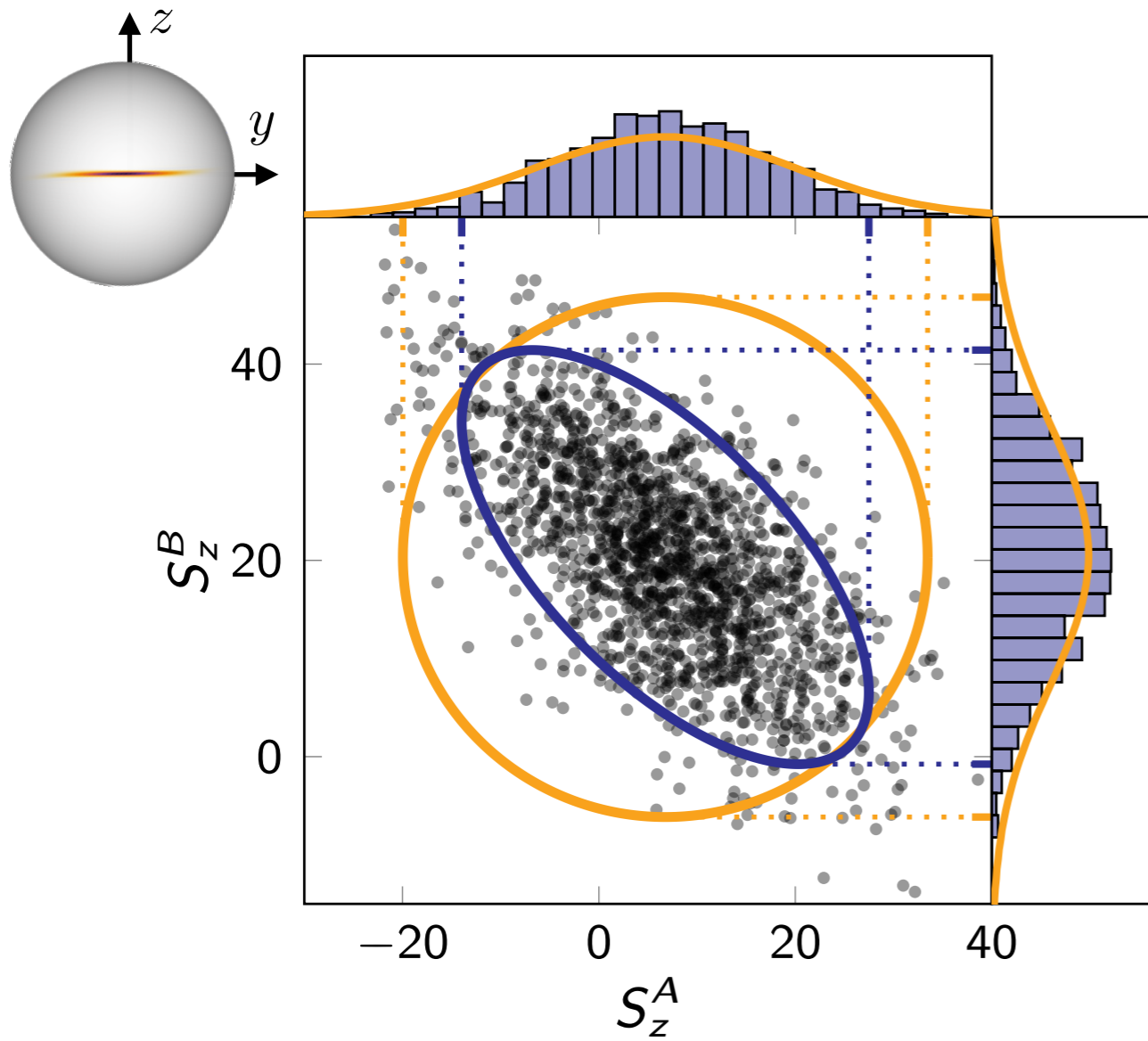
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