Electric dipole moments in effective field theory

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Searching for New Physics at the Quantum Technology Frontier
Congressi Stefano Franscini (CSF), Ascona

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Outline

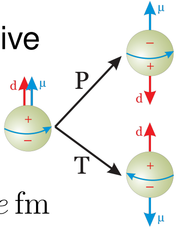
Overview

1. Introduction
2. A tower of EFTs
3. Leptonic dipole moments
4. Neutron EDM in the low-energy EFT
5. Matching to lattice QCD
6. Summary

CP violation: a case for new physics

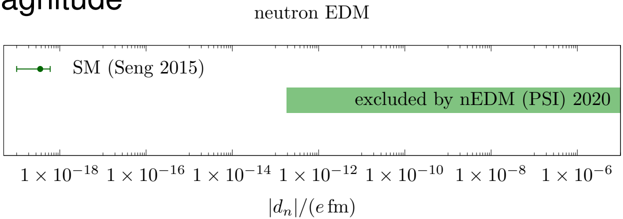
- **baryon asymmetry** in the universe requires more CP violation than Standard Model (SM) can provide
- so far **no direct evidence** of physics beyond the SM
- two options:
  - light new physics is very well hidden (weakly coupled)
  - **new physics is heavy**, with masses well above the electroweak scale
- focus here on the second option

Electric dipole moments

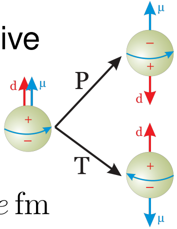


- **electric dipole moments** (EDMs) are sensitive probes of CP violation
- **SM (CKM) contribution** tiny
- current experimental limit: $|d_n| < 1.8 \times 10^{-13}\, e\,\mathrm{fm}$

  → nEDM Collaboration, PRL **124** (2020) 081803
- n2EDM (PSI) will improve sensitivity by **two orders** of magnitude



Electric dipole moments



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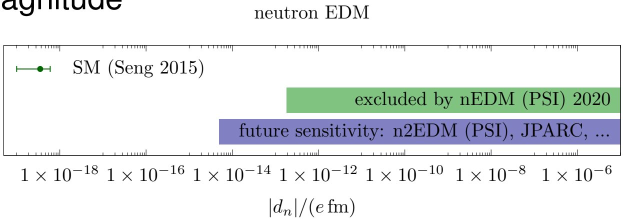


Electric dipole moments

- non-observation leads to strong **constraints on CP-violating sources**
- observation would be a clear **signal of physics beyond the SM** or QCD θ-term

Theory challenges

- non-observation: how to turn experimental bounds into **best generic constraints** on new physics?
- observation: how to **disentangle different possible sources** of CP violation?

⇒ work with generic, **model-independent** framework

⇒ **accuracy of theoretical description** needs to match experimental precision

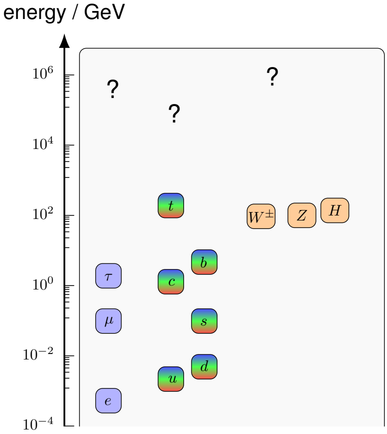
⇒ **control uncertainties**, in particular non-perturbative aspects

Overview

Effective field theories (EFTs)

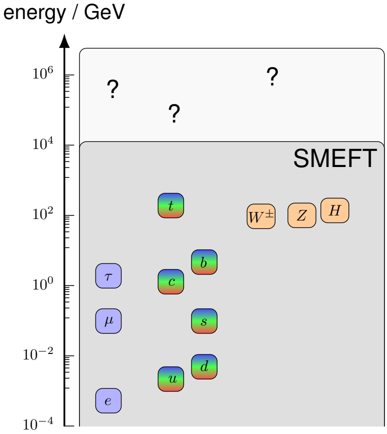
- ideal to deal with widely separated scales:
  $m_N \ll v \ll \Lambda_{\mathrm{UV}}$
- based on a **small set of assumptions**
- **generic framework**, can be used ‘stand-alone’ or in connection with a broad range of specific models
- work with the relevant degrees of freedom at a particular energy $\Rightarrow$ **simplify calculations**
- connect different energy regimes, **avoid large logs**

Effective field theories (EFTs)



- new physics expected at high energies
- its low-energy quantum effects described by **effective field theory**, containing only SM particles (SMEFT)
- **low-energy EFT** (LEFT): only light SM particles

Effective field theories (EFTs)



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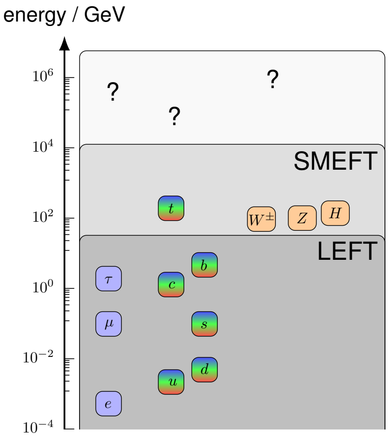
Effective field theories (EFTs)



- new physics expected at high energies
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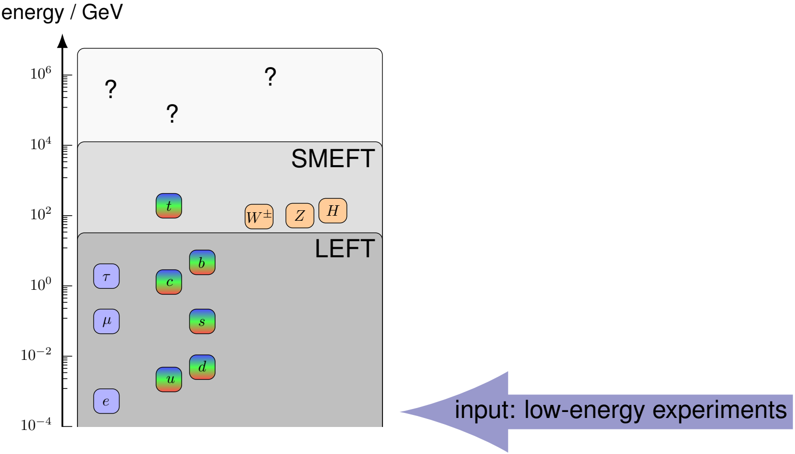
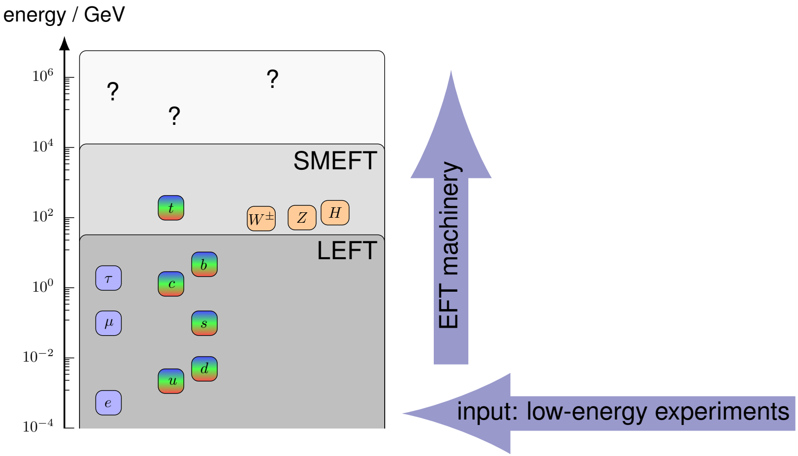
Effective field theories (EFTs)

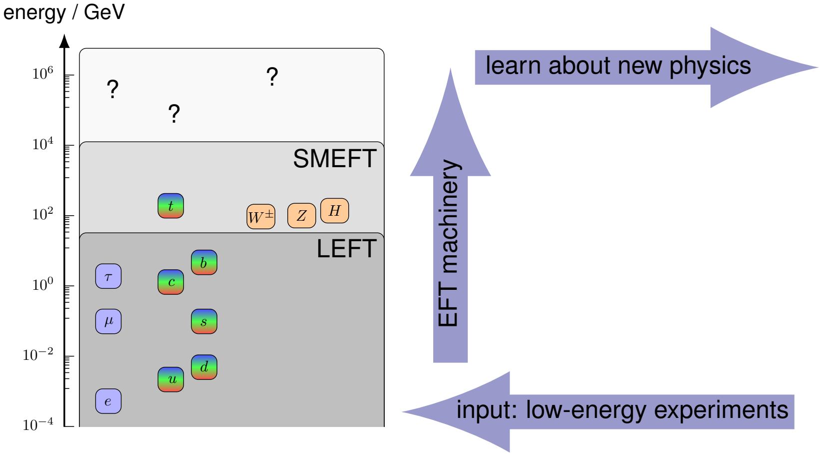


input: low-energy experiments

Effective field theories (EFTs)



Effective field theories (EFTs)

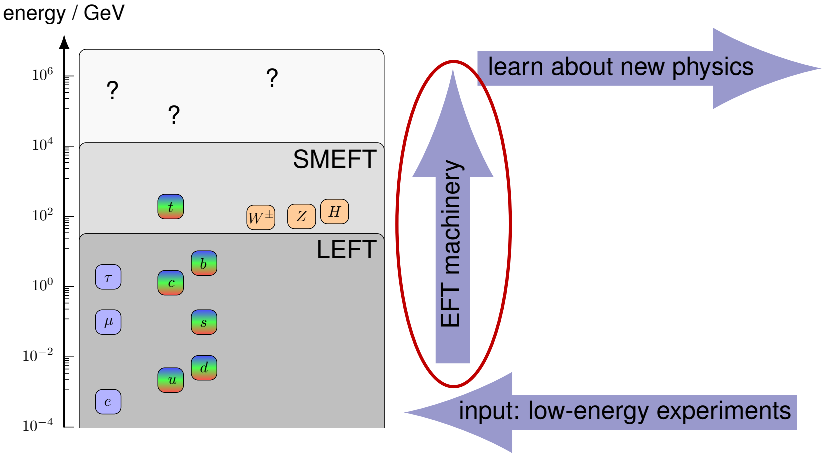


learn about new physics

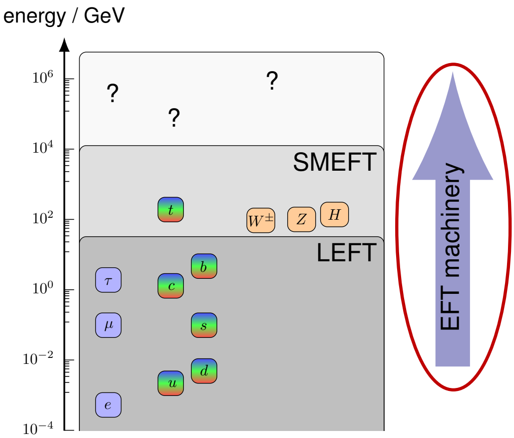
EFT machinery

input: low-energy experiments

Effective field theories (EFTs)



Effective field theories (EFTs)



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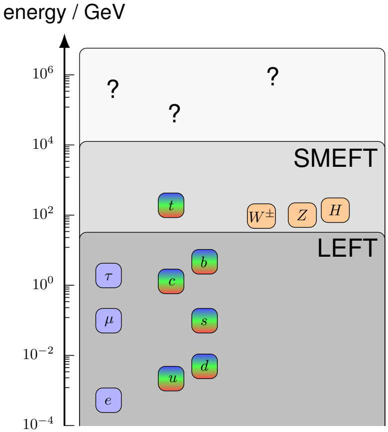

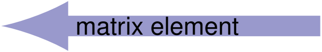

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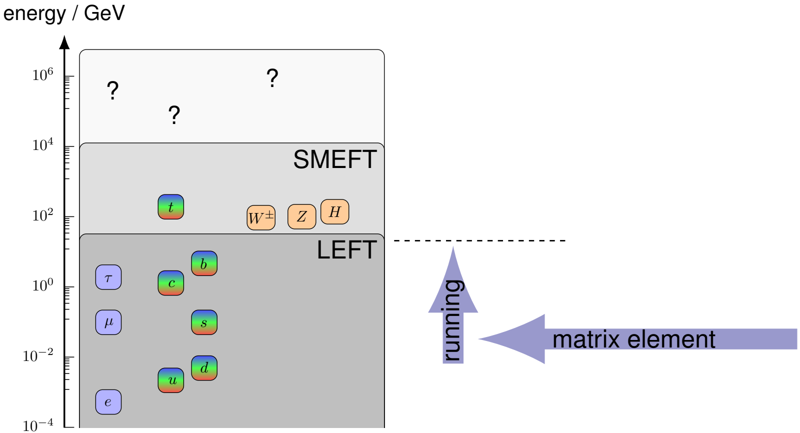
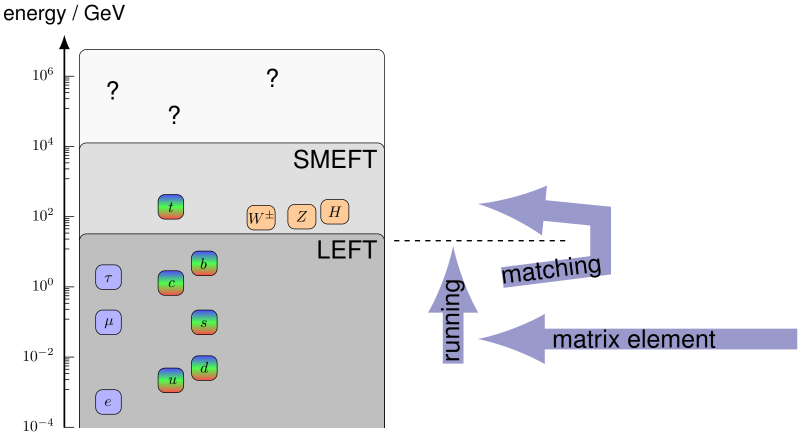


→ Jenkins, Manohar, Stoffer JHEP **01** (2018) 084

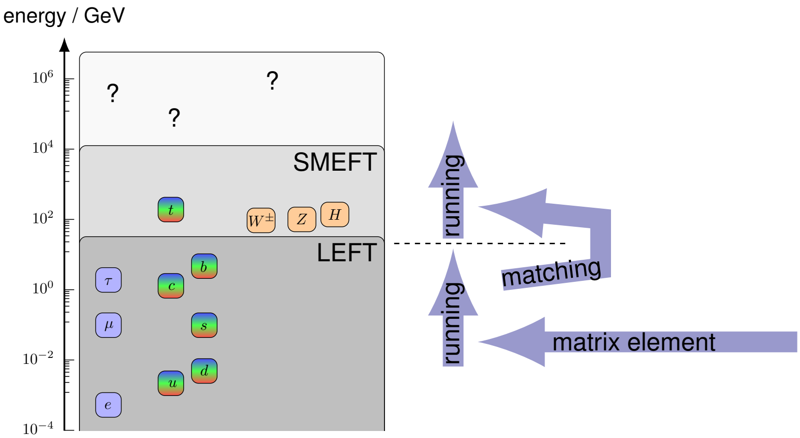
Effective field theories (EFTs)



→ Jenkins, Manohar, Stoffer JHEP **03** (2018) 016

→ Dekens, Stoffer, JHEP **10** (2019) 197

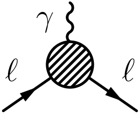
Effective field theories (EFTs)



→ Jenkins et al. (2013, 2014)
→ Alonso et al. (2014)

Overview

Form factors



$$= \quad ie\,\bar{u}(p')\Gamma^\mu(p,p')u(p)\ , \quad k = p' - p$$

form-factor decomposition of vertex function:

$$\Gamma^\mu(p,p') = \gamma^\mu F_E(k^2) + i\frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}F_M(k^2) + \frac{\sigma^{\mu\nu}k_\nu}{2m_\ell}\gamma_5 F_D(k^2) + \frac{k^2\gamma^\mu - k^\mu \not{k}}{m_\ell^2}\gamma_5 F_A(k^2)$$

anomalous magnetic moment: $a_\ell = F_M(0)$

electric dipole moment: $d_\ell = \dfrac{e}{2m_\ell}F_D(0)$

Dipole operators

- leptonic **dipole operators**

$$\mathcal{L}_{\text{LEFT}} \supset L_{\substack{e\gamma \\ pr}} (\bar{e}_{Lp} \sigma^{\mu\nu} e_{Rr}) F_{\mu\nu} + \text{h.c.}$$

give tree-level contribution to dipole moments:

$$a_\ell = \frac{g_\ell - 2}{2} = 4 \frac{m_\ell}{e} \text{Re} L_{\substack{e\gamma \\ \ell\ell}} , \quad d_\ell = -2 \text{Im} L_{\substack{e\gamma \\ \ell\ell}}$$

- real/imaginary parts of same Wilson coefficients, but no model-independent relation
- many more operators contribute at loop level:

→ Panico, Pomarol, Riembau, JHEP **04** (2019) 090

→ Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer, JHEP **07** (2021) 107

→ Brod, Polonsky, Stamou, arXiv:2306.12478

Leptonic EDMs

→ dedicated talks by Eric Hessels, Lorenz Willmann, Chavdar Dutsov

- tiny SM contributions
- electron EDM: → Roussy et al., arXiv:2212.11841

$$|d_e| < 4.1 \times 10^{-17}\, e\,\mathrm{fm}$$

- best direct limit on muon EDM: → BNL, PRD **80** (2009) 052008

$$|d_\mu| < 1.5 \times 10^{-6}\, e\,\mathrm{fm}$$

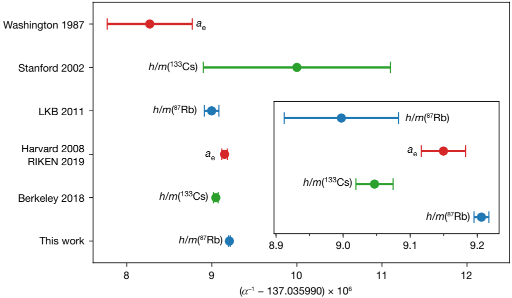
- indirect bound on muon EDM from $^{199}\mathrm{Hg}$ and ThO EDMs: → Ema, Gao, Pospelov, PRL **128** (2022) 13, 131803

$$|d_\mu(^{199}\mathrm{Hg})| < 6.4 \times 10^{-7}\, e\,\mathrm{fm}$$

$$|d_\mu(\mathrm{ThO})| < 1.9 \times 10^{-7}\, e\,\mathrm{fm}$$

Electron anomalous magnetic moment

- as opposed to EDMs, need to control SM prediction: a_e **limited** by knowledge of α_{QED}
- tension between ${}^{133}\mathrm{Cs}$ vs. ${}^{87}\mathrm{Rb}$ recoil measurements **above 5σ**: → talk by Pierre Cladé



→ Morel, Yao, Cladé, Guellati-Khélifa, Nature **588** (2020) 7836, 61

Muon anomalous magnetic moment

- as opposed to EDMs, need to control SM prediction: $\boldsymbol{a_\mu}$ **limited** by knowledge of **hadronic contributions**
- multiple tensions:
  - BNL/FNAL vs. SM 2020 White Paper: 4.2σ
  - BMWc lattice QCD vs. BNL/FNAL: 1.5σ
  - pre-2023 e^+e^- hadronic cross-section data vs. BMWc lattice QCD: 2.1σ
  - intermediate Euclidean-time window: 3.7σ
  - pre-2023 e^+e^- hadronic cross-section data vs. CMD-3 (dispersive fit below $1\,\mathrm{GeV}$): 3.7σ