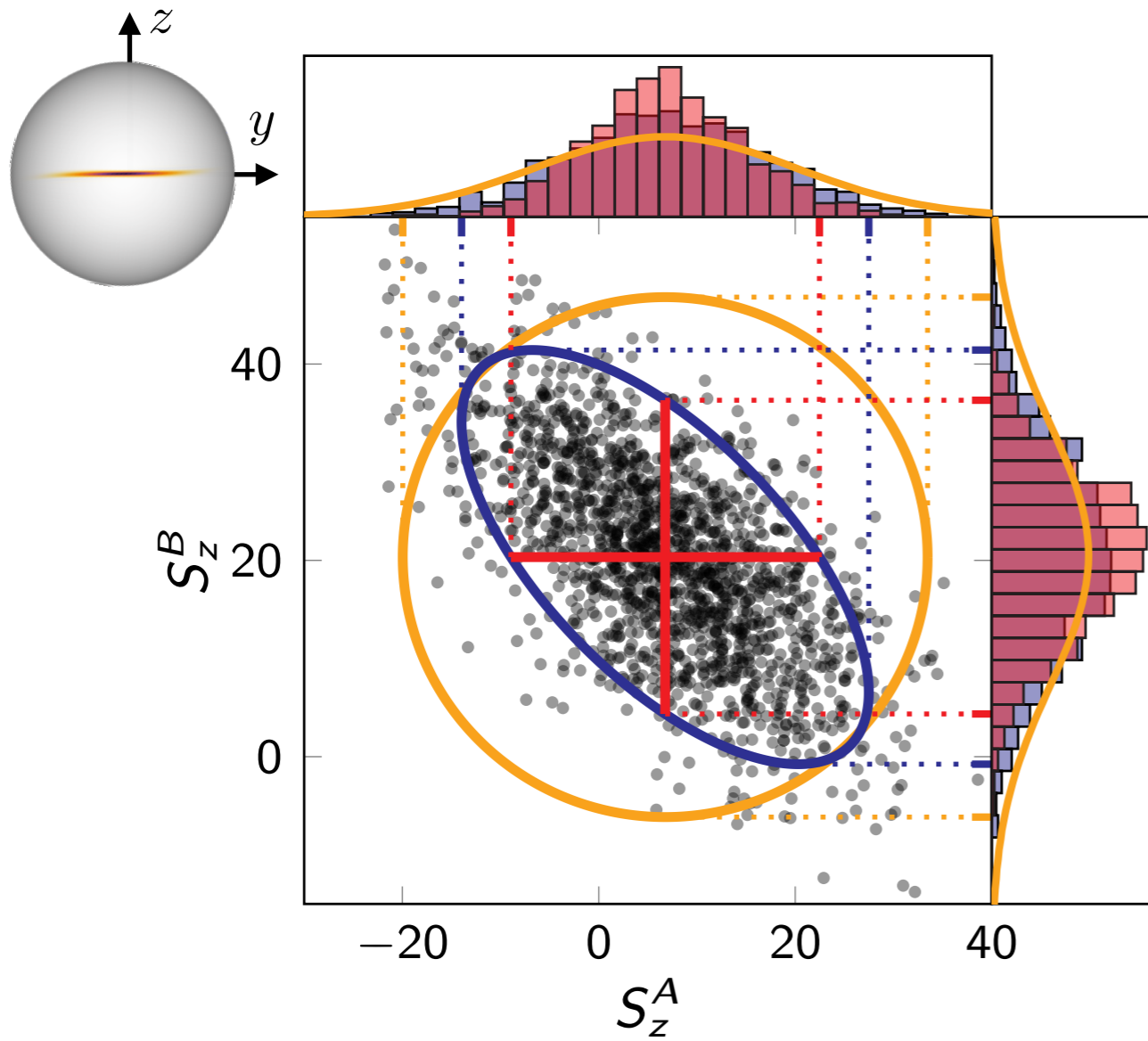
Entanglement criterion:

$$\mathcal{E}_{\text{Ent}} = \frac{4 \text{Var}(g_z \hat{S}_z^A + \hat{S}_z^B) \text{Var}(g_y \hat{S}_y^A + \hat{S}_y^B)}{\left(|g_z g_y| |\langle \hat{S}_x^A \rangle| + |\langle \hat{S}_x^B \rangle| \right)^2} \geq 1$$

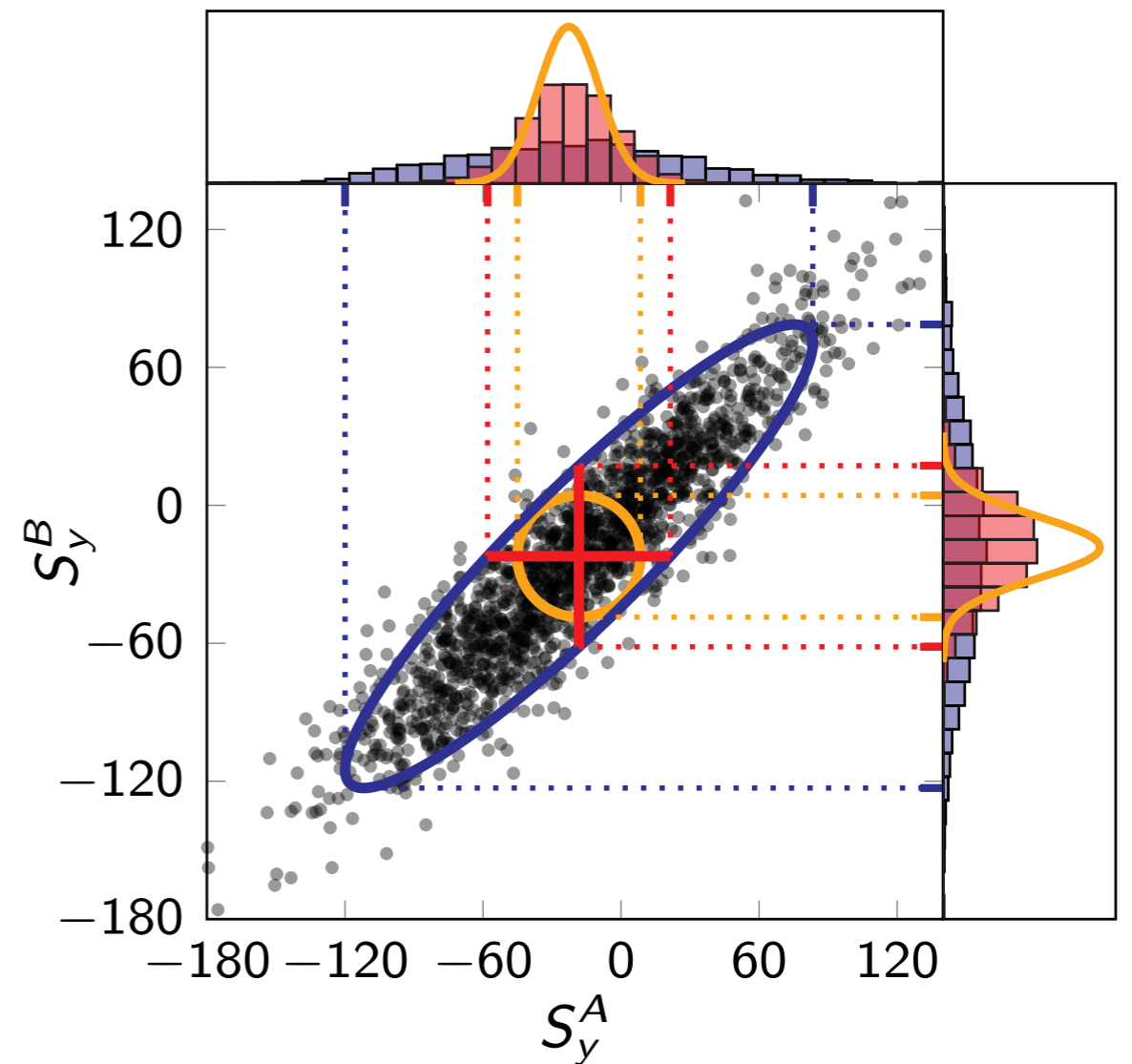
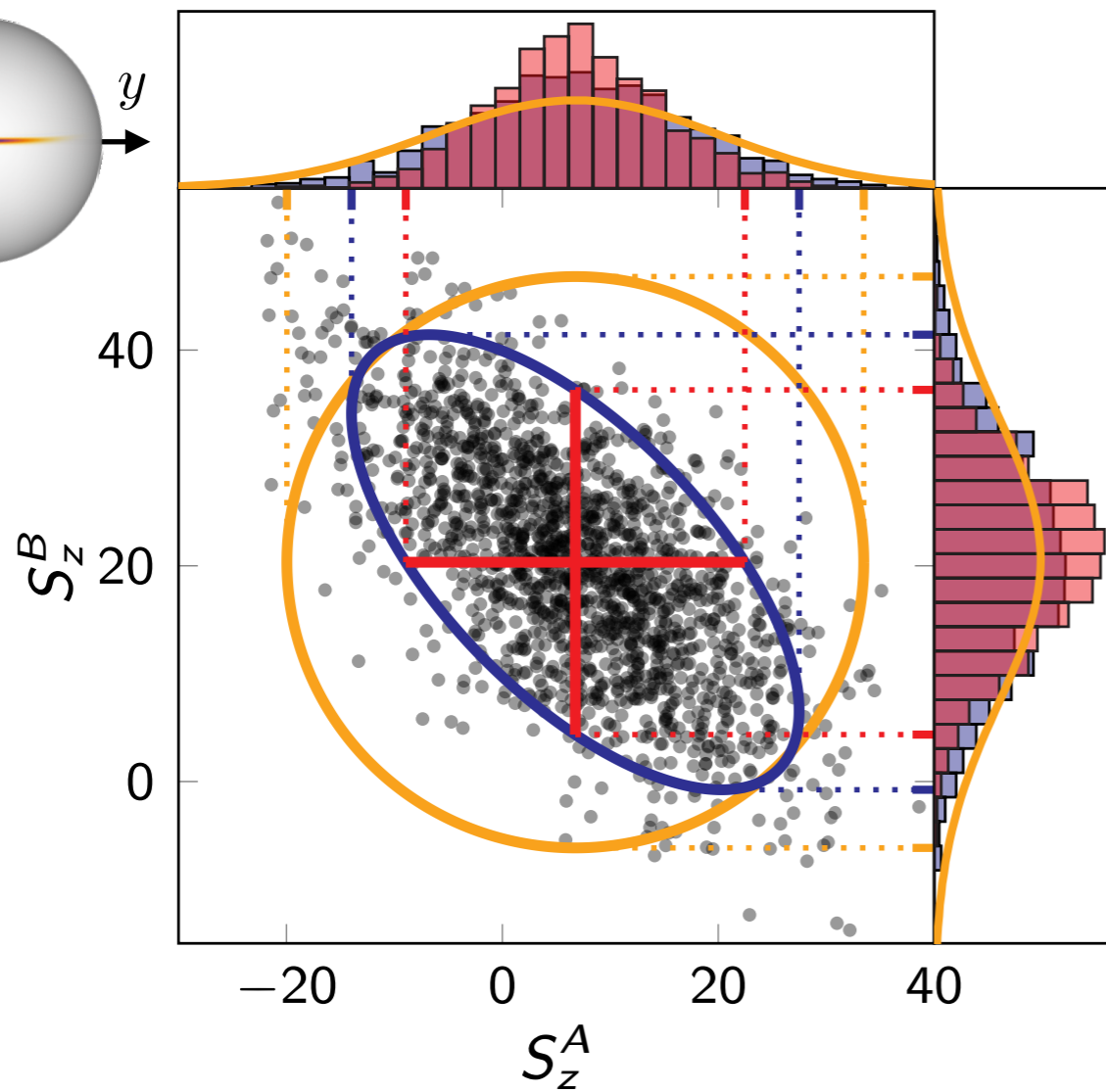
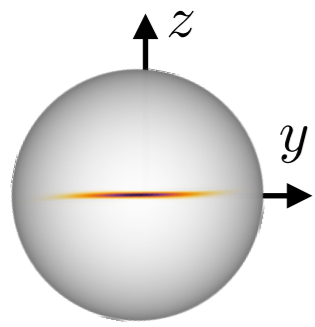
Correlations reveal entanglement between the two BECs



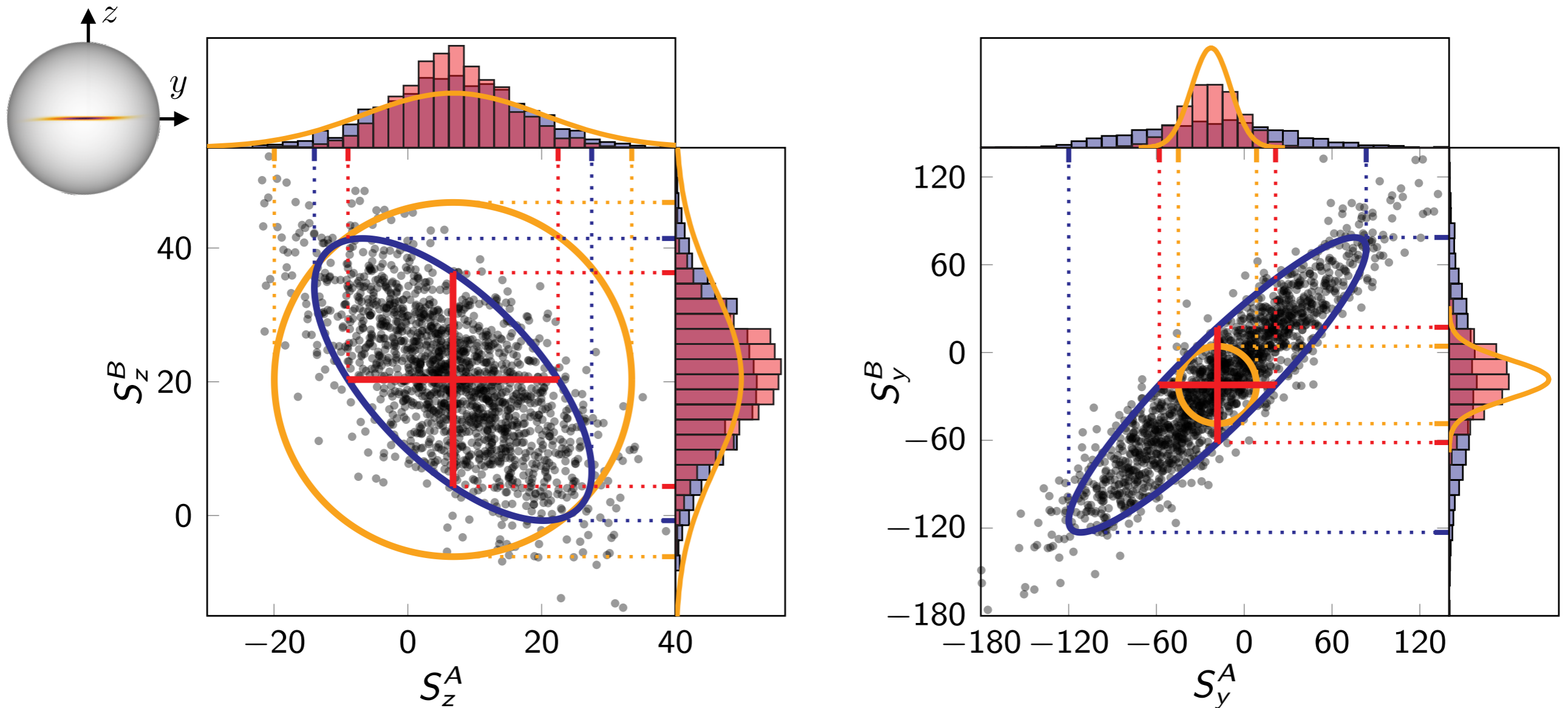
Correlations reveal entanglement between the two BECs



Correlations reveal entanglement between the two BECs



Correlations reveal entanglement between the two BECs

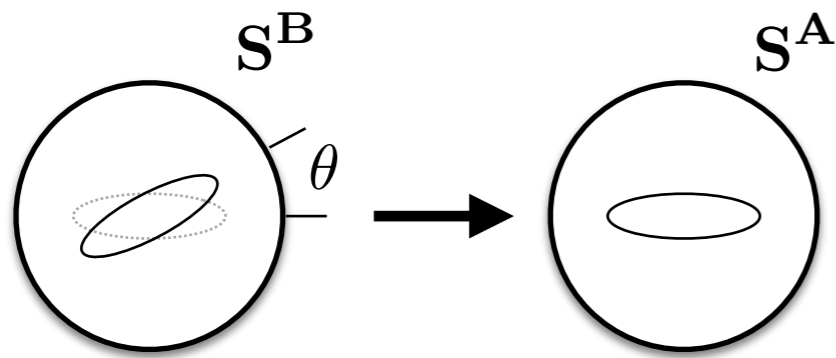


- initial spin squeezing $\xi^2 \approx -7$ dB
- split spins are still squeezed
- contrast in S_x^A and S_x^B about 96%

Entanglement and EPR steering detected:

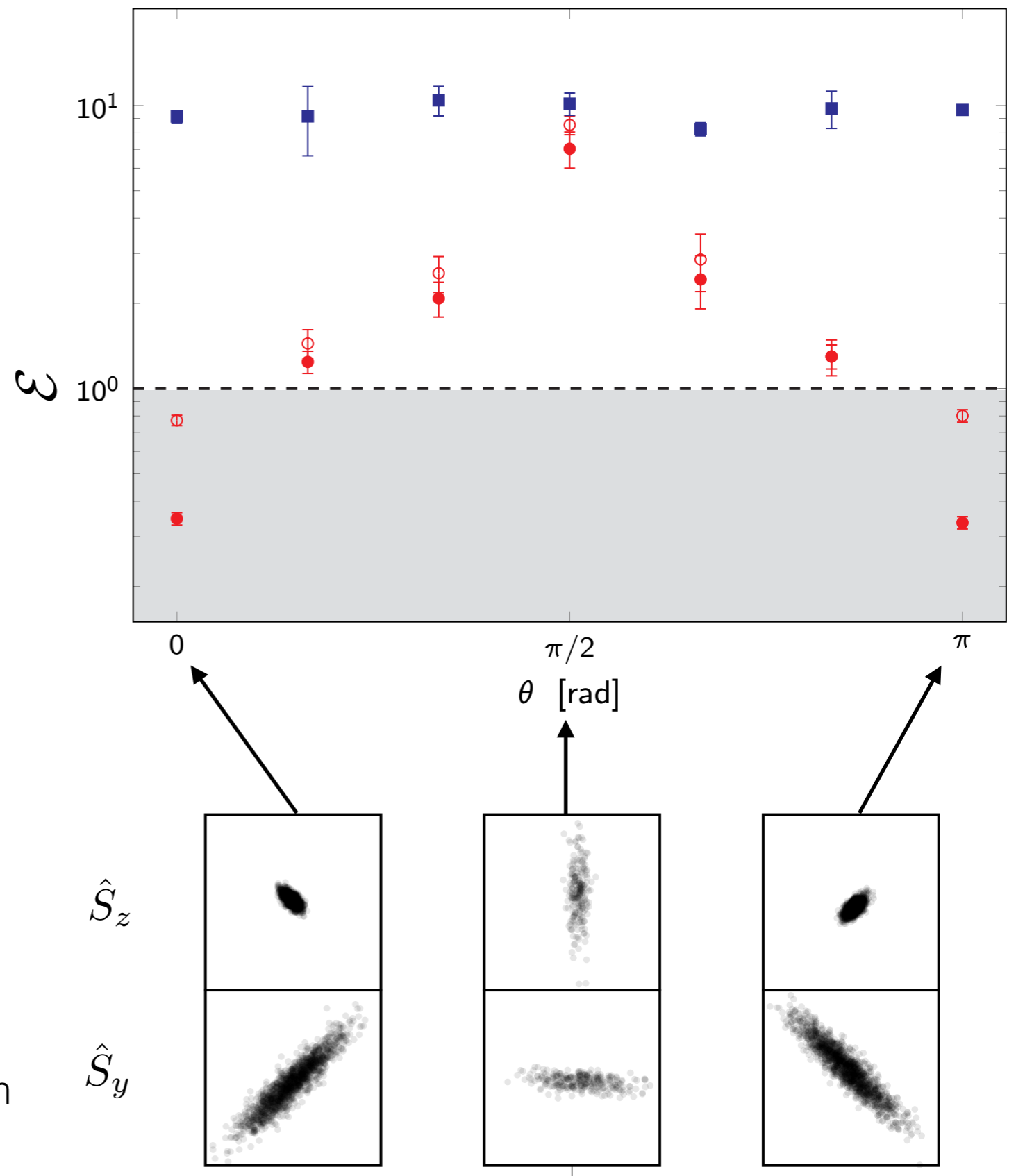
	$A \rightarrow B$	$B \rightarrow A$
\mathcal{E}_{Ent}	0.35 ± 0.01	
\mathcal{E}_{EPR}	0.81 ± 0.03	0.77 ± 0.03
$H.l.$	10.2 ± 0.4	9.2 ± 0.5