Muon anomalous magnetic moment

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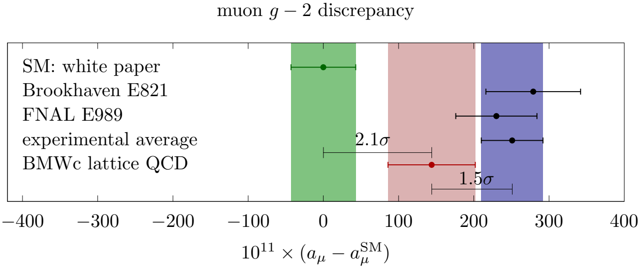

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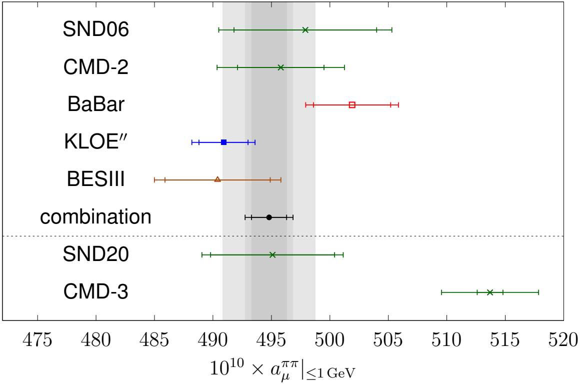
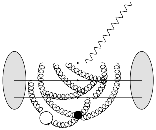


Overview

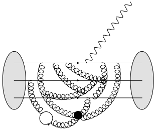
Neutron EDM in LEFT

- contribution schematically given as

$d_N \sim$  $= \sum_i L_i \, \langle N|\mathcal{O}_i|N\gamma\rangle$

Neutron EDM in LEFT

- contribution schematically given as

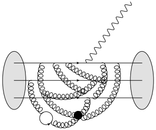
$d_N \sim$  $= \sum_i L_i \, \langle N|\mathcal{O}_i|N\gamma\rangle$

LEFT operator coefficients

Neutron EDM in LEFT

- contribution schematically given as

$d_N \sim$



$$= \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$$

hadronic matrix element

Neutron EDM in LEFT

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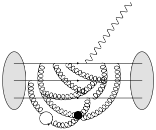
$d_N \sim$  $= \sum_i L_i \, \langle N|\mathcal{O}_i|N\gamma\rangle$

- calculate matrix element in LEFT at a renormalization scale of $\mu \sim 2 \ldots 3\,\mathrm{GeV}$
- at present, **large uncertainties** on matrix elements dilute experimental sensitivity
- aim for $10 - 25\%$ precision to avoid cancellations

→ Alarcon et al., arXiv:2203.08103

Neutron EDM in LEFT

- hadronic EDMs (nEDM) complicated: **QCD is non-perturbative** at low energies
- any P-odd, CP-odd flavor-conserving operator **contributes non-perturbatively** to nEDM:
  - QCD θ-term
  - dimension-five quark (C)EDM operators
  - dimension-six three-gluon operator
  - dimension-six P/CP-odd four-fermion operators