Individual manipulation of the two BECs



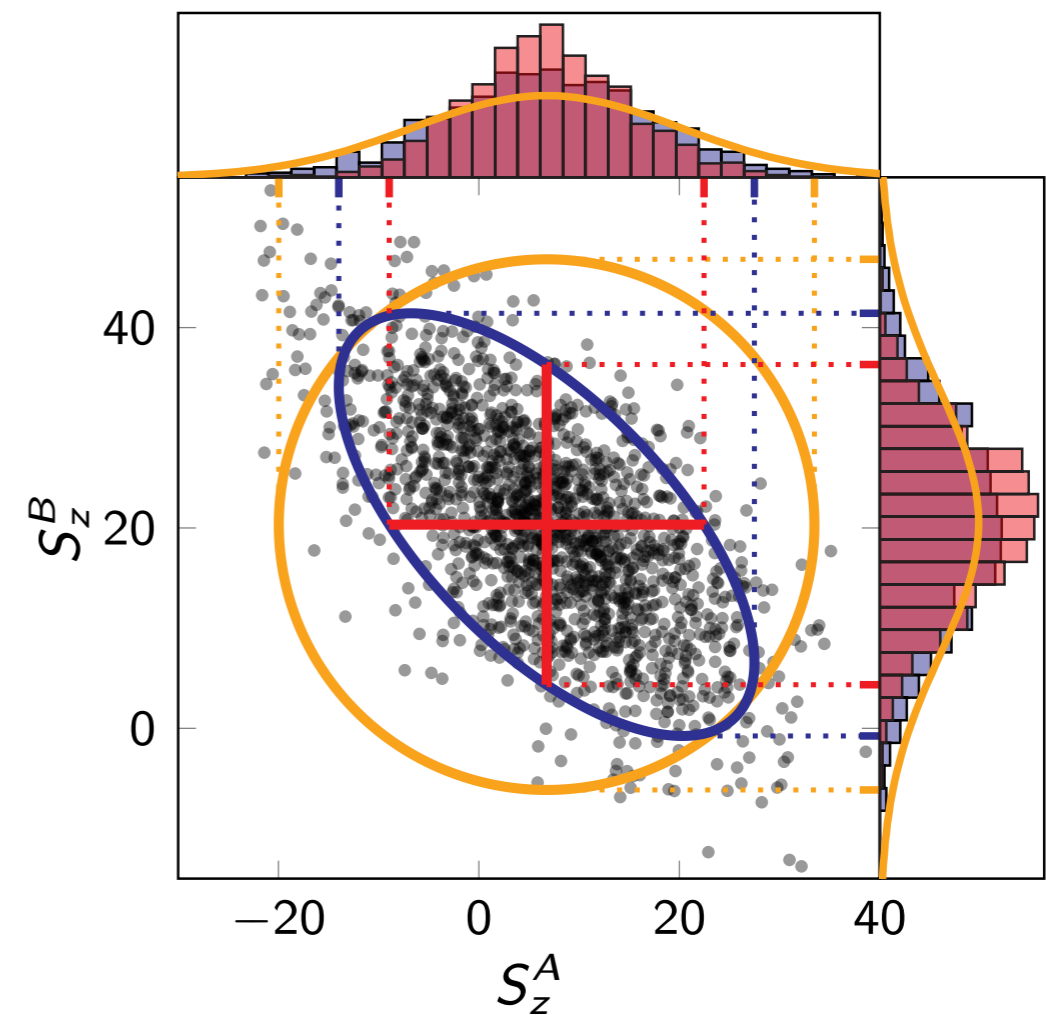
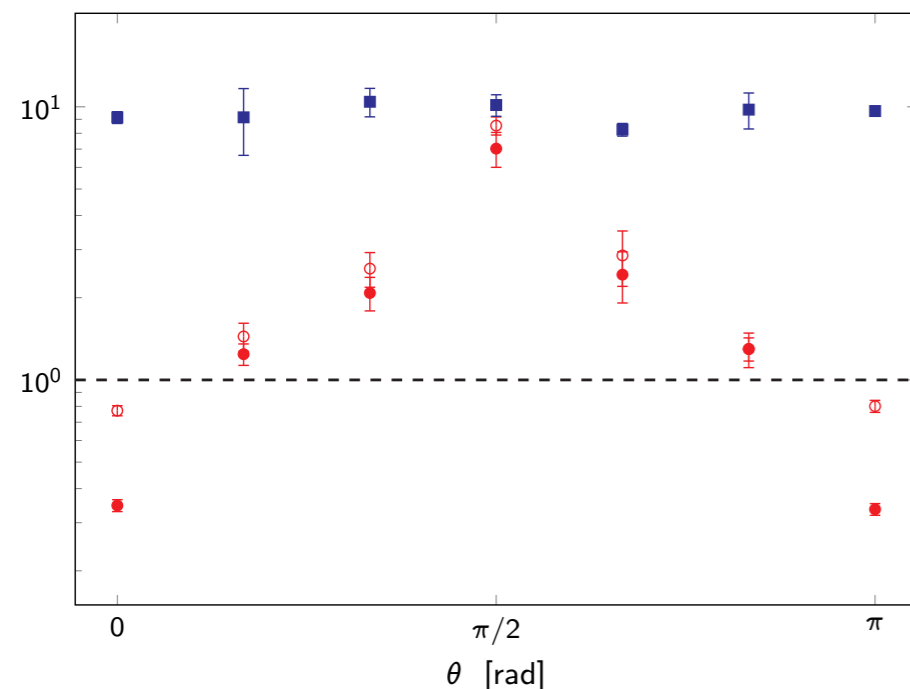
\hat{S}_z	θ	0
\hat{S}_y	$\pi/2 + \theta$	$\pi/2$

- correlations vanish for $\theta = \pi/2$
- for $\theta = \pi$ correlations become anticorrelations and vice versa (gains g_z, g_y change sign)
- manipulation of spin B does not destroy the entanglement
- individual control important for: quantum key distribution, teleportation etc.



Summary

- EPR Paradox between two Bose-Einstein condensates containing each about 700 atoms
- microwave control to realize a BEC beam splitter



- individual control of two macroscopic spins
- future applications in quantum metrology

PRX 13, 021031 (2023)
M.D. Reid, Physics 16, 92 (2023)

Quantum Atom Optics group in Basel



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Yifan Li



Lex Joosten



Philipp Treutlein