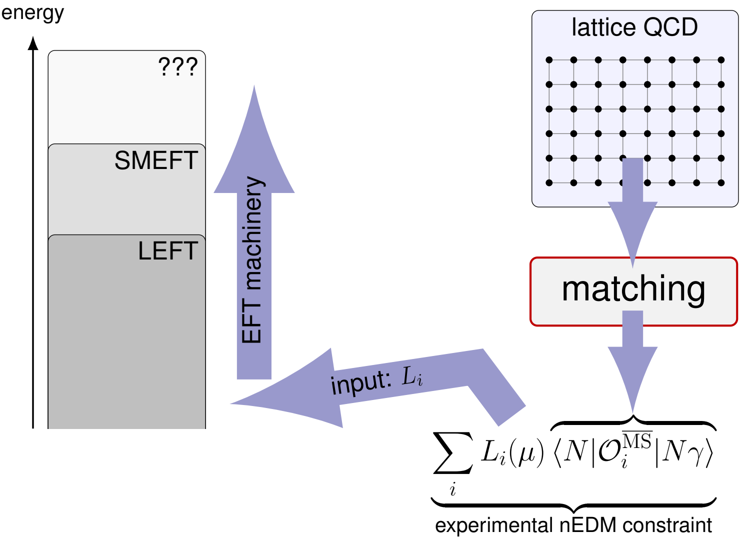
Neutron EDM in LEFT

$$
\begin{aligned}
d_N = &- (1.5 \pm 0.7) \times 10^{-3}\, \bar{\theta}\, e\, \mathrm{fm} \\
&- (0.20 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s \\
&- (0.55 \pm 0.28) e\, \tilde{d}_u - (1.1 \pm 0.55) e\, \tilde{d}_d + (??) e\, \tilde{d}_s \\
&+ (50 \pm 40) \mathrm{MeV}\, e\, \tilde{d}_G + (??)\ \text{four-quark}
\end{aligned}
$$

→ Alarcon et al., arXiv:2203.08103

- ideally use **lattice QCD** to compute matrix elements
- problem with lattice and EFT: $d_N \sim \sum_i L_i(\mu) \langle N | \mathcal{O}_i^{\overline{\mathrm{MS}}} | N\gamma \rangle$
  $\overline{\mathrm{MS}}$ cannot be implemented on the lattice!
- requires a **matching calculation**

Neutron EDM in LEFT



$$\underbrace{\sum_i L_i(\mu)\,\overbrace{\langle N|\mathcal{O}_i^{\overline{\mathrm{MS}}}|N\gamma\rangle}}_{\text{experimental nEDM constraint}}$$

Overview

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5. **Matching to lattice QCD**
6. Summary

General procedure

- $\overline{\mathrm{MS}}$: subtraction of $1/\varepsilon$ poles in dimensional regularization
- **define renormalized operators** in a scheme amenable to lattice computations
- compute their matrix elements in lattice QCD
- calculate relation between $\overline{\mathrm{MS}}$ and lattice scheme **in perturbation theory** (at $\mu \sim 2 \ldots 3\,\mathrm{GeV}$)
- use this matching to derive matrix elements of $\overline{\mathrm{MS}}$ operators

RI schemes

- **R**egularization-**I**ndependent (**S**ymmetric) **MOM**entum-subtraction scheme

  → Martinelli et al. (1995), Sturm et al. (2010)
- impose renormalization conditions on truncated **off-shell Green's functions** for Euclidean momenta
- RI-SMOM: **insert momentum** into operator to suppress unwanted IR effects
- calculation in a **fixed R_ξ gauge**

Matching $\overline{\mathrm{MS}}$ and RI-SMOM

- matching for dimension-5 quark (C)EDM operators:

  → Bhattacharya et al., PRD **92** (2015) 11, 114026

- dimension-6 three-gluon operator $GG\widetilde{G}$:

  → Cirigliano, Mereghetti, Stoffer, JHEP **09** (2020) 094

- complications:
  - **huge set of operators** (34 for three-gluon operator), including unphysical ones
  - requires calculation of **many matrix elements**
  - **power divergences** in lattice spacing difficult to tackle

A more promising scheme: gradient flow

→ Lüscher, JHEP **08** (2010) 071, JHEP **04** (2013) 123

- **gradient flow**: introduce new artificial dimension: flow time t (not related to ordinary time)
- boundary condition: ordinary QCD at $t = 0$, $B_\mu(t = 0) = G_\mu$, $\chi(t = 0) = \psi$
- gauge-invariant **flow equations**:

$$\partial_t B_\mu = D_\nu G_{\nu\mu} \,, \quad \partial_t \chi = D^2 \chi$$

- flow acts as a UV regulator

Gradient flow: advantages

- “flowed operators” automatically **UV finite**, apart from quark-field (+ coupling & mass) renormalization
- connect flowed operators with $\overline{\mathrm{MS}}$ operators in perturbation theory
- **gauge-invariant** results
- on the lattice: **continuum limit** for fixed t possible
- **power divergences** no longer in $1/a$, but in $1/t$ $\Rightarrow$ **disentangled from continuum limit**

Gradient-flow matching: current status

- dimension-5 quark (C)EDM matched at one loop:

  → Mereghetti, Monahan, **Rizik**, Shindler, Stoffer, JHEP **04** (2022) 050

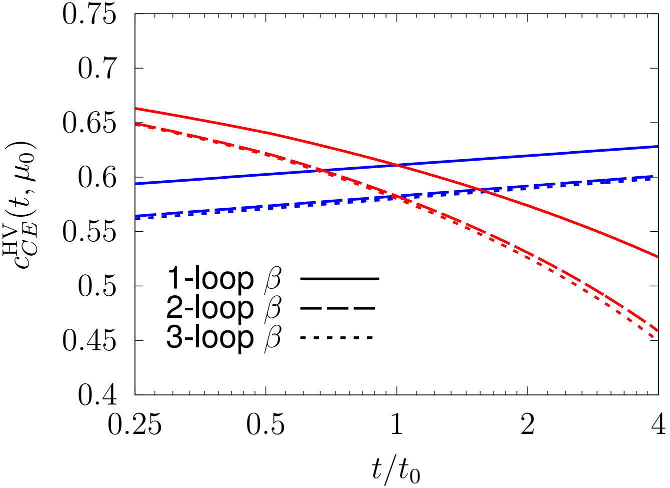
- dimension-6 four-quark operators:

  → **Bühler**, Stoffer, arXiv:2304.00985 [hep-lat]

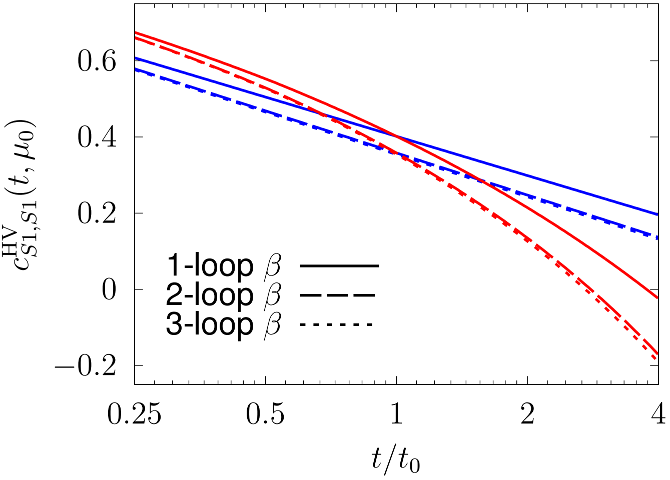
- dimension-6 CP-odd three-gluon operator:

  → **Lara Crosas**, Mereghetti, Monahan, **Rizik**, Shindler, Stoffer, in progress

Quark CEDM matching coefficient



Four-quark matching coefficient (scalar singlet)



Overview

Theory challenges

low-energy precision searches pose interesting **theory challenges**:

- given the experimental progress: reach appropriate **theoretical accuracy**
- **model-independent and robust** connection between low-energy physics and UV theories: EFT provide ideal framework, need to control (perturbative) running and mixing effects
- problem at low energies are (huge) **hadronic uncertainties**

Theory challenges

- if using lattice QCD for matrix elements
  $\Rightarrow$ **matching calculation** to appropriate scheme
- traditional RI-SMOM schemes very challenging
- recent progress with **gradient flow**: dimension 5 and dimension-6 four-quark completed at one loop
- **matching equations up to dimension 6** nearly completed at one loop
- in some cases, two-loop coefficients would be useful